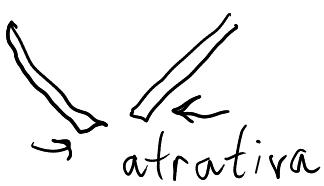
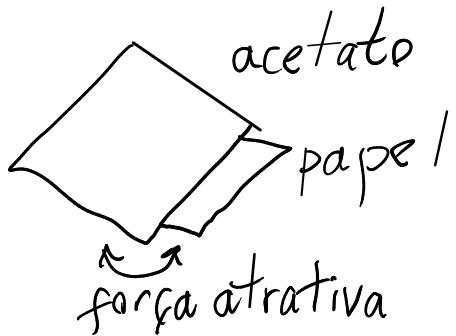
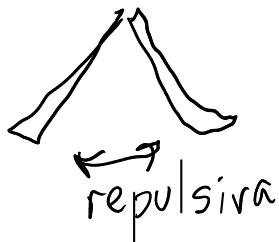


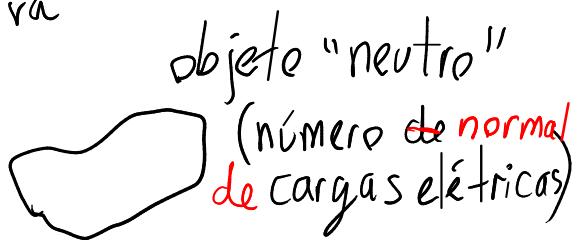
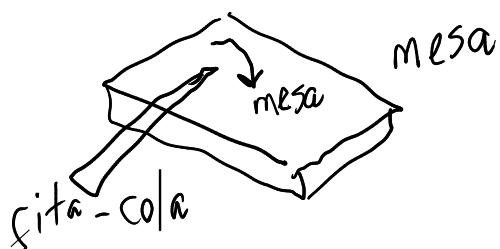
1. CAMPO ELÉTRICO

aula 1 . 2020-09-21

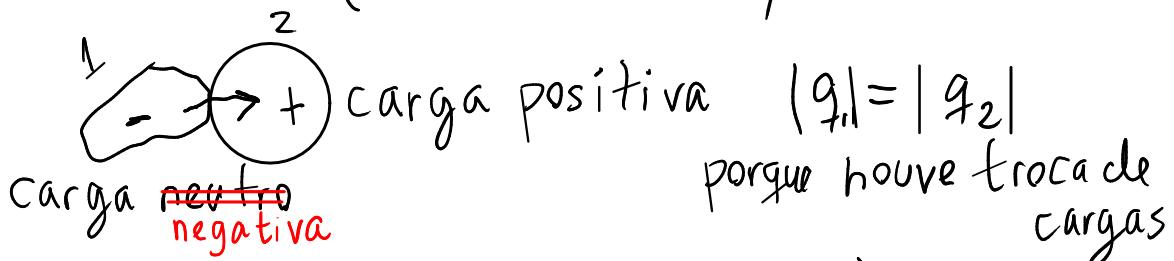
Força elétrica



séc. XVIII : B. Franklin



série triboelétrica (tabela 1.1 do livro)



carga positiva $|q_1| = |q_2|$

porque houve troca de cargas

UNIDADE SI DE CARGA $\rightarrow 1 \text{ C}$ (coulomb)

exemplos : $q_1 = +3.5 \text{ C}$ $q_2 = -2.8 \text{ C}$

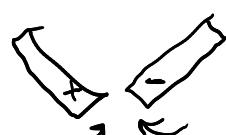
cargas do mesmo sinal

↓
força repulsiva

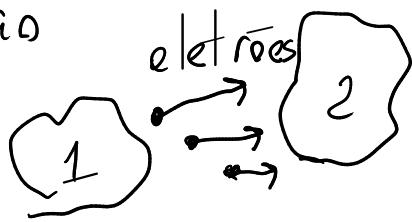
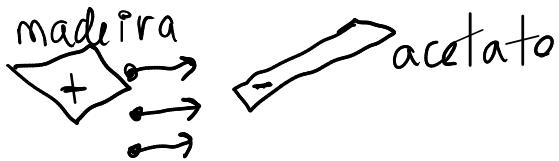


cargas de sinais opostos

↓
força atrativa

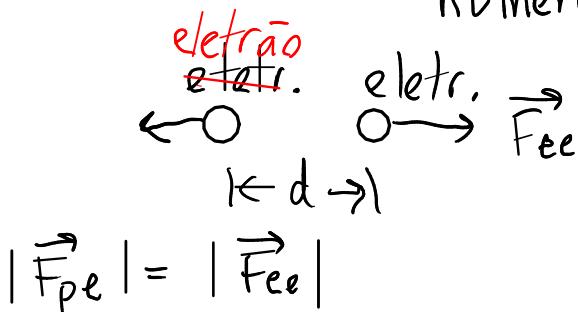
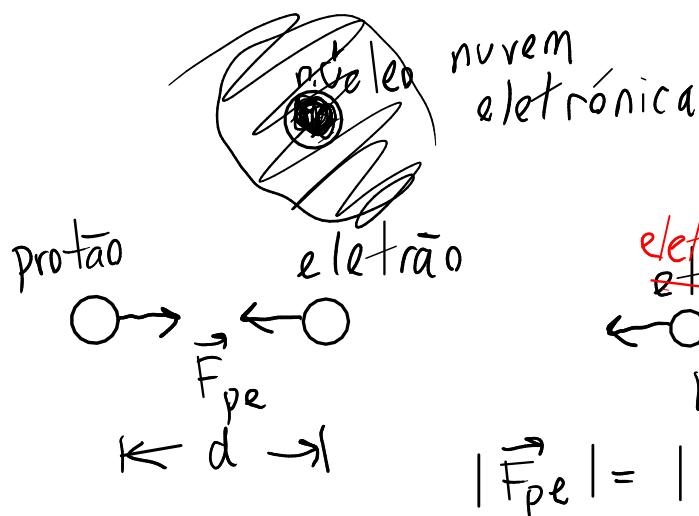


1897 → descoberta do eletrão



⇒ Qeletrão é negativa

MODELO ATÓMICO



$$q_{\text{protão}} > 0$$

$$q_{\text{eletrão}} < 0$$

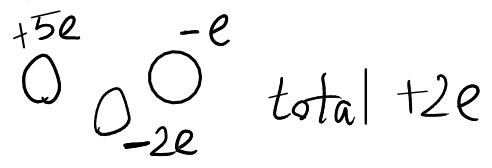
$$|q_{\text{prot.}}| = |q_{\text{eletrão}}| \\ \approx \text{carga elementar} \\ (e)$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

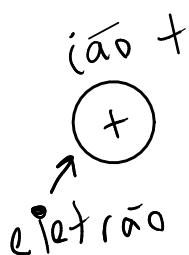
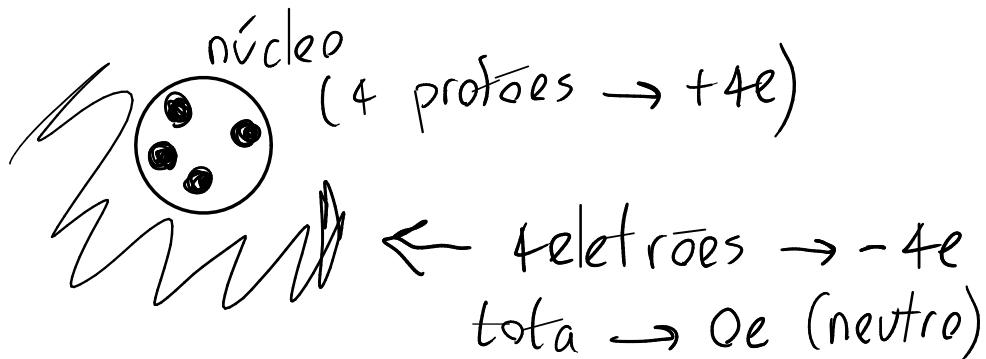
(1.6×10^{-19} com 3 algarismos)

Propriedades da carga elétrica

- ① Quantização. Todas as cargas que podem existir, são múltiplos inteiros de e
exemplos: $-3e$, $+5e$ (não $5.2e$)
- ② Conservação. As cargas não podem ser modificadas (totais)
exemplos desintegração

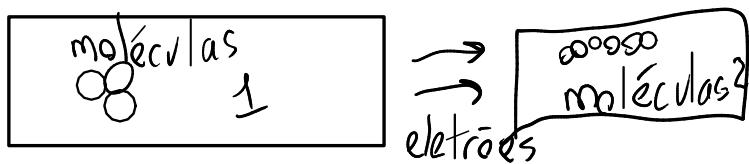
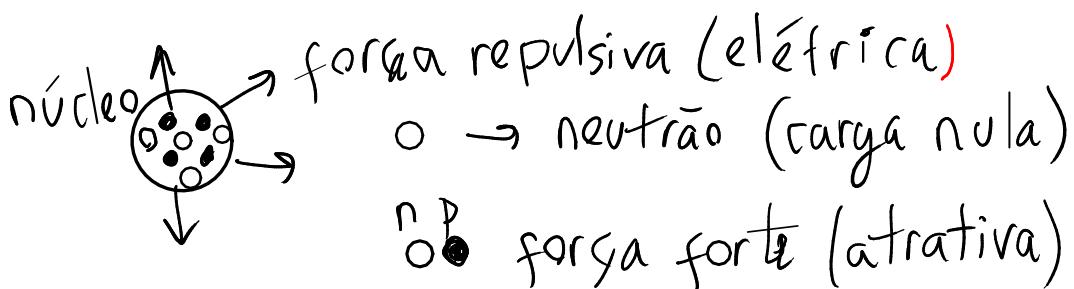


Átomo



$$q_{total} = -e \rightarrow (5 \text{ eletrões})$$

$$q_{total} = +e \rightarrow (\text{apenas } 3 \text{ eletrões})$$



a massa do eletrão é que é 2000 menor que a do protão ou neutrão

(a carga do eletrão tem o mesmo valor absoluto da do protão)

LEI DE COULOMB

2 cargas "pontuais" q_1 e q_2 , a uma distância d

q_2

q_1

d

força elétrica de módulo F_{12}

$$F_{12} = k \frac{|q_1| |q_2|}{d^2}$$

em unidades SI : $k = 8.998 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$

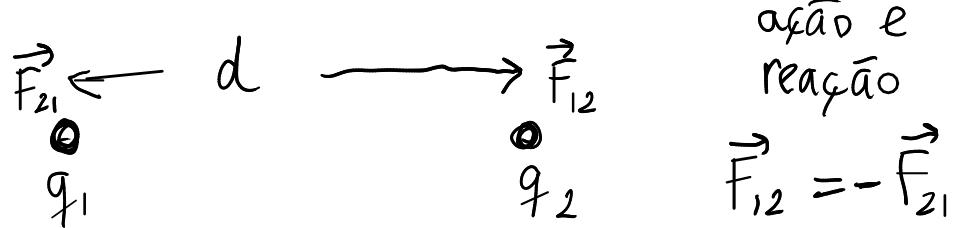
($k = 9 \times 10^9$, com 3 algarismos)

cargas típicas $\rightarrow nC = 10^{-9} C \approx 10^{10}$ eletróes

$3.5678\dots nC$
3.5678 em vez de $n e$
inteiro

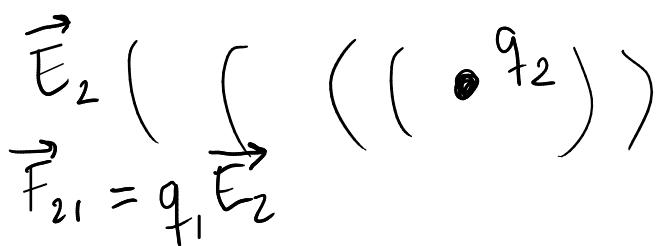
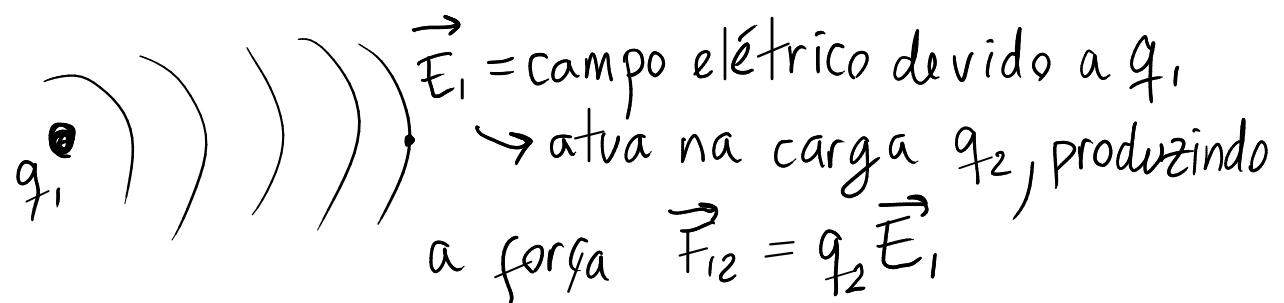
aula 2. 2020-09-23

Força elétrica



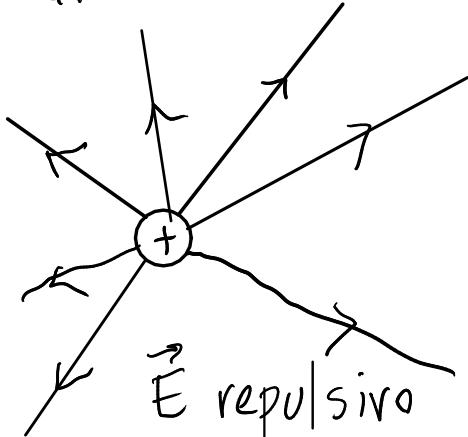
$$\Rightarrow F_{12} = F_{21} = k \frac{|q_1| |q_2|}{d^2}$$

Otra interpretación

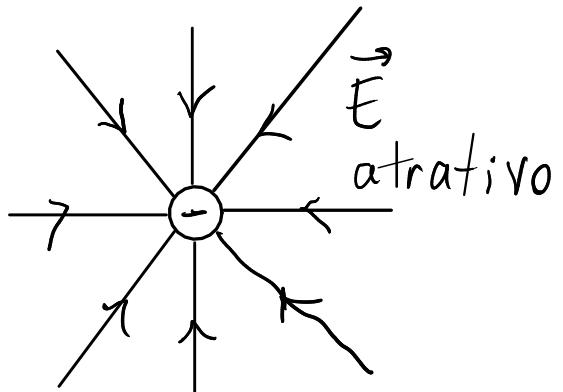


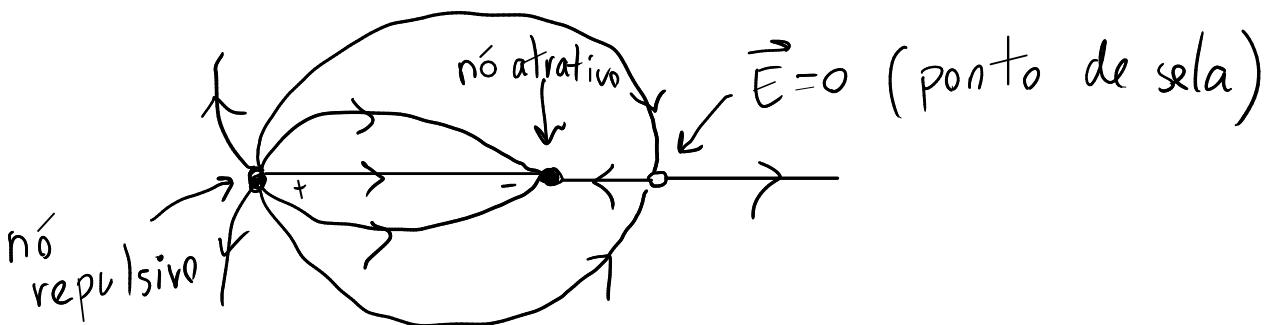
$$\text{cargas pontuais } q_i \rightarrow E_i = \frac{k|q_i|}{d^2}$$

$$q_i > 0$$



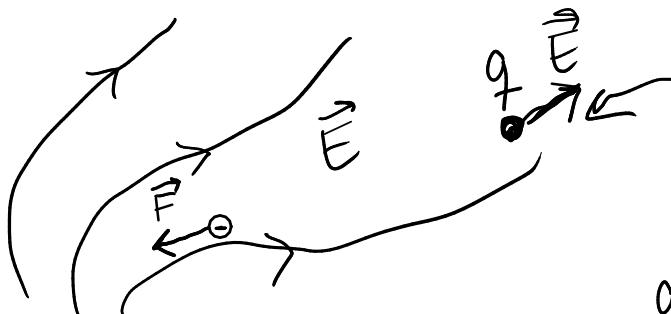
$$q_i < 0$$





$$\vec{E}_1 \leftarrow \vec{E}_2 \rightarrow \vec{E}_i$$

$$\vec{E} = \sum_{i=1}^n \vec{E}_i$$



q (pontual) num P:
 ponto

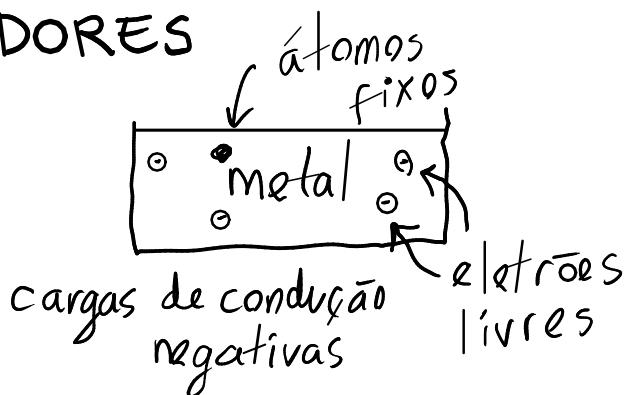
$$\vec{F}_{elétrica} = \begin{cases} 0 & \Rightarrow \vec{E}_P = \vec{0} \\ \vec{F}_{el.} \neq \vec{0} & \Rightarrow \vec{E}_P = \frac{\vec{F}_{el.}}{q} \end{cases}$$

$$\boxed{\vec{F}_q = q \vec{E}}$$

CONDUTORES E ISOLADORES

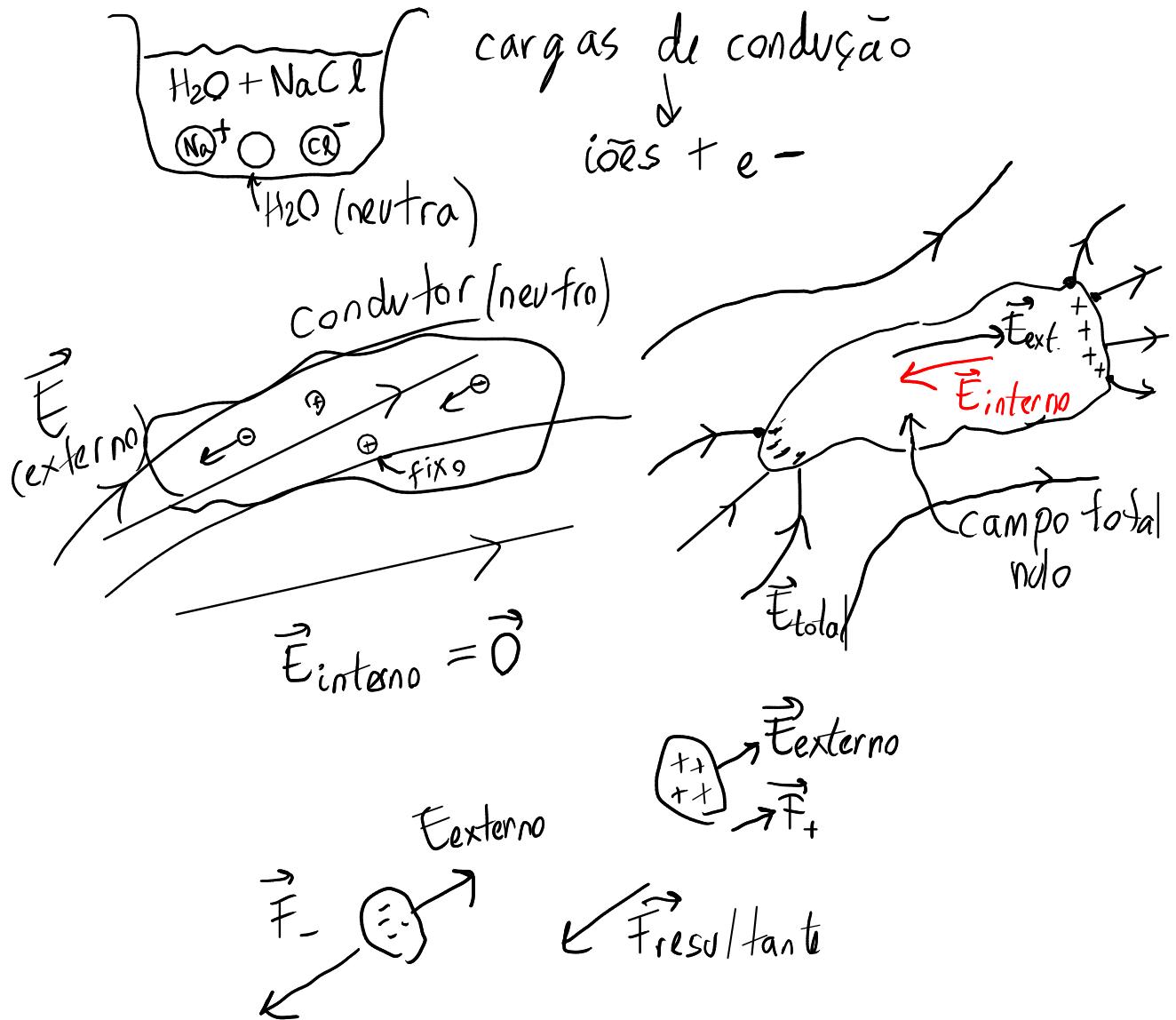
1. Condutores

cargas livres de se deslocarem no material

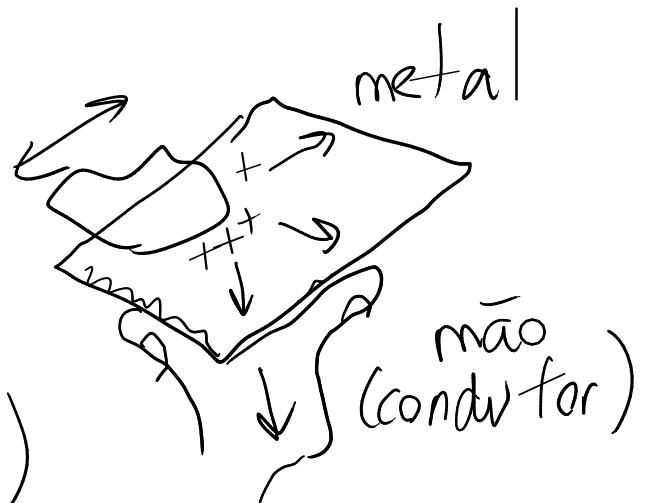
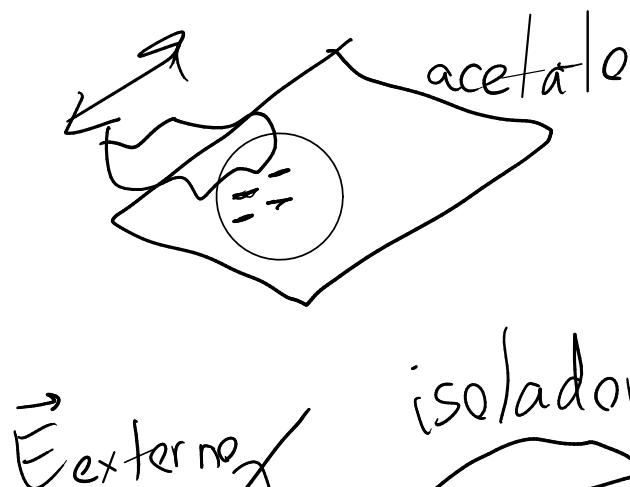


cargas de condução negativas

soluções



2. Isoladores. Não há cargas de condução
ex: papel, madeira, plástico, acetato, ...



cada átomo forma
um dipolo

$$|q_1| = |q_2| \quad q_1 + q_2 = 0$$

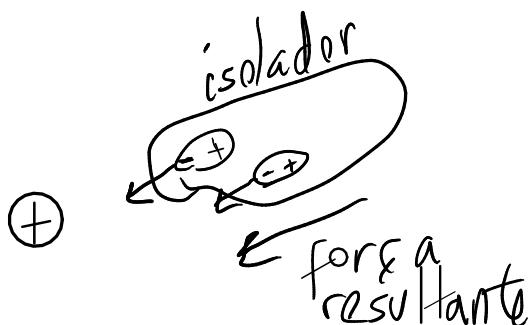


$K = \text{constante dielétrica}$
 \downarrow
 número (sem unidades) > 1

Campo de uma carga pontual q dentro de
um isolador

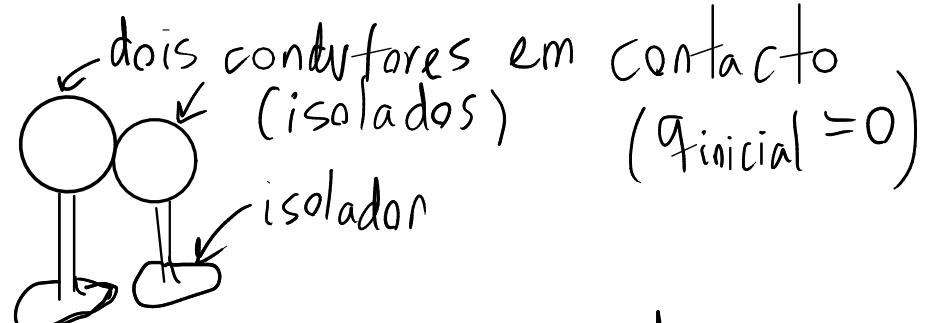
q óleo ou
água destilada
(isoladores)

$$E = \frac{k|q|}{Kd^2}$$

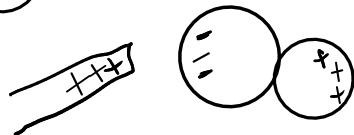


Eletrização por indução

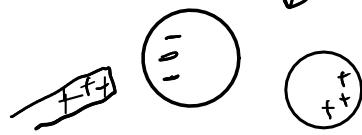
Exemplo: ①



②



③

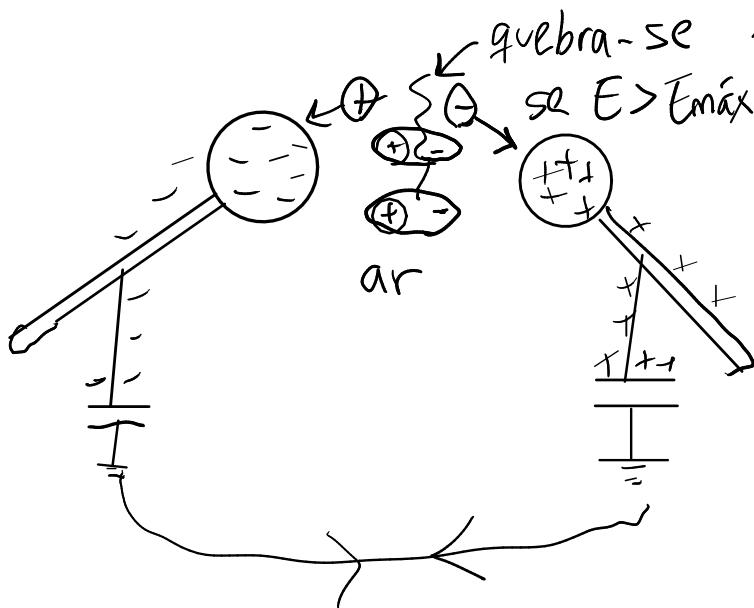


④



todo ISOLADOR tem
um campo máximo
chamado:

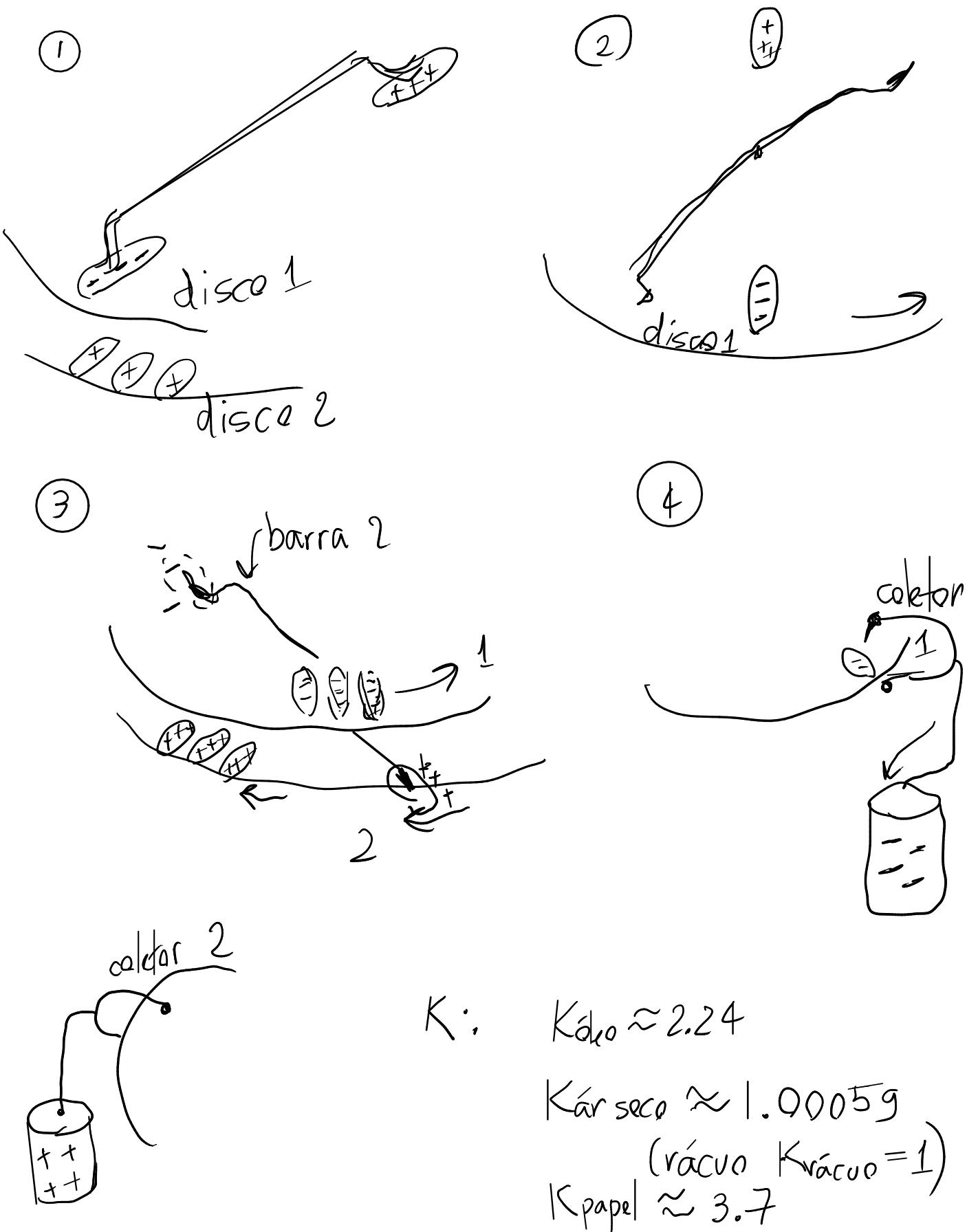
Gerador de Wimshurst



plasma = gás ionizado
rigidez dielettrica
no caso do ar

$$E_{\text{máx}} = 3 \times 10^6 \frac{\text{N}}{\text{C}}$$

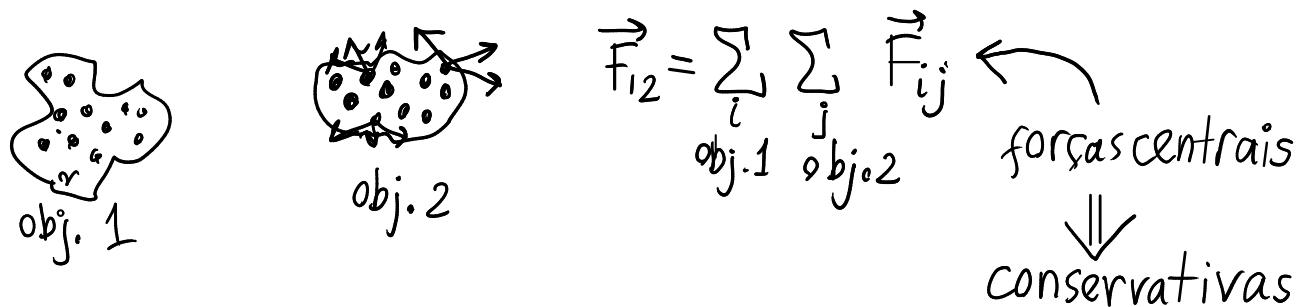




$E \text{rigidez do ar seco} = 3 \times 10^6 \frac{\text{N}}{\text{C}}$ ar húmido \rightarrow rigidez menor

2. VOLTAGEM E CORRENTE

Força elétrica entre 2 objetos



$\Rightarrow \vec{F}_{12}$ é uma força conservativa

Energia potencial elétrica de uma carga pontual

q no ponto P

$$U_p = - \int_{P_0}^P \vec{F}_e \cdot d\vec{r} \quad (\text{qualquer percurso})$$

$$\vec{F}_e = q \vec{E} \quad \Rightarrow \quad U_p = q \left(- \int_{P_0}^P \vec{E} \cdot d\vec{r} \right)$$

"potencial"

$$U_p = q V_p$$

V_p = potencial em P

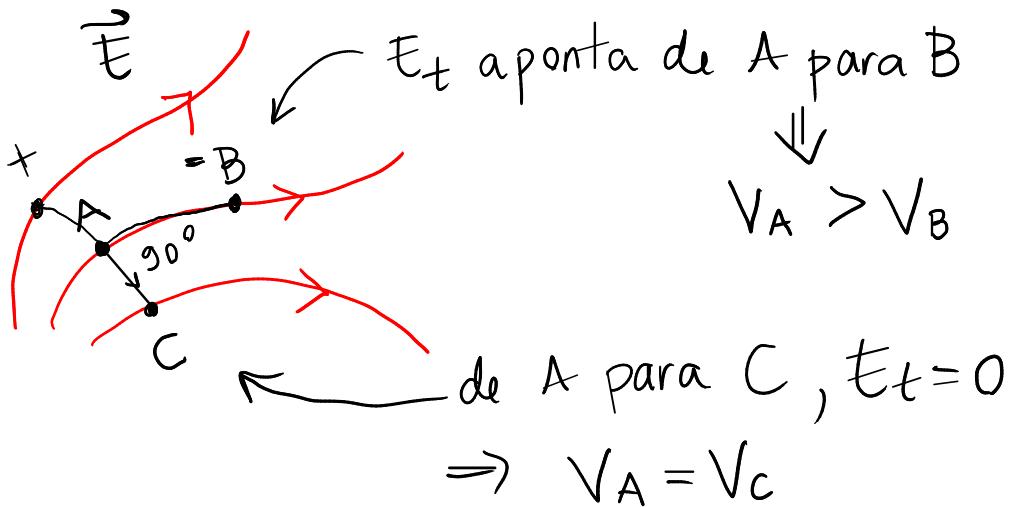
U (em J, no S.I.) \rightarrow propriedade das cargas

V (em volt = $V = \frac{J}{C}$) \rightarrow propriedade do espaço

Diferença de potencial (voltagem)

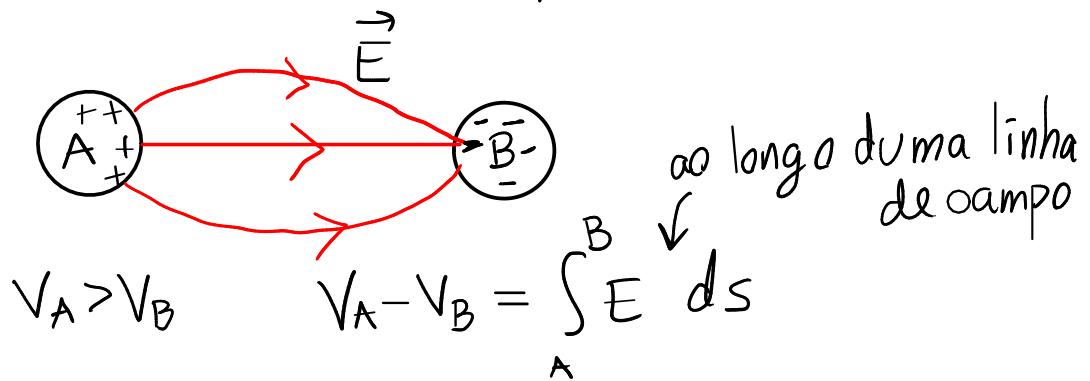
$$V_A - V_B = \int_A^B E_t ds$$

$(E_t = \text{componente tangente ao percurso})$



V diminui na direção e sentido das linhas de campo elétrico.

Exemplo: Gerador de Wimshurst



$$\Delta V = V_A - V_B = \bar{E} d_{AB}$$

Se $\bar{E} > E_{\text{máx do ar}} = 3 \times 10^6 \frac{V}{m}$ ($\frac{V}{m} = \frac{N}{m} = \text{unidades de } \bar{E}$)

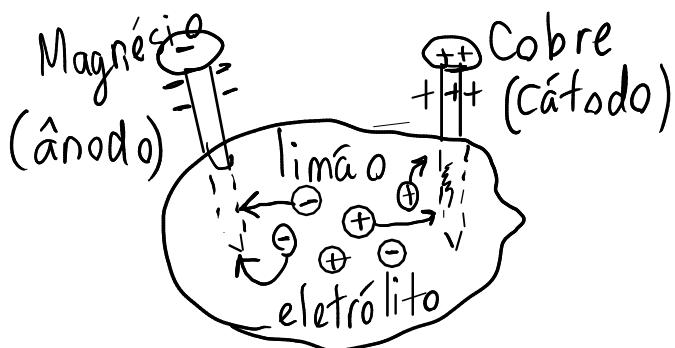
\Downarrow
 passam cargas para A e B

$$\text{se } d_{AB} = 3 \text{ cm} \quad (3 \times 10^{-2} \text{ m})$$

$$\Delta V_{\text{máx}} = 3 \times 10^6 \times 3 \times 10^{-2} = 90000 \text{ V}$$

a descarga faz diminuir q_+, q_- e ΔV

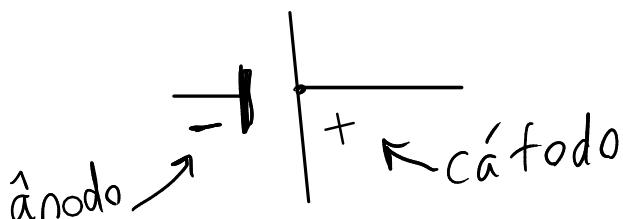
PILHAS QUÍMICAS



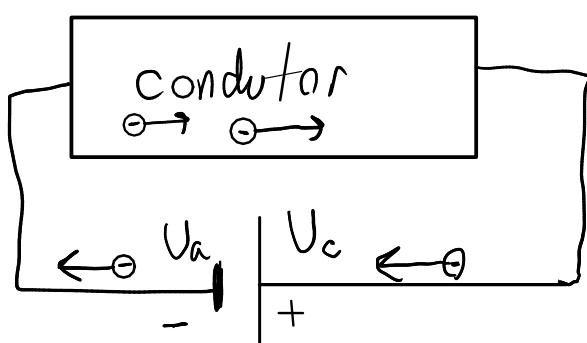
→ um eletrólito (solução)
+ 2 eletródos de metais diferentes

energias das reações químicas:
Ucátodo e Uânode

↓ diagrama de circuito $U_{\text{cátodo}} > U_{\text{ânode}}$



Condutor ligado à pilha



cada elétron que circula pelo circuito reduz uma carga " e^- " no ânodo, e no cátodo, que são recuperadas pelas reações químicas no eletrólito

Quando não houver mais iões no eletrólito, a pilha fica descarregada

cada eletrão de condução: sai do ânodo com energia:

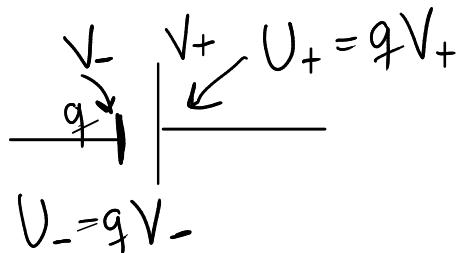
$$-eU_a$$

entra no cátodo com energia $-eU_c$ ($< -eU_a$)

$$\Delta U = -eU_a - (-eU_c) = e(U_c - U_a) > 0$$

energia fornecida pelas reações químicas

$$\frac{\Delta U}{e} = V_+ - V_- = \text{voltagem da pilha}$$



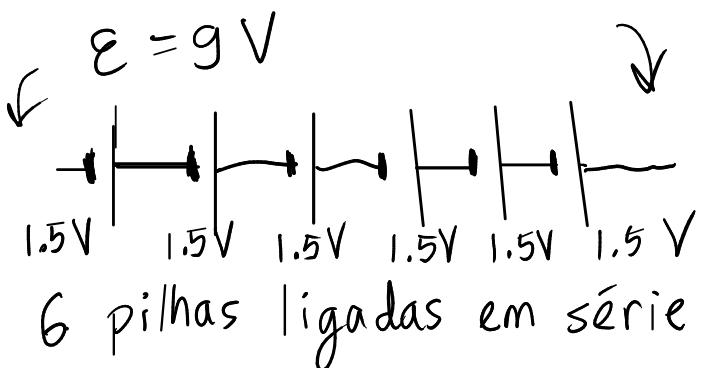
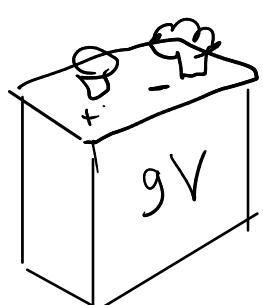
diferença das energias das reações químicas, por unidade de carga.

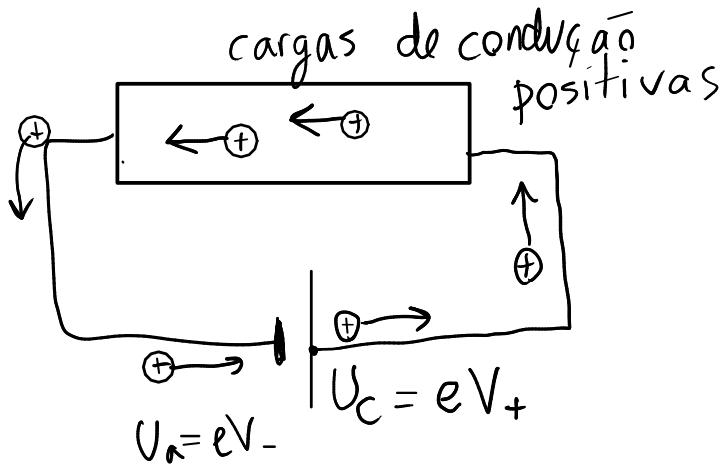
propriedade do eletrólito e os eletródos usados

força eletromotriz duma pilha (f.e.m.)

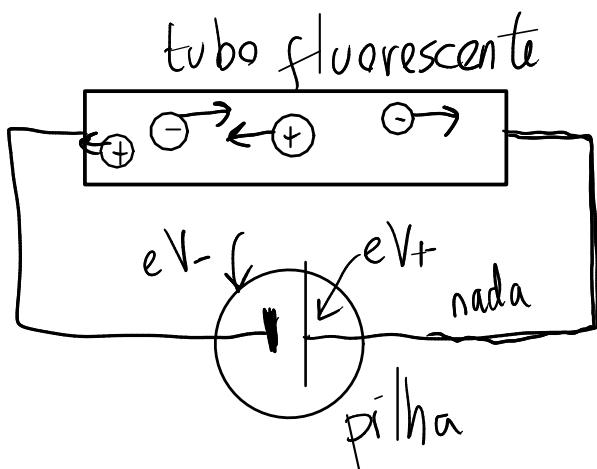
$$\mathcal{E} = V_+ - V_- \quad (\text{não depende da carga da pilha})$$

para a maior parte dos materiais \mathcal{E} está entre 1 V e 2 V.





$$\begin{aligned}\Delta U &= U_c - U_a \\ &= eV_+ - eV_- \\ &= eE\end{aligned}$$



por cada e :

$\Delta U = eE$ (fornecida pela pilha)

está a diminuir a carga no eletrólito (um ião + menos e um ião - menos)

$$\begin{aligned}Q_{\max} (\text{pilha}) &= \text{carga total dos iões} + \\ &= -\text{carga total dos iões} -\end{aligned}$$

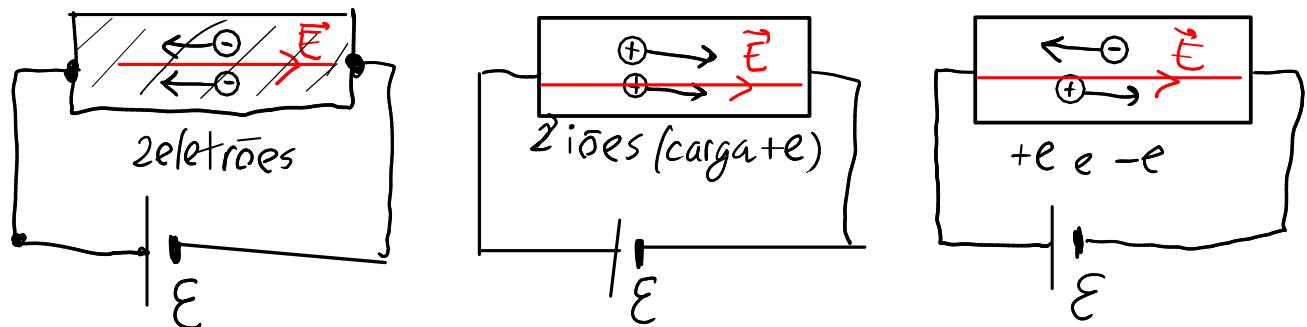
Energia máxima da pilha:

$$U_{\max} = Q_{\max} E$$



$$\begin{aligned}Q_{\max} &= 2300 \text{ mA} \cdot \text{h} \\ &= 2300 \text{ mA} \cdot (3600 \text{ s}) \\ &= 2300 \times 10^{-3} \times 3600 \text{ C}\end{aligned}$$

Condutor ligado a uma pilha.
3 situações equivalentes



Nos três casos:

Eletrolito \rightarrow { 2 iões (+e) reagem no cátodo (cátions)
{ 2 iões (-e) reagem no ânodo (aníons)

Pilha → q diminui em 2e

L'energia diminui $ze E$

passa energia potencial elétrica no sentido de \vec{E}

\Rightarrow convertida em energia cinética das $q_{\text{condução}}$

Condução: forças dissipativas dos íons fixos nas cargas de condução

$$V_{\text{conduta}} \rightarrow V_{\text{limite}} \quad f \rightarrow ((O))((O))$$

fixos

ΔU (energia fornecida pela pilha) \rightarrow calor no condutor

$\Delta U \rightarrow$ calor \rightarrow aumento da temperatura
 \downarrow
 \rightarrow luz
 \rightarrow polarização ...

CORRENTE ELÉTRICA

$$\vec{I} = \begin{cases} \text{direção e sentido do campo } \vec{E} \text{ (maior } V \text{ para menor)} \\ \text{módulo } I = \frac{|\Delta q|}{\Delta t} \end{cases}$$

Δq : carga transferida num intervalo Δt

corrente instantânea:

$$I = \left| \frac{dq}{dt} \right| \quad q(t) \text{ carga que passa por qualquer parte do circuito no instante } t$$

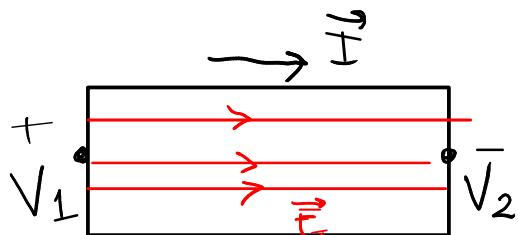
$$|\Delta q| = \int_{t_1}^{t_2} I dt \rightarrow |\Delta q| \text{ num intervalo } [t_1, t_2]$$

Unidade SI de corrente:

$$1 \frac{C}{S} = 1 A \text{ (ampere)} \quad (A \cdot S = C)$$

$$A \cdot h = 3600 C$$

POTÊNCIA ELÉTRICA DISSIPADA NUM CONDUTOR



$$V_1 > V_2$$

num intervalo infinitesimal dt passa carga

$$|dq| = I dt$$

energia elétrica dissipada: $dU = |dq|V_1 - |dq|V_2$
 $= |dq|(V_1 - V_2) > 0$

$$dU = |dq| \Delta V = \overset{\text{voltagem}}{I} \Delta V dt$$

Potência instantânea (dissipada no condutor)

$$P = \frac{dU}{dt} = I \Delta V$$

UNIDADES

corrente $\rightarrow A \left(\frac{C}{s} \right) \quad (Ah)$

carga $\rightarrow C = A \cdot S, A \cdot h = 3600 C, e = 1.6 \times 10^{-19} C$

campo $\vec{E} \rightarrow \frac{N}{C} = \frac{V}{m} \quad \left(\frac{\text{força}}{\text{carga}}, \frac{\Delta V}{\Delta S} \right)$

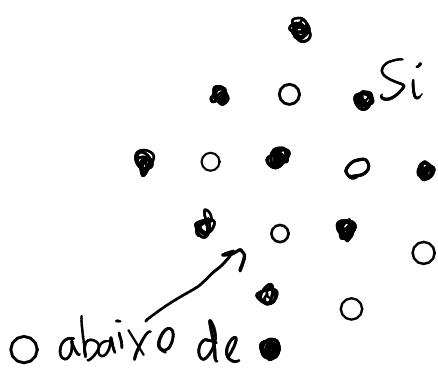
potência $\rightarrow W = \frac{J}{s} = V \cdot A \quad \left(\frac{dU}{dt}, \Delta V I \right)$

energia $\rightarrow J = W \cdot s = C \cdot V \quad (U_e = qV)$

eletônio-volt: $eV = 1.6 \times 10^{-19} J \quad (kWh)$
 $W \cdot h = 3600 J \quad kW \cdot h = 3.6 \times 10^6 J$

SEMICONDUTORES

cristais de Si_{14} ou Ge_{32} (valência 4)

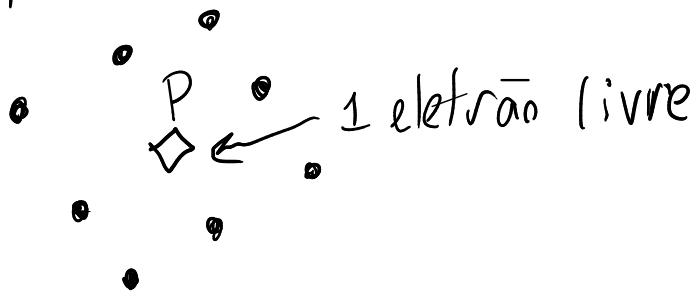


cristal FCC (face centered cubic)

ligação covalente
2 elétrões
(força magnética)

Semicondutor de tipo N (negativo)

impurezas de P_{15} ou As_{33} (valência 5)

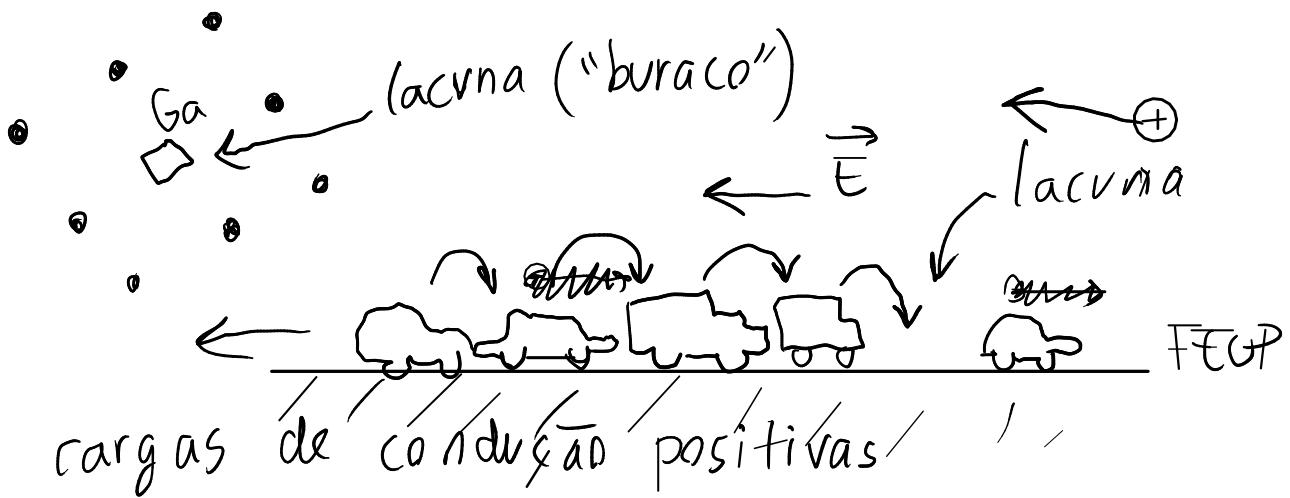


condutor
 \oplus \ominus

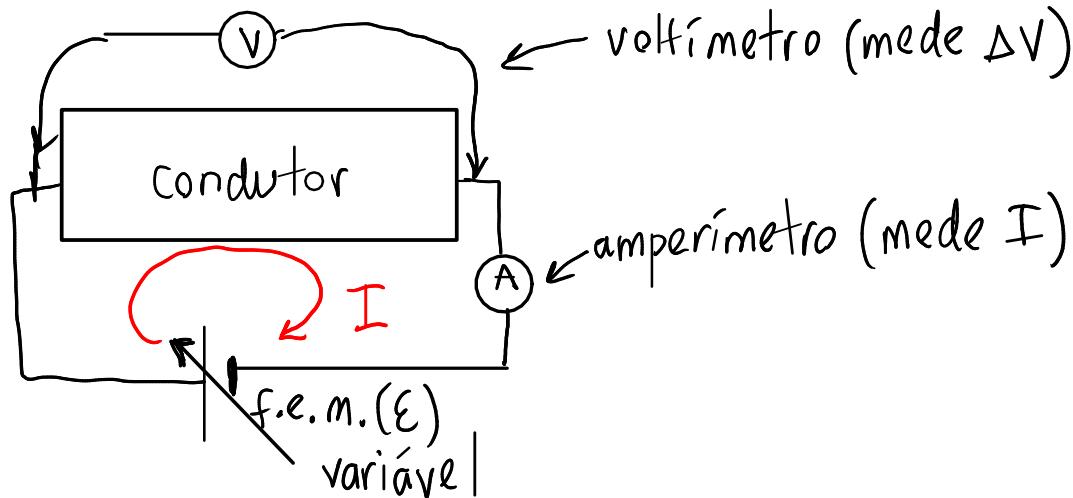
cargas condução negativas

Semicondutor tipo P(positivo)

impurezas de Ga_{31} ou In_{49} (valência 3)

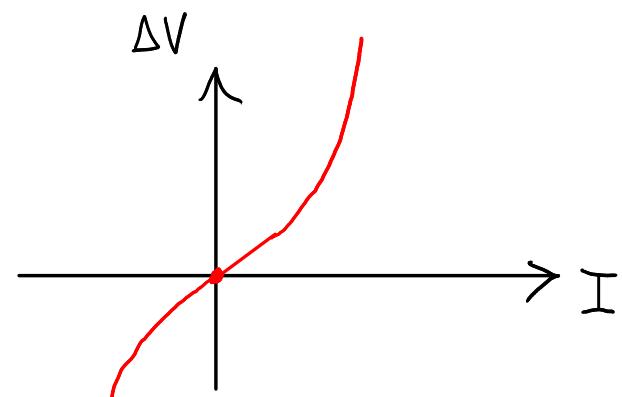
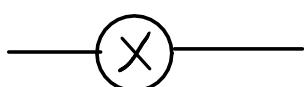


CURVAS DE VOLTAGEM-CORRENTE (características)

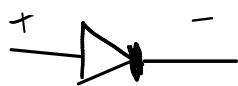


Exemplos:

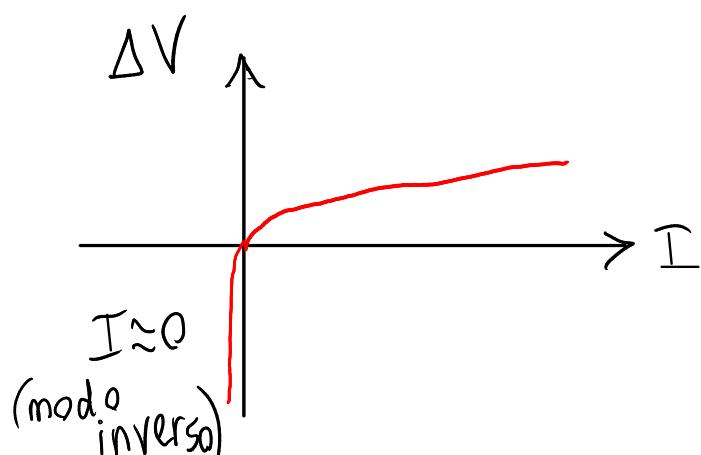
① Lâmpada incandescentes



② Díodos

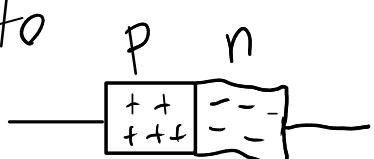


$I > 0$
(modo direto)



deixa passar corrente no modo direito

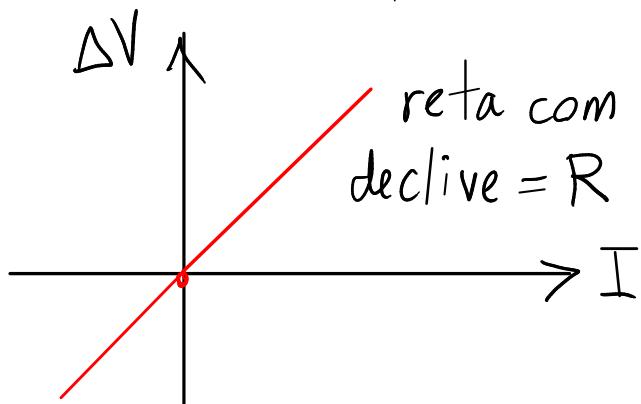
No modo inverso, $I \approx 0$



③ Resistências (em inglês resistor) (dispositivo)



R = resistência
(propriedade física
(em inglês resistance))



Exemplos: metais, grafite, ... (condutores em que toda a energia elétrica dissipada é convertida em calor)

Lei de Ohm

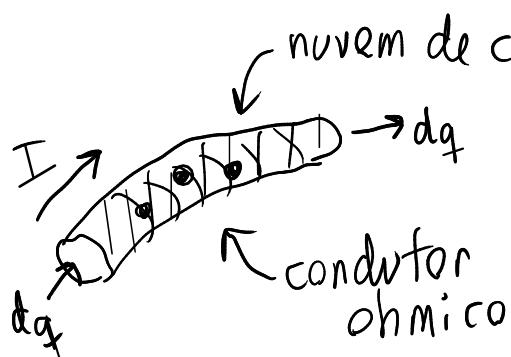
$$\Delta V = R I$$

R = resistência

Unidade SI de resistência:

$$1 \frac{V}{A} = 1 \Omega \text{ (ohm)}$$

RESISTIVIDADE



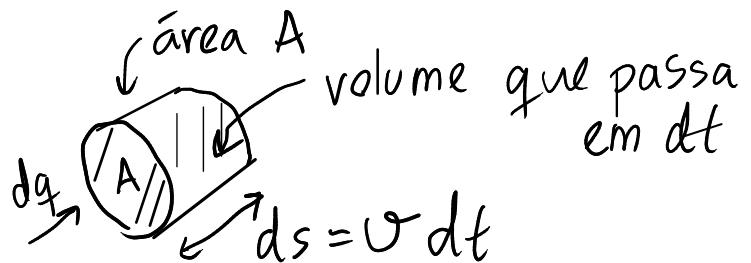
nuvem de cargas de condução (\approx líquido incompressível)
com n cargas elementares por unidade de volume

$$I = \frac{dq}{dt}$$

$dq = I dt$ carga transferida num intervalo dt

$dq = (ne) \times \text{volume que passa no intervalo } dt$

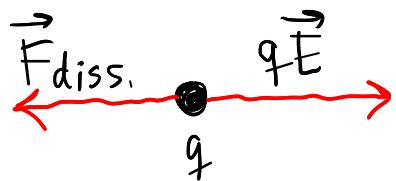
↑
carga por
unidade de
volume



$$dq = ne(A V dt)$$

$$I = neA V$$

Forças sobre cada carga de condução



$$F_{\text{diss}} = k \nu \quad \begin{matrix} (\text{número de}) \\ (\text{Reynolds}) \\ \text{baixo} \end{matrix}$$

$$\Rightarrow \nu_{\text{limite}}: k \nu = q E$$

$$\nu = \frac{q}{k} E \Rightarrow I = neA \left(\frac{q}{k} E \right)$$

$$E = \frac{\Delta V}{L}$$

↑
média

↑
médio

L = comprimento do condutor ΔV = voltagem no condutor

constante R^{-1}

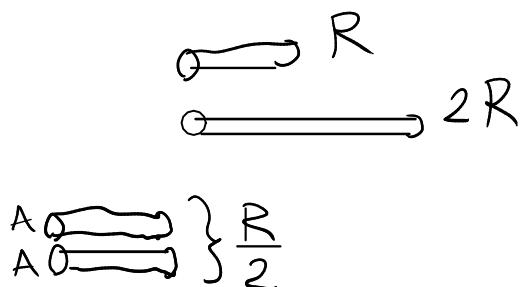
$$I = \left(\frac{neq}{k} \right) \frac{A}{L} \Delta V$$

lei de Ohm

$$I = \frac{\Delta V}{R}$$

$$\rho = \frac{k}{\text{neg}} = \text{resistividade do material}$$

$$R = \frac{\rho L}{A}$$



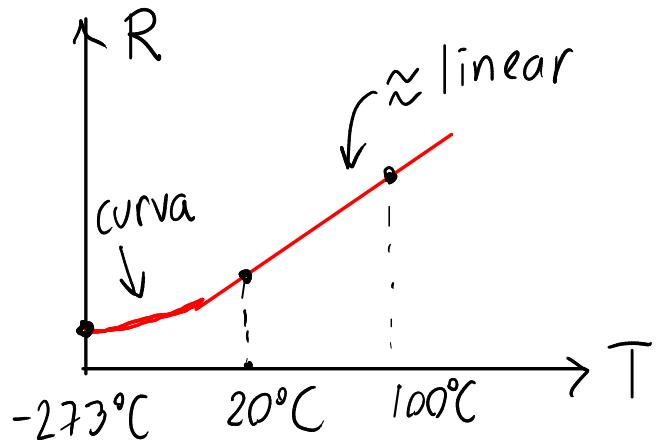
$\rho \rightarrow$ depende do material (propriedade física)

$R \rightarrow$ propriedade geométrica/física

ρ também depende da temperatura T
se T aumenta, as forças dissipativas aumentam,

RESISTÊNCIA VS TEMPERATURA

Numa resistência:



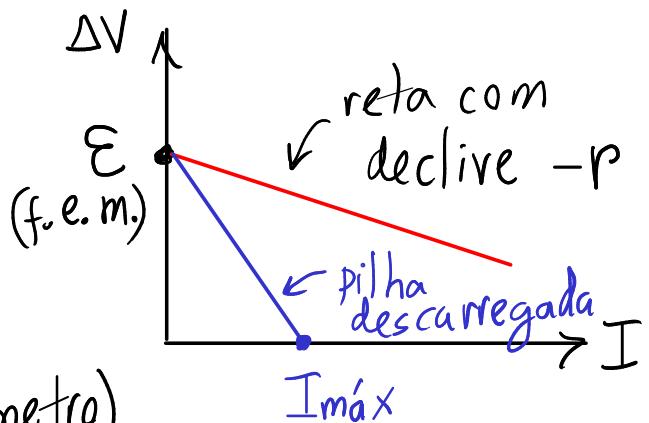
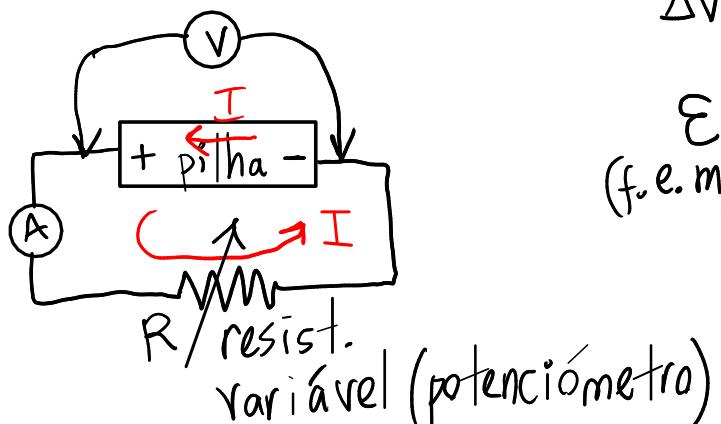
$$R(T) = R_{20} (1 + \alpha_{20}(T - 20))$$

valor a 20°C

declive = $R_{20} \alpha_{20}$

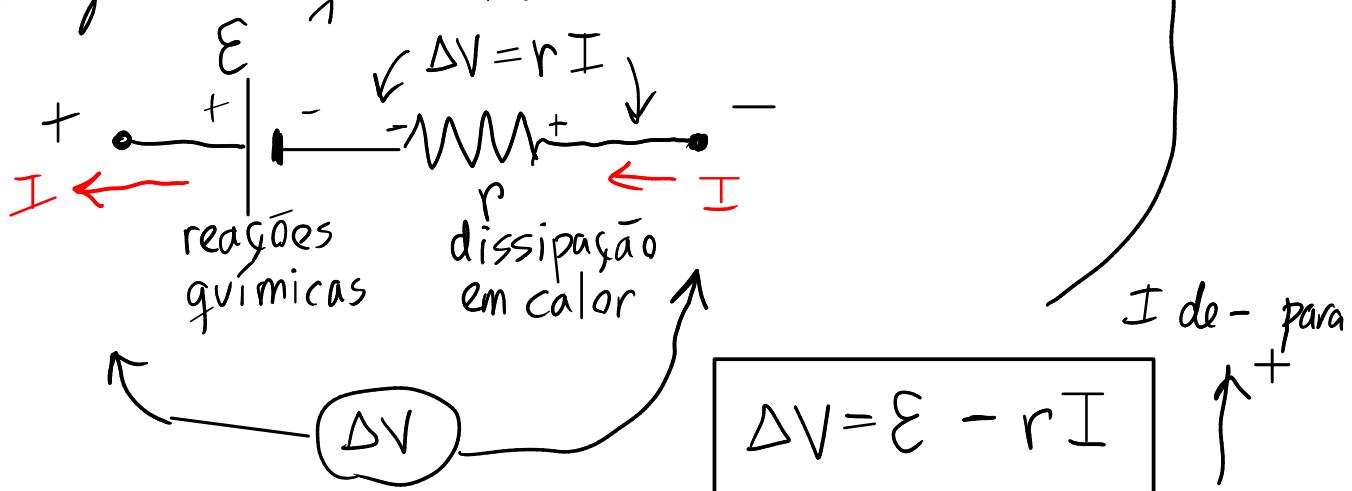
α_{20} = coeficiente de temperatura do material ($^\circ\text{C}^{-1}$)

CARACTERÍSTICA DE UMA BATERIA (pilha)



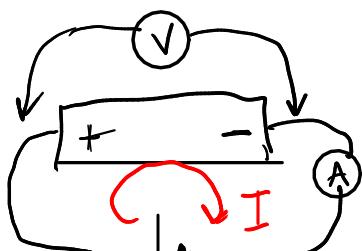
r = resistência interna da pilha
(baixa se a pilha tiver carga máxima)

Diagrama equivalente



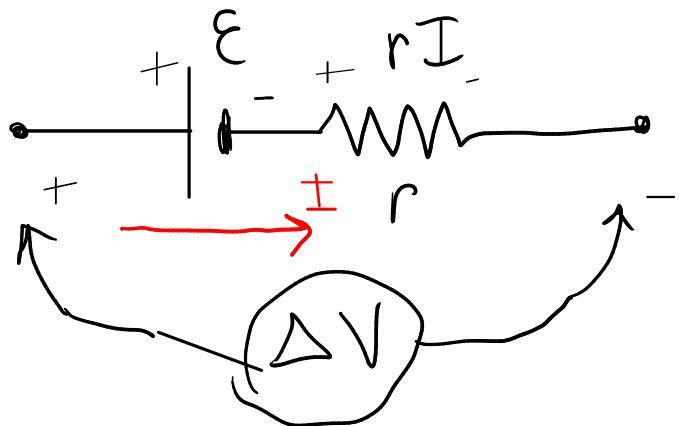
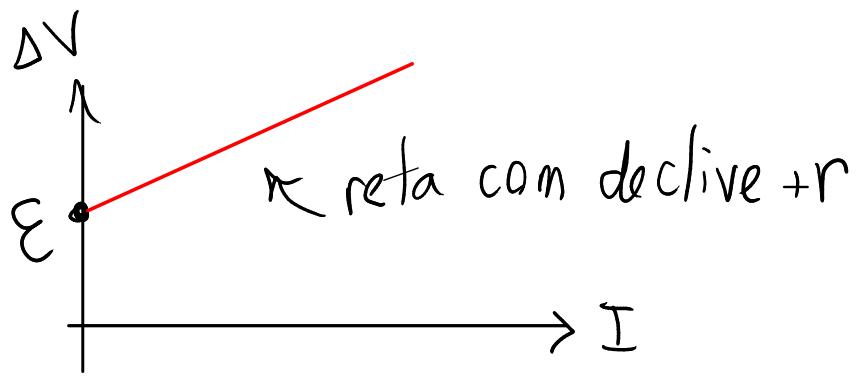
pilha em modo gerador

Modo receptor (pilha recarregável)



I de + para -

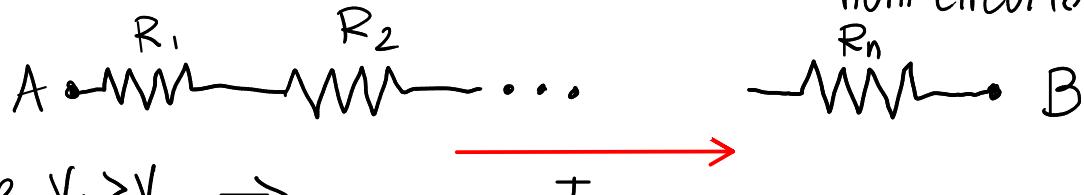
fonte externa com f.e.m. maior que a da pilha



$$\Delta V = E + r I$$

RESISTÊNCIAS EM SÉRIE

n resistências num mesmo "ramo" (percurso único entre 2 pontos num circuito)



se $V_A > V_B \Rightarrow I$

$$I_1 = I_2 = \dots = I_n = I$$

$$\Delta V = V_A - V_B = \Delta V_1 + \Delta V_2 + \dots + \Delta V_n$$

$$= R_1 I_1 + R_2 I_2 + \dots + R_n I_n = R_1 I + R_2 I + \dots + R_n I$$

$$\Delta V = (R_1 + R_2 + \dots + R_n) I$$

lei de Ohm

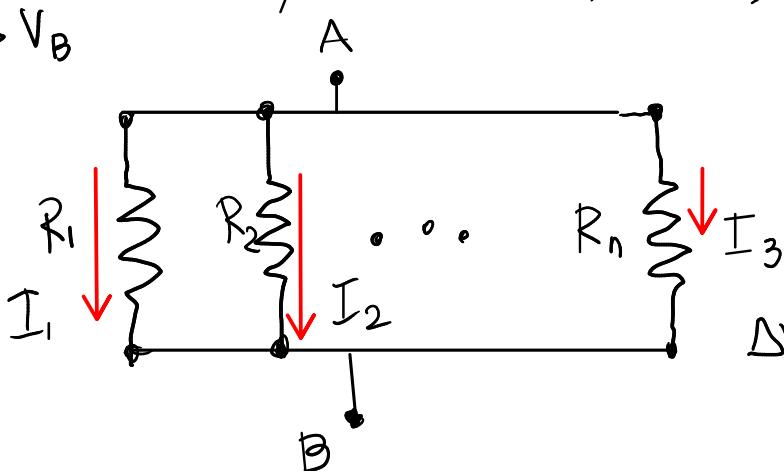
resistência equivalente: $R_s = R_1 + R_2 + \dots + R_n$



RESISTÊNCIAS EM PARALELO

n resistências, em ramos diferentes, entre os 2 mesmos pontos

$$V_A > V_B$$



$$\Delta V_1 = \Delta V_2 = \dots = \Delta V_n = \Delta V \quad (V_A - V_B)$$

$$I = I_1 + I_2 + \dots + I_n = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + \dots + \frac{\Delta V}{R_n}$$

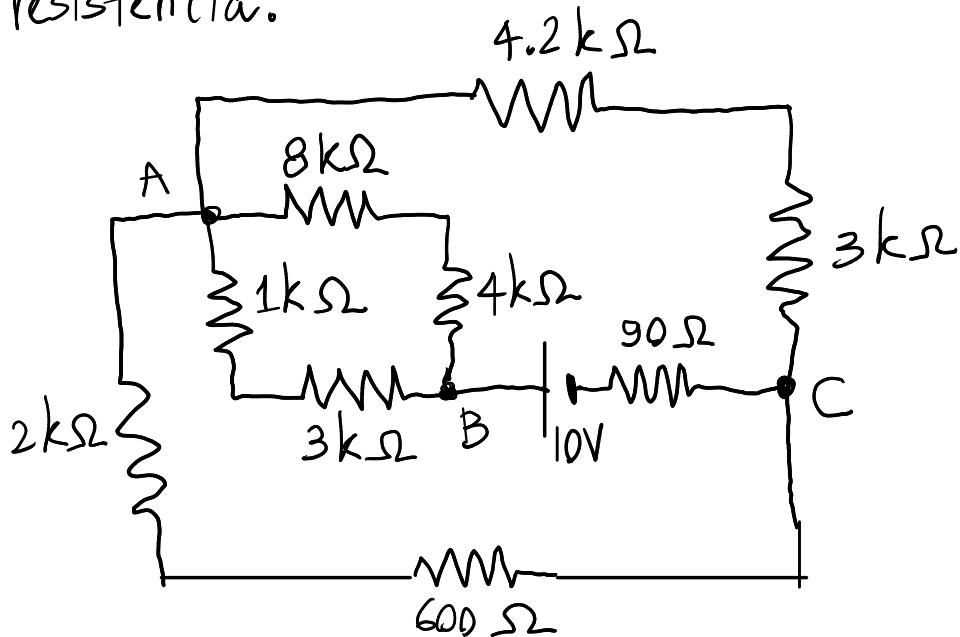
$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right) \Delta V$$

lei de Ohm

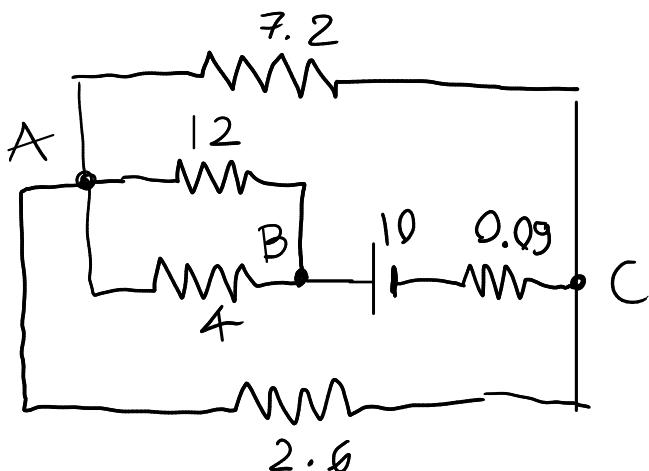
resistência equivalente: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$

caso particular $\rightarrow n=2$: $R_p = \frac{R_1 R_2}{R_1 + R_2}$

Exemplo: Determine a voltagem e corrente em cada resistência.

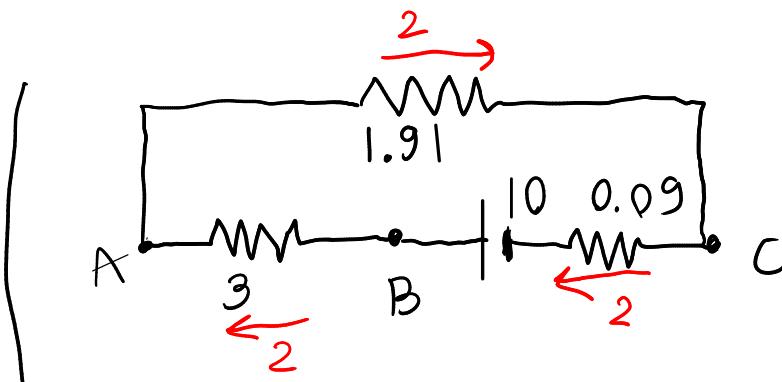


Unidades: $\Delta V \rightarrow V$, $R \rightarrow k\Omega \Rightarrow (I = \frac{\Delta V}{R})$ mA



$$\frac{4 \times 12}{4 + 12} = \frac{48}{16} = 3$$

$$\frac{7.2 \times 2.6}{7.2 + 2.6} = 1.91$$

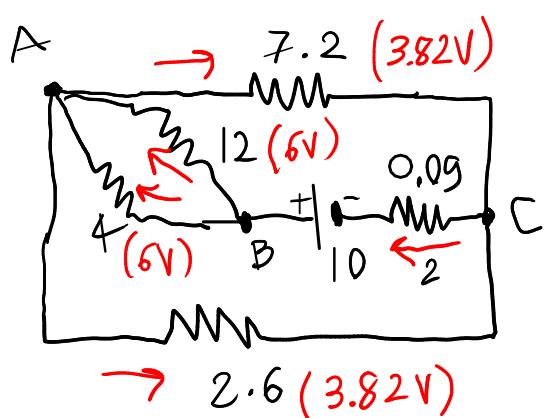
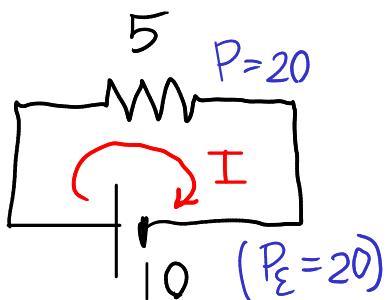


$$\Delta V_{1.91} = 1.91 \times 2 = 3.82$$

$$\Delta V_3 = 6$$

$$\Delta V_{0.09} = 0.18$$

$$I = \frac{10}{5} = 2 \text{ mA}$$



$$I_{7.2} = \frac{3.82}{7.2} = 0.53$$

$$I_{2.6} = \frac{3.82}{2.6} = 1.47$$

$$I_{12} = \frac{6}{12} = 0.5$$

$$I_4 = \frac{6}{4} = 1.5$$

$$I_{4.2} = I_3 = 0.53 \quad \Delta V_{4.2} = 4.2 \times 0.53 = 2.23 \text{ V}$$

$$\Delta V_3 = 3 \times 0.53 = 1.59$$

$$I_2 = I_{0.6} = 1.47 \Rightarrow \Delta V_2 = 2.94 \text{ V}, \Delta V_{0.6} = 0.88 \text{ V}$$

$$I_4 = I_8 = 0.5 \Rightarrow \Delta V_4 = 2 \text{ V}, \Delta V_8 = 4 \text{ V}$$

$$I_1 = I_3 = 1.5 \Rightarrow \Delta V_1 = 1.5 \text{ V}, \Delta V_{1.5} = 4.5 \text{ V}$$

Na f.e.m. $I = 2 \text{ mA}$ (de menor para maior V)
 \Rightarrow modo gerador (fornecendo energia)

POTÊNCIAS

$$\text{condutor: } P = \Delta V I$$

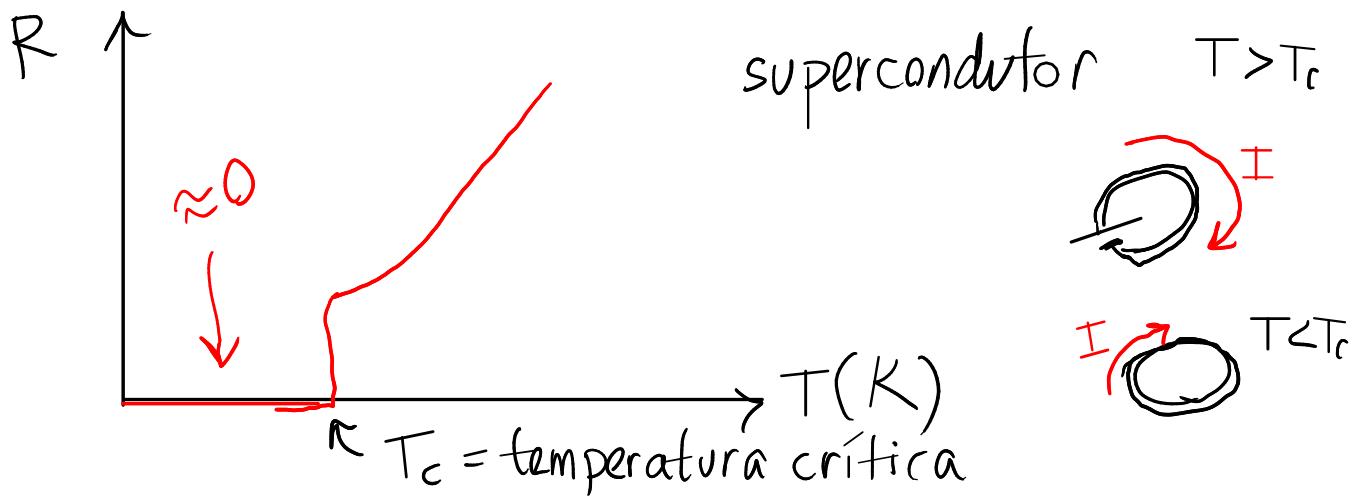
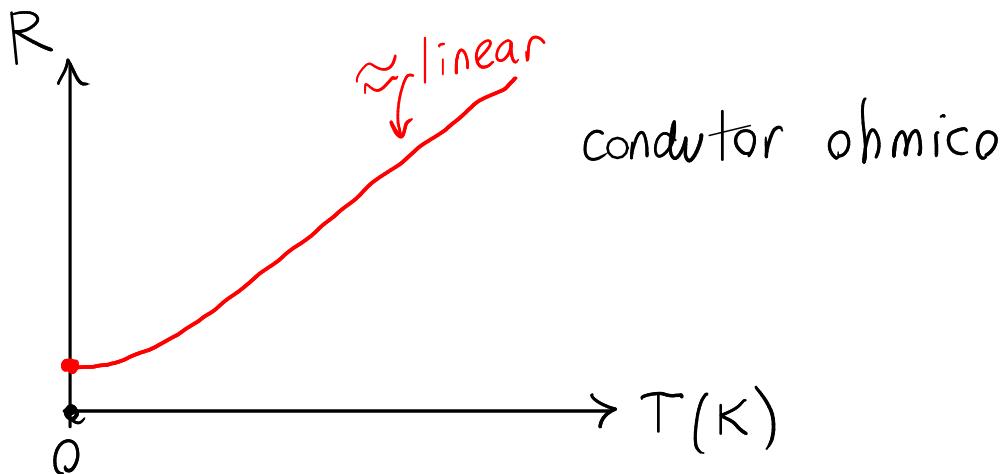
$$\text{resistências: } P = RI^2 = \frac{\Delta V^2}{R}$$

a fonte fornece $P_E = \epsilon I_E = 10 \times 2 = 20 \text{ mW}$

= soma das potências dissipadas (em calor) nas 9 resistências.

fonte no modo receptor \rightarrow potência absorvida $= P_E = \epsilon I$

SUPERCONDUTIVIDADE



primeiro supercondutor (séc. XX) \rightarrow mercúrio

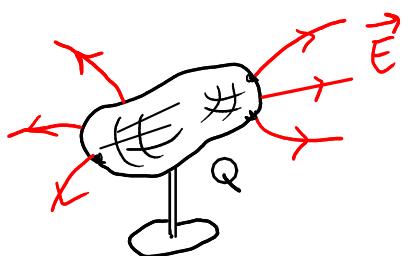
$T_c \approx 4.2\text{ K}$ (hélio líquido $\rightarrow T < 4.2\text{ K}$)

supercondutores de alta temperatura.



CAPACIDADE ELÉTRICA

condutor isolado com carga Q



arbitra-se $V=0$ no infinito

$$V_{\text{condutor}} = \int_{\text{condutor}}^{\infty} \vec{E} \cdot d\vec{r}$$

em cada posição \vec{r} , \vec{E} é diretamente proporcional a Q

$$\Rightarrow V_{\text{condutor}} = \frac{Q}{C} \quad \text{constante (capacidade do condutor)}$$

quanto maior for C , maior será a carga no condutor, quando $V_{\text{condutor}} \neq 0$

C não depende nem de $V_{\text{cond.}}$ nem de Q .
(propriedade geométrica do condutor)

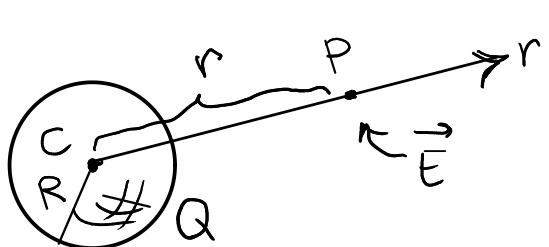
ítalica

Unidade SI de capacidade

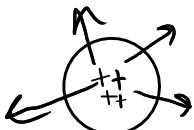
$$1 \text{ F (farad)} = 1 \frac{C}{V} = 1 \frac{C^2}{N \cdot m}$$

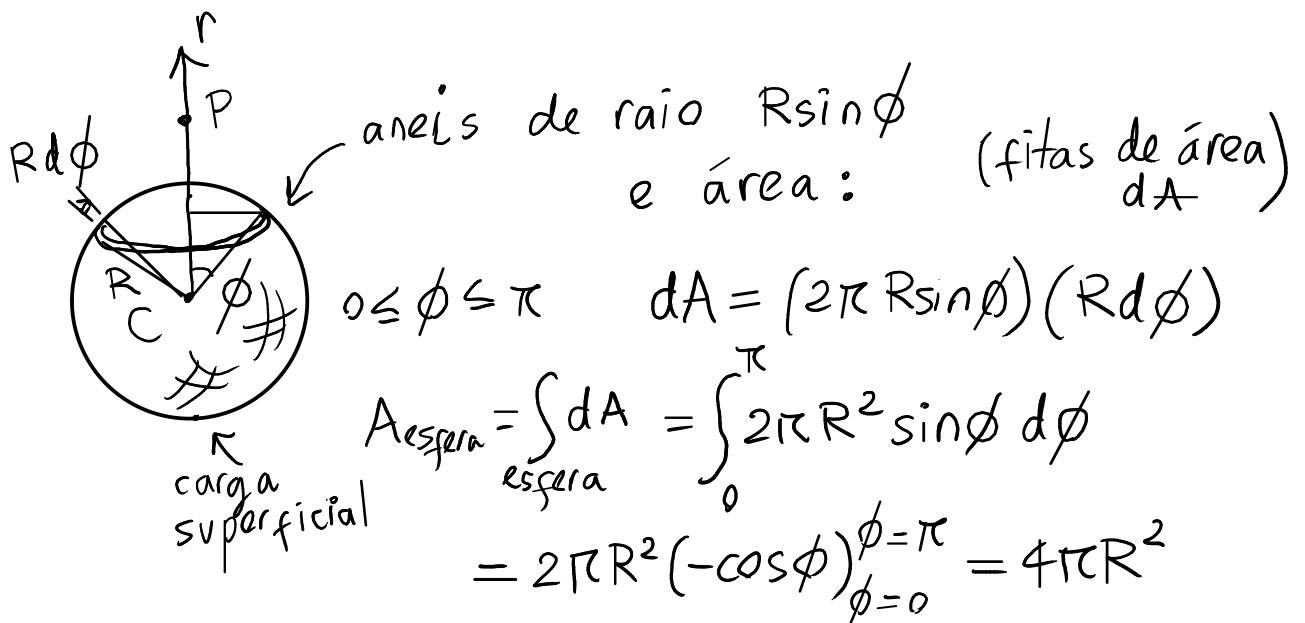
$$k = 9 \times 10^9 \frac{N \cdot m^2}{C^2} = 9 \times 10^9 \frac{m}{F}$$

Esfera condutora de raio R



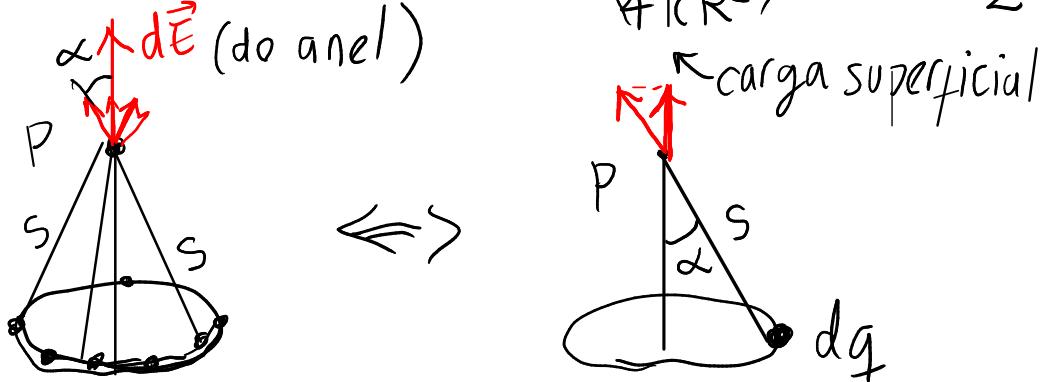
calcular \vec{E} a uma distância r do centro, em função Q





$Q \rightarrow$ distribui-se uniformemente na superfície da esfera

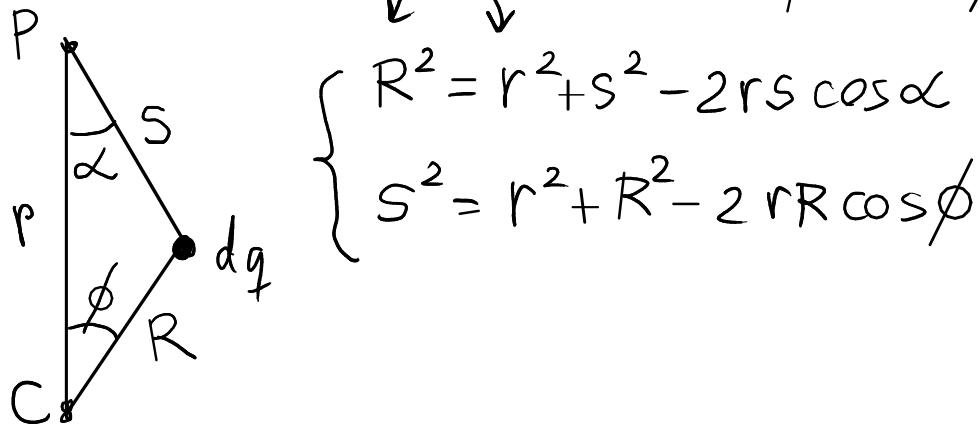
$$\Rightarrow \text{carga no anel} = dq = \left(\frac{Q}{4\pi R^2}\right) dA = \frac{Q}{2} \sin\phi d\phi$$



$$dE = \left(\frac{k dq}{S^2}\right) \cos\alpha = \frac{k Q}{2} \left(\frac{\cos\alpha \sin\phi d\phi}{S^2}\right)$$

α e S dependem de ϕ

lei do cosseno constantes (não dependem de ϕ)

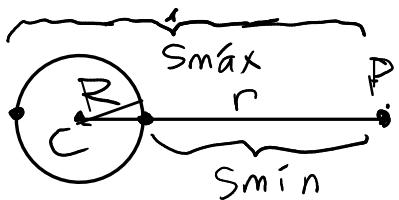


$$\Rightarrow \left\{ \begin{array}{l} \cos \alpha = \frac{r^2 + s^2 - R^2}{2rs} \\ 2sds = 0 + 0 + 2rR \sin \phi d\phi \rightarrow \sin \phi d\phi = \frac{sds}{rR} \end{array} \right.$$

$$\Rightarrow dE = \frac{kQ}{2} \left(\frac{1}{s^2} \right) \left(\frac{r^2 + s^2 - R^2}{2rs} \right) \left(\frac{sds}{rR} \right) = \frac{kQ}{4Rr^2} \left(\frac{r^2 + s^2 - R^2}{s^2} \right) ds$$

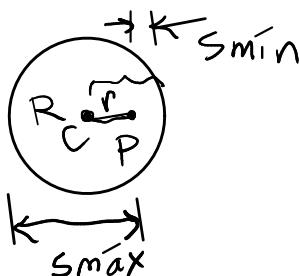
$$E = \int dE = \frac{kQ}{4Rr^2} \int_{s_{\min}}^{s_{\max}} \frac{r^2 + s^2 - R^2}{s^2} ds$$

a) $r > R$ (P fora da esfera)



$$s_{\min} = r - R \quad s_{\max} = R + r$$

b) $r < R$ (dentro da esfera)



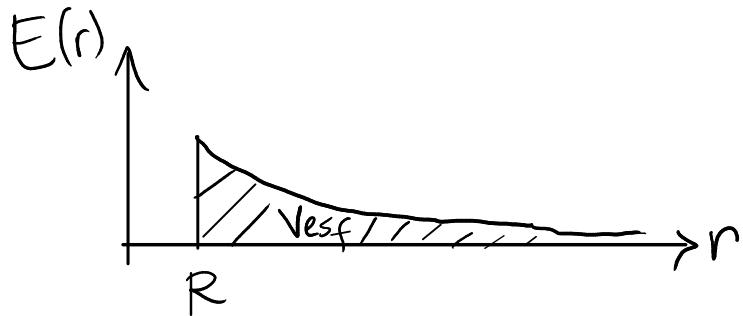
$$s_{\min} = R - r \quad s_{\max} = R + r$$

$$\Rightarrow I = \begin{cases} 4R, & r > R \\ 0, & r < R \end{cases}$$

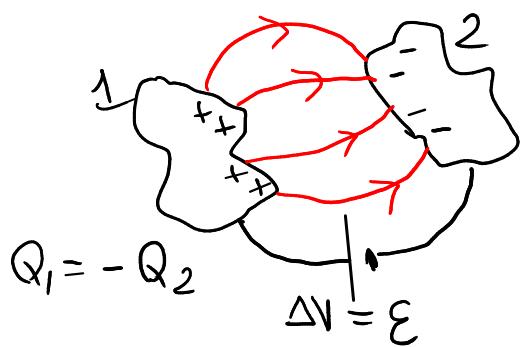
$$\Rightarrow E(r) = \begin{cases} \frac{kQ}{r^2}, & r > R \\ 0, & r < R \end{cases} \quad \begin{matrix} \text{(como se Q estivesse)} \\ \text{toda no centro} \end{matrix}$$

$$V_{esfera} = \int_R^\infty E dr \quad \vec{E} \cdot d\vec{r} = kQ \int_R^\infty \frac{dr}{r^2} = \frac{kQ}{R}$$

$$C_{\text{sfera}} = \frac{Q}{V_{\text{sfera}}} = \frac{R}{k} \quad (k = 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2})$$



CONDENSADORES

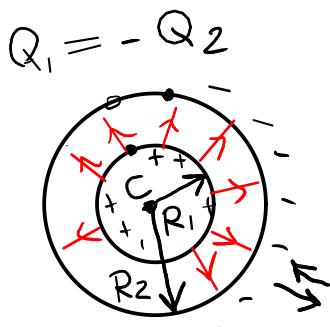


armaduras
dois condutores isolados,
próximos entre si

$$\Delta V = \frac{Q}{C} \quad \begin{array}{l} \leftarrow \text{valor absoluto de } Q_1 \\ \text{e } Q_2 \end{array}$$

$C = \frac{Q}{\Delta V}$ \leftarrow constante que depende da geometria do condensador

Condensador esférico



duas esferas condutoras, isoladas, concêntricas, de raios

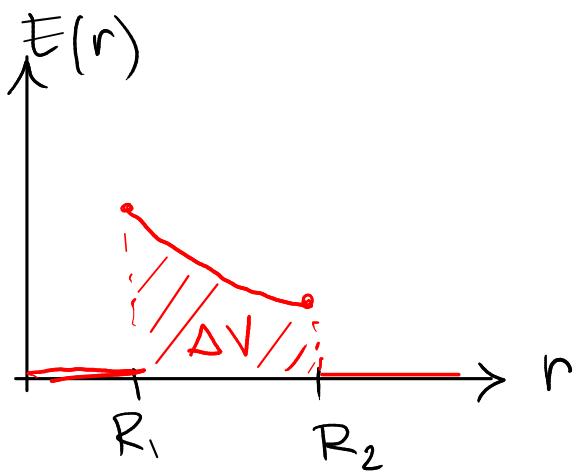
$$R_1 < R_2$$

$$E_1 = \begin{cases} \frac{kQ_1}{r^2}, & r > R_1 \\ 0, & r < R_1 \end{cases} \quad E_2 = \begin{cases} \frac{kQ_2}{r^2}, & r > R_2 \\ 0, & r < R_2 \end{cases}$$

$$r < R_1 \Rightarrow E_1 = E_2 = 0$$

$$R_1 < r < R_2 \Rightarrow E_{\text{total}} = E_1 \quad (E_2 = 0)$$

$$r > R_2 \Rightarrow E_{\text{total}} = 0 \quad |\vec{E}_1| = |\vec{E}_2| \quad e \quad \vec{E}_1 = -\vec{E}_2$$



$$\Delta V = \int_{R_1}^{R_2} \frac{kQ}{r^2} dr$$

$$Q_1 = -Q_2 \quad Q_1 + Q_2 = 0$$

$$Q = |Q_1| = |Q_2|$$

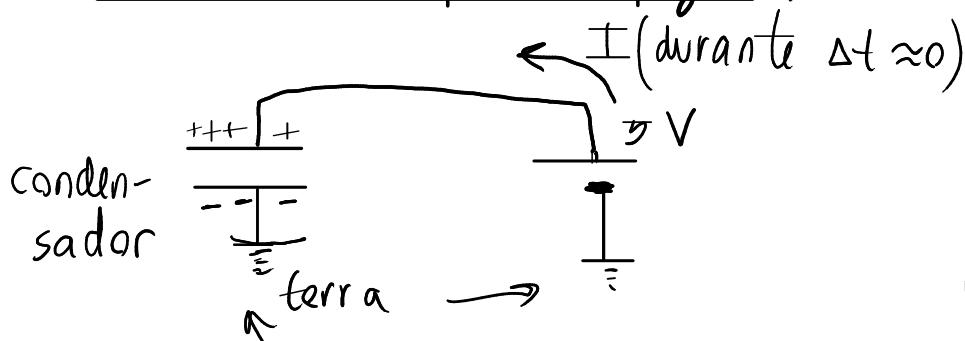
$$\Delta V = kQ \left(-\frac{1}{r} \right) \Big|_{r=R_1}^{r=R_2} = kQ \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$C = \frac{Q}{\Delta V} \Rightarrow C_{\text{esférico}} = \frac{1}{k \left(\frac{R_2 - R_1}{R_1 R_2} \right)}$$

condensador esférico →

$$C = \frac{R_1 R_2}{k(R_2 - R_1)}$$

Comentários (respostas às perguntas)



$$\begin{array}{c} ++/+ \\ \hline --/- \end{array} \begin{array}{l} Q \\ \hline \Delta V \end{array}$$

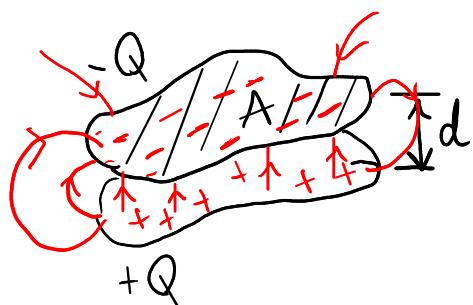
Memória dum computador

$$\begin{array}{c} \boxed{} \\ \hline \boxed{} \end{array} \begin{array}{l} Q=0 \rightarrow 0 \\ Q \neq 0 \rightarrow 1 \end{array}$$

$$Q = C \Delta V$$

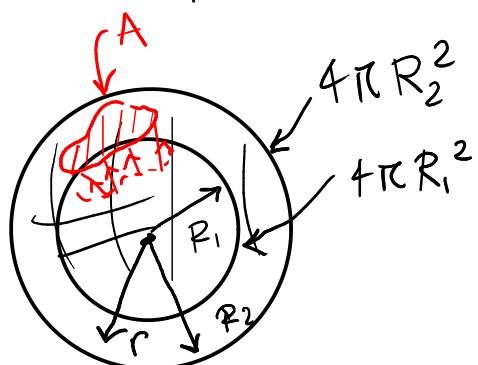
$$I = \frac{dQ}{dt} = C \frac{d\Delta V}{dt}$$

CONDENSADOR PLANO



Duas superfícies condutoras planas, identicas e paralelas, com área A , à uma distância d .

Aproximação: condensador esférico



carga distribuída uniformemente

a aproximação é boa no limite:

$$R_1 \rightarrow \infty, R_2 \rightarrow \infty$$

$$(R_2 - R_1 = d)$$

Q_e = carga na esfera.
 Q = carga na área A

$$\frac{Q}{Q_e} = \frac{A}{4\pi R_1^2} \Rightarrow Q_e = \frac{4\pi R_1^2}{A} Q$$

dentro do condensador esférico

$$E(r) = \frac{k Q_e}{r^2} = \frac{k}{r^2} \left(\frac{4\pi R_1^2}{A} \right) Q \quad R_1 < r < R_2$$

$$R_1, R_2 \rightarrow \infty \quad r \rightarrow R_1 \quad (r \rightarrow R_2)$$

$$\Rightarrow E(r) = \frac{4\pi k Q}{A} \quad (\text{constante!})$$

$$\Delta V = \int_{\text{arm. 1}}^{\text{armad. 2}} E(r) dr = \frac{4\pi k Q}{A} \int_1^2 dr = \frac{4\pi k Q}{A} d$$

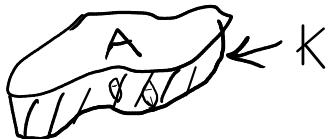
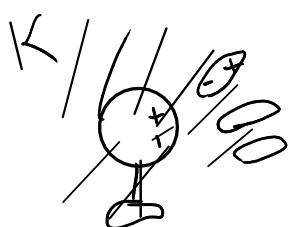
$$C = \frac{Q}{\Delta V} \Rightarrow$$

$$C_{\text{plano}} = \frac{A}{4\pi k d}$$



entre as armaduras costuma colocar-se um isolador (dielétrico) para evitar o contacto entre as armaduras, e aumentar a capacidade.

Capacidade com dielétrico (com constante K)



$$\text{sem dielétrico: } Q \rightarrow E_0 \rightarrow \Delta V_0 \rightarrow C_0 = \frac{Q}{\Delta V_0}$$

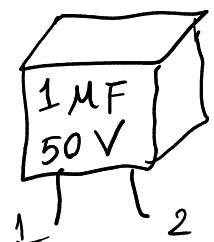
$$\text{com dielétrico: } Q \rightarrow E = \frac{E_0}{K} \rightarrow \Delta V = \frac{\Delta V_0}{K} \rightarrow C = \frac{Q}{\Delta V}$$

$$C = K C_0 \quad K \geq 1 \Rightarrow C \geq C_0$$

outra vantagem: rigidez dielétrica do dielétrico maior que a do ar.

$$E_{\text{máx}} \rightarrow \Delta V_{\text{máx}} \rightarrow Q_{\text{máx}} = C \Delta V_{\text{máx}}$$

se $\Delta V > \Delta V_{\text{máx}}$ \Rightarrow queima-se o dielétrico



ENERGIA ARMAZENADA NUM CONDENSADOR

Estado inicial



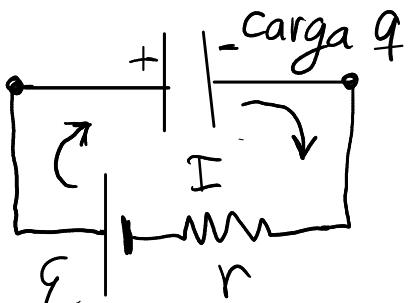
$$Q=0 \Rightarrow \Delta V=0$$

$V=0$ (energia armazenada)

avmento da energia.

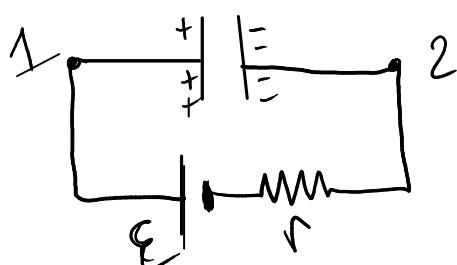
$$\delta U = \delta q V_+ - \delta q V_- = \delta q (V_+ - V_-) = \Delta V \delta q$$

(>0)



num intervalo dt
 entrar carga $+dq = Idt$
 numa armadura, e carga
 $-dq = Idt$ na outra armadura
 $(\Delta V$ aumenta)

Estado estacionário :



$$\Delta V = E \Rightarrow I=0$$

Q permanece constante

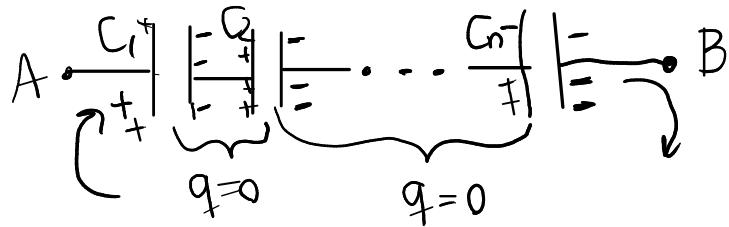
$$Q = CE$$

$$U = \int_0^{\text{final}} dU = \int_0^Q \Delta V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \left(\frac{q^2}{C} \right)_{q=0}^{q=Q}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C \Delta V^2 = \frac{1}{2} Q \Delta V$$

em qualquer estado

CONDENSADORES EM SÉRIE (no mesmo ramo)



$$Q_1 = Q_2 = \dots = Q_n$$

$$\Delta V = |V_A - V_B| = \Delta V_1 + \Delta V_2 + \dots + \Delta V_n$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \dots + \frac{Q}{C_n} \Rightarrow \Delta V = Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} \right)$$

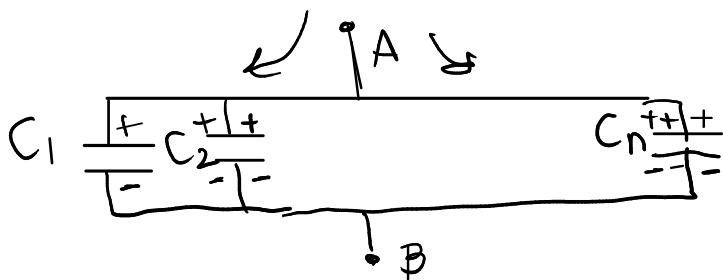
$$\Delta V = \frac{Q}{C_s}$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$C \rightarrow \Delta V = \frac{Q}{C}$$

$$R \rightarrow \Delta V = RI$$

CONDENSADORES EM PARALELO . n ramos entre os mesmos 2 pontos



$$Q_1 \neq Q_2 \neq \dots \neq Q_n$$

$$\Delta V = \Delta V_1 = \Delta V_2 = \dots = \Delta V_n$$

$$Q = Q_1 + Q_2 + \dots + Q_n = C_1 \Delta V + C_2 \Delta V + \dots + C_n \Delta V$$

$$Q = (C_1 + C_2 + \dots + C_n) \Delta V \Rightarrow$$

$$C_p = \sum_{i=1}^n C_i$$

ULTRACONDENSADORES (supercapacitor)

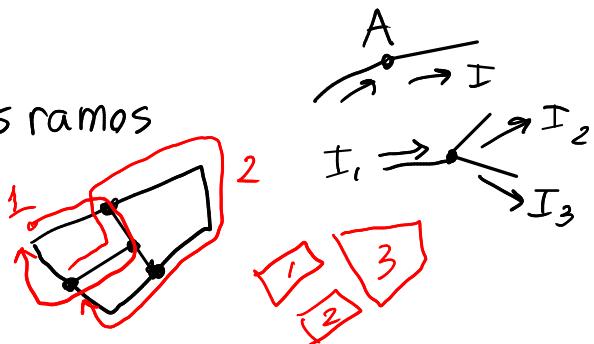
capacidades de kF

CIRCUITOS DE CORRENTE CONTÍNUA

Leis Kirchhoff

nó = ponto comum a 3 ou mais ramos

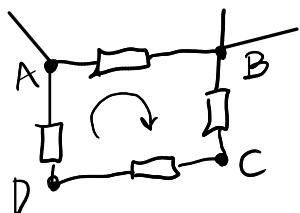
malha = percurso fechado



① Lei das malhas (voltagens).

Em qualquer malha, a soma algébrica das voltagens é nula.

Exemplo:



$$V_A = V_A \quad V_A - V_A = 0$$

$$(V_A - V_B) + (V_B - V_A) = 0$$

$$\Delta V_{BA} + \Delta V_{AB} = 0$$

$$\Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} + \Delta V_{DA} = 0$$

ΔV_{PQ} ← voltagem de P para Q

$$\longrightarrow \Delta V_{PQ} = V_Q - V_P$$

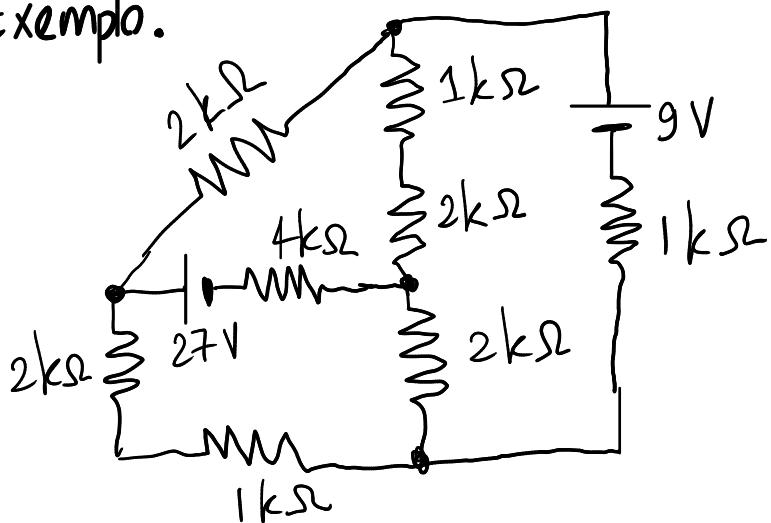
$$\Delta V_{PQ} = \begin{cases} >0, & V_P < V_Q \\ <0, & V_P > V_Q \\ =0, & V_P = V_Q \end{cases}$$

② Lei dos nós (correntes)

Em qualquer nó, a soma algébrica das correntes é nula

$$\begin{cases} I_j > 0 & \text{se entrar corrente no nó pelo ramo } j \\ I_j < 0 & \text{se sair corrente do nó } " " " " \\ & (\text{ou ao contrário}) \end{cases}$$

Exemplo.



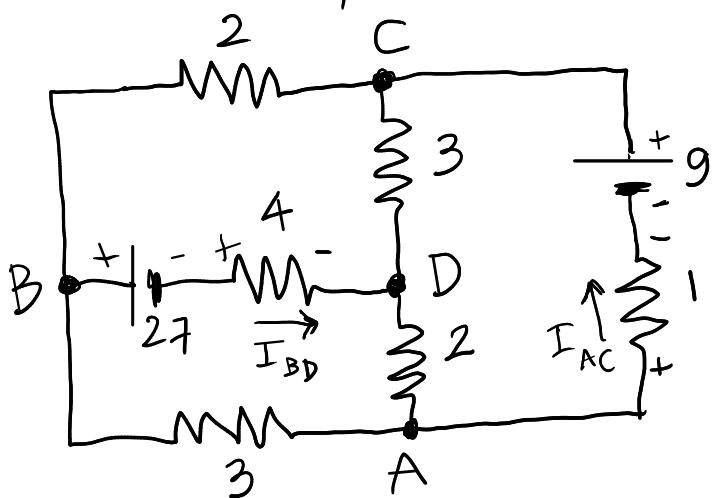
Unidades:

$$\Delta V \rightarrow V$$

$$R \rightarrow k\Omega$$

$$I \rightarrow mA$$

círculo equivalente:



Método dos nós.

lei de Ohm

$$\left\{ \begin{array}{l} V_A - V_B = 3 I_{AB} \\ V_B - V_C = 2 I_{BC} \\ V_A - V_D = 2 I_{AD} \\ V_C - V_D = 3 I_{CD} \\ V_A - V_C = I_{AC} - 9 \\ V_B - V_D = 4 I_{BD} + 27 \end{array} \right.$$

$$\underbrace{V_A, V_B, V_C, I_{AB}, I_{BC}, I_{AD}, I_{CD}, I_{AC}, I_{BD}}$$

3 nós independentes
3 malhas independentes
6 ramos (12 equações)
(eq. de Ohm)
12 variáveis
6 correntes nos ramos
+ 6 voltagens nas 6 resistências

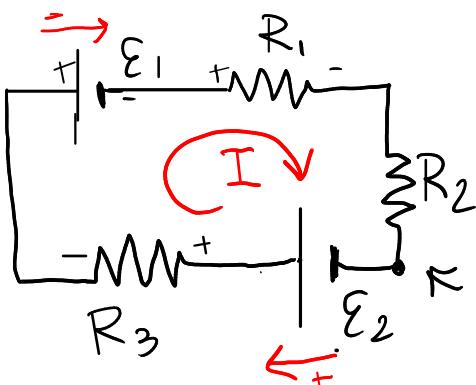
arbitrar um dos potenciais 0
exemplo: $V_D = 0$

lei dos nós

A	$I_{AB} + I_{AD} + I_{AC} = 0$
B	$-I_{AB} + I_{BC} + I_{BD} = 0$
C	$-I_{AC} - I_{BC} + I_{CD} = 0$

Método das malhas

1 malha



eq. da malha:

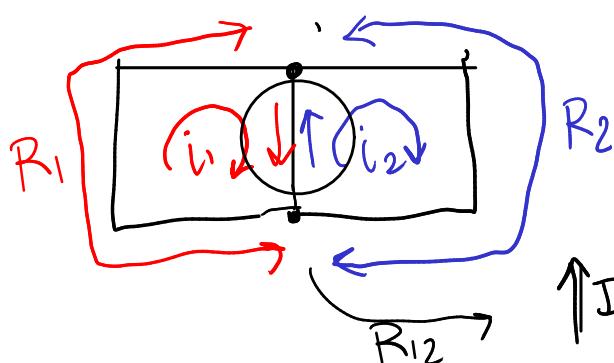
$$E_2 - R_3 I - E_1 - R_1 I - R_2 I = 0$$

$$(R_1 + R_2 + R_3) I = E_2 - E_1$$

série

↑ para | | para ↑

duas malhas



i_1 e i_2 = correntes das malhas
(no mesmo sentido)

$$\uparrow I = i_2 - i_1 \quad \downarrow I = i_1 - i_2$$

2 eq. de malhas:

$$\begin{cases} R_1 i_1 + R_{12} (i_1 - i_2) = E_1 \\ R_2 i_2 + R_{12} (i_2 - i_1) = E_2 \end{cases}$$

↖ forte total nas malha 1 e 2

$$\begin{bmatrix} R_{11} & -R_{12} \\ -R_{12} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$\uparrow R \vec{i} = \vec{E}$

$$R_{11} = R_1 + R_{12}$$

$=$ resist. total
da malha 1

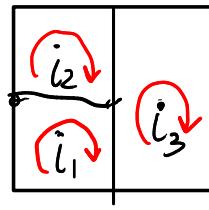
$$R_{22} = R_2 + R_{12} = R_{\text{malha 2}}$$

$$\Rightarrow \vec{i} = R^{-1} \vec{E}$$

$$R_{n \times n} \quad \vec{i}_{n \times 1} \quad \vec{E}_{n \times 1}$$

n malhas

No exemplo acima,

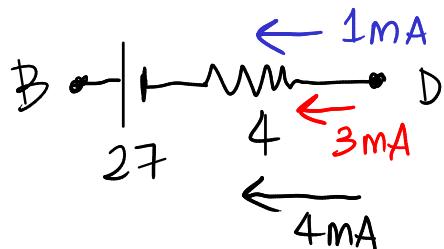


$$R = \begin{bmatrix} 9 & -4 & -2 \\ -4 & 9 & -3 \\ -2 & -3 & 6 \end{bmatrix} \quad E = \begin{bmatrix} -27 \\ 27 \\ -9 \end{bmatrix}$$

$$i = R^{-1} E$$

i: invert(matrix([9,-4,-2],[-4,9,-3],[-2,-3,6])) . [27,27,-9];

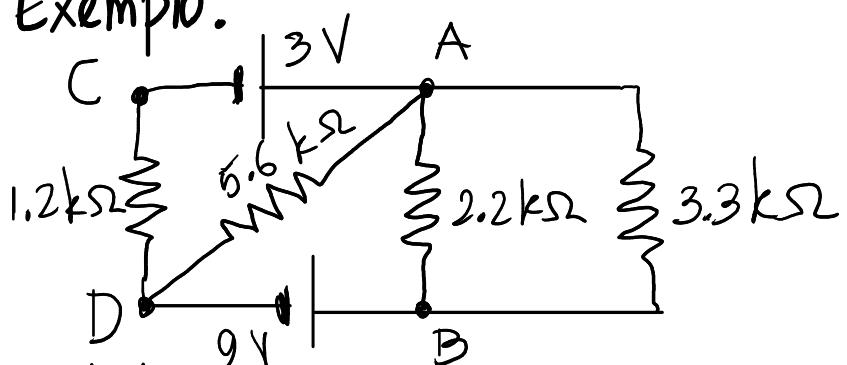
$$\hookrightarrow \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} \quad i_1 = -3, \quad i_2 = 1, \quad i_3 = -2$$



MÉTODO DE SOBREPOSIÇÃO

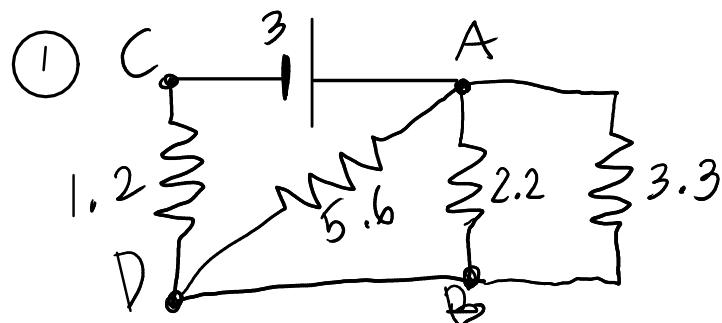
Circuito com $n (> 1)$ f.e.m.'s \Leftrightarrow sobreposição de n circuitos com apenas uma das f.e.m. e as outras em curto-círcuito.

Exemplo.

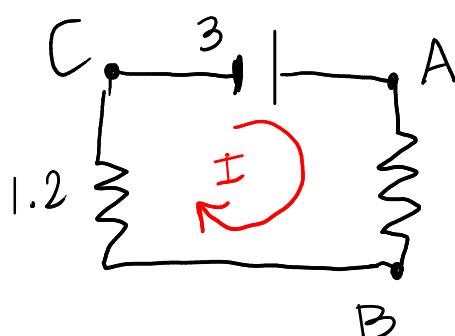


Determine a corrente e voltagem EM cada uma das resistências.

Unidades: $R \rightarrow k\Omega$, $\Delta V \rightarrow V \Rightarrow I \rightarrow mA$
dois circuitos mais simples:



$$V_B = V_D$$



3 resistências em paralelo entre A e B

$$R_P = \left(\frac{1}{2.2} + \frac{1}{3.3} + \frac{1}{5.6} \right)^{-1} = 1.068$$

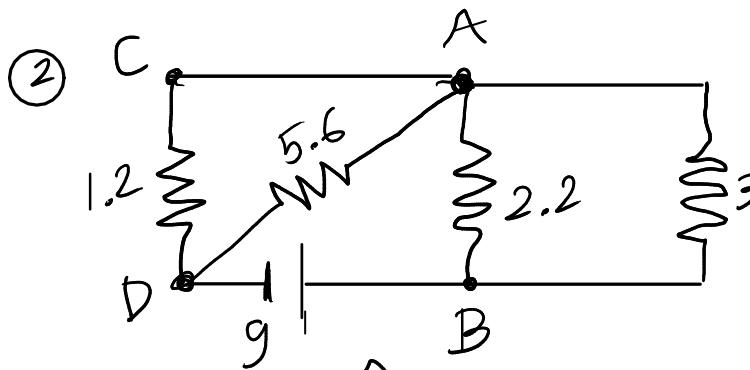
$$I_{1.2} = I = \frac{3}{1.2 + 1.068} = 1.323$$

$$V_{AB} = -I \times 1.068 = -1.413$$

$$V_{BC} = -I \times 1.2 = -1.587$$

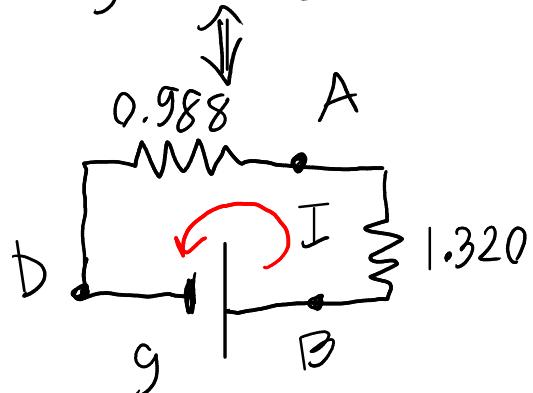
$$I_{5.6} = -\frac{V_{AB}}{5.6} = 0.253$$

$$I_{3.3} = -\frac{V_{AB}}{3.3} = 0.428 \quad I_{2.2} = -\frac{V_{AB}}{2.2} = 0.642$$



$$R_{AB} = \left(\frac{1}{2.2} + \frac{1}{3.3} \right)^{-1} = 1.320$$

$$R_{AD} = \left(\frac{1}{1.2} + \frac{1}{5.6} \right)^{-1} = 0.988$$



$$I = \frac{9}{0.988 + 1.32} = 3.899$$

$$V_{AB} = +1.32I = -5.147$$

$$V_{AD} = -0.988I = -3.853$$

$$I_{1.2} = \frac{3.853}{1.2} = 3.211 \quad I_{5.6} = \frac{3.853}{5.6} = 0.688 \quad I_{2.2} = \frac{5.147}{2.2} = 2.339$$

$$I_{3.3} = \frac{5.147}{3.3} = 1.560$$

Resultado final (sobreposição):

$$I_{3.3} = 1.560 - 0.428 = 1.132 \text{ mA (de B para A)}$$

$$I_{2.2} = 1.697 \text{ mA (de B para A)}$$

$$I_{5.6} = 0.941 \text{ mA (de A para D)}$$

$$I_{1.2} = 1.888 \text{ mA (de C para D)}$$

$$V_{BA} = 3.734 \text{ V}$$

$$V_{AD} = \cancel{0.941} \rightarrow -5.266 \text{ V}$$

$$V_{CD} = 2.266 \text{ V}$$

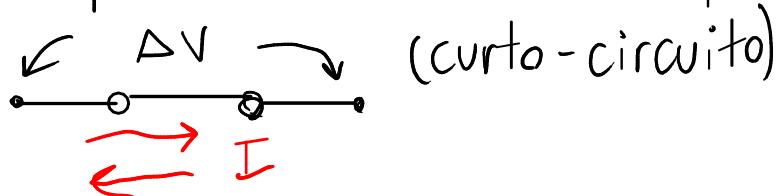
CIRCUITOS COM RESISTÊNCIAS E CONDENSADORES

Condensadores.

① descarregado: $Q=0 \Rightarrow \Delta V = \frac{Q}{C} = 0$

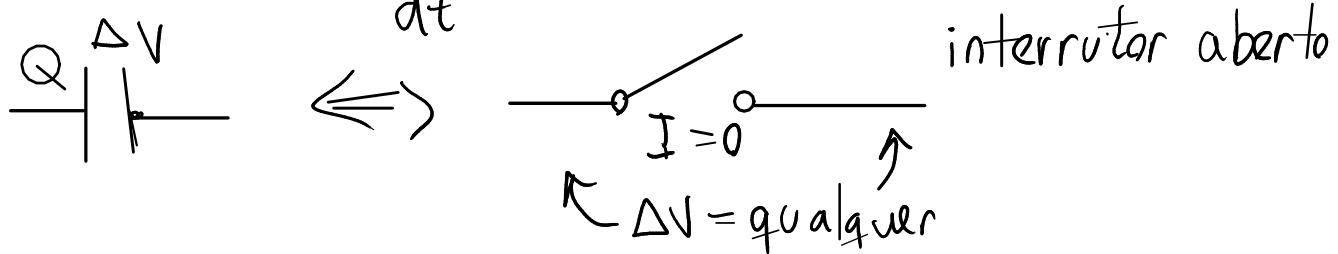
mas $\frac{dQ}{dt} \neq 0$ ($I = \frac{dQ}{dt}$) $I = \text{qualquer valor}$

Equivalente a um interruptor fechado

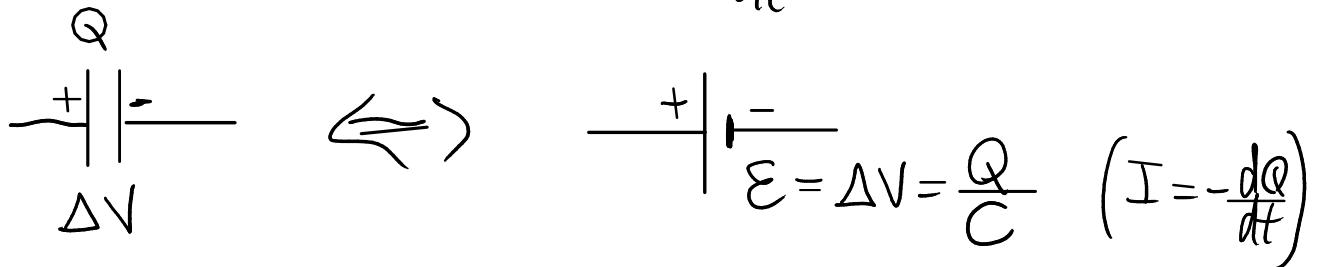


② estado estacionário: $Q = \text{constante}$ ($\Delta V = \text{constante}$)

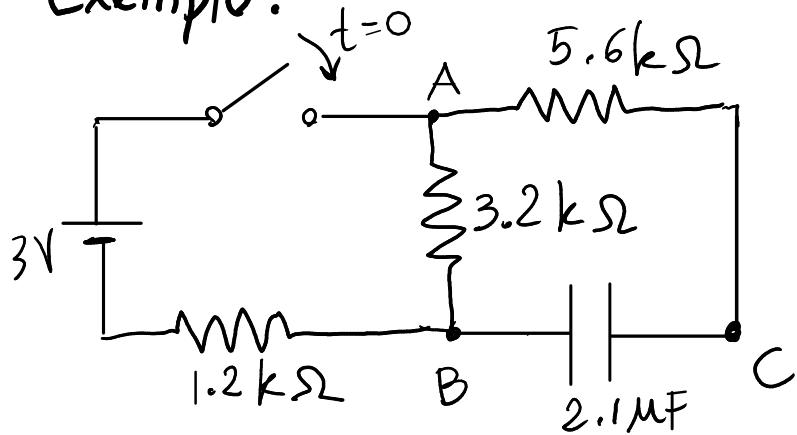
$$\Rightarrow I = \frac{dQ}{dt} = 0$$



③ estado transitório: $I = \frac{dQ}{dt} \neq 0 \quad Q \neq 0, \Delta V \neq 0$



Exemplo.

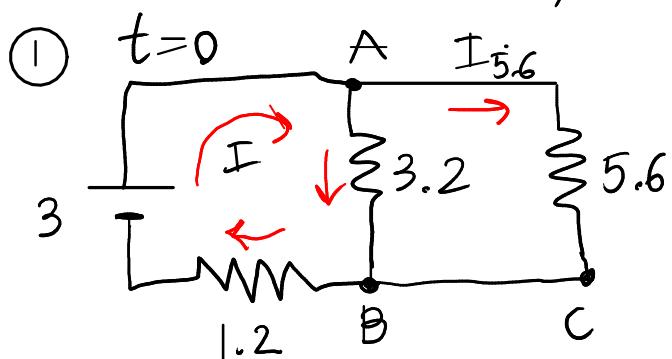


Em $t=0$, o condensador está descarregado e fecha-se o interruptor.

Num instante t_1 posterior (muito maior que 0), abre-se novamente

o interruptor. Determine a corrente na resistência de $5.6 \text{ k}\Omega$, em $t=0$ e t_1 .

unidades: $R \rightarrow \text{k}\Omega$, $\Delta V \rightarrow \text{V}$, $I \rightarrow \text{mA}$



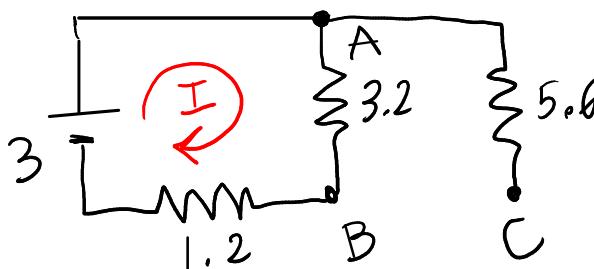
$$R_{AB} = \left(\frac{1}{3.2} + \frac{1}{5.6} \right)^{-1} = 2.036$$

$$I = \frac{3}{1.2 + 2.036} = 0.927$$

$$V_{BA} = 2.036 I = 1.888 \text{ V}$$

$$I_{5.6} = \frac{V_{BA}}{5.6} = 0.337 \text{ mA} \quad (\text{de } A \text{ para } C)$$

② $t \rightarrow t_1$ ($t < t_1$) \Rightarrow estado estacionário

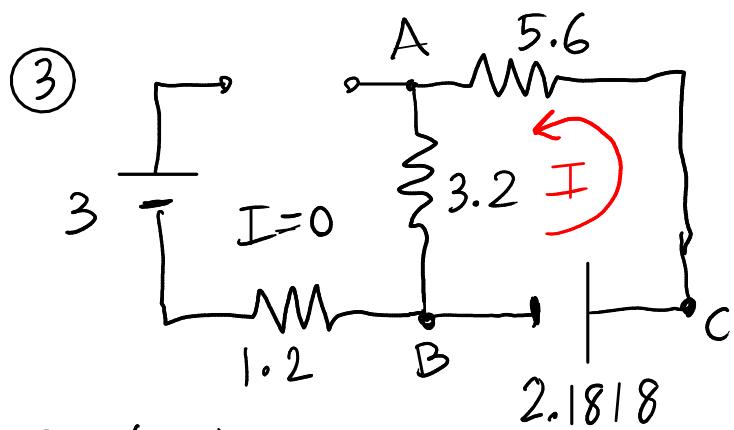


$$I_{5.6} = 0 \quad V_A = V_C$$

$$I = \frac{3}{4.4} = 0.618$$

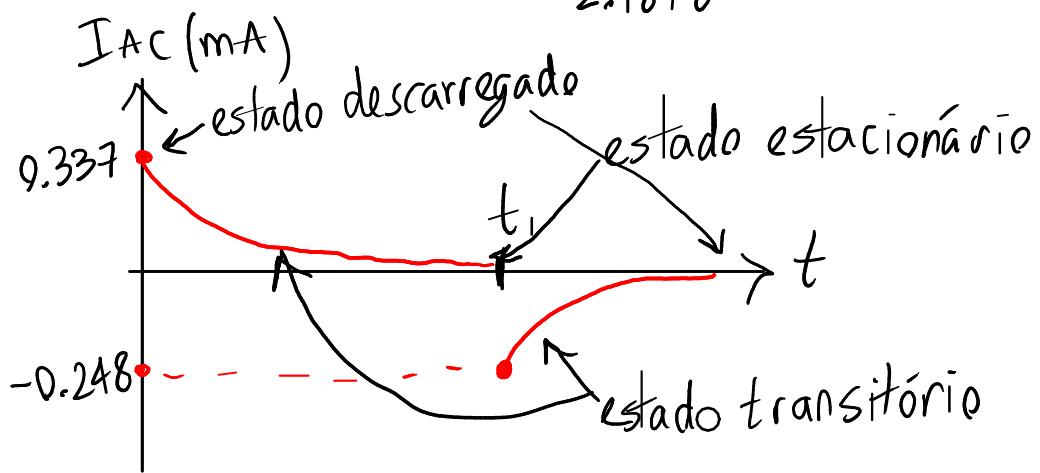
$$V_{BA} = V_{BC} = 3.2 I = 2.1818 \text{ V}$$

\uparrow
voltage no condensador (carga positiva em C)

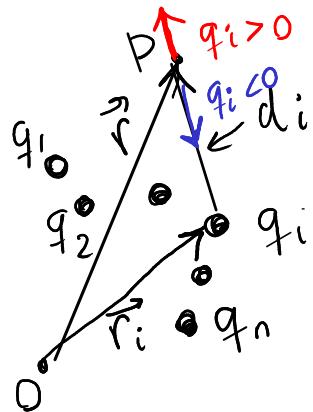


$$I = \frac{2.1818}{8.8} = 0.248 \text{ mA}$$

(de C para A)



CAMPO ELÉTRICO DE CARGAS PONTUAIS



$$E_i = \frac{k|q_i|}{d_i^2} \quad d_i = |\vec{r} - \vec{r}_i|$$

direção de $\vec{r} - \vec{r}_i$ e o mesmo sentido se $q_i > 0$, o sentido oposto, se $q_i < 0$

$$\hat{e}_{d_i} = \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|}$$

$$\vec{E}_i = \frac{kq_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

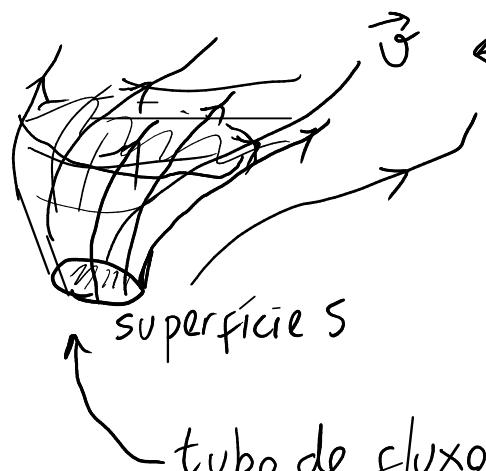
$$\vec{E}(\vec{r}) = \sum_{i=1}^n \frac{kq_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

coord. cartesianas

$$E_x = \sum_{i=1}^n \frac{kq_i(x - x_i)}{((x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2)^{3/2}}$$

semelhante para E_y e E_z

FLUXO ELÉTRICO



campo de velocidades dum fluido incompressível

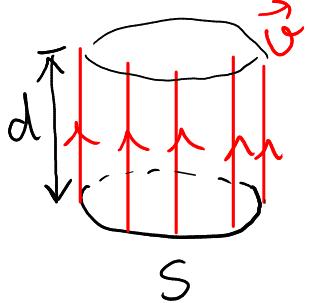
ψ_S = fluxo através de S = volume de fluido que passa através de S , por unidade de t.

tubo de fluxo da superfície S .

Se no tubo de fluxo não há nem entrada nem saída de fluido $\Rightarrow \psi_S$ é o mesmo em qualquer do tubo.



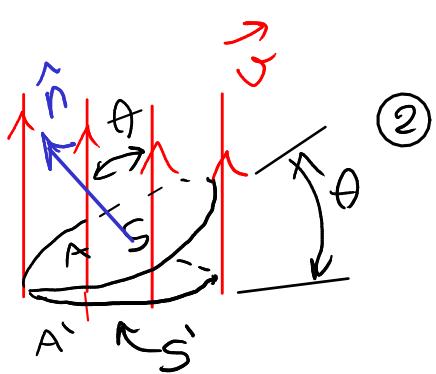
$$\psi_{S_2} = \psi_{S_1}$$



① caso de S perpendicular ao campo, e campo constante.

$$\Rightarrow \gamma_S = Ad \quad (A = \text{área de } S) \quad (d = \text{distância que o fluido percorre em } t=1)$$

$$d = v t$$



$$\boxed{\gamma_S = A v}$$

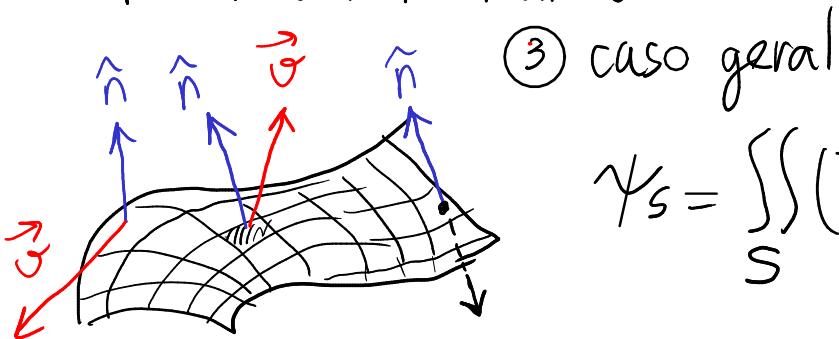
② \vec{v} constante e S inclinada θ em relação a \vec{v} .

$$\gamma_S = \gamma_{S'} = A' v$$

$$A' = A \cos \theta \quad \theta = \angle(\vec{v}, \hat{n})$$

$$\gamma_S = A v \cos \theta = A \vec{v} \cdot \hat{n}$$

\hat{n} = versor normal a S



③ caso geral

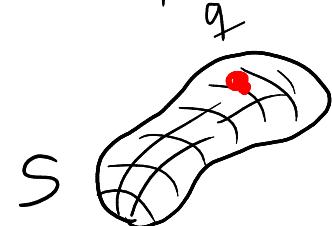
$$\gamma_S = \iint_S (\vec{v} \cdot \hat{n}) dA$$

fluxo de qualquer campo vetorial contínuo

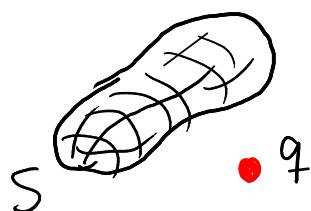
$$\boxed{\gamma_S = \iint_S (\vec{E} \cdot \hat{n}) dA}$$

LEI DE GAUSS

válida para superfícies fechadas S . Seja uma carga pontual q

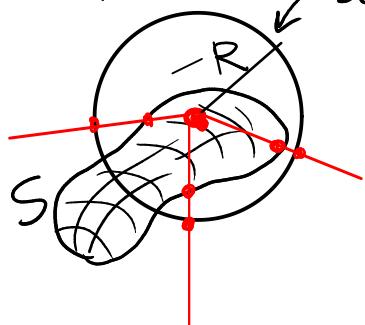


(a) q dentro



(b) q fora

$$\textcircled{a} \quad \psi_S = ?$$



Se = superfície esférica de raio R , com centro em q .

Se no mesmo tubo de fluxo

se S for fechada, define-se $\psi_S > 0$, se for para fora, e $\psi_S < 0$ se o fluxo for para dentro

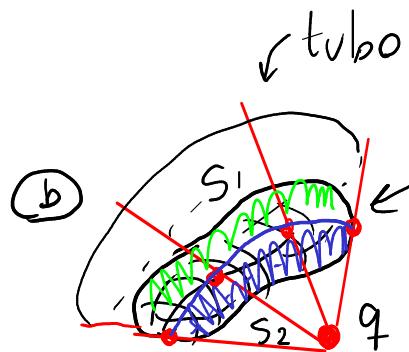
$$q = \begin{cases} > 0 & \rightarrow \psi_S > 0 \\ < 0 & \rightarrow \psi_S < 0 \end{cases} \quad \psi_S = \psi_{Se}$$

módulo de \vec{E} em Se : $E = \frac{k|q|}{R^2}$
e \vec{E} é perpendicular a Se .

$$\Rightarrow \psi_{Se} = \pm E A_{Se} = \pm \left(\frac{k|q|}{R^2} \right) (4\pi R^2)$$

$$\boxed{\psi_S = 4\pi k q}$$

(q dentro S)



linhas de campo tangentes a S

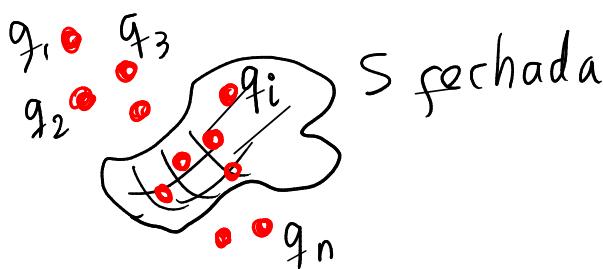
$$|\psi_{S_1}| = |\psi_{S_2}| \quad (\text{mesmo tubo})$$

S

$$\psi_{S_1} = -\psi_{S_2}$$

$$\Rightarrow \psi_S = \psi_{S_1} + \psi_{S_2} = 0$$

$$\boxed{\psi_S = 0} \quad q \text{ fora de } S$$



$$\psi_S = \iint_S (\vec{E} \cdot \hat{n}) dA$$

$$\vec{E} = \sum_{i=1}^n \vec{E}_i$$

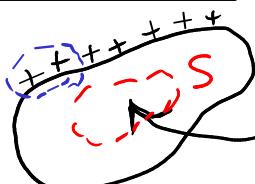
$$\psi_S = \sum_{i=1}^n \psi_i \quad \psi_i = \iint_S (\vec{E}_i \cdot \hat{n}) dA = \begin{cases} 0, q_i \text{ fora de } S \\ 4\pi k q_i, q_i \text{ dentro} \end{cases}$$

$$\Rightarrow \psi_S = \sum_{\substack{q_i \text{ dentro de} \\ S}} \psi_i = 4\pi k \sum_{\substack{q_i \text{ dentro} \\ \text{de } S}} q_i$$

lei de
Gauss

$$\boxed{\psi_S = 4\pi k q_{\text{int}}}$$

$q_{\text{int}} = \text{carga no interior de } S$



condutor com carga total Q

$\vec{E} = 0$ dentro do condutor

S , fechada dentro do condutor $\Rightarrow \psi_S = 0 \quad (\vec{E} = \vec{0})$

lei de Gauss $\Rightarrow q_{int} = 0 \Rightarrow Q$ apenas na fronteira do condutor.

toda a carga Q distribui-se na superfície do condutor

se houver dielétrico com constante K em S :

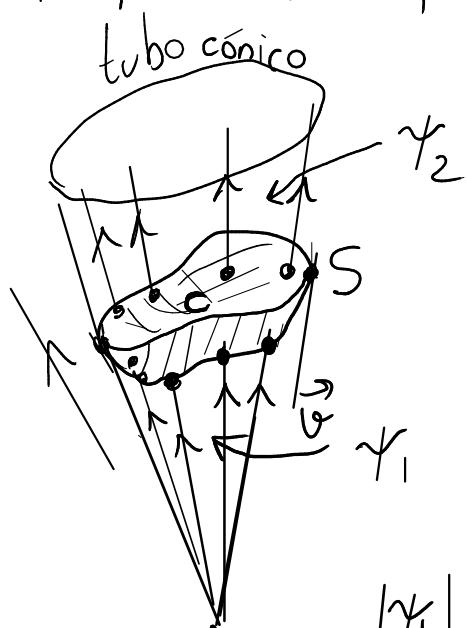
$$k \rightarrow \frac{k}{K}$$

$$\Rightarrow \gamma_s = \frac{4\pi k}{K} q_{int}$$

Pergunta: porque é que no caso b divide-se S assim?

Resposta: para demonstrar que o fluxo total é nulo.

Pense no caso dum fluido. Cada linha de campo que atravessa S entra em S num ponto (fluxo negativo) e sai noutra ponto (fluxo positivo). As linhas que tocam S em apenas um ponto (sem atravessar) definem uma curva C que separa os pontos onde há linhas a sair, dos pontos onde há linhas a entrar.



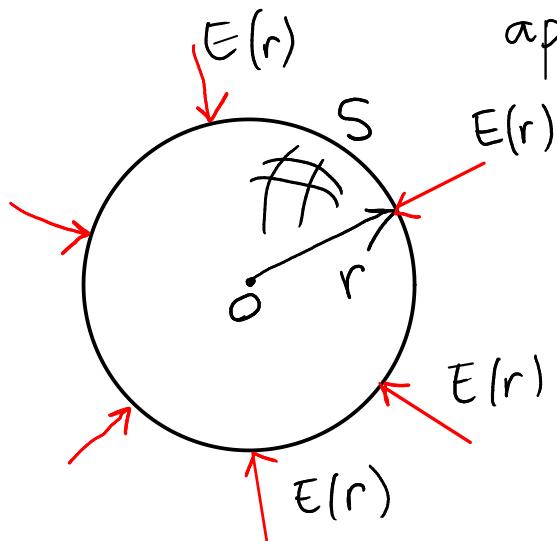
$\gamma_1 < 0$ (entra fluido em S)

$\gamma_2 > 0$ (sai fluido de S)

$|\gamma_1| = |\gamma_2|$ dentro do mesmo tubo

DISTRIBUIÇÕES SIMÉTRICAS DE CARGA

Simetria esférica. É perpendicular a qualquer esfera centrada na origem e E depende apenas de r.



$$\begin{aligned}\gamma_S &= \iint_S (\vec{E} \cdot \hat{n}) dA = \iint_S E dA \\ &= E \iint_S dA \\ \Rightarrow \gamma_S &= EA\end{aligned}$$

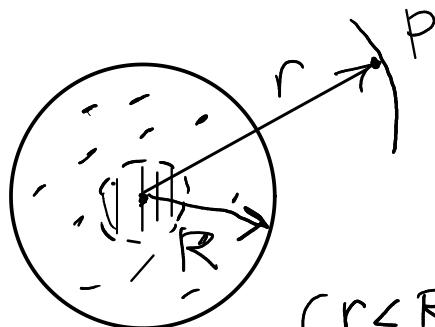
S (esfera; r const.)

mas, pela lei de Gauss $\Rightarrow \gamma_S = 4\pi k q_{\text{int}}$.

$$\Rightarrow \boxed{E(r) = \frac{4\pi k q_{\text{int}}}{A}} \quad 0 \leq r$$

① Esfera condutora, isolada, de raio R e com carga Q.

A carga distribui-se uniformemente na superfície



\Rightarrow simetria esférica.

$$E(r) = \frac{4\pi k q_{\text{int}}}{A}$$

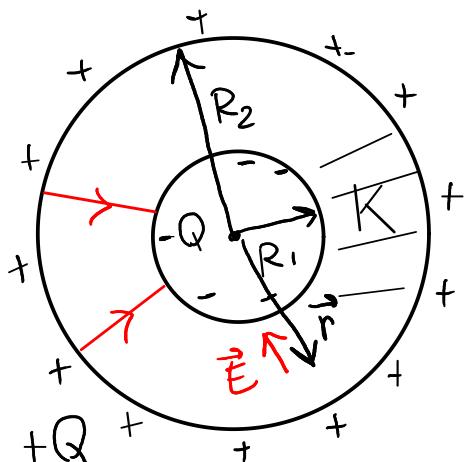
esfera de raio r concêntrica

$$\left\{ \begin{array}{l} r < R \Rightarrow q_{\text{int}} = 0 \Rightarrow E(r) = 0 \\ r > R \Rightarrow q_{\text{int}} = Q \Rightarrow E(r) = \frac{4\pi k Q}{4\pi r^2} \end{array} \right.$$

$$E(r) = \begin{cases} 0, & r < R \\ \frac{kQ}{r^2}, & r > R \end{cases} \quad (\text{como se } Q \text{ estivesse no centro})$$

\vec{E} radial

- ② Condensador esférico. 2 esferas condutoras, de raios R_1 e R_2 ($R_1 < R_2$) e cargas $+Q$ e $-Q$.



\Rightarrow simetria esférica

$$E(r) = \frac{4\pi k q_{\text{int}}}{K A}$$

\curvearrowleft dieletílico em r

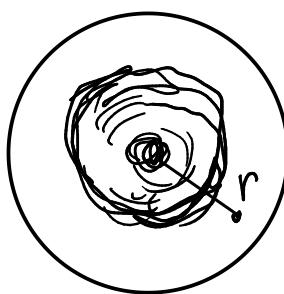
a) $r < R_1 \Rightarrow q_{\text{int}} = 0 \Rightarrow E(r) = 0$

b) $R_1 < r < R_2 \Rightarrow q_{\text{int}} = -Q \Rightarrow E(r) = \frac{4\pi k(-Q)}{K(4\pi r^2)}$

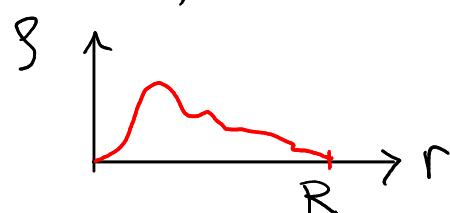
$$E(r) = -\frac{kQ}{Kr^2} \quad (- \text{ indica sentido o oposto a } \vec{r})$$

c) $r > R_2 \Rightarrow q_{\text{int}} = -Q + Q = 0 \Rightarrow E(r) = 0$

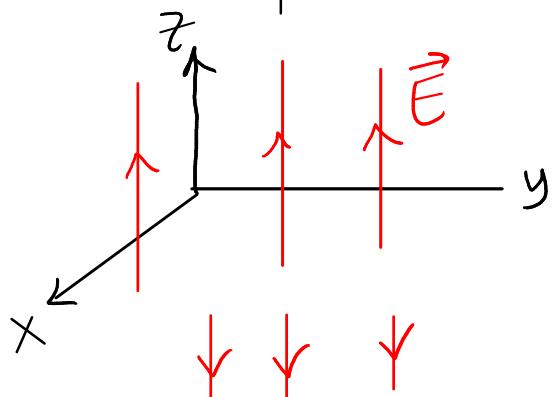
- ③ esfera isoladora, com carga distribuída em função de r



$$s = \text{carga volúmica} \\ = f(r)$$



Simetria plana. É perpendicular a um plano (linhas de campo paralelas) e E depende apenas da distância até o plano.



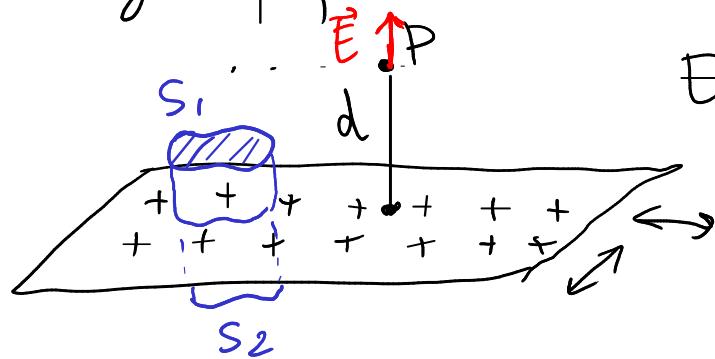
se o plano for xy

$$\Rightarrow \vec{E} = E(z) \hat{k}$$

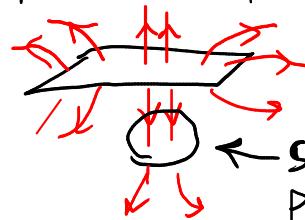
$$E \geq 0 \\ \text{ou } E < 0$$

① Plano "infinito", com carga distribuída uniformemente

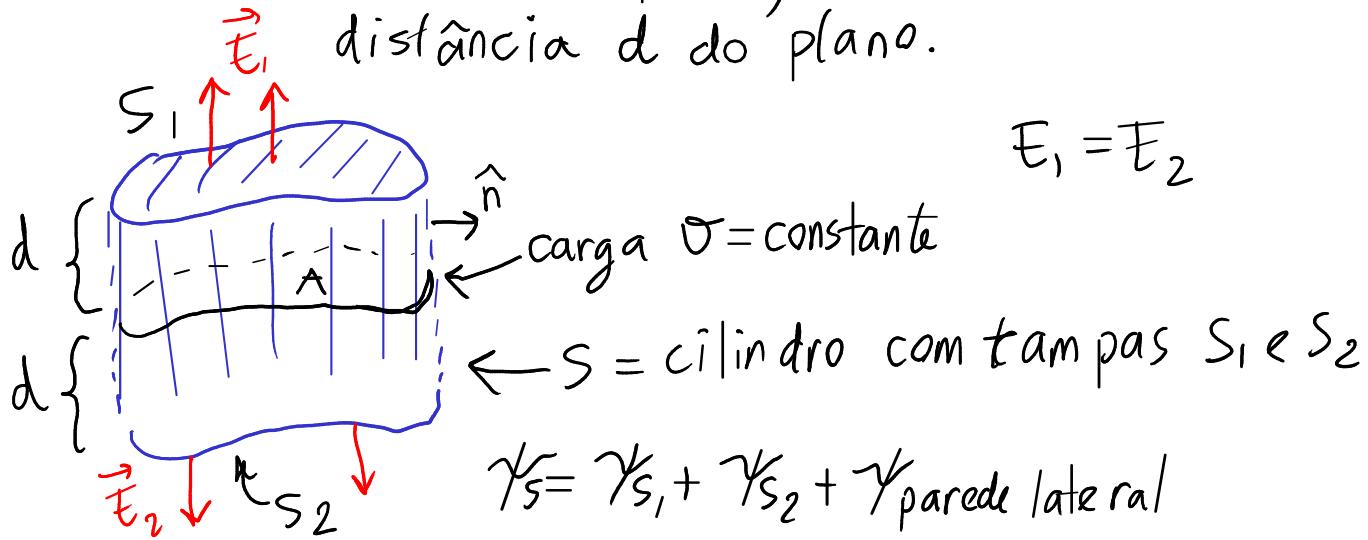
carga superficial $= \sigma = \text{constante}$ \Rightarrow simetria plana



$E(d)$ se d for muito menor que o tamanho do plano \Rightarrow plano \approx infinito



S_1 e S_2 = superfícies iguais nos dois lados do plano, à mesma distância d do plano.



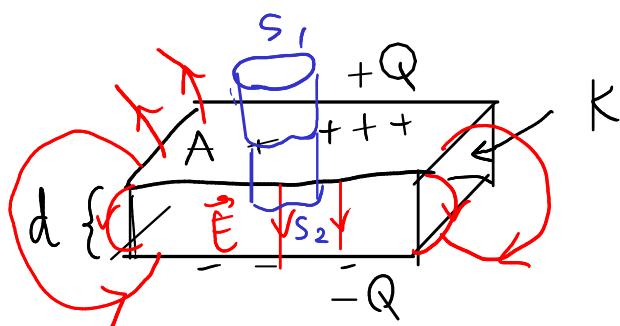
$$\gamma_s = EA + EA + 0 \quad \vec{E} \text{ perpendicular a } \hat{n}$$

$$\gamma_s = 2EA(d)A \quad q_{int} = \sigma A$$

lei de Gauss: $2EA = 4\pi k(\sigma A)$

$$E_{plano} = 2\pi k \sigma$$

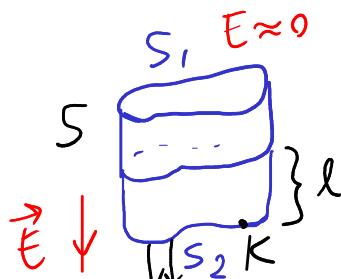
② Condensador plano



Se d for muito menor que as arestas das armaduras
 \approx simetria plana

S = cilindro com tampas S_1 e S_2 a uma distância l da armadura com carga $+Q$

a) $l < d$



$$\gamma_{s2} = EA \quad \gamma_{s1} = 0$$

$$\gamma_{lateral} = 0$$

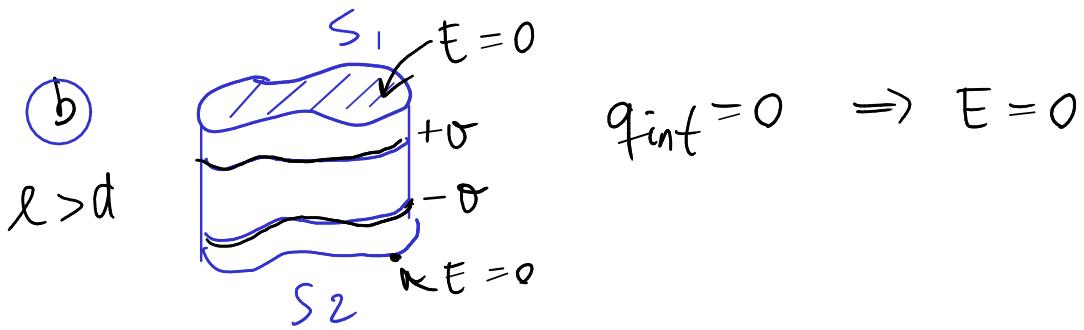
$$\gamma_s = EA \quad q_{int} = \sigma A$$

$$\Rightarrow EA = \frac{4\pi k(\sigma A)}{K} \quad \Rightarrow$$

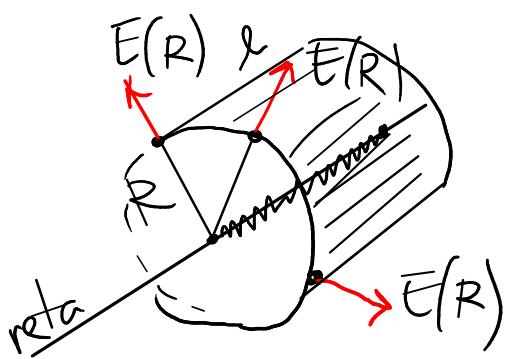
$$E = \frac{4\pi k \sigma}{K}$$

$$\Delta V = \int_1^2 Eds = Ed = \frac{4\pi k \sigma d}{K} = \frac{4\pi k Qd}{KA}$$

$$C = \frac{Q}{\Delta V} = \frac{KA}{4\pi kd}$$



Simetria cilíndrica. \vec{E} perpendicular a uma reta e E depende apenas da distância R até essa reta.



$S =$ cilindro com comprimento l , raio R , e eixo na reta.

$$\gamma_s = \gamma_{\text{tampas}} + \gamma_{\text{paralelafera}}$$

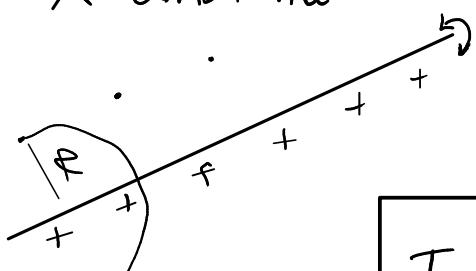
$$= \underset{\text{0}}{\downarrow} + \iint_{\text{parede}} E(R) dA$$

$$\gamma_s = E(R)(2\pi R l)$$

lei de Gauss: $2\pi R l E = 4\pi k q_{int}$

$$E = \frac{2k q_{int}}{R l}$$

Exemplo: fio reto, muito comprido, com carga linear λ constante.



\Rightarrow simetria cilíndrica

$q_{int} =$ carga num pedaço do fio de compr. $l = \lambda l$

$$E_{\text{fio}} = \frac{2k \lambda}{R}$$

POTENCIAL ELETROSTÁTICO

$$\begin{matrix} P \\ \cdot \\ V_P \end{matrix} \quad \begin{matrix} Q \\ \cdot \\ V_Q \end{matrix}$$

V : função da posição
cada ponto do espaço tem um valor de V

$$V_Q - V_P = - \int_P^Q \vec{E} \cdot d\vec{r} \quad (\text{qualquer percurso de integração})$$

\vec{E} é conservativo, $\int_P^Q \vec{E} \cdot d\vec{r}$ não depende do percurso de P até Q

Seja: $P = (x, y, z)$ e $Q = (x + \Delta x, y, z)$ e $d\vec{r} = \hat{i} dx$ (reta de P até Q)

$$V_Q - V_P = - \int_{(x, y, z)}^{(x + \Delta x, y, z)} (\vec{E} \cdot \hat{i}) dx = - \int_{(x, y, z)}^{(x + \Delta x, y, z)} E_x dx = - \bar{E}_x \Delta x$$

↑ valor médio

$$\bar{E}_x = - \frac{V(x + \Delta x, y, z) - V(x, y, z)}{\Delta x}$$

$$E_x(x, y, z) = \lim_{\Delta x \rightarrow 0} - \frac{V(x + \Delta x, y, z) - V(x, y, z)}{\Delta x} = - \frac{\partial V}{\partial x}$$

derivada parcial (com y e z constantes)

Da mesma forma: $E_y = - \frac{\partial V}{\partial y}$, $E_z = - \frac{\partial V}{\partial z}$

$$\vec{E}(x, y, z) = - \frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k} = - \text{gradiente de } V$$

$V(x, y, z)$ deverá ser função continua.

OPERADOR NABLA

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

$$\vec{E} = -\vec{\nabla} V$$

\vec{E} é conservativo

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \quad \left(= -\frac{\partial^2 V}{\partial x \partial y} = -\frac{\partial^2 V}{\partial y \partial x} \right)$$

$$\frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y} \quad \left(= -\frac{\partial^2 V}{\partial y \partial z} = -\frac{\partial^2 V}{\partial z \partial y} \right)$$

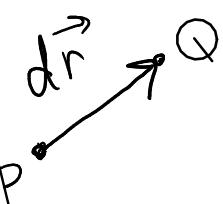
$$\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x} \quad \left(= -\frac{\partial^2 V}{\partial z \partial x} = -\frac{\partial^2 V}{\partial x \partial z} \right)$$

$$\Leftrightarrow \vec{\nabla} \times \vec{E} = \vec{0} \quad (\text{rotacional de } \vec{E})$$

$$\begin{aligned} \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \times (E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) &= \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{i} \\ &\quad + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{j} \\ &\quad + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \hat{k} \\ &= 0 \hat{i} + 0 \hat{j} + 0 \hat{k} = \vec{0} \end{aligned}$$

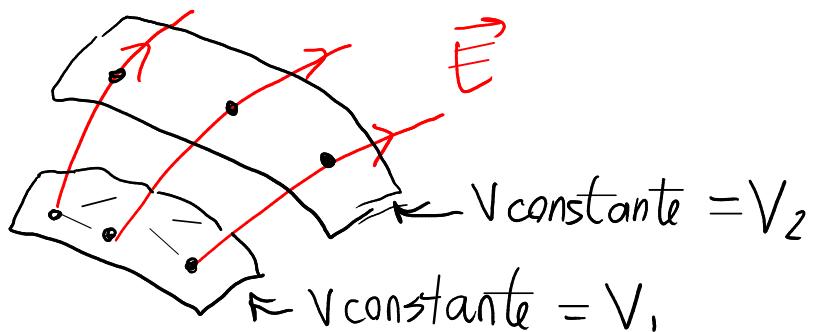
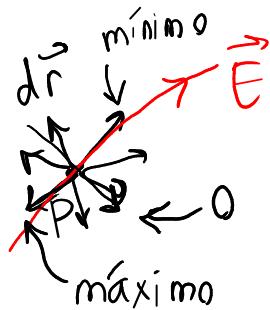
em qualquer ponto P

SUPERFÍCIES EQUIPOTENCIAIS



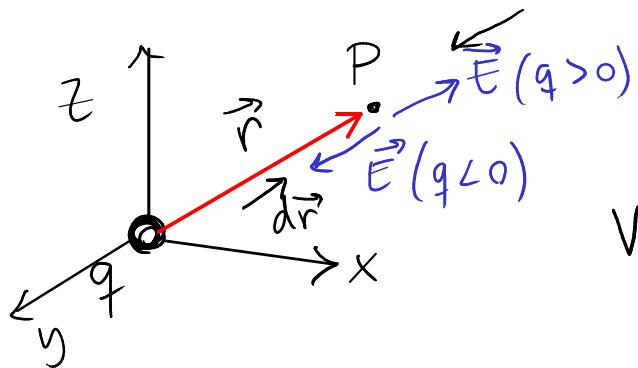
$$V_Q - V_P = dV = -\vec{E} \cdot d\vec{r}$$

$dV = -\vec{E} \cdot d\vec{r} = \begin{cases} 0, & \text{se } \vec{E} \text{ for perpendicular a } d\vec{r} \\ \text{máximo, se } \vec{E} \text{ for na direção de } d\vec{r}, \text{ no sentido o oposto} \\ \text{mínimo, se } \vec{E} \text{ for na direção e sentido de } d\vec{r} \end{cases}$



Superfícies equipotenciais = superfícies perpendiculares às linhas de campo \vec{E}

POTENCIAL DE CARGAS PONTUAIS



arbitra-se $V=0$ se $r \rightarrow \infty$

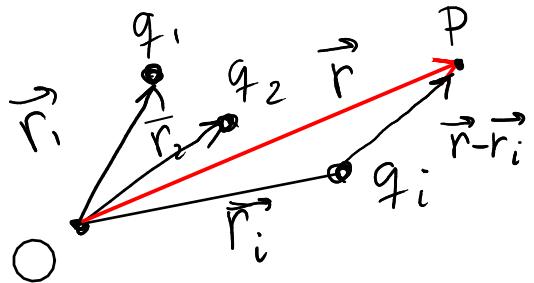
$$V_P = - \int_{\infty}^P \vec{E} \cdot d\vec{r} \quad \begin{array}{l} (\text{usa-se o percurso radial}) \\ \downarrow \end{array}$$

$$\begin{aligned} \vec{E} \cdot d\vec{r} &= \pm \left(\frac{k|q|}{r^2} \right) dr \\ &= \frac{kq}{r^2} dr \end{aligned}$$

$$V(x, y, z) = - \int_{\infty}^{|r|} \frac{kq}{r^2} dr = + \frac{kq}{r} \Big|_{r \rightarrow \infty}^r = \frac{kq}{r}$$

$$V(x, y, z) = \frac{kq}{\sqrt{x^2 + y^2 + z^2}}$$

n cargas pontuais.



$$V(\vec{r}) = \sum_{i=1}^n \frac{k q_i}{|\vec{r} - \vec{r}_i|}$$

Caso particular: cargas num plano (plano Xy)

potencial no plano Xy ($\vec{r} = x\hat{i} + y\hat{j}$)

$$V(x, y) = \sum_{i=1}^n \frac{k q_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2}}$$

Exemplo. Determine o potencial, no plano Xy , produzido pelas seguintes 3 cargas (no plano Xy):

carga (uC)	x_i (cm)	y_i (cm)
-4	20	10
+3	10	-30
+2	-30	0

Resolução: unidades $\rightarrow q \rightarrow \mu C$, $(x, y) \rightarrow dm$

$$k = 9 \times 10^9 \frac{N \cdot m^2}{C^2} = 9 \times 10^9 \frac{V \cdot m}{C} = 9 \times 10^9 \frac{V \cdot (10 \text{ dm})}{10^6 \mu C}$$

$$k = 90 \frac{kV \cdot dm}{\mu C}$$

$$\begin{aligned} q_1 &= -4 \\ q_2 &= +3 \\ q_3 &= +2 \end{aligned}$$

$$\begin{aligned} \vec{r}_1 &= (2, 1) \\ \vec{r}_2 &= (1, -3) \\ \vec{r}_3 &= (-3, 0) \end{aligned}$$

$$V(x,y) = -\frac{360}{\sqrt{(x-2)^2 + (y-1)^2}} + \frac{270}{\sqrt{(x-1)^2 + (y+3)^2}} + \frac{180}{\sqrt{(x+3)^2 + y^2}}$$

No Maxima:

```

File Edit View Search Terminal Help
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1) norma(v) := sqrt(v.v)$

(%i2) q: [-4,3,2]$

(%i3) p: [[2,1],[1,-3],[-3,0]]$

(%i4) V: sum(90*q[i]/norma([x,y]-p[i]),i,1,3);
          270           180
(%o4) ----- + ----- -
          sqrt((y + 3)  + (x - 1) )   sqrt(y  + (x + 3) )
                                         360
                                         -----
                                         2           2
                                         sqrt((y - 1)  + (x - 2) )
(%i5) ploteq(V,[x,-5,5],[y,-5,5])$

(%i6) ploteq(V,[x,-50,50],[y,-50,50])$

(%i7) ploteq(V,[x,-500,500],[y,-500,500])$

(%i8) 
```

POTENCIAL DE DISTRIBUIÇÕES SIMÉTRICAS DE CARGA

Exemplo. Esfera de raio R e carga Q , distribuída uniformemente dentro do seu volume.
(problema 5 do capítulo)

$\rho = \text{constante} \Rightarrow \text{simetria esférica}$

$$\Rightarrow E(r) = \begin{cases} \left(\frac{kQ}{R^3}\right)r & , r \leq R \quad (\text{dentro}) \\ \frac{kQ}{r^2} & , r \geq R \quad (\text{fora}) \end{cases}$$

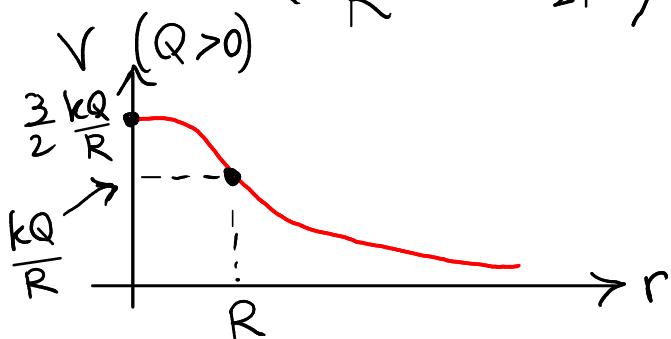
$$V_{\infty} = 0 \Rightarrow V(r) = - \int_{\infty}^r E(r) dr$$

$$\textcircled{a} \quad r \geq R \Rightarrow V(r) = - \int_{\infty}^r \frac{kQ}{r^2} dr = kQ \left(\frac{1}{r}\right)_{\infty}^r = \frac{kQ}{r}$$

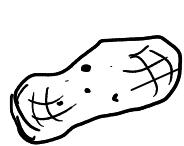
$$\textcircled{b} \quad r \leq R \Rightarrow V(r) = - \int_{\infty}^R E dr - \int_R^r E dr = -kQ \int_{\infty}^R \frac{dr}{r^2} - \left(\frac{kQ}{R^3}\right) \int_R^r r dr$$

$$= \frac{kQ}{R} + \frac{kQ}{2R^3} (R^2 - r^2) = \frac{3kQ}{2R} - \frac{kQ}{2R^3} r^2$$

$$V(r) = \begin{cases} \frac{kQ}{r} & , r \geq R \quad (\text{fora}) \\ \frac{kQ}{R} \left(\frac{3}{2} - \frac{r^2}{2R^2}\right) & , r \leq R \quad (\text{dentro}) \end{cases}$$



CONDUTORES EM EQUILÍBRIO ELETROSTÁTICO



condutor isolado
com carga Q

① $\vec{E} = \vec{0}$ no interior do condutor

② $V_1 - V_2 = - \int_2^1 \vec{E} \cdot d\vec{r}$ se 1 e 2 estiverem dentro do condutor, $\vec{E} = \vec{0}$ no integral
 $\Rightarrow V_1 - V_2 = 0$

V tem valor constante em todo o condutor

③ $q_{\text{int}} = \frac{\gamma_s}{4\pi k}$ numa região dentro do condutor,
 $\gamma_s = 0$ ($\vec{E} = \vec{0}$) $\Rightarrow q_{\text{int}} = 0$

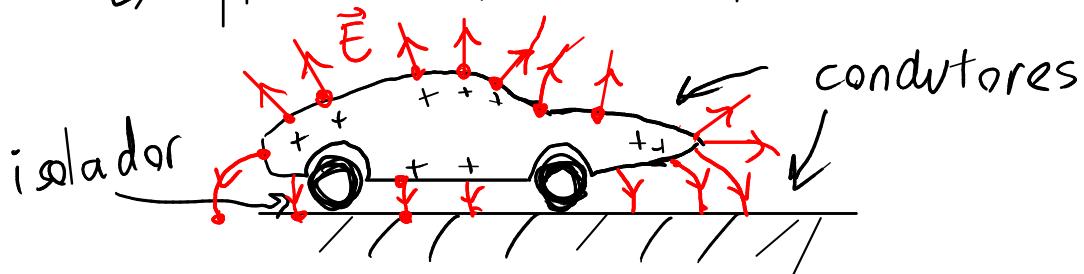
existe carga unicamente na superfície do condutor

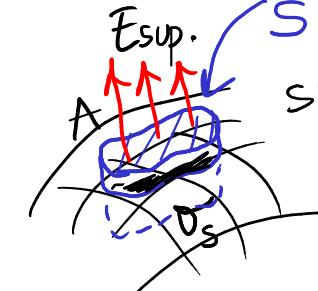
$\sigma_{\text{superfície}}$ = carga superficial ($Q = \iint_{\text{sup.}} dA$)

④ 2 implica \rightarrow a superfície do condutor é superfície equipotencial

\Rightarrow o campo \vec{E} é perpendicular à superfície do condutor

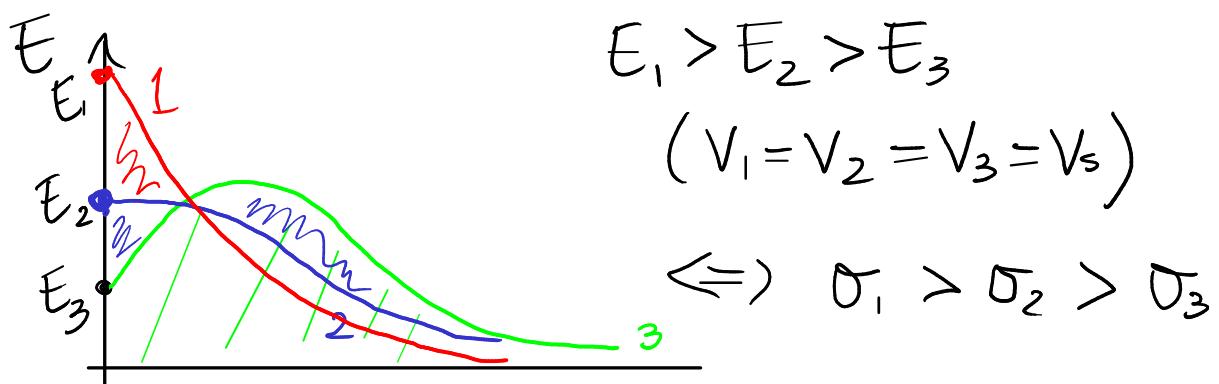
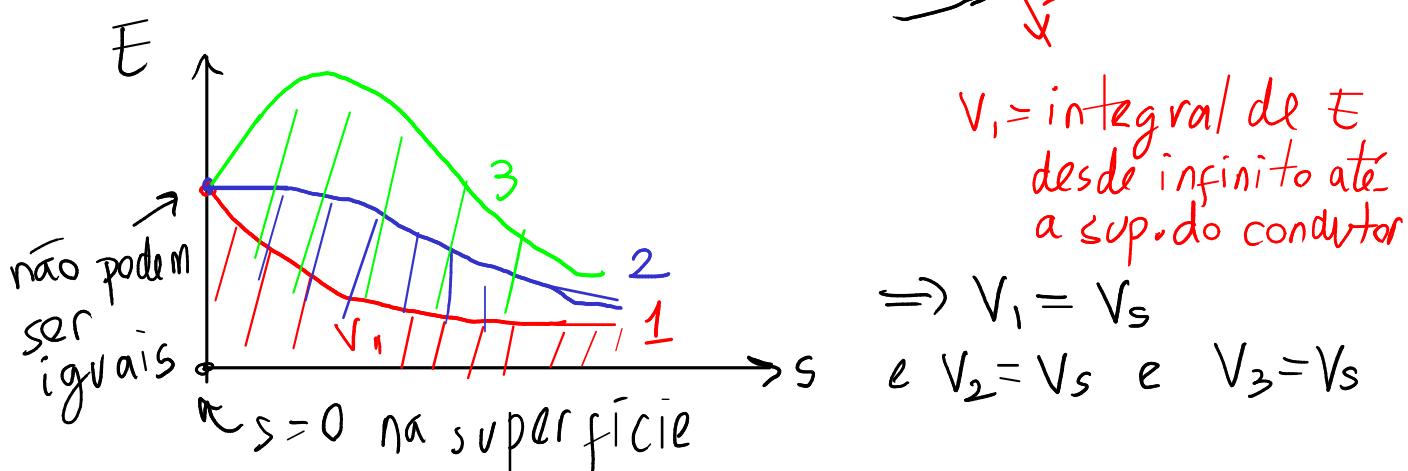
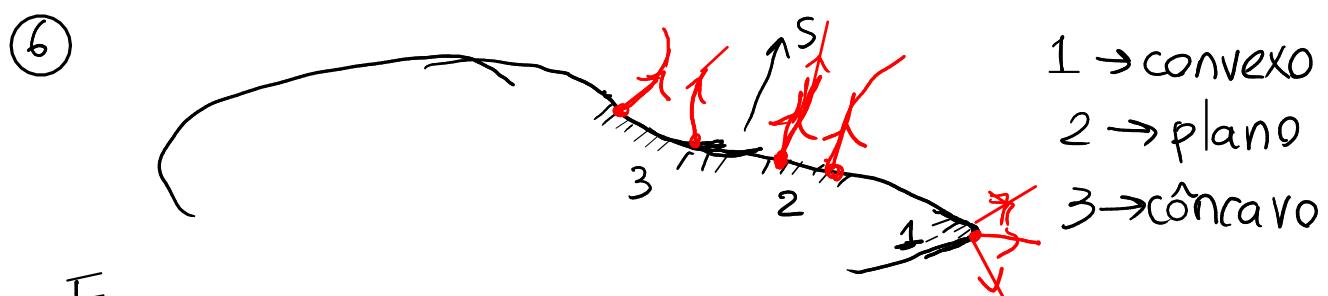
Exemplo: automóvel = condutor isolado



5  $S = \text{cilindro} \perp \text{à superfície (infinitesimal)}$
 SUPERFÍCIE

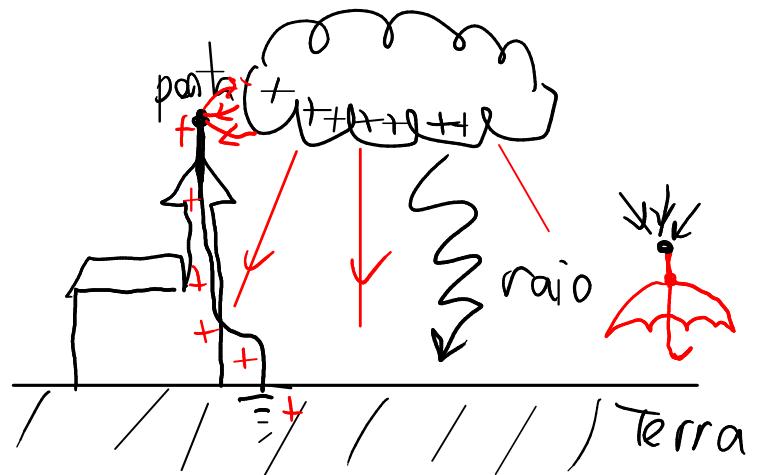
$$\left\{ \begin{array}{l} \gamma_S = E_{\text{sup}} A \\ \gamma_S = 4\pi k q_{\text{int}} \end{array} \right. \begin{array}{l} \text{carga na} \\ \text{área } A \\ = \sigma_s A \end{array}$$

$$E_s A = 4\pi k (\sigma_s A) \Rightarrow \boxed{\sigma_s = \frac{E_s}{4\pi k}}$$



Maior acumulação de carga nas regiões convexas.
 (menor nas regiões côncavas) (poder das pontas)

pára-raios



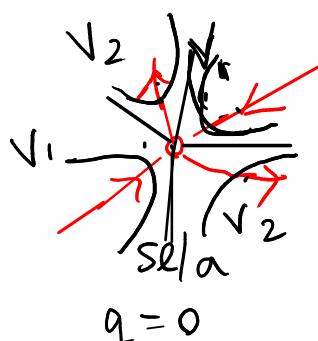
PONTOS DE EQUILÍBRIO DO CAMPO \vec{E}
($\vec{E} = \vec{0}$)

matriz jacobiana de $\vec{E}(x, y, z) = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

$$J(\vec{E}) = \begin{bmatrix} \frac{\partial E_x}{\partial x} & \frac{\partial E_x}{\partial y} & \frac{\partial E_x}{\partial z} \\ \frac{\partial E_y}{\partial x} & \frac{\partial E_y}{\partial y} & \frac{\partial E_y}{\partial z} \\ \frac{\partial E_z}{\partial x} & \frac{\partial E_z}{\partial y} & \frac{\partial E_z}{\partial z} \end{bmatrix}$$

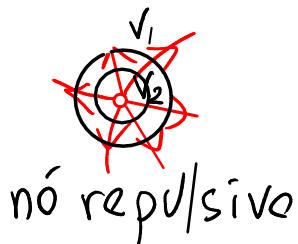
= matriz simétrica
↓
valores próprios reais
↓

ponto de equil., $\vec{E} = 0$ } {
 carga nula \rightarrow ponto de sela ($\lambda_1 = -\lambda_2$)
 carga positivo \rightarrow nó repulsivo ($\lambda_i > 0$)
 carga negativo \rightarrow nó atrativo ($\lambda_i < 0$)



$$q = 0$$

$$V_1 > V_2$$



$$V_1 < V_2$$

máximo local

$$q > 0$$



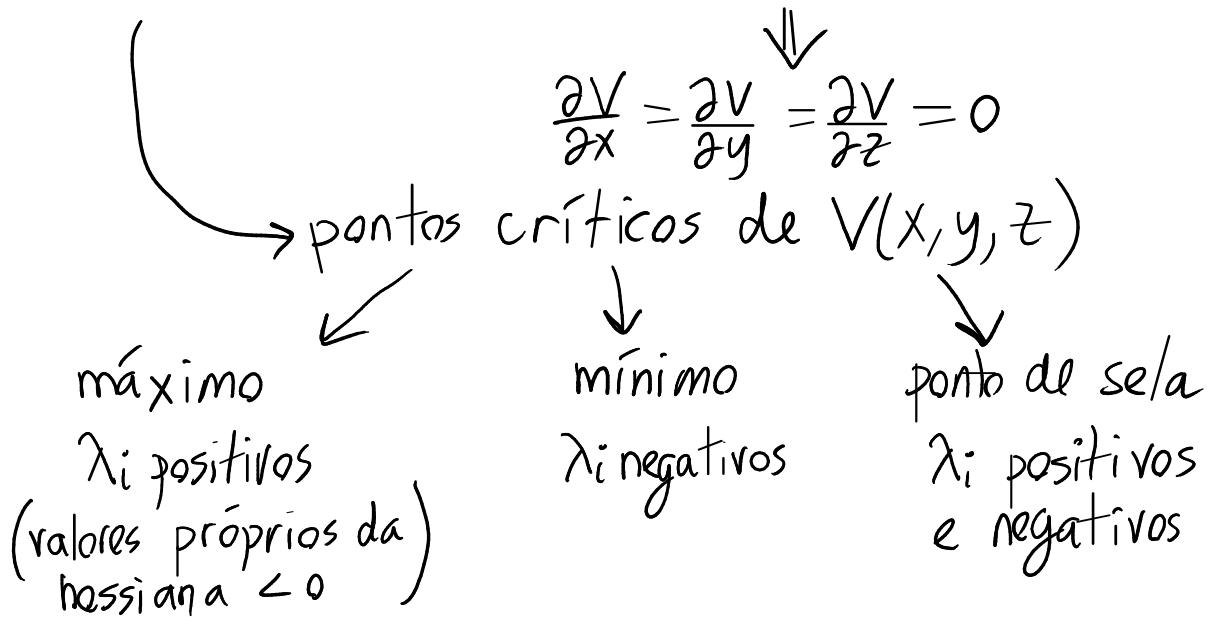
$$V_1 > V_2$$

mínimo local

$$q < 0$$

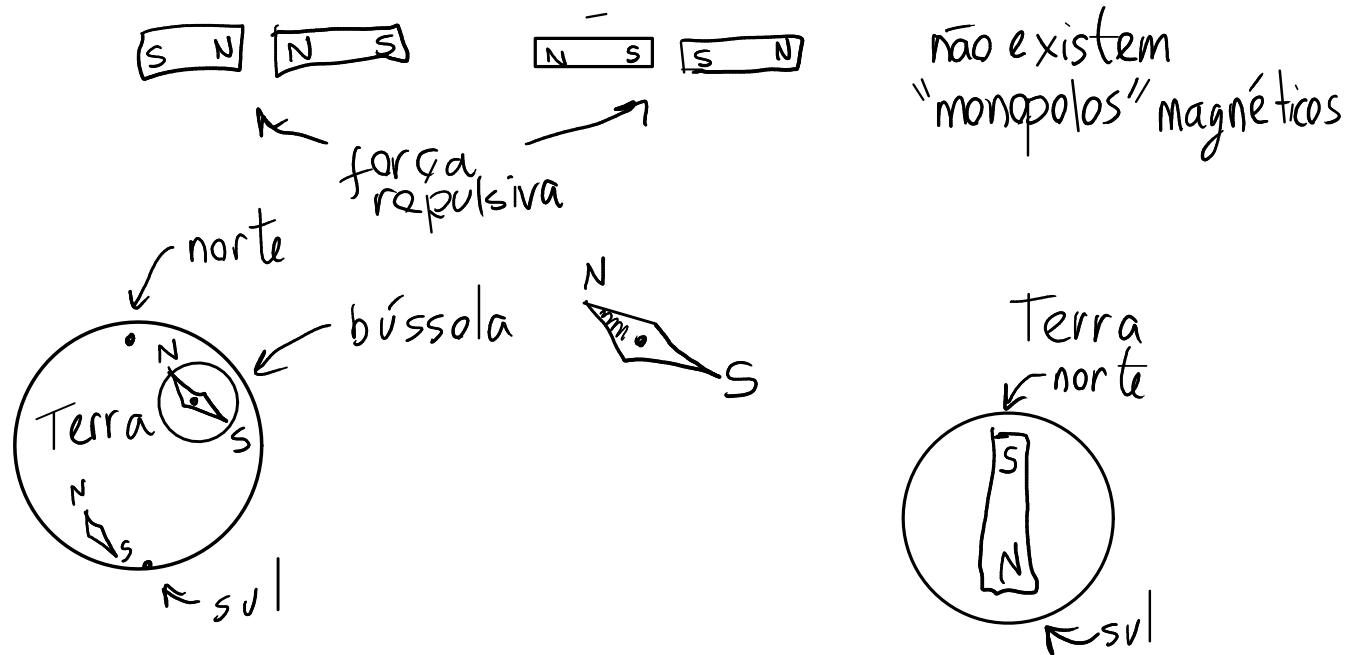
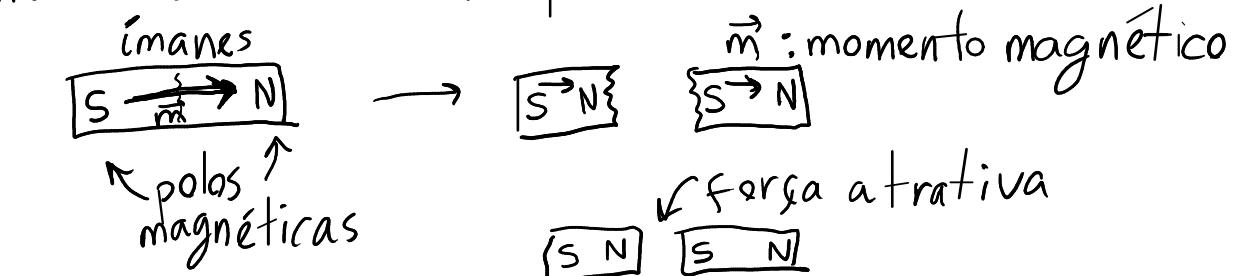
$$\mathbb{J}(\vec{E}) = \begin{bmatrix} -\frac{\partial^2 V}{\partial x^2} & -\frac{\partial^2 V}{\partial x \partial y} & -\frac{\partial^2 V}{\partial x \partial z} \\ -\frac{\partial^2 V}{\partial x \partial y} & -\frac{\partial^2 V}{\partial y^2} & -\frac{\partial^2 V}{\partial y \partial z} \\ -\frac{\partial^2 V}{\partial x \partial z} & -\frac{\partial^2 V}{\partial y \partial z} & -\frac{\partial^2 V}{\partial z^2} \end{bmatrix} = -\text{matriz Hessiana de } V(x, y, z)$$

pontos de equilíbrio $\rightarrow \vec{E} = \vec{0}, E_x = E_y = E_z = 0$

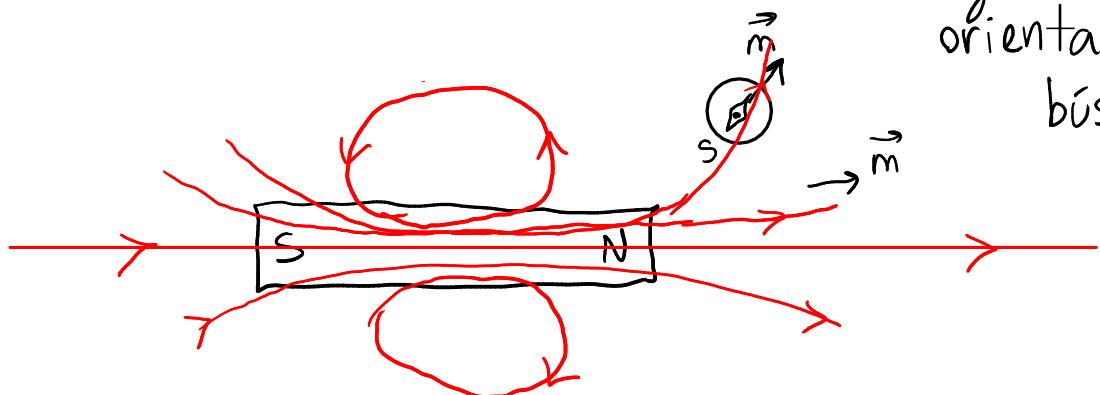


FORÇA MAGNÉTICA

entre ímanes, atrativas ou repulsivas



LINHAS DE CAMPO MAGNÉTICO \vec{B} . seguem a orientação duma bússola



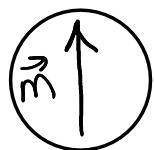
Existem linhas de campo \vec{B} fechadas

$\Rightarrow \oint \vec{B} \cdot d\vec{r} \neq 0 \Rightarrow \vec{B}$ não é conservativo

percurso fechado C

FONTES DO CAMPO MAGNÉTICO

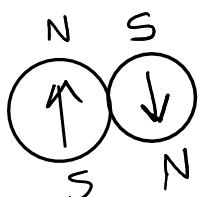
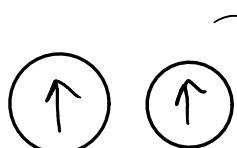
①



partícula
elementar

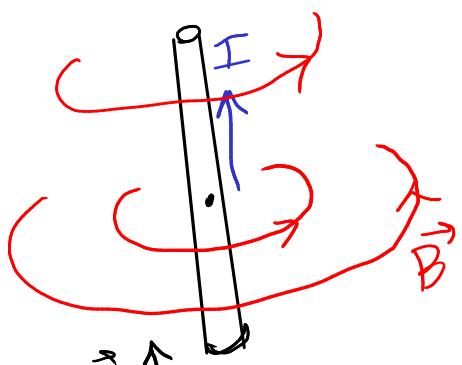
\vec{m} = momento magnético da partícula = spin

dois elétrões



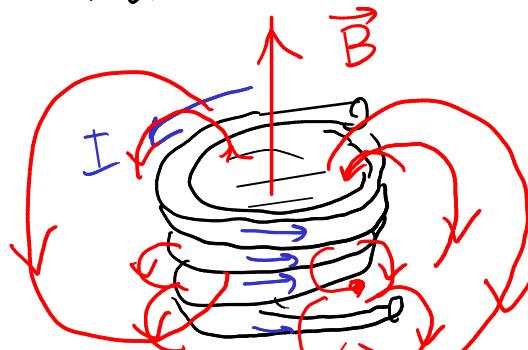
colados pela
força magnética

② Corrente elétrica

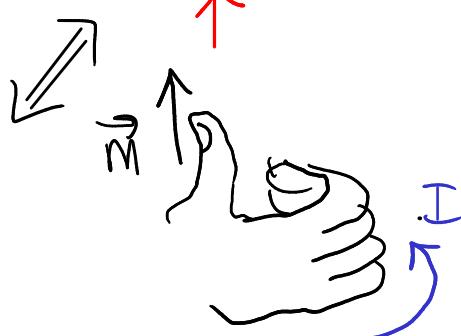
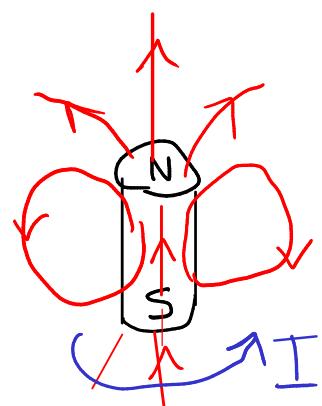


linhas de campo \vec{B} : circumferências perpendiculares ao condutor com centro nele.

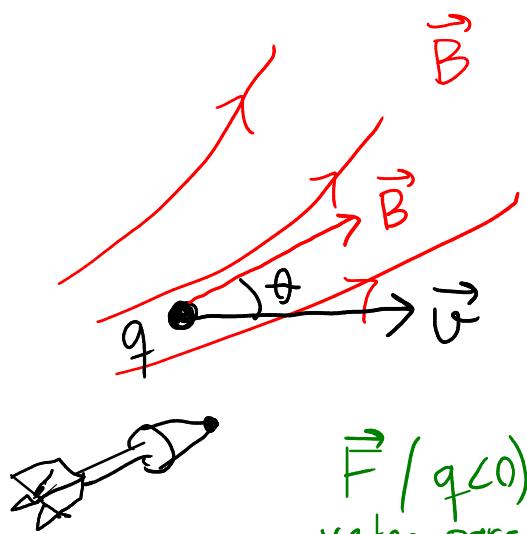
Orientadas no sentido da regra da mão direita.



bobina com
várias
"espiras"



FORÇA MAGNÉTICA SOBRE UMA CARGA PONTUAL (sem spin)



\vec{F} = força magnética sobre
a "partícula" (sem spin)
é perpendicular a \vec{B} e a \vec{v}

$\vec{F} / q < 0$)
vetor para lá

A diagram illustrating a vector \vec{B} originating from a point X on a two-dimensional Cartesian coordinate system. The horizontal axis is labeled x and the vertical axis is labeled y . A red arrow labeled \vec{B} originates from point X and points upwards and to the right.

$q > 0 \rightarrow \vec{F}$ no sentido da regra da mão direita de \vec{v} para \vec{B}
 $q < 0 \rightarrow \vec{F}$ " " " " " " " " de \vec{B} para \vec{v}

módulo da força, $|F|$, depende de $|\vec{v}|$, $|q|$, $\sin\theta$

$$\theta = 0 \text{ ou } \theta = 180^\circ \Rightarrow \vec{F} = \vec{0}$$

$$\theta = 90^\circ \text{ ou } \theta = 270^\circ \Rightarrow |\vec{F}| \text{ m\'aximo}$$

$$|\vec{F}| = \left(|\vec{v}| (q |\sin \theta|) \right) |\vec{B}|$$

$$\Rightarrow \boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

↓
Unidades SI

The diagram shows a vector \vec{B} at an angle θ to a current I . The vector \vec{B} is decomposed into two components: one parallel to \vec{B} and one perpendicular to \vec{B} . The perpendicular component is labeled $\vec{B} \sin \theta$. The parallel component is labeled $\vec{B} \cos \theta$. A curved arrow indicates the direction of the perpendicular component, and a straight arrow indicates the direction of the parallel component.

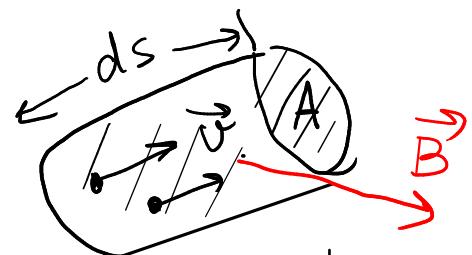
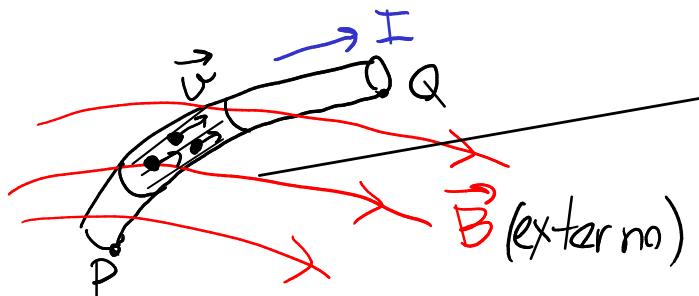
$$N = C \cdot \left(\frac{m}{s}\right) \cdot T$$

tesla (unidad SI)

$$1 \text{ T} = \frac{N \cdot S}{C \cdot m} = \frac{N}{A \cdot m}$$

outra unidade de \vec{B} : $1G = 10^{-4} T$ (gauß)

FORÇA MAGNÉTICA SOBRE CONDUTORES COM CORRENTE



parte infinitesimal do condutor

(J e B a proximadamente constantes no volume Ad_s)

força nessa parte: carga volumica

$$d\vec{F} = dq (\vec{J} \times \vec{B}) = \rho (Ad_s) (\vec{J} \times \vec{B}) = (\underbrace{\rho A \vec{J}}_{I}) \times \vec{B} ds$$

$$\Rightarrow d\vec{F} = (\vec{I} \times \vec{B}) ds$$

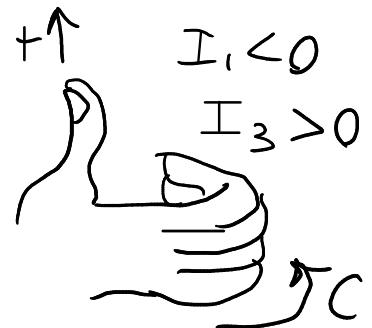
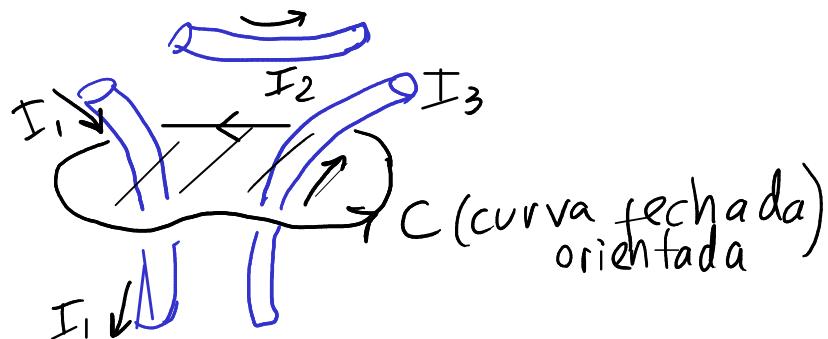
no condutor:

$$\vec{F} = \int_P^Q d\vec{F} = \int_P^Q (\vec{I} \times \vec{B}) ds \quad \begin{matrix} \text{(integral de} \\ \text{linha no condutor)} \end{matrix}$$

caso particular: condutor retílineo, de comprimento l , num campo \vec{B} uniforme

$$\Rightarrow \vec{F} = (\vec{I} \times \vec{B}) \int_P^Q ds \quad \Rightarrow \boxed{\vec{F}_{fio_ret.} = (\vec{I} \times \vec{B}) l}$$

LEI DE AMPÈRE



$$\oint_C \vec{B} \cdot d\vec{r} = 4\pi k_m I_{int}$$

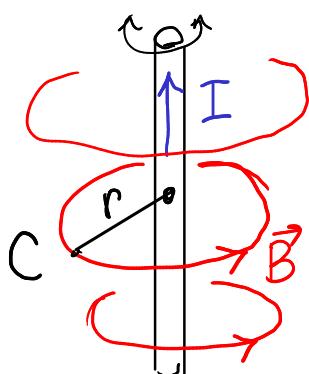
I_{int} = corrente total
através de C , no
sentido da regra
da mão direita

neste exemplo, $I_{int} = -I_1 + I_3$

$$k_m = 10^{-7} \frac{T \cdot m}{A}$$

constante magnética

CAMPO DE UM CABO RETILÍNEO

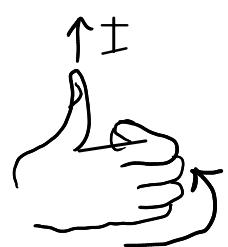


Simetria cilíndrica ($\nabla \cdot \vec{B} = 0$)

não existem monopólos

linhas de campo \vec{B} circulares, perpendiculares ao cabo e centradas nele. ($B(r)$)

$$\left\{ \begin{array}{l} \oint_C \vec{B} \cdot d\vec{r} = B \oint_C ds = 2\pi r B \\ \oint_C \vec{B} \cdot d\vec{r} = +4\pi k_m I \end{array} \right.$$



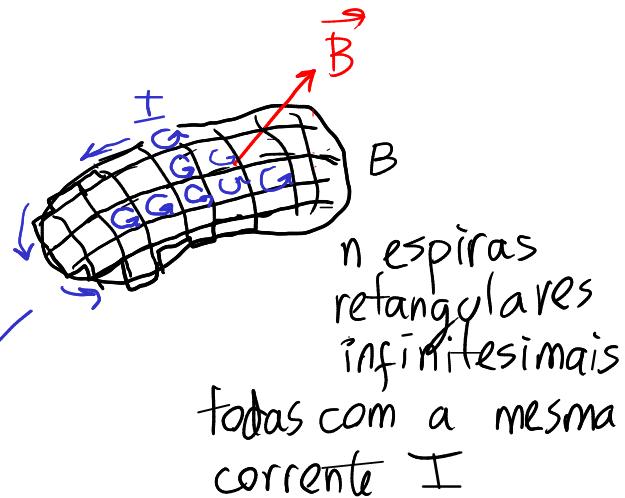
$$B(r) = \frac{2k_m I}{r}$$

cabo retilíneo (infinito)

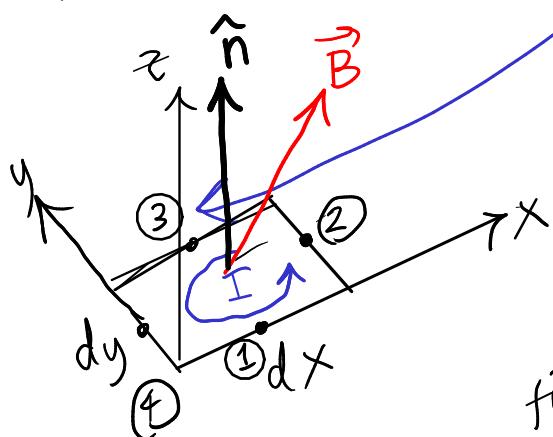
MOMENTO MAGNÉTICO



espira num campo
 \vec{B} externo



n espiras
retangulares
infinitesimalis
todas com a mesma
corrente I



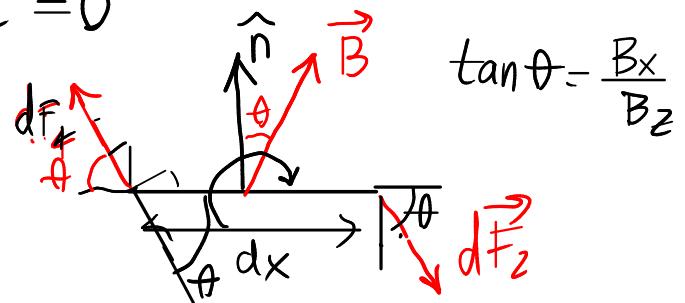
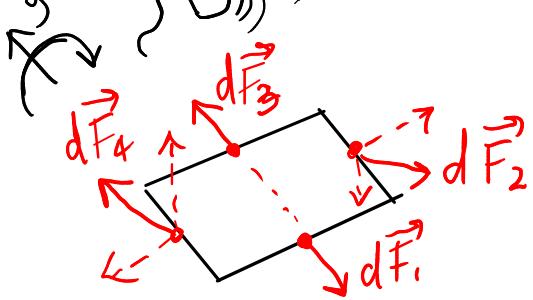
$$\vec{B} \approx \text{constante} = B_x \hat{i} + B_z \hat{k}$$

$$\text{versor normal} = \hat{n} = \hat{k}$$

$$\text{fio rect., } \vec{B} \text{ constante: } \vec{F} = (\vec{I} \times \vec{B}) l$$

- ① $\vec{I}_1 = I\hat{i}$, $l_1 = dx \Rightarrow d\vec{F}_1 = -IB_z\hat{j}dx$
- ② $\vec{I}_2 = I\hat{j}$, $l_2 = dy \Rightarrow d\vec{F}_2 = IB_z\hat{i}dy - IB_x\hat{k}dy$
- ③ $\vec{I}_3 = -I\hat{i}$, $l_3 = dx \Rightarrow d\vec{F}_3 = +IB_z\hat{j}dx$
- ④ $\vec{I}_4 = -I\hat{j}$, $l_4 = l_2 \Rightarrow d\vec{F}_4 = -IB_z\hat{i}dy + IB_x\hat{k}dy$

$$\sum d\vec{F}_i = \vec{0}$$

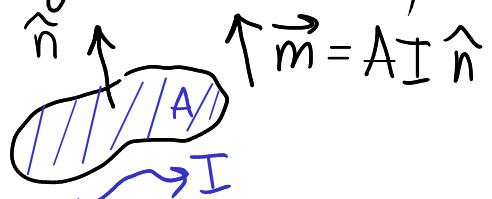


Binário: $|d\vec{F}_2| \sin\theta dx$

$$d\vec{M} = +IB_x dx dy \hat{j} = \frac{I dx dy}{I dA} (\hat{k} \times \vec{B})$$

$$d\vec{m} = (I dA) \hat{n} \quad \text{Momenato magnético da espira}$$

$$d\vec{M} = d\vec{m} \times \vec{B}$$



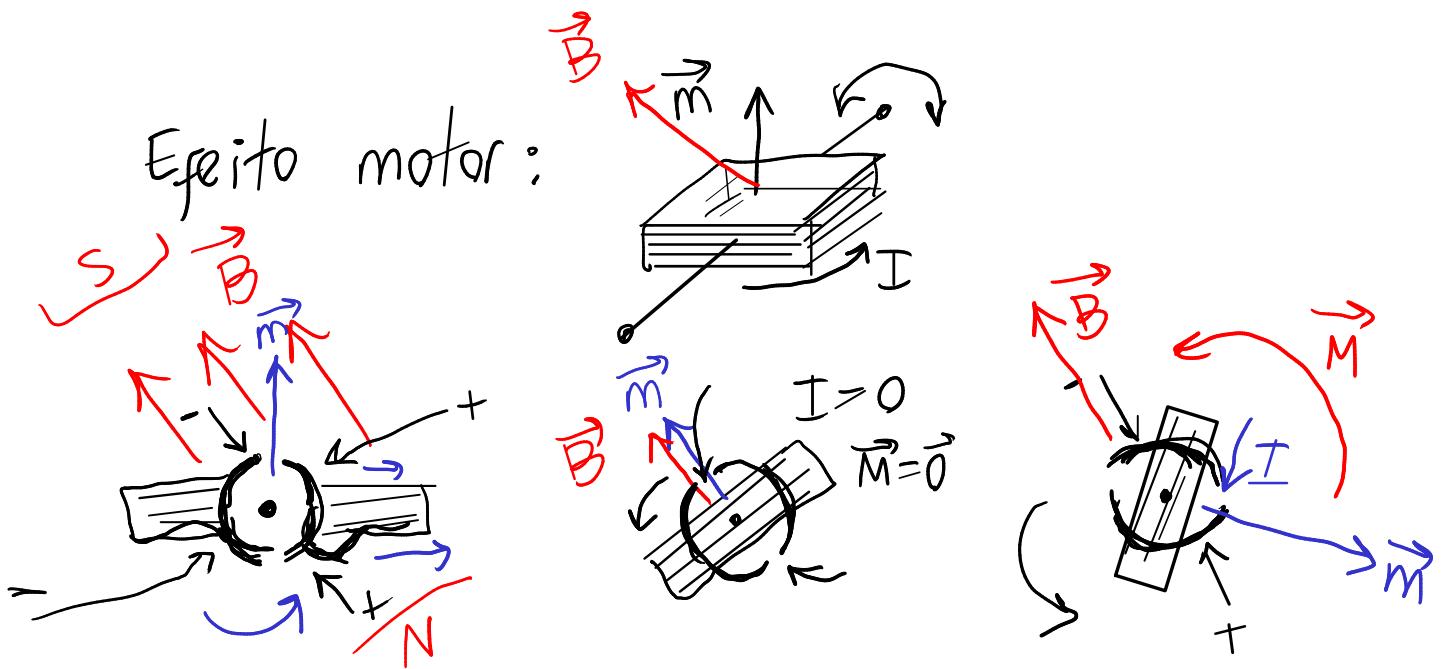
$$\sin\theta = \begin{cases} +1, & \theta = 90^\circ \\ -1, & \theta = 270^\circ \\ 0, & \theta = 0, 180^\circ \end{cases}$$

Bobina com N espiras, todas com área A:

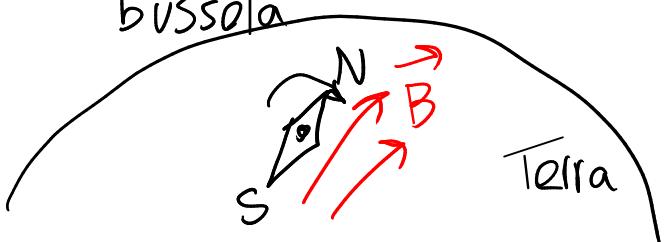


$$\vec{m} = NAI\hat{n}$$

Efeito motor:



bússola



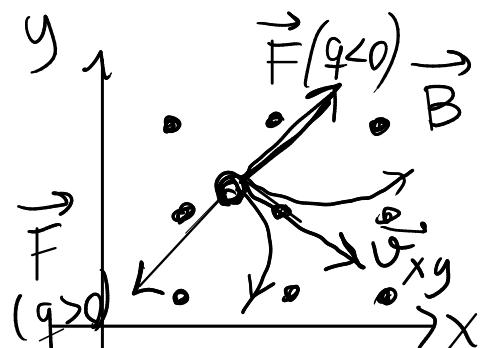
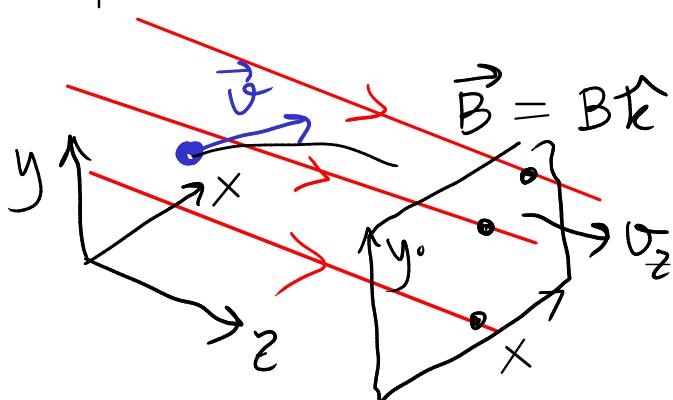
$\vec{B} \approx \text{constante}$ (na bússola)
 $\Rightarrow \vec{F} = \vec{0} \quad \vec{M} \neq \vec{0}$

partícula elementar com spin:



MOVIMENTO DE CARGAS PONTUAIS NO CAMPO MAGNÉTICO.

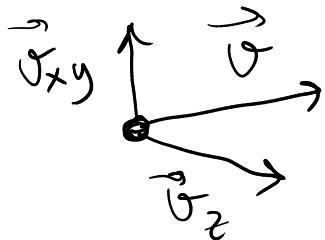
campo \vec{B} constante



$$\vec{F} = q(\vec{v} \times \vec{B}) \text{ perpendicular a } \vec{v} \text{ e a } \hat{e}_t$$

$$\Rightarrow F_t = 0 \Rightarrow a_t = 0 \Rightarrow \frac{dv}{dt} = 0$$

$v = \text{constante}$ (\vec{F} não realiza trabalho)

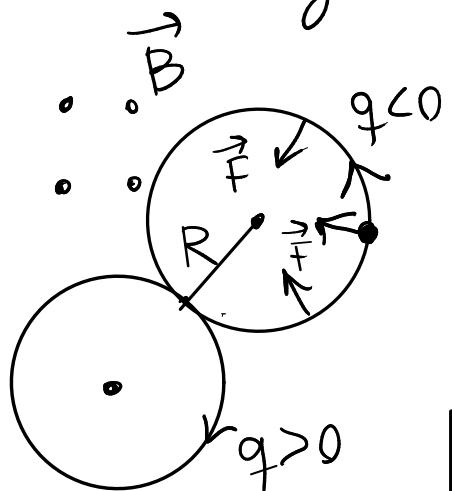


$$\vec{F} = q(\vec{v} \times \vec{B}) = q(v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) \times B \hat{k}$$

$$= q(v_y B \hat{i} - v_x B \hat{j}) \quad (F_z = 0)$$

$$a_z = 0 \Rightarrow v_z = \text{constante}$$

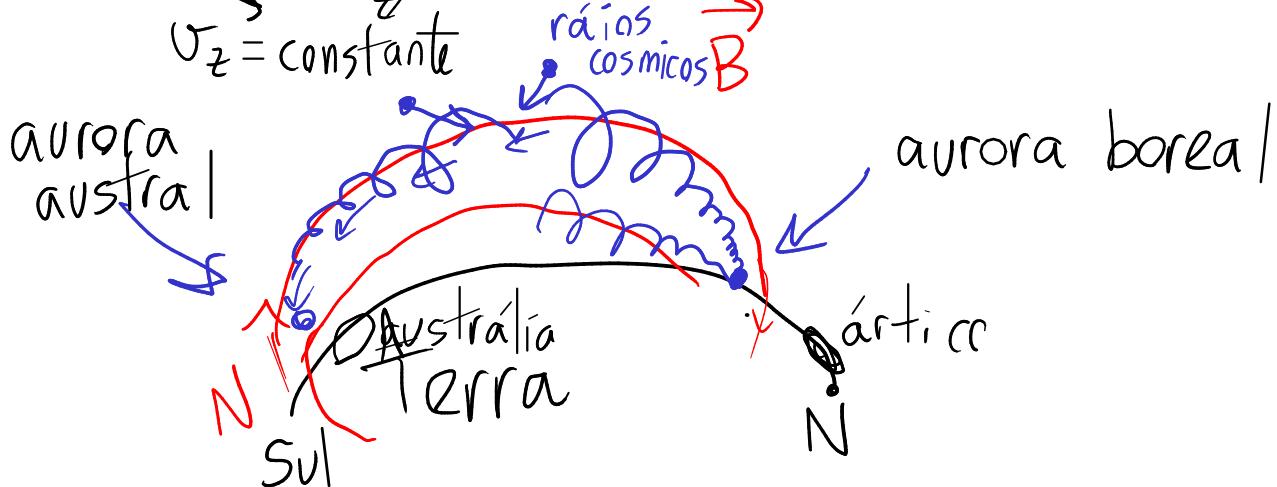
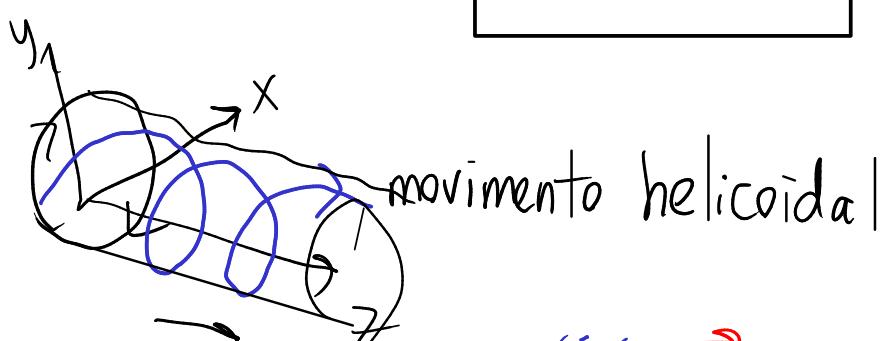
movimento segundo z uniforme + movimento circular no plano xy uniforme



$$|F| = |q| |\vec{v}_{xy}| B \quad (\theta = 90^\circ)$$

$$F_n = \frac{mv_{xy}^2}{R} = |q| |\vec{v}_{xy}| B$$

$$R = \frac{mv_{xy}}{|q| B}$$



FORÇA MAGNÉTICA ENTRE CABOS PARALELOS COM CORRENTE

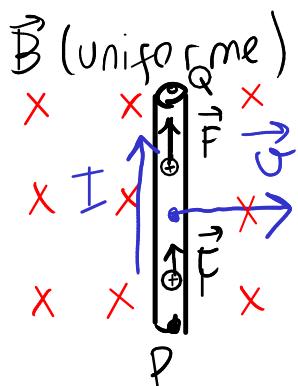
Dois cabos com correntes I_1 e I_2 , com comprimento l , distanciados d . $\Rightarrow I_1 \rightarrow \vec{B}_1 \rightarrow F_{12} = (\vec{I}_2 \times \vec{B}_1) l$

$$F_{12} = \frac{2k_m I_1 I_2 l}{d}$$

Dois Casos



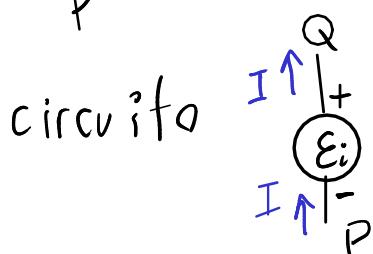
INDUÇÃO ELETROMAGNÉTICA



$$\vec{F} = q(\vec{v} \times \vec{B})$$

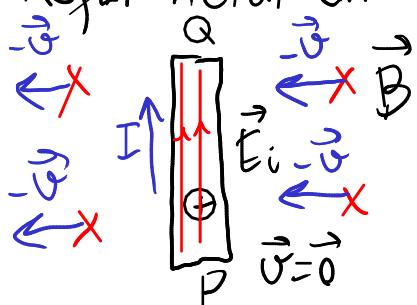
$$\text{f.e.m. induzida} = \mathcal{E}_i = P \int_P^Q \vec{F} \cdot d\vec{r}$$

$$\mathcal{E}_i = |\vec{v} \times \vec{B}| \int_P^Q ds = vB l \leftarrow \frac{q}{PQ} \text{ distância}$$



$$\vec{\mathcal{E}}_i = \vec{0}$$

Referencial em movimento com o condutor



$\vec{F}_m = \vec{0}$
campo elétrico \vec{E}_i (induzido)
devida à variação de \vec{B}

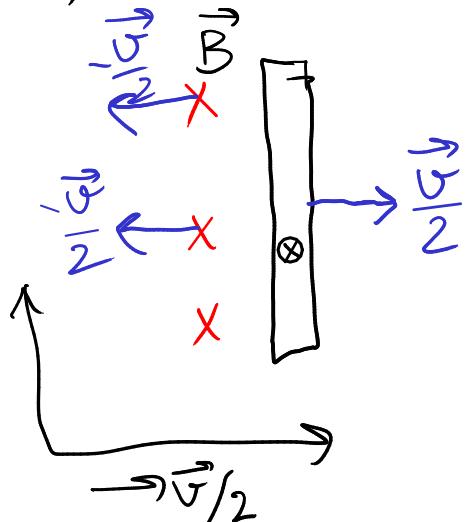
campo eletromagnético (\vec{E}, \vec{B})

$$\vec{F} = q\vec{E}_i \quad \text{como} \quad \vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{E}_i = \vec{v} \times \vec{B}$$

$$E_i = \int_P^Q \vec{E}_i \cdot d\vec{r}$$

Referencial com velocidade $\frac{\vec{v}}{2}$

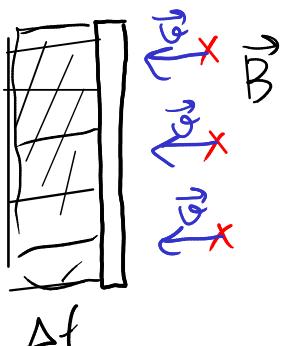


$$\vec{B}, \vec{E}_i = \frac{1}{2}(\vec{v} \times \vec{B})$$

$$\vec{F} = \frac{q}{2}(\vec{v} \times \vec{B}) + \frac{q}{2}(\vec{v} \times \vec{B}) \\ = q(\vec{v} \times \vec{B})$$

Fluxo magnético : $\gamma_s = \iint_S (\vec{B} \cdot \hat{n}) dA$
referencial em mov. com o condutor

$$\begin{aligned} \text{área} &\rightarrow \text{área} = l(v\Delta t) \\ &= l(v\Delta t) \end{aligned}$$



$$\Delta \gamma_s = l v \Delta t B$$

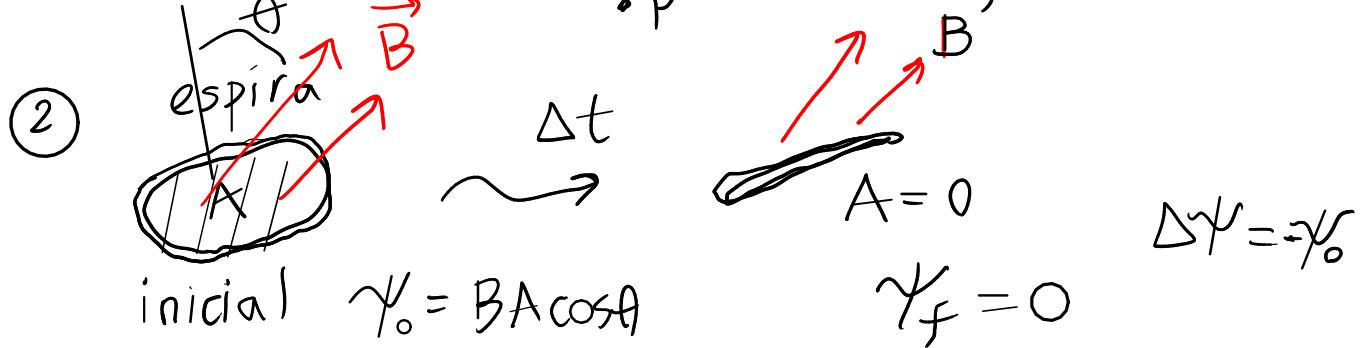
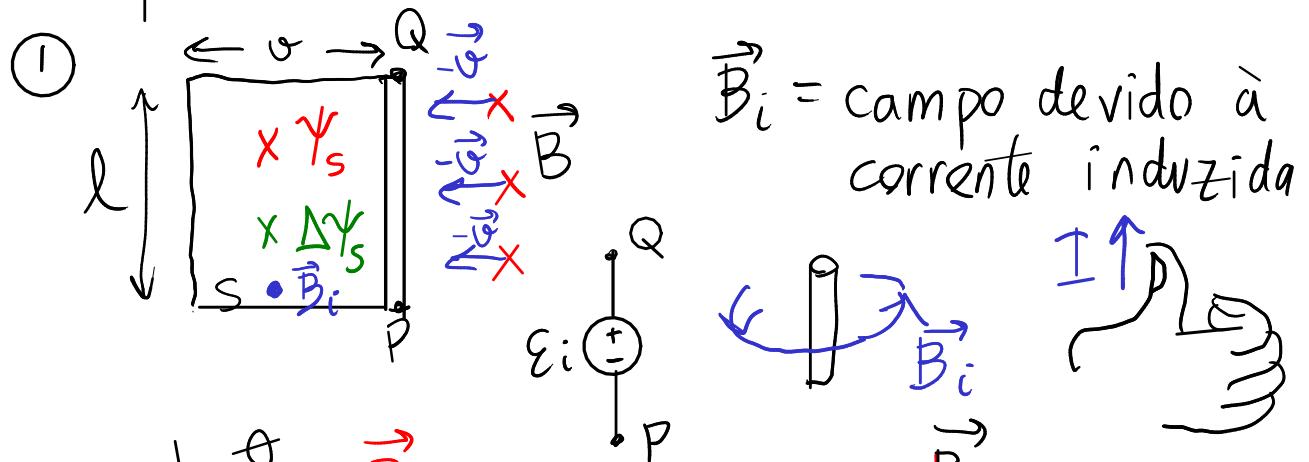
$$E_i = \frac{\Delta \gamma_s}{\Delta t}$$

LEI DE FARADAY

$$E_i = - \frac{d\gamma_s}{dt}$$

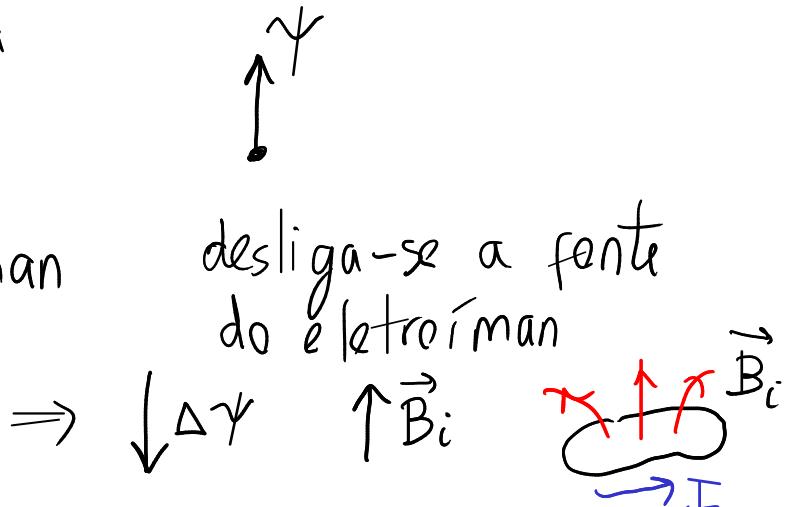
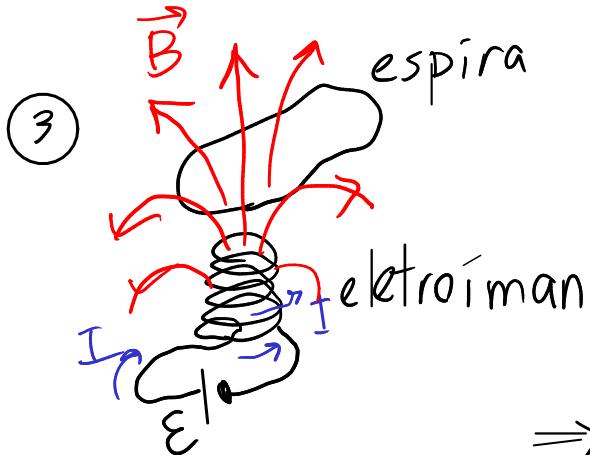
Lei de Lenz: quando existir variação de ψ_s , a f.e.m. induzida produz corrente induzida que contraria essa variação de ψ_s

Exemplos:

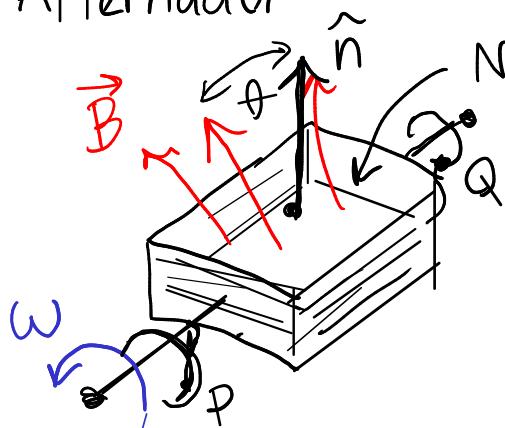


$$\varepsilon_i = \frac{-BA \cos\theta}{\Delta t}$$

$$I = \frac{\varepsilon_i}{R_{\text{Respira}}}$$



④ Alternador



N espiras de área A

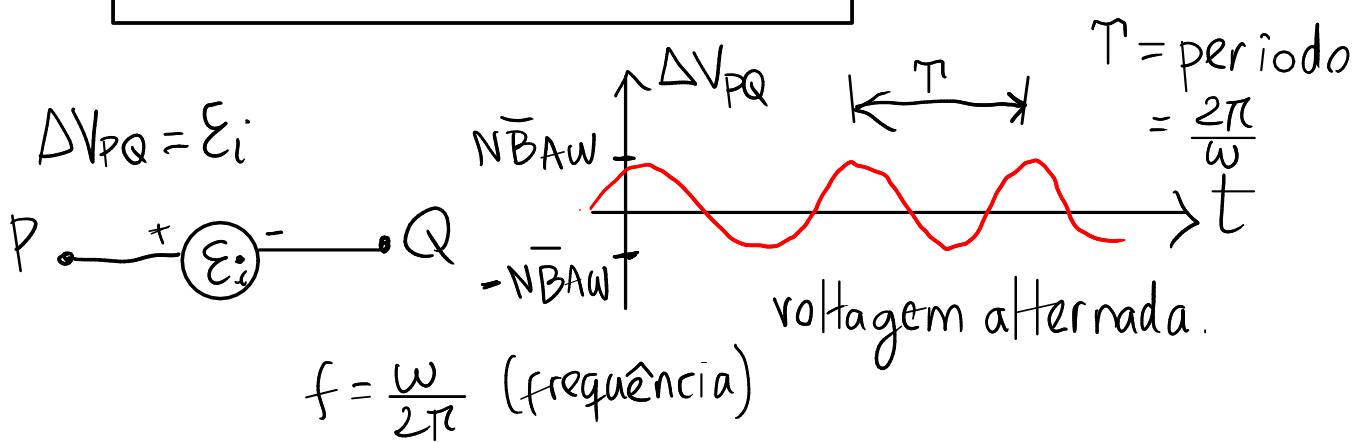
$$\begin{aligned}\psi_{\text{bobina}} &= N \iint_{\text{espira}} (\vec{B} \cdot \hat{n}) dA \\ &= (N \bar{B} \cos \theta) A\end{aligned}$$

$$E_i = - \frac{d\psi_{\text{bobina}}}{dt} = - \bar{N} \bar{B} A \frac{d}{dt} (\cos \theta)$$

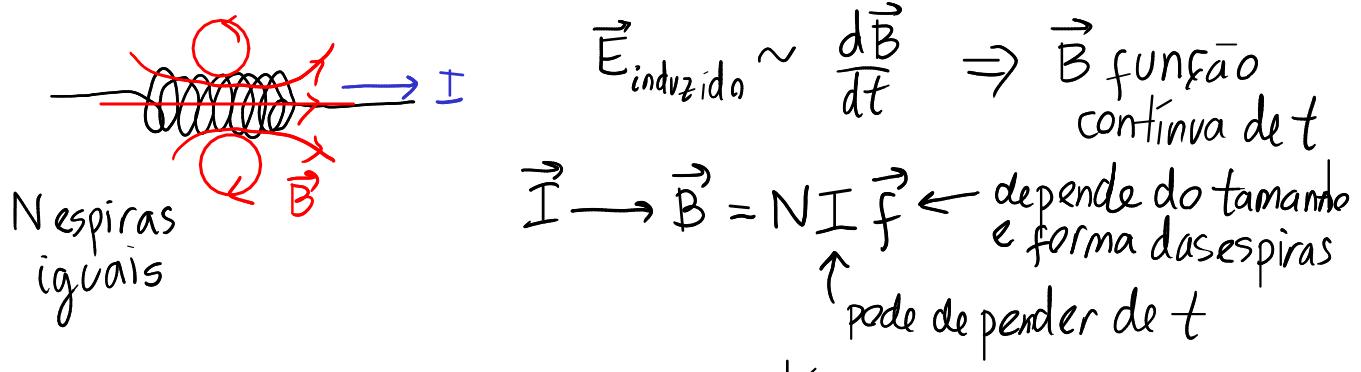
$$= (\bar{N} \bar{B} A \sin \theta) \frac{d\theta}{dt}$$

$$E_i = \bar{N} \bar{B} A \omega \sin(\omega t + \theta_0)$$

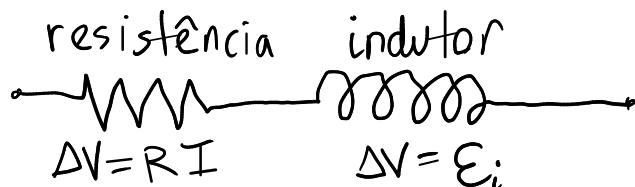
(se $\omega = \text{constante}$)



AUTO INDUÇÃO



$\Rightarrow I(t) = \text{função contínua}$
representação nos circuitos



$$\mathcal{E}_i = - \frac{d\psi}{dt}$$

(efeito Joule) (lei de Faraday)

ξ constante geométrica

$$\psi = N \iint_{\text{espira}} (\vec{B} \cdot \hat{n}) dA = N \iint_{\text{espira}} NI (\vec{f} \cdot \hat{n}) dA = N^2 \xi I$$

em geral, em qualquer dispositivo com corrente I

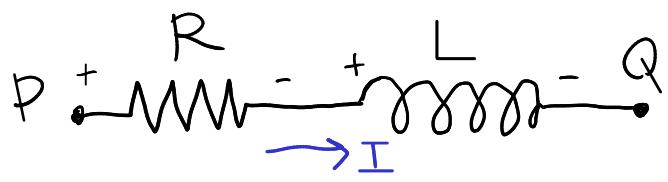
$$\boxed{\psi = LI}$$

L = indutância do dispositivo.

$$\boxed{\mathcal{E}_i = - \frac{d\psi}{dt} = -L \frac{dI}{dt}}$$

Unidade SI de indutância

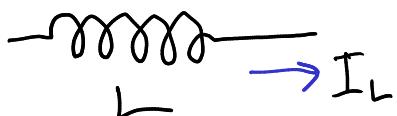
$$\text{Henry: } 1H = 1 \frac{V \cdot S}{A}$$



$$V_p - V_Q = RI - L \frac{dI}{dt}$$

↑ ↑
Ohm Faraday

CIRCUITOS COM INDUTORES



① $I_L = 0$ (corrente nula). $\Delta V = -L \frac{dI_L}{dt}$ (qualquer valor)

círculo equivalente:

② Estado estacionário: $\frac{dI}{dt} = 0$ (corrente constante)

I pode ter qualquer valor

$$\Delta V = -L \frac{dI}{dt} = 0$$

círculo equivalente:



③ Estado transitório. $I \neq 0, \frac{dI}{dt} \neq 0$

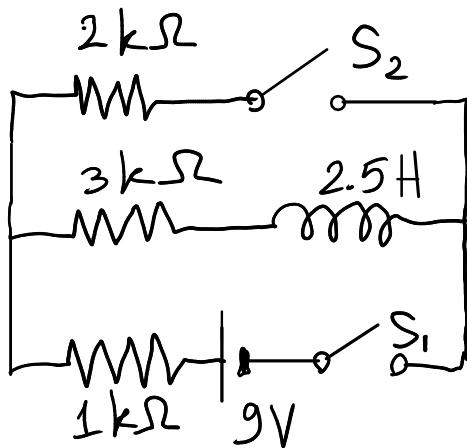
círculo equivalente:



I continua, mas
 ΔV não

$$\Delta V = \mathcal{E}_i = -L \frac{dI}{dt}$$

Exemplo.

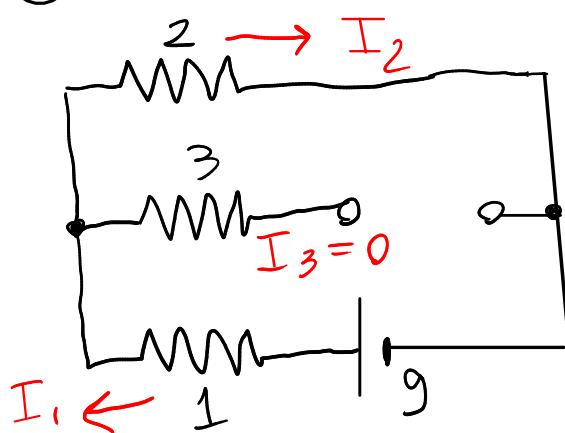


inicialmente S_1 e S_2 estão abertos.
Fecham-se os dois interruptores em $t=0$.
Em $t_1 > 0$ (muito elevado)
abre-se S_2 , mantendo S_1 fechado.

Determine as correntes nas 3 resistências, em $t=0$, $t=t_1$ e $t \rightarrow \infty$

Resolução. Unidades: $R \rightarrow k\Omega$, $\Delta V \rightarrow V$, $\Rightarrow I \rightarrow mA$

a) em $t=0$



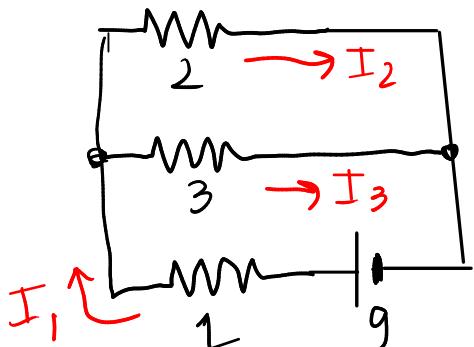
$$I_3 = 0$$

$$I_1 = I_2 = \frac{9}{2+1} = 3 \text{ mA}$$

em $t \rightarrow t_1^-$ $I_3 \neq 0$

$L \rightarrow$ estado estacionário

b) Estado estacionário no limite $t \rightarrow t_1^-$



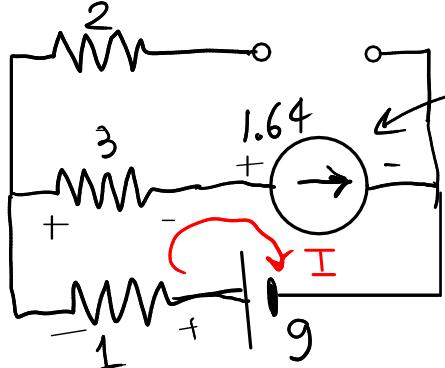
$$I_1 = \frac{9}{1 + \frac{6}{5}} \leftarrow 3 \text{ em paralelo com } 2$$

$$I_1 = \frac{45}{11} = 4.09 \text{ mA}$$

$$\Delta V_2 = \Delta V_3 = \frac{6}{5} \times \frac{45}{11} = \frac{54}{11}$$

$$I_2 = \frac{\Delta V_2}{2} = 2.45 \text{ mA} \quad I_3 = \frac{\Delta V_3}{3} = 1.64 \text{ mA}$$

C) em $t=t_1$



Uma malha com corrente

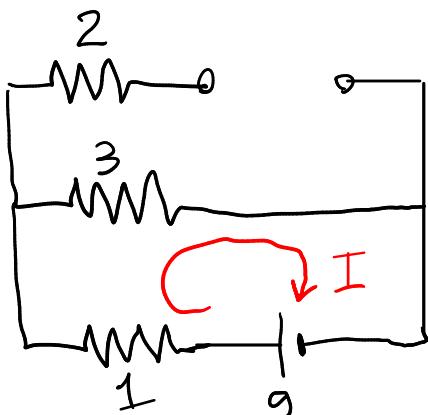
$$I = 1.64 \text{ mA}$$

$$I_2 = 0, \quad I_1 = I_3 = 1.64 \text{ mA}$$

$$\Delta V_L = 9 - (3+1)1.64$$

$$= 9 - 6.56 = 2.44 \text{ V}$$

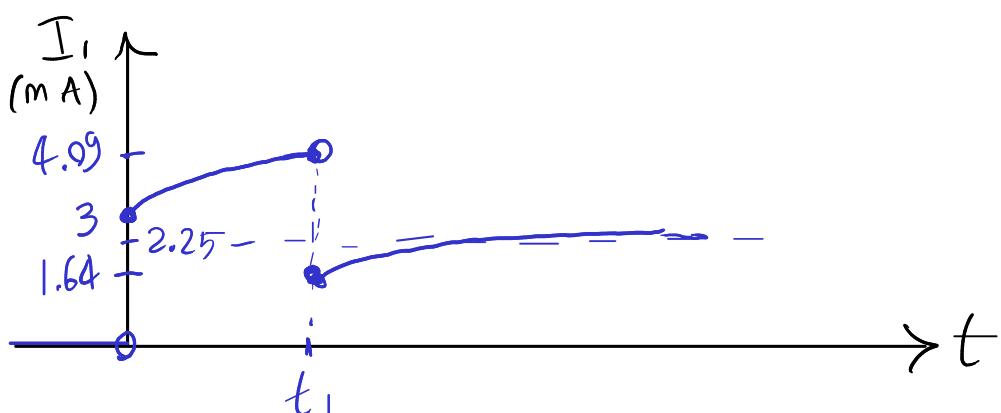
d) $t \rightarrow \infty$

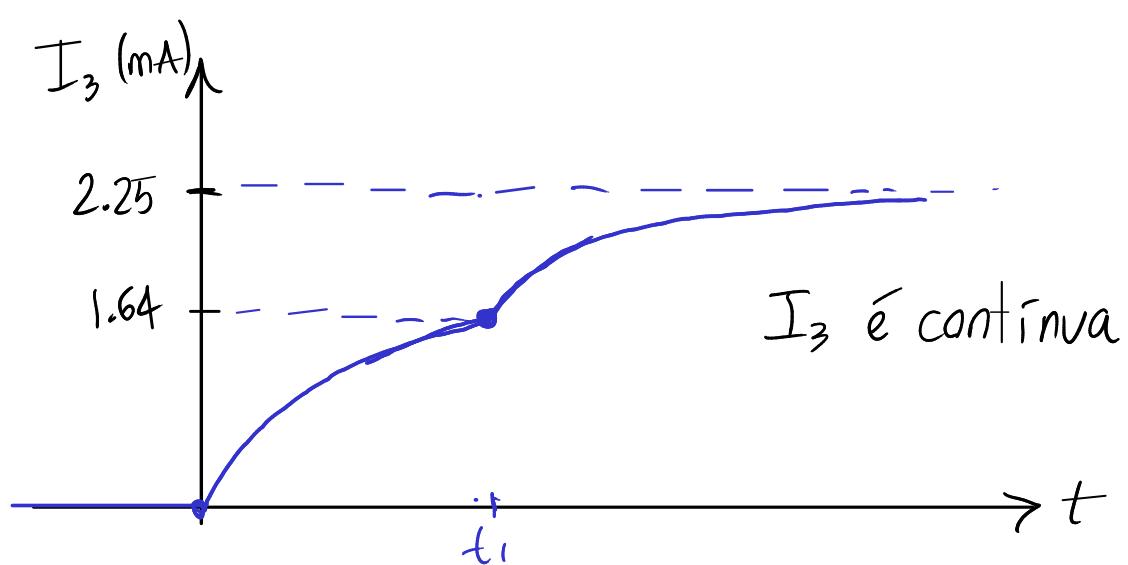
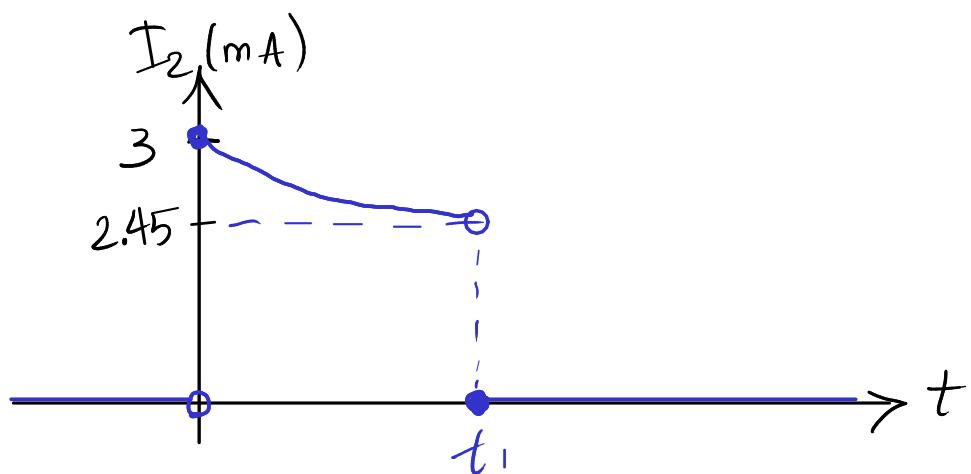


$$9 - 4I = 0 \quad I = \frac{9}{4} = 2.25 \text{ mA}$$

$$I_2 = 0, \quad I_1 = I_3 = 2.25 \text{ mA}$$

Gráficos de $I_1(t)$, $I_2(t)$ e $I_3(t)$





PROCESSAMENTO DE SINAIS

sinal \rightarrow função que depende de t (voltage $V(t)$ ou corrente $I(t)$)



Sistemas de processamento de sinais: circuito elétrico com apenas uma fonte (sinal de entrada) e várias resistências, condensadores e indutores.



$$V(t) = V_+ - V_-$$

$I(t)$ depende do circuito



$$I_e(t)$$

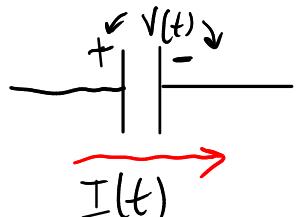
$V(t)$ depende do circuito

① Resistências:



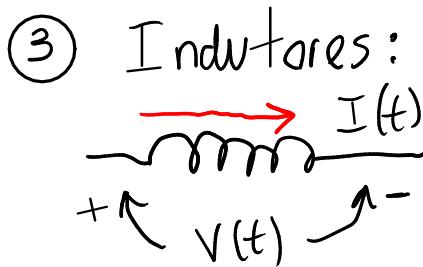
$$V(t) = R I(t)$$

② Condensadores:



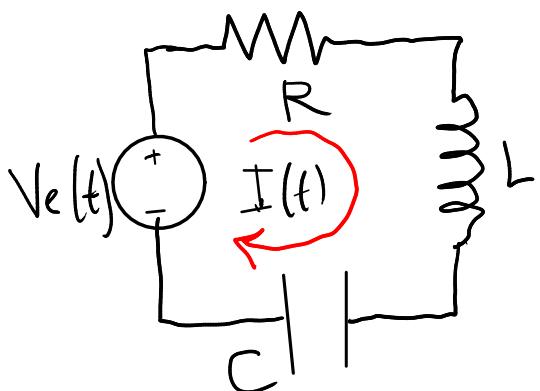
$$Q(t) = C V(t) \quad (I = \dot{Q})$$

$$I(t) = C \dot{V}(t)$$



$$V(t) = +L \frac{dI}{dt}$$

Exemplo 1. Circuito RLC, em série, com fonte de tensão variável.



Entrada $\rightarrow V_e(t)$

Saída $\rightarrow I(t)$

uma malha com equação:

$$L \frac{dI}{dt} + RI + \frac{Q}{C} = V_e \quad (I = \dot{Q})$$

$$\Rightarrow L \ddot{I} + RI + \frac{\dot{I}}{C} = \dot{V}_e$$

permite determinar $I(t)$
para uma $V_e(t)$ dada

E.D.O. linear, 2ª ordem, com coeficientes constantes

Resolução por transformada de Laplace

$$\mathcal{L}\{I(t)\} = \tilde{I}(s) = \int_0^\infty I(t) e^{-st} dt \quad \begin{matrix} \text{unidades de s} \\ = t^{-1} \end{matrix}$$

$$\Rightarrow \mathcal{L}\{\dot{I}\} = \int_0^\infty \dot{I} e^{-st} dt = s\tilde{I} - I_0 \quad \leftarrow I(t=0)$$

$$\mathcal{L}\{\ddot{I}\} = s\mathcal{L}\{\dot{I}\} - \dot{I}_0 = s^2 \tilde{I} - sI_0 - \dot{I}_0$$

$$\text{E.D.O.} \rightarrow L(s^2 \tilde{I} - sI_0 - \dot{I}_0) + R(s\tilde{I} - I_0) + \frac{\tilde{I}}{C} = s\tilde{V}_e - V_e(0)$$

equação algébrica

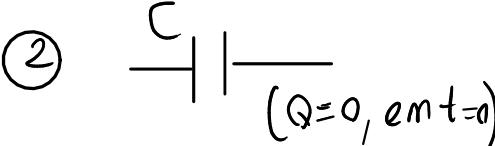
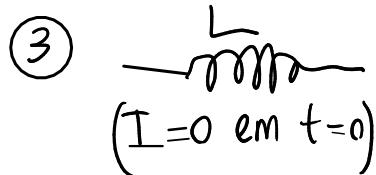
$$(Ls^2 + Rs + \frac{1}{C}) \tilde{I} = s\tilde{V}_e - V_e(0) + LsI_o + L\dot{I}_o + RI_o$$

$$\Rightarrow \tilde{I} = \frac{s\tilde{V}_e - V_e(0) + LsI_o + L\dot{I}_o + RI_o}{Ls^2 + Rs + \frac{1}{C}}$$

← função de s

a transformada inversa $\mathcal{L}^{-1}\{\dots\}$ dá $I(t)$

DOMÍNIO DA FREQUÊNCIA S $\leftarrow (Hz = s^{-1})$

	domínio de t	domínio de s
① 	$V(t) = R I(t)$	$\tilde{V} = R \tilde{I}$
② 	$\dot{V} = \frac{I}{C}$	$s\tilde{V} = \frac{\tilde{I}}{C}$
③ 	$V = L\dot{I}$	$\tilde{V} = Ls\tilde{I}$

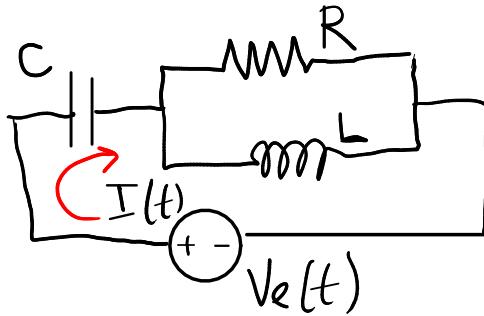
IMPEDÂNCIA

$$\tilde{V} = Z(s) \tilde{I}$$

lei de Ohm generalizada
 $Z(s)$ = impedância

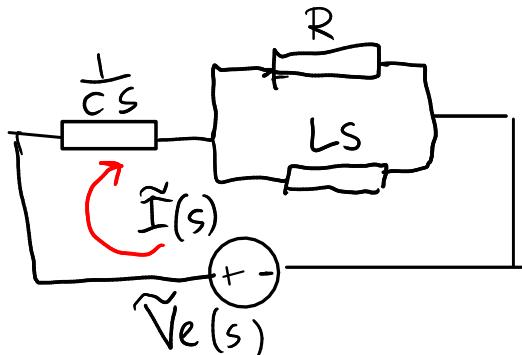
$$Z(s) = \begin{cases} R, \text{ nas resistências} & \text{unidade SI} \\ \frac{1}{Cs}, \text{ nos condensadores} & \downarrow \\ Ls, \text{ nos indutores} & \frac{1}{F \cdot Hz} = \Omega \\ & H \cdot Hz = \Omega \end{cases}$$

Exemplo 2.



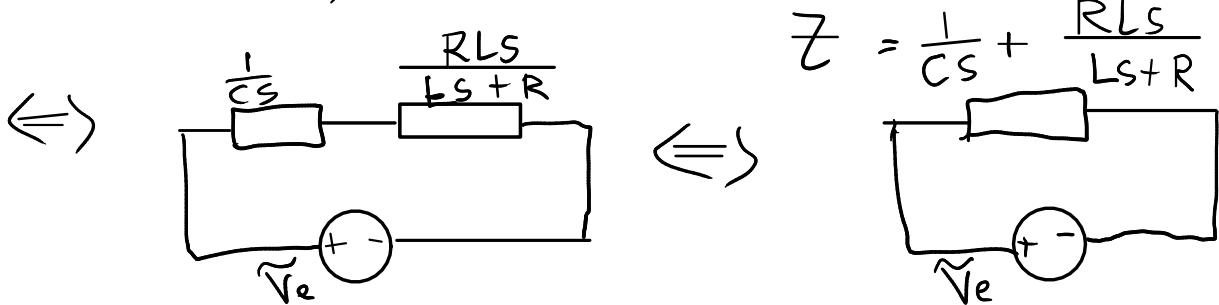
$$\text{em } t=0, \\ Q=0, \quad I=0$$

Círculo no domínio da frequência



$$Z_p = \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)^{-1}$$

$$Z_s = Z_1 + Z_2$$



$$\tilde{I} = \frac{\tilde{V}_e}{Z} = \frac{\tilde{V}_e (LCS^2 + RCS)}{RLCS^2 + LS + R} \rightarrow I(t) = \mathcal{L}^{-1} \{ \tilde{I} \}$$

equação diferencial do circuito:

$$(RLCS^2 + LS + R) \tilde{I} = (LCS^2 + RCS) \tilde{V}_e$$

↓

$$\begin{aligned} \tilde{I} &\rightarrow I(t) \\ \tilde{s}\tilde{I} &\rightarrow \dot{I}(t) \\ \tilde{s}^2\tilde{I} &\rightarrow \ddot{I}(t) \end{aligned}$$

$$RLC\ddot{I} + L\dot{I} + RI = LC\ddot{V}_e + RC\dot{V}_e$$

Unidades:

$$\begin{array}{l} L \rightarrow Rt \\ C \rightarrow \frac{t}{R} \\ S \rightarrow \frac{1}{t} \end{array}$$

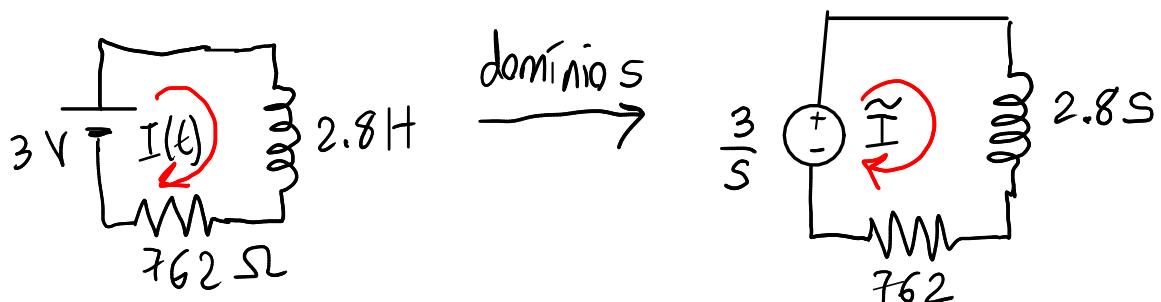
$$LCS^2 \rightarrow \cancel{(Rt)} \left(\frac{t}{R} \right) \left(\frac{1}{t} \right) \quad RCS \rightarrow \cancel{R} \left(\frac{t}{R} \right) \left(\frac{1}{t} \right)$$

$$RLC \rightarrow R(Rt) \left(\frac{t}{R} \right) \rightarrow Rt^2 \quad RLCS^2 \rightarrow R$$

$$LS \rightarrow (Rt) \left(\frac{1}{t} \right) \rightarrow R$$

Exemplo 1. Uma bobina com $L = 2.8 \text{ H}$ e $R = 762 \Omega$ liga-se a uma fonte de tensão, ideal, com $\mathcal{E} = 3 \text{ V}$. Determine a corrente na bobina 1 ms após ter sido ligada à fonte.

Resolução (SI)



$$\tilde{I} = \frac{3/s}{2.8s + 762} = \frac{\frac{3}{s}}{s(s + \frac{762}{2.8})} = \frac{\frac{15}{14}}{s(s + \frac{1905}{7})} = \frac{A}{s} + \frac{B}{s + \frac{1905}{7}}$$

$$\frac{15}{14} = A\left(s + \frac{1905}{7}\right) + Bs$$

$$s=0 \quad A\left(\frac{1905}{7}\right) = \frac{15}{14} \quad A = \frac{15}{2 \times 1905} = \frac{1}{254}$$

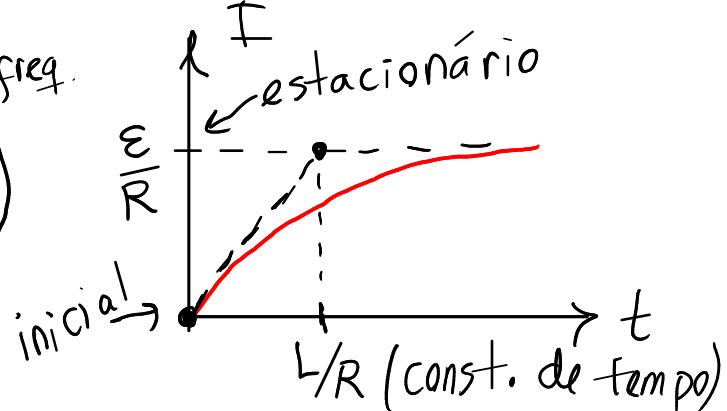
$$s = -\frac{1905}{7} \quad B\left(-\frac{1905}{7}\right) = \frac{15}{14} \quad B = -\frac{1}{254}$$

$$\Rightarrow I(t) = \frac{1}{254} \left(1 - e^{-\frac{1905}{7}t} \right) \quad I(0.001) = \frac{1 - e^{-\frac{1.905}{7}}}{254}$$

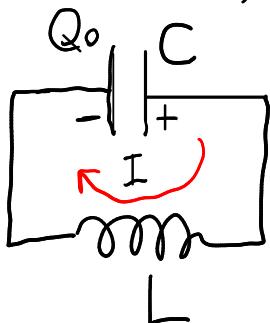
$$I(0.001) = 0.938 \text{ mA}$$

$$I(t) = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{R}{L}t} \right)$$

círculo LR

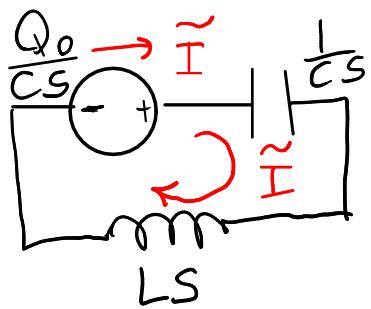


Exemplo 2. Um condensador, com carga inicial Q_0 , liga-se a um indutor ideal. Determine a corrente no circuito, em função do tempo.



$$V_C(t) = \frac{Q}{C} = \frac{1}{C} \left(\int_0^t I dt - Q_0 \right)$$

$$\tilde{V}_C = \mathcal{L}\{V_C\} = \frac{\tilde{I}}{CS} - \frac{Q_0}{CS}$$



$$\frac{\tilde{I}}{CS} + \tilde{I} LS - \frac{Q_0}{CS} = 0$$

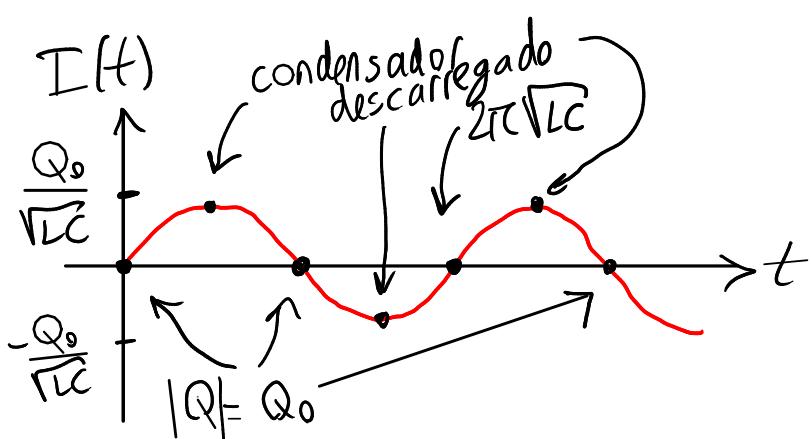
$$\tilde{I} = \frac{Q_0/CS}{LS + \frac{1}{CS}}$$

$$I(t) = \mathcal{L}^{-1}\{\tilde{I}\} = \frac{Q_0}{\sqrt{LC}} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

função seno com frequência angular

$$\omega = \frac{1}{\sqrt{LC}}$$

freq.
angular
do circuito
 LC

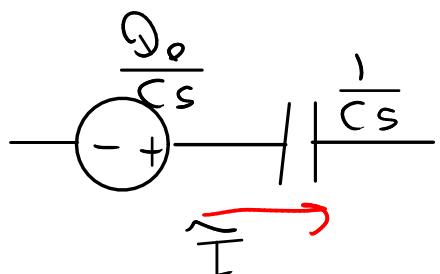


círculo
oscilador

voltage no indutor: $\tilde{V}_L = Z_L \tilde{I} = LS \tilde{I}$

$$V_L(t) = \frac{Q_0}{C} \cos\left(\frac{t}{\sqrt{LC}}\right)$$

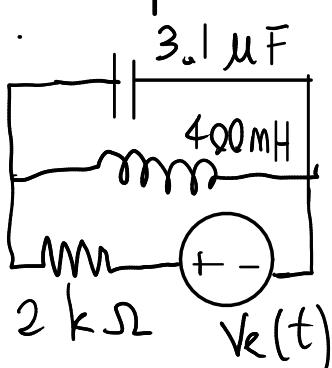
voltagem no condensador



$$\tilde{V}_c = \frac{\tilde{I}}{Cs} - \frac{Q_0}{Cs}$$

$$V_c(t) = -\frac{Q}{C} \cos\left(\frac{t}{\sqrt{LC}}\right) = -V_L(t)$$

Exemplo 3.

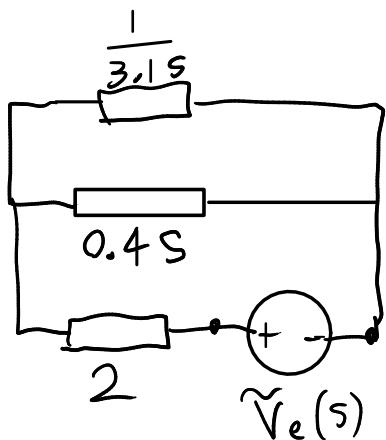


$$V_e(t) = 5(1 - e^{-600t}) \text{ (SI)}$$

$$\text{em } t=0, Q_c=0, I_L=0$$

determine $I(t)$ na resistência.

Resolução. $\Delta V \rightarrow V$, $R \rightarrow k\Omega$, $C \rightarrow \mu F$ $\frac{1}{Cs} = 2$
 $\Rightarrow Z \rightarrow k\Omega$, $I \rightarrow mA$, $s \rightarrow kHz$, $t \rightarrow ms$ $S = \frac{1}{CZ}$
 $L \rightarrow H$ $Z = LS$



$$t \rightarrow ms : V_e(t) = 5(1 - e^{-0.6t})$$

$$\tilde{V}_e(s) = \mathcal{L}\{V_e(t)\}$$

$$Z_t = 2 + \left(3.1s + \frac{1}{0.4s}\right)^{-1}$$

$$\tilde{I} = \frac{\tilde{V}_e}{Z_t} \rightarrow I(t) = \mathcal{L}^{-1}\{\tilde{I}\}$$

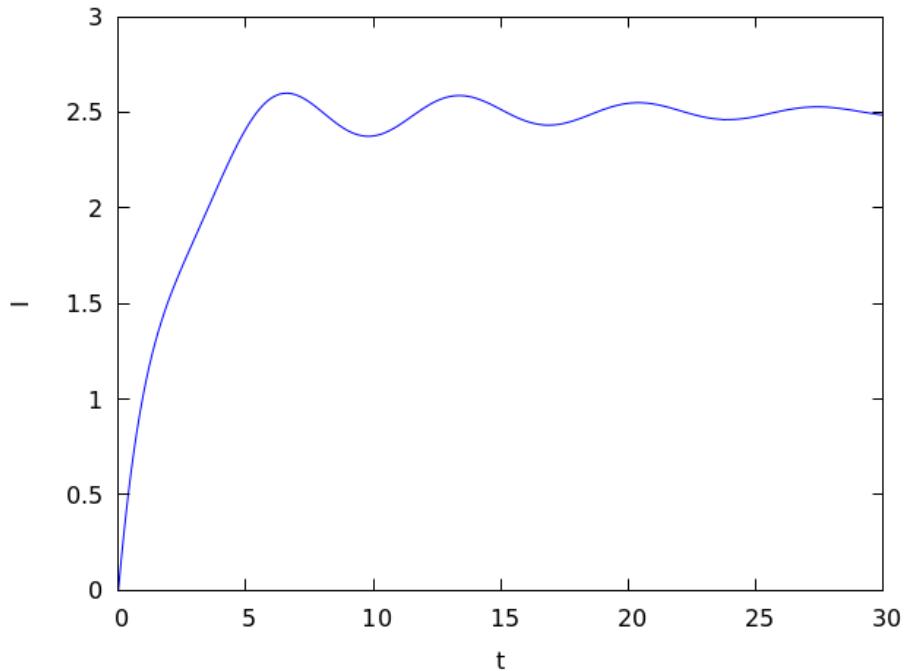
```

(%i1) Ve: 5*(1-exp(-0.6*t));
(%o1)                               5 (1 - %e- 0.6 t)
(%i2) ve: laplace(Ve, t, s);
(%o2)      1      1
           5 (- - -----)
                  s      s + 0.6
(%i3) z: ratsimp(2+1/(3.1*s+1/0.4/s));
(%o3)      2
           62 s + 10 s + 50
-----  

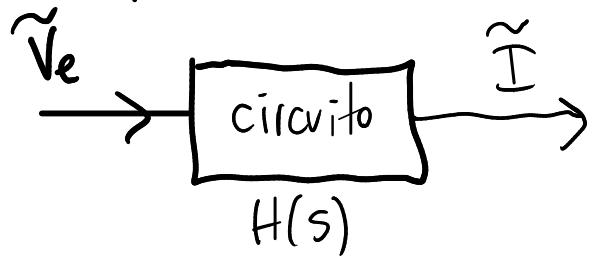
                  2
           31 s + 25
(%i4) i: ratsimp(ve/z);
(%o4)      2
           465 s + 375
-----  

                  4      3      2
           310 s + 236 s + 280 s + 150 s
(%i5) I: ilt(i,s,t)$
(%i6) plot2d(I,[t,0,30],[ylabel,"I"])$

```



FUNÇÃO DE TRANSFERÊNCIA ($H(s)$)



$$\tilde{I} = H(s) \tilde{V}_e$$

forma geral
nos circuitos
lineares

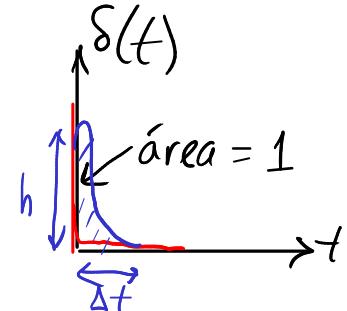
No exemplo 3, $H(s) = \frac{1}{Z_L} = \frac{31s^2 + 25}{62s^2 + 10s + 50}$ (s em $\frac{1}{k\text{ Hz}}$)

IMPULSO UNITÁRIO (delta de Dirac)

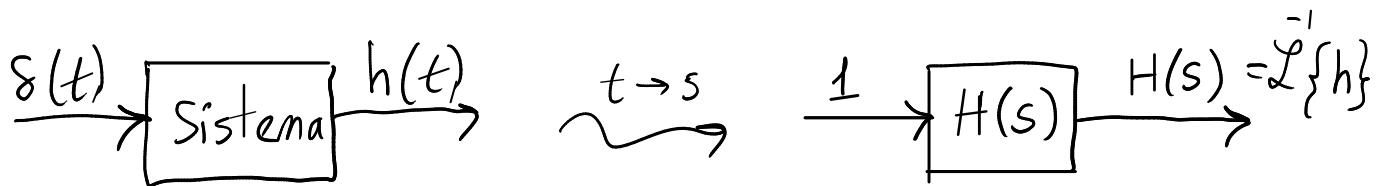
$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t=0 \end{cases}$$

$$\int_0^\infty \delta(t) f(t) dt = f(0) \Rightarrow \int_0^\infty \delta(t) dt = 1$$

$$\delta(t-4) = \begin{cases} 0, & t \neq 4 \\ \infty, & t=4 \end{cases}$$



$$\mathcal{L}\{\delta(t)\} = \int_0^\infty \delta(t) e^{-st} dt = 1$$



Maxima: $\delta(t) \rightarrow \text{delta}(t)$

```
(%i1) laplace(delta(t),t,s);
(%o1)
(%i2) ilt(1,s,t);
(%o2)
(%i3) f: (3*s^2+5*s-8)/(6*s^2+2*s-4)$
(%i4) ilt(f,s,t);
(%o4)
(%i5) partfrac(f,s);
(%o5)
(%i6) ilt(%-1/2,s,t);
(%o6)
(%i7) %+ delta(t)/2;
(%o7)
```

$$\begin{aligned} & \text{laplace calcua } \mathcal{L}\{\delta(t)\} \\ & \text{mas } \text{ilt} \text{ n\~ao calcula} \\ & \mathcal{L}^{-1}\{1\} \end{aligned}$$

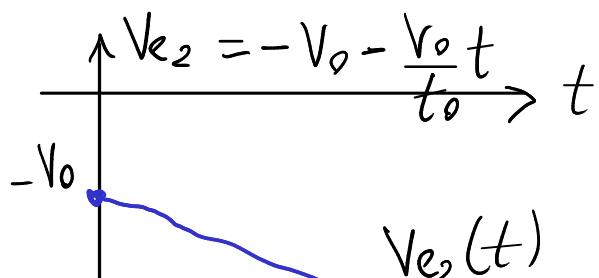
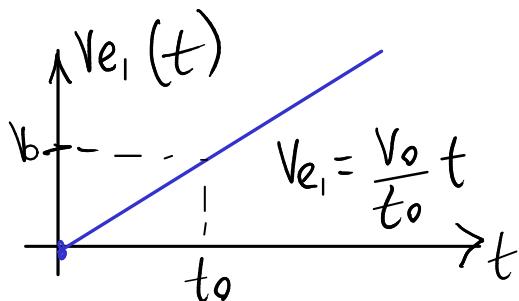
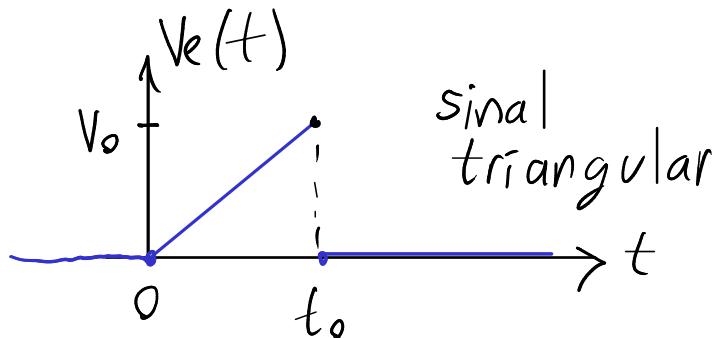
$$\begin{aligned} & \text{para obter } \mathcal{L}^{-1}\{ \} \\ & \text{duma fun\~ao} \\ & \text{raciona\~al com} \\ & \text{polin\'omios do} \\ & \text{mesmo grau no} \\ & \text{numerador e} \\ & \text{denominador,} \\ & \text{podemos usar} \\ & \text{partfrac()} \end{aligned}$$

SINAIS PARCELARMENTE CONTÍNUOS

Exemplo 10.2 (livro).



filtro passa-alto

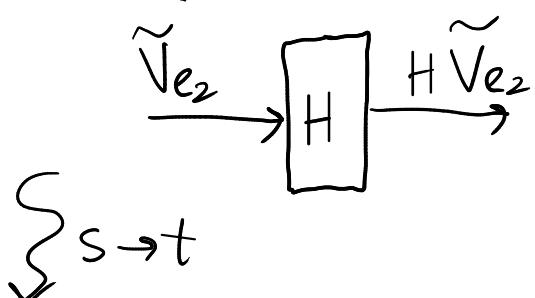
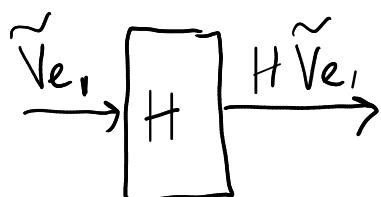


dois sinal continuos em $t \in [0, \infty[$

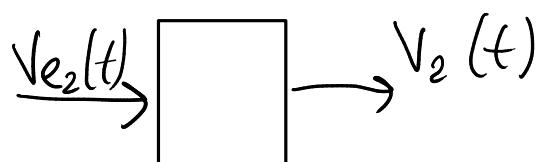
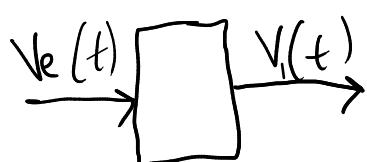
$$V_e(t) = V_{e1}(t) + u(t-t_0) V_{e2}(t-t_0)$$

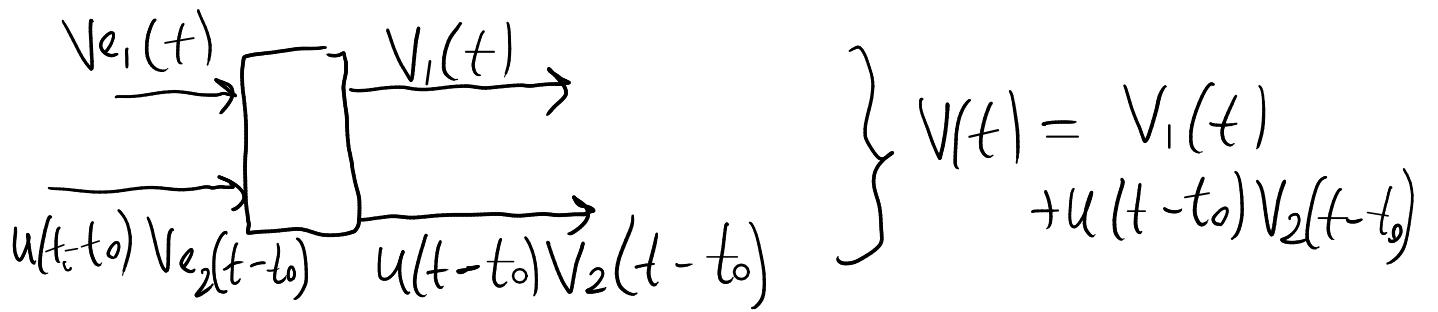
$$u(t-t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t \geq t_0 \end{cases}$$

degrau unitário
(função de Heaviside)



$\brace s \rightarrow t$





$$Z = R + \frac{1}{CS} \quad \tilde{I} = \frac{\tilde{V}_e}{Z} = \frac{CS}{RCS+1} \tilde{V}_e$$

$$\tilde{V} = R \tilde{I} = \frac{RCS}{RCS+1} \tilde{V}_e \Rightarrow H(s) = \frac{RCS}{RCS+1}$$

(%i8) H: RC*s/(RC*s+1);

(%o8)

$$\frac{RC}{RC + 1}$$

(%i9) Ve1: V0*t/t0\$

(%i10) vel: laplace(Ve1,t,s);

(%o10)

$$\frac{V_0}{s^2}$$

(%i11) v1: H*vel;

(%o11)

$$\frac{RC V_0}{s (RC + 1)}$$

(%i12) V1: ilt(v1,s,t);

(%o12)

$$\frac{-t/RC}{t_0}$$

(%i13) Ve2: -V0-V0*t/t0\$

(%i14) ve2: laplace(Ve2,t,s);

(%o14)

$$\left(\frac{V_0}{s^2} - \frac{V_0}{s} \right)$$

(%i15) v2: H*ve2;

(%o15)

$$\frac{RC s \left(\frac{V_0}{s^2} - \frac{V_0}{s} \right)}{RC s + 1}$$

(%i16) V2: ilt(v2,s,t);

(%o16)

$$\left(\frac{-t/RC}{RC t_0} - \frac{(RC V_0 t_0 - RC V_0)^2}{RC t_0} \right) - \frac{RC V_0}{t_0}$$

```
(%i17) V: V1 + unit_step(t-t0)*subst(t=t-t0,V2);
          t - t0
          -
          2
          (RC V0 t0 - RC V0) %e      RC V0
(%o17) unit_step(t - t0) ((- -----) - -----)
          RC t0                  t0
          - t/RC
          RC V0 %e      RC V0
          - ----- + -----
          t0          t0
```

(Capítulo 12. Não entra no exame)
4 equações de Maxwell

- ① Lei de Gauss $\oint_S (\vec{E} \cdot \hat{n}) dA = 4\pi k q_{int}$
- ② $\oint_C \vec{E} \cdot d\vec{r} = - \frac{\partial}{\partial t} \iint_S (\vec{B} \cdot \hat{n}) dA$ (Lei de Faraday)
- ③ $\iint_S (\vec{B} \cdot \hat{n}) dA = Q$ (não há monopólos mag.)
- ④ Lei de ampére: $\oint_C \vec{B} \cdot d\vec{r} = 4\pi k_m I_{int} + \downarrow$
termos de Maxwell
 $\vec{B} \text{ e } \vec{E} \neq \vec{0}$ (onda eletrromagnética)
 $(\frac{\partial \vec{E}}{\partial t})$
com velocidade $v = \sqrt{\frac{k}{k_m}}$

$$v = \sqrt{\frac{9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}{10^{-7} \text{ ...}}} = 3 \times 10^8 \frac{\text{m}}{\text{s}} \quad (= \text{velocidade da luz!})$$