EXERCISES ABOUT PUSH-DOWN AUTOMATA (PDAS)

1 Consider the context-free language over the alphabet $\Sigma=\{a,b\}$ defined by: $L = \{a^n b^k \mid n \le k \le 2n\}$. **[SELECTED]**

a) Write a context-free grammar (CFG) for L.

b) Propose a PDA for recognizing L.

c) Show the sequences of instantaneous descriptions of that automaton for the input string *aabbb*.

d) What does happen when the input string is *aaabb*? Justify using the instantaneous descriptions.

2 Consider the CFG G= ({S, A}, {0, 1}, P, S) with productions: **[SELECTED]**

$$S \rightarrow A\ 1\ A$$

$$A \rightarrow 1A \mid 0\ A \mid \varepsilon$$

Propose a PDA, accepting by empty stack, which recognizes the language of grammar G.

3 Consider the CFG G= ({S}, {i, e}, P, S) with the following productions:

$$S \rightarrow S\ S \mid i\ S \mid i\ S\ e\ S \mid \varepsilon$$

Show a PDA, accepting by empty stack, which recognizes the language of the grammar G.

4 Consider the following PDA, which accepts by empty stack: **[SELECTED]**

$P = (\{p, s\}, \{0, 1\}, \{Z, 0, 1\}, \delta, p, Z)$.

Function δ is defined as follows:

$\delta(p, 0, Z) = \{ (p, 0Z) \}$
$\delta(p, 0, 1) = \{ (p, \varepsilon) \}$
$\delta(p, 0, 0) = \{ (p, 00) \}$
$\delta(p, \varepsilon, Z) = \{ (s, \varepsilon) \}$
$\delta(p, 1, Z) = \{ (p, 1Z) \}$
$\delta(p, 1, 0) = \{ (p, \varepsilon) \}$
$\delta(p, 1, 1) = \{ (p, 11) \}$

a) Show the sequence of reachable configurations when starting from configuration (p, 1100, Z).

b) Is the string 1100 recognized by the automaton? Why?

c) What does happen with the sequence of reachable configurations when starting from configuration (p, 101, Z)?

5 "A context-free grammar is ambiguous if there exist a leftmost and a rightmost derivation of at least a string recognized by the grammar". Is this statement true? Justify. **[SELECTED]**