PROOF BY INDUCTION EXERCISES

1 Prove that, for every natural number, $1+3+5+ ...+(2n+1) = (n+1)^2$. **[SELECTED]**

2 Consider the following definition of a tree. **[SELECTED]**

Definition of tree:

1. A simple node is a tree.
2. If $T_1, T_2, ..., T_k$ are trees, the structure that results from taking a new node N and connecting with an edge each of the trees $T_1, T_2, ..., T_k$ to N is a tree.
3. Nothing else is a tree other than that obtained from 1. and 2.

Prove that a tree has a number of nodes larger than the number of edge E by one unit.

3 A palindrome is a string that can be read in the same way both from left-to-right and from right-to-left. Consider the following inductive definition of the related concept pal: **[SELECTED]**

1. Each character of the alphabet is a pal.
2. If α is a pal so is the result of concatenating any character before and after α.
3. Nothing is a pal except if obtained from 1. and 2.

Prove by induction that every pal is a palindrome. Is the converse true? If so, prove it. Otherwise, correct the definition so that it becomes true.

4 An ATM has only 20€ and 50€ bills. Prove by induction that this machine can supply any amount that is a multiple of 10€, equal or larger than 40€.

Define the structure over which you will make your proof. Show the different steps of the proof. **[SELECTED]**

5 Prove that, for every natural number $n\geq2$, $(1-1/2)(1-1/3)\ldots(1-1/n) = 1/n$.

6 Prove that, for every natural number $n\geq1$, 2 is a factor of n^2+n.

7 Prove that the sum of the first *n* perfect cubes is a perfect square. [Note: you need to prove a stronger proposition (Inventor's Paradox).]

8 A binary tree can be recursively defined in the following way: *"a binary tree is constituted by one node and 0, 1 or 2 sub-trees connected by edges to that node. A full node is a node that has two sub-trees. A leaf is a node that has no sub-trees."*

> *We want to prove, using mathematical induction, that "for every binary tree the number of leaves equals the number of full nodes plus one".*

Present the structure over which you will apply the proof by induction and explain its inductive definition.