Theory of Computation

L.EIC, 2nd Year

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Outline

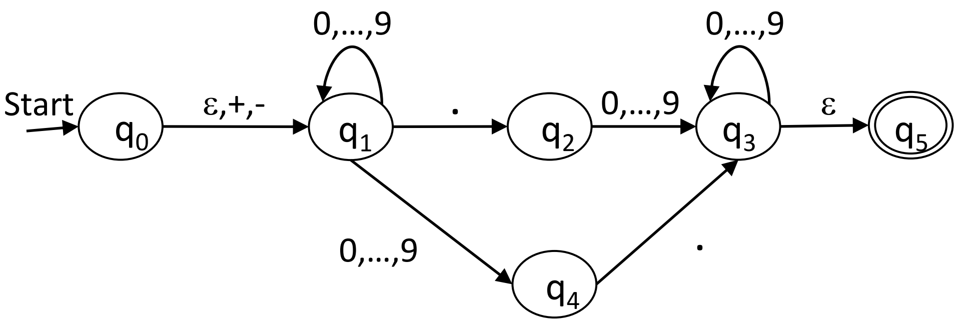
- Non-Deterministic Finite Automata with ε transitions (ε-NFAs)
- Conversion of ε-NFAs into DFAs

Finite Automata with ε Transitions

- What is an ε transition?
- A spontaneous transition (empty-string transition) that can be followed without receiving/consuming/processing input symbols (i.e., for any input symbol)
- An ε-NFA is an NFA with ε transitions

Finite Automata with ε Transitions (cont.)

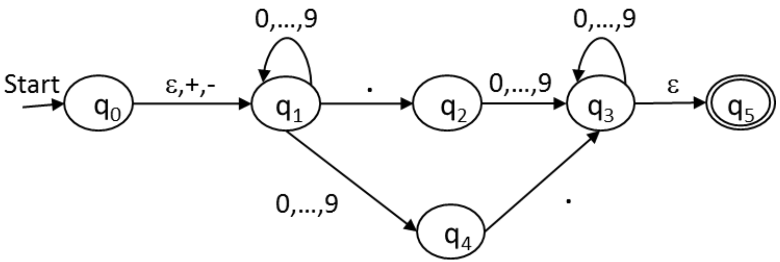
- Example: ε-NFA that recognizes decimal numbers
  - Signal + or – optional
  - Sequence of digits
  - A decimal point
  - Another sequence of digits
  - At least one of the sequences of digits is non-empty



Exercise 1

- Modify the previous state diagram in order to not recognize inputs like: .5, +.1, and -.1 (i.e., before the '.' there must be at least one digit)
- More precise, this new definition of a decimal number is:
  - Signal + or – optional
  - A sequence of digits with length greater or equal 1
  - A decimal part consisting of a '.' followed by an optional sequence of digits x, such that $|x| \geq 0$.

Formal Notation ε-NFA



- ε-NFA $E = (Q, \Sigma, \delta, q_0, F)$
  - The major difference is in the transition function δ to deal with ε
    - $\delta(q, a)$: state $q \in Q$ and $a \in \Sigma \cup \{\varepsilon\}$
- Example: $E = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{.,+,-,0,..,9\}, \delta, q_0, \{q_5\})$
- The symbol representing the empty-string, ε, is not visible in the sequence of digits
  - It represents spontaneous transitions
  - We deal with it in the same way as with the non-determinism, i.e., considering that the automaton can be in all the states before and after the ε transition
- To know which are the states we can reach from a state q with ε, we calculate the ε-close(q)
  - ε-close(q_0)= $\{q_0,q_1\}$; ε-close(q_3)= $\{q_3,q_5\}$

δ	ε	+,-	.	0,..,9
$\to q_0$	$\{q_1\}$	$\{q_1\}$	$\varnothing$	$\varnothing$
q_1	$\varnothing$	$\varnothing$	$\{q_2\}$	$\{q_1\ q_4\}$
q_2	$\varnothing$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_3	$\{q_5\}$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_4	$\varnothing$	$\varnothing$	$\{q_3\}$	$\varnothing$
$*q_5$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Extended Transitions

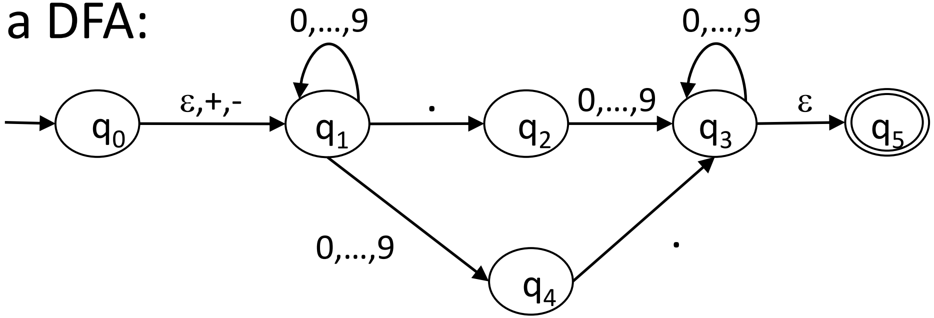
- *ε-close(q) or Eclose(q)*
  - Basis: State *q* is in *EClose(q)*
  - Induction: if *p* is in *EClose(q)* and exists an ε-transition from *p* to *r*, then *r* is also in *EClose(q)*
- Extended transition $\hat{\delta}$
  - Basis: $\hat{\delta}(q,\varepsilon) = EClose(q)$
  - Induction: *w=xa*, $a \in \Sigma$ (thus, $a \neq \varepsilon$)
    - 1. let's $\hat{\delta}(q,x)=\{p_1, p_2, \ldots, p_k\}$
    - 2. $\bigcup_{i=1}^{k} \delta(p_i, a) = \{r_1, \ldots, r_m\}$
    - 3. $\hat{\delta}(q,w) = \bigcup_{j=1}^{m} Eclose(r_j)$
  - (1.) gives the states reached from *q* following a path representing **x** that can include (and/or terminate in) one or more ε

Eliminating ε Transitions

- Given an ε-NFA E there exists always an equivalent DFA D
  - E and D accept the same language
- Technique of subsets construction
  - ε-NFA E = $(Q_E, \Sigma, \delta_E, q_0, F_E)$ → DFA D = $(Q_D, \Sigma, \delta_D, q_D, F_D)$
- Q_D is the set of subsets of Q_E closed in ε
  - $Q_D = EClose(Q_E)$
- Start state: $q_D = EClose(q_0)$
- $F_D = \{S \mid S \text{ is in } Q_D \text{ and } S \cap F_E \neq \emptyset\}$
- Transition $\delta_D(S,a)$, with a in Σ and S in Q_D
  - $S=\{p_1, p_2, \ldots, p_k\}$
  - Calculate $$\bigcup_{i=1}^{k} \delta_E(p_i, a) = \{r_1, \ldots, r_m\}$$
  - Terminate with $$\delta_D(S,a) = \bigcup_{j=1}^{m} EClose(r_j)$$

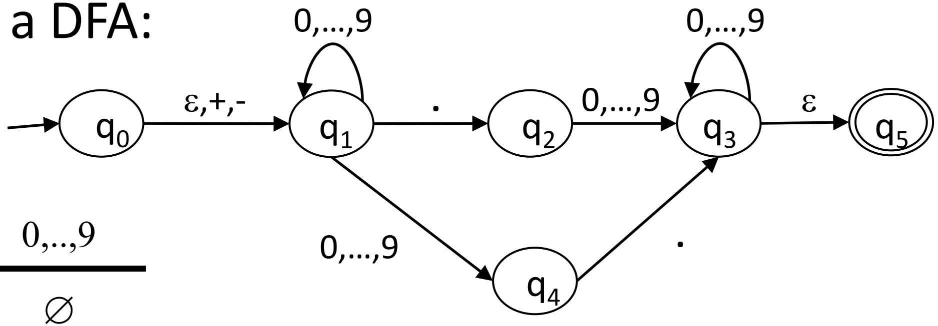
Example of the Recognizer of Decimals

- Convert the ε-NFA to a DFA:



Example of the Recognizer of Decimals

- Convert the ε-NFA to a DFA:



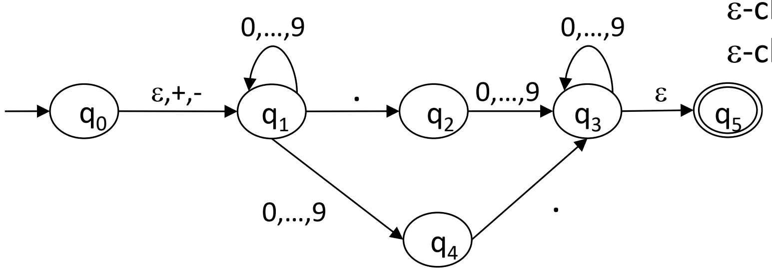
δ	ε	+,-	.	0,..,9
$\rightarrow q_0$	$\{q_1\}$	$\{q_1\}$	$\varnothing$	$\varnothing$
q_1	$\varnothing$	$\varnothing$	$\{q_2\}$	$\{q_1\ q_4\}$
q_2	$\varnothing$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_3	$\{q_5\}$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_4	$\varnothing$	$\varnothing$	$\{q_3\}$	$\varnothing$
$*q_5$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

ε-close(q0)={q0, q1}
ε-close(q1)={q1}
ε-close(q2)={q2}
ε-close(q3)={q3, q5}
ε-close(q4)={q4}
ε-close(q5)={q5}

Example of the Recognizer of Decimals

ε-close(q0)={q0, q1}
ε-close(q1)={q1}
ε-close(q2)={q2}
ε-close(q3)={q3, q5}
ε-close(q4)={q4}
ε-close(q5)={q5}

- Convert the ε-NFA to a DFA:

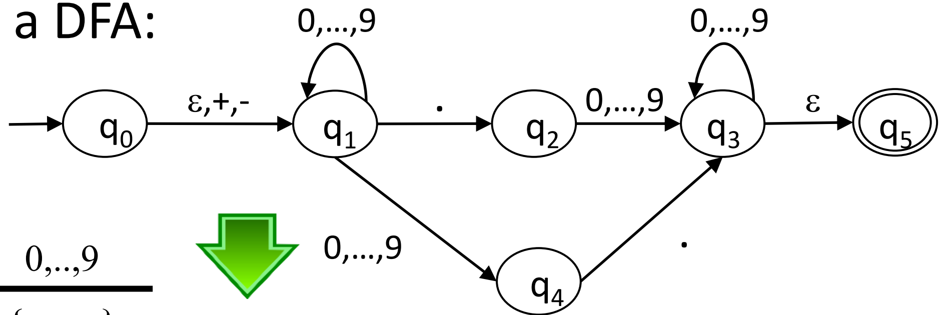


δ	ε	+,-	.	0,..,9
$\rightarrow q_0$	$\{q_1\}$	$\{q_1\}$	$\varnothing$	$\varnothing$
q_1	$\varnothing$	$\varnothing$	$\{q_2\}$	$\{q_1\ q_4\}$
q_2	$\varnothing$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_3	$\{q_5\}$	$\varnothing$	$\varnothing$	$\{q_3\}$
q_4	$\varnothing$	$\varnothing$	$\{q_3\}$	$\varnothing$
$*q_5$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

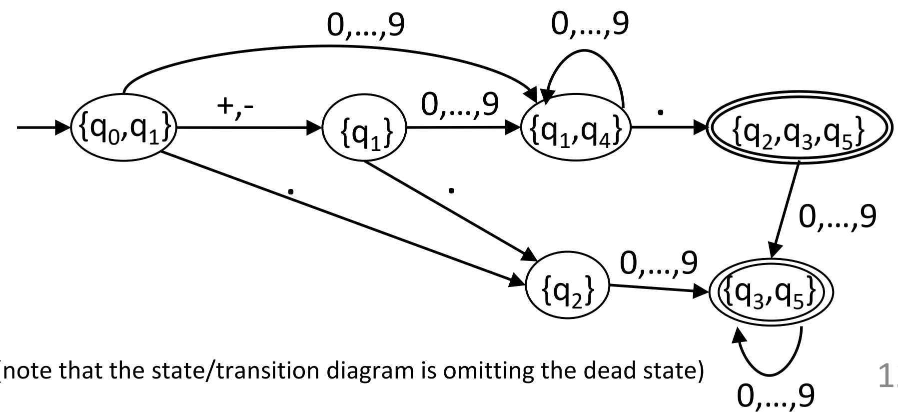
δ	+,-	.	0,..,9
$\rightarrow\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$	$\{q_1, q_4\}$
$\{q_1\}$	$\varnothing$	$\{q_2\}$	$\{q_1, q_4\}$
$\{q_2\}$	$\varnothing$	$\varnothing$	$\{q_3, q_5\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3, q_5\}$	$\{q_1, q_4\}$
$*\{q_3, q_5\}$	$\varnothing$	$\varnothing$	$\{q_3, q_5\}$
$*\{q_2, q_3, q_5\}$	$\varnothing$	$\varnothing$	$\{q_3, q_5\}$
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

Example of the Recognizer of Decimals

- Convert the ε-NFA to a DFA:



δ	+,-	.	0,..,9
$\rightarrow\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$	$\{q_1, q_4\}$
$\{q_1\}$	$\varnothing$	$\{q_2\}$	$\{q_1, q_4\}$
$\{q_2\}$	$\varnothing$	$\varnothing$	$\{q_3, q_5\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3, q_5\}$	$\{q_1, q_4\}$
$*\{q_3, q_5\}$	$\varnothing$	$\varnothing$	$\{q_3, q_5\}$
$*\{q_2, q_3, q_5\}$	$\varnothing$	$\varnothing$	$\{q_3, q_5\}$
$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$



(note that the state/transition diagram is omitting the dead state)

Exercise 2

- Consider the following ε-NFA:

	ε	a	b	c
$\rightarrow p$	$\varnothing$	$\{p\}$	$\{q\}$	$\{r\}$
q	$\{p\}$	$\{q\}$	$\{r\}$	$\varnothing$
$*r$	$\{q\}$	$\{r\}$	$\varnothing$	$\{p\}$

- Calculate the ε-close for each state
- Indicate all the strings with length ≤ 3 accepted by the automaton
- Convert the ε-NFA into a DFA