Turing Machine Examples

L.EIC, 2nd Year

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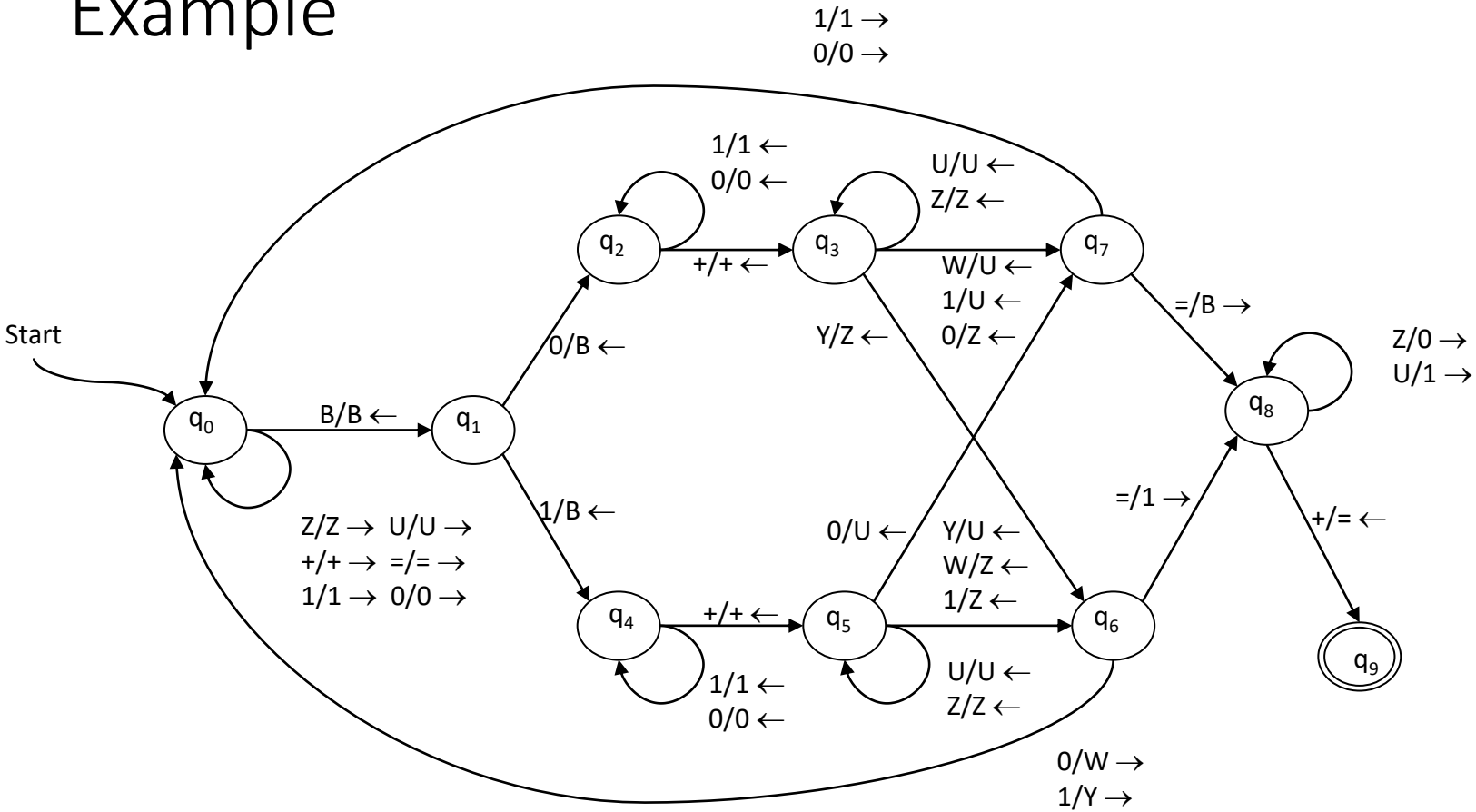
Example

- TM to add two binary numbers in the form: =n1+n2
  - The numbers have the same number of bits
  - The result should be on the left of the assignment (‘=‘)
  - There is no need to maintain the right-hand side of the assignment

  - Example:
    - =011+010 $\rightarrow$ 101=

Example


1/1 →
0/0 →
1/1 ←
0/0 ←
U/U ←
Z/Z ←
q2
q3
q7
+/+ ←
W/U ←
1/U ←
0/Z ←
=/B →
Start
Y/Z ←
Z/0 →
U/1 →
0/B ←
q0
B/B ←
q1
q8
1/B ←
=/1 →
+/= ←
Z/Z → U/U →
+/+ → =/= →
1/1 → 0/0 →
0/U ←
Y/U ←
W/Z ←
1/Z ←
q4
+/+ ←
q5
q6
q9
1/1 ←
0/0 ←
U/U ←
Z/Z ←
0/W →
1/Y →

Description

- Strategy
  - Add n_1 to n_2, substituting n_1, bit by bit, starting by the LSB
  - In an iteration add one bit (Z=0, U=1), coding eventual carry in the next bit (W=0+carry, Y=1+carry)
  - Finishes when detects =
- State meaning
  - q_0: beginning of the iteration to add one bit; traverse the tape to the rightmost position
  - q_1: read the LSB of n_2 and erases it; transition q_2-q_3 if it is 0 and q_4-q_5 if it is 1
  - q_2: (if bit=0) goes to left until the beginning of n_1
  - q_3: traverse the already processed bits of n_1 an process one more; go to q_6 if there is carry and to q_7 if there is no carry
  - q_4 and q_5 equal to q_2 and q_3 in case of bit=1
  - q_6: use the carry in the following bit seguinte and finish the iteration; detects end of the number
  - q_7: finish the iteration, w/o carry, and detects end of the number
  - q_8: = is transferred from left to right of the number and Z's and U's are substituted by 0's and 1's, respectively
  - q_9: final state

Computing Trace

U: one
Z: zero
W: one
Y: zero

- Example =011+010
  - q_0=011+010 ⊢* =011+010q_0B ⊢ =011+01$q_1$0 ⊢ =011+0$q_2$1 ⊢ =011+$q_2$01 ⊢ =011q_2+01 ⊢ =01$q_3$1+01 ⊢ =0$q_7$1U+01 ⊢ =01q_0U+01 ⊢* =01U+01q_0B ⊢ =01U+0$q_1$1 ⊢ =01U+$q_4$0 ⊢ =01Uq_4+0 ⊢ =01q_5U+0 ⊢ =0$q_5$1U+0 ⊢ =$q_6$0ZU+0 ⊢ =Wq_0ZU+0 ⊢* =WZU+0q_0B ⊢ =WZU+$q_1$0 ⊢ =WZUq_2+ ⊢ =WZq_3U+ ⊢ =Wq_3ZU+ ⊢ =q_3WZU+ ⊢ q_7=UZU+ ⊢ q_8UZU+ ⊢* 101q_8+ ⊢ 10$q_9$1=