P02: Exercises about DFAs [SELECTED]

Solutions for the selected exercises: 1, 2, 3a), 5, 4

1.

a)

	A	B	C	0	1	2
q0	q0	q1	q2	q0	q1	q2
q1	q0	q1	q2	q0	q1	q2
q2	q0	q1	q2	q0	q1	q2

b) Each state represents the level where the elevator is in.

2.

	0…9	-
→ q0	q1	∅
q1	q2	q5
q2	q3	q5
q3	q4	q5
q4	∅	q5
q5	q6	∅
q6	q7	q8
q7	∅	q8
q8	q9	∅
* q9	q10	∅
* q10	∅	∅
∅	∅	∅

(The previous DFA does not verifies if the input date in the specified format is a valid date or not.)

3a)

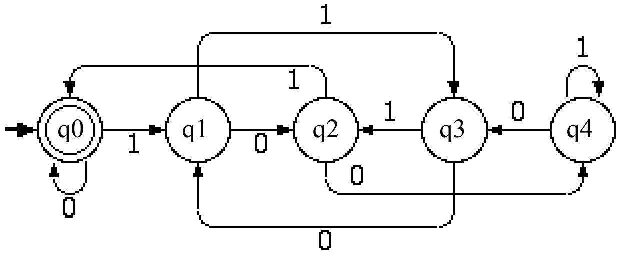
	0	1
→ q0	q1	q0
q1	q2	q0
* q2	q2	q0

5.

a)

- 5 states: one for each possible remain value (q0 identifies the rest: 0; q1 rest 1, q2, rest 2; q3, rest 3, and q4, rest 4):

	0	1
→ * q0	q0	q1
q1	q2	q3
q2	q4	q0
q3	q1	q2
q4	q3	q4



(in the previous DFA we are considering that the chain with length 0, ε, represents a multiple of 5, but would be easy to avoid the acceptance of ε!)

- Force a start with 1: add an initial state that links itself to the "dead" state case the chain start with 0, and links to q1 case starts by 1:

	0	1
→ s	d	q1
* q0	q0	q1
q1	q2	q3
q2	q4	q0
q3	q1	q2
q4	q3	q4
d	d	d

b)

Observations:

- We can just invert all the transitions!
- The previous start state becomes the only accept state (as before).
- The previous accept state becomes possible start states (since the only accept state was already the start state, this configuration if kept).

	0	1
→ * q0	q0	q2
q1	q3	q0
q2	q1	q3
q3	q4	q1
q4	q2	q4

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4)

Hypothesis: $\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y)$

Case base: $y = \varepsilon$

$\hat{\delta}(q, x) = \hat{\delta}(\hat{\delta}(q, x), \varepsilon)$

From the definition bases $\hat{\delta}$:

We can say that $\hat{\delta}(q, x) = p$, and we know that $\hat{\delta}(p, \varepsilon) = p$

Inductive step: since |y| = n, consider strings of length n+1 generated from the concatenation of a letter (identified as 'a') with y: |ya| = n+1

$\hat{\delta}(q, xya) = \hat{\delta}(\hat{\delta}(q, x), ya)$

Demonstration steps:

Expressions	Reason
$\hat{\delta}(\hat{\delta}(q, x), ya)$	Departure
$\hat{\delta}(\hat{\delta}(\hat{\delta}(q, x), y), a)$	By the definition of $\hat{\delta}$, considering $\hat{\delta}(q, x)$ as a state
$\hat{\delta}(\hat{\delta}(q, xy), a)$	For hypothesis
$\hat{\delta}(q, xya)$	By the definition of $\hat{\delta}$

On this way, we obtain $\hat{\delta}(q, xya)$ *qed*