Elementary constructions

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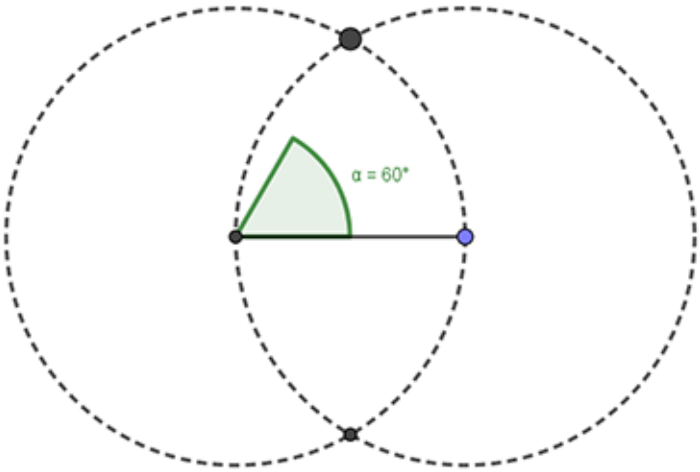
The use of the compass means drawing circles around points and intersecting.

The first construction is

- Drawing a circle around a point on a line
- Drawing a second circle on the intersection of the first circle and the line
- Intersecting the two circles

The angle between the line and the intersection will be of 60°, or $\frac{1}{6}$ of a circle.



A circle is the most elementary continuous form; an infinite curve.
Whereas a polygon has a finite number of sides, a line has no sides; a circle must have sides, otherwise it would be a line.
Then, a circle has “infinite” sides.

But then, a larger circle must have more sides than a smaller circle, or otherwise they would be the same size.
Then, we conclude there are different infinities at hand. That is, in the continuous realm.

The circle, then, contains the passage to the uncountable, and it’s only one of the three elementary geometric forms (line, triangle, circle) from which basic theorems in geometry are derived.
These three elementary forms already contain the problematique of a lot of later Mathematics. As such, they can be seen as an expression of this problematique in a particular form or language, the language of Geometry, not superior, inferior, older or newer than other theories. In Euclid we were already in front of this thousands of years ago. In the meantime, the theory got larger, but the root problematique remains the same.

The passage from a polygon to a circle represents the passage from discrete to continuous.

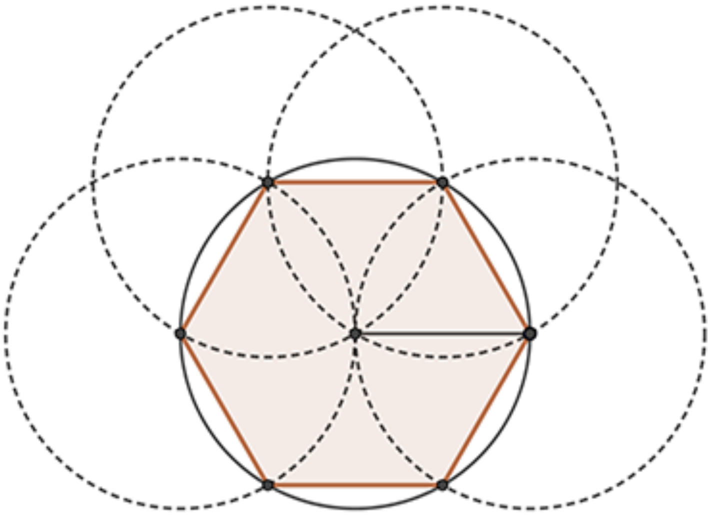
To illustrate that using a compass is working with circles, we’ll draw full compass circles.

For example, since the angle is $\frac{1}{6}$ of a circle, we can trivially inscript a 6-sided polygon inside the circle.

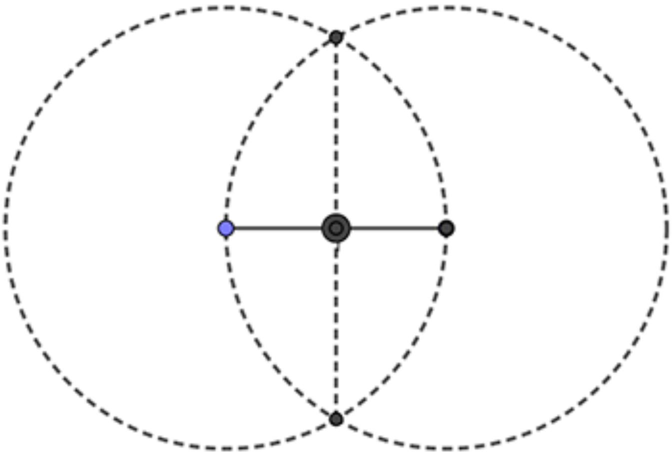
For each intersection:

- Use the intersection as the center of a new circle
- Intersect the new circle again

Doing this four times is enough because each circle defines two intersection points. So with four circles the two extra points not in common between them are taken care of.

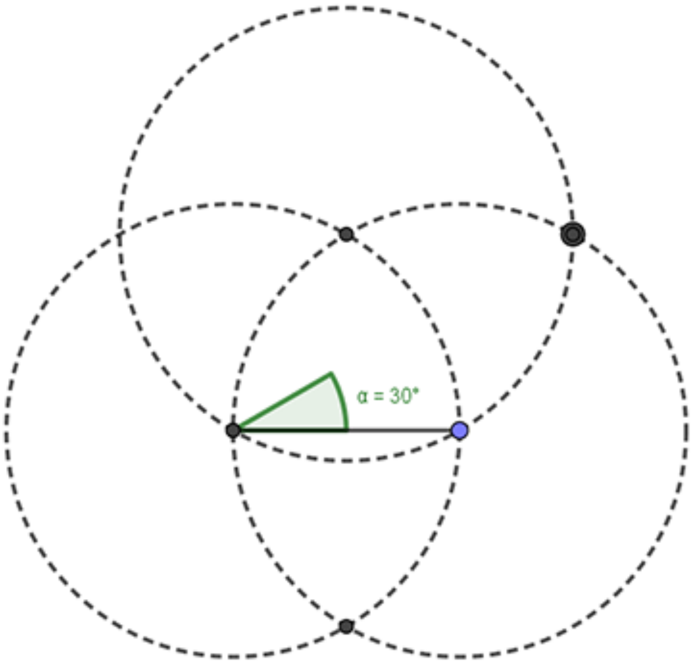


We have begun with a point and found a symmetric polygon with it as the center.
But, beginning with a segment, we can find its middle point.



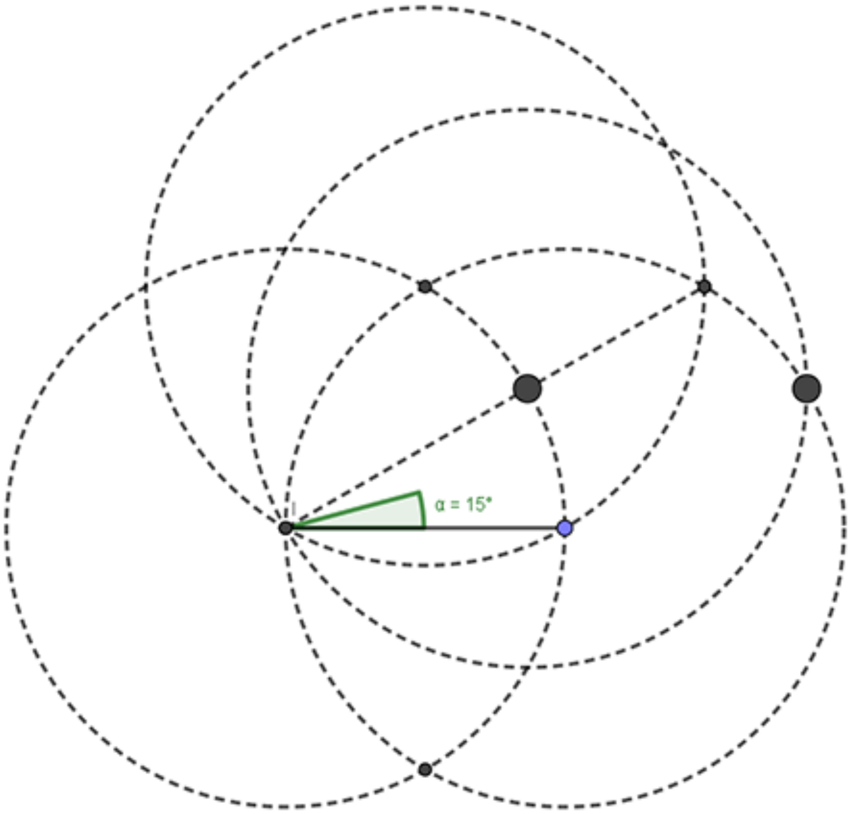
Two points in a circle forming a 60° angle can be used to find a 30° angle.

- Draw one further circle with center at the intersection
- Intersect the new circle with the previous one

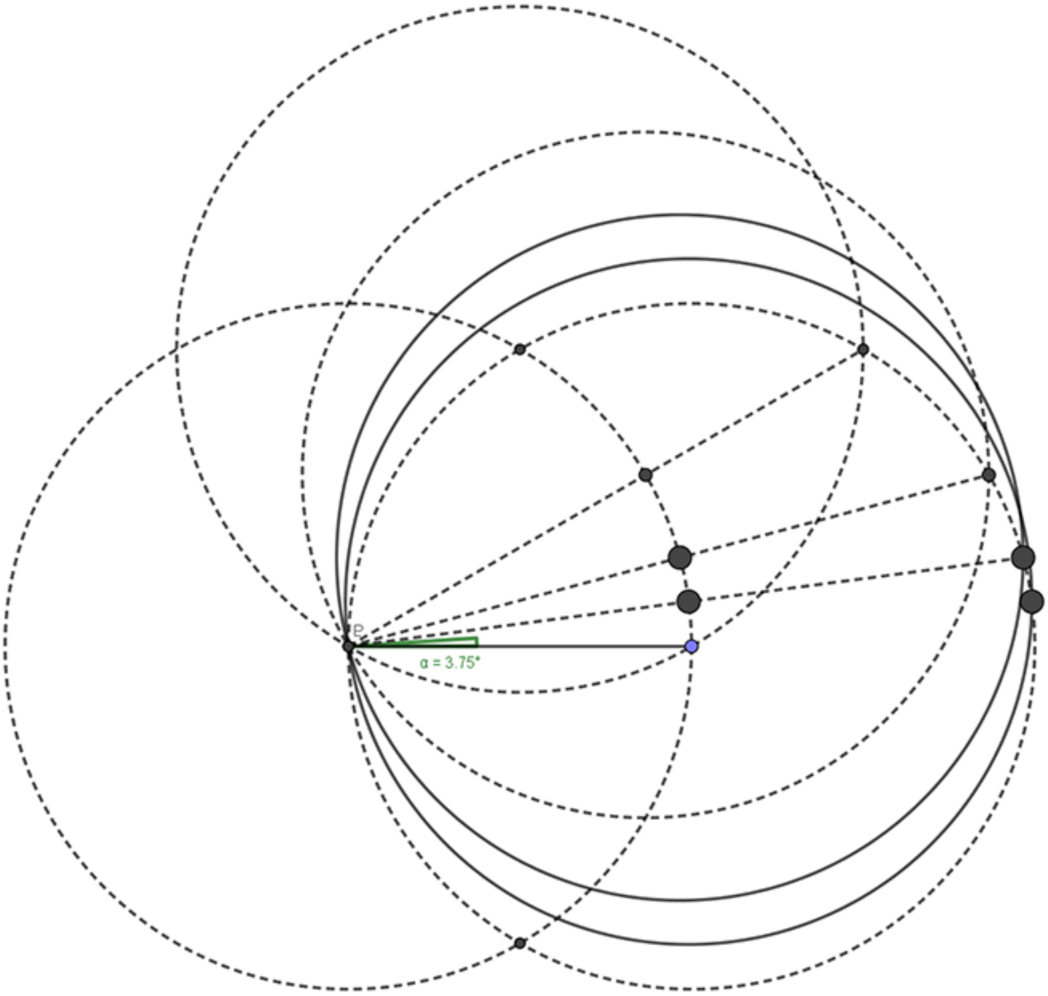


To get half of the 30° angle, intersecting between circles is not enough. It is necessary to intersect a circle and a segment:

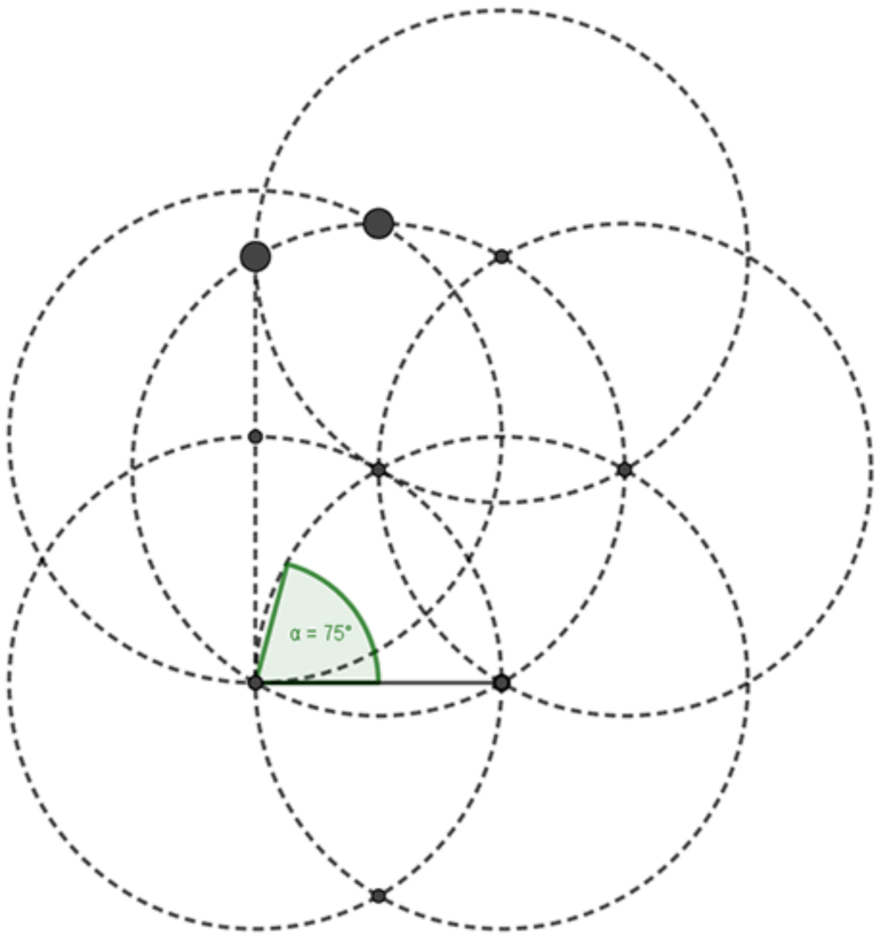
- Create a segment from the origin point to the 30° intersection point
- The intersection of the segment with the first circle defines a 30° point on the first circle
- Use it as a center for another circle of the same diameter
- The intersection of the new circle with the circle which defines the 30° angle is then at 15°



Let’s take it two steps further to generate a 3.75° angle.



This can be used to generate arbitrary fractional angles from 60° by picking different "initial" circles. For example, in the next graphic we obtain the 75° angle by using an intersection with a segment angled at 90°:



Curiously, the same process doesn't work for the 60° construction, to find a 30° angle: