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Exemplos

$A=(0,1)$.

$x=0 \Rightarrow x \in A'$?
$0=\inf A \Rightarrow$
$\forall \varepsilon > 0, \varepsilon \in \mathbb{R}\colon \exists a \in A | 0 < a < \varepsilon \Leftrightarrow$
$\forall \dot{O}(0)\colon \dot{O}(0) \cap A \neq \varnothing \Rightarrow$
$x \in A'$.

$x=1 \Rightarrow x \in A'$?
$1=\sup A \Rightarrow$
$\forall \varepsilon > 0, \varepsilon \in \mathbb{R}\colon \exists a \in A | 1-\varepsilon < a < 1 \Leftrightarrow$
$\forall \dot{O}(1)\colon \dot{O}(1) \cap A \neq \varnothing \Rightarrow$
$x \in A'$.

$x \in A \Rightarrow x \in A'$?
$x \in A \Rightarrow$
$\forall \dot{O}(x)\colon \dot{O}(x) \cap A \neq \varnothing \Rightarrow$
$x \in A'$.

Exercícios

Ex 1. $A=\mathbb{R} \Rightarrow A'=?$

Suponha $A' \neq A$. Então $\exists x \notin A=\mathbb{R}$, absurdo.
$A'=\mathbb{R}$.

Ex 2. $A=\mathbb{Q} \Rightarrow A'=?$

$A'=\mathbb{Q}$.

Ex 3. $A=\mathbb{N} \Rightarrow A'=?$

$\forall a,b \in \mathbb{N}\colon \exists c \in [a,b] | \dot{O}(c)=\varnothing$. Então $A'=\varnothing$.

Ex 4. $A=\left\{\frac{1}{n}, n \in \mathbb{N}\right\} \Rightarrow A'=?$

$A=\left\{\frac{1}{1}, \frac{1}{2}, \ldots.\right\}, \lim_{n\to\infty}\left\{\frac{1}{n}\right\}=0.$
$A=(0,1] \Rightarrow A'=[0,1]$.

Ex 5. $A \subset [a,b]$, A é conj. infinito. Provar que $\exists$ ao menos um ponto limite de A que $\in [a,b]$.

$x \in A' \Rightarrow$
$\forall \varepsilon > 0, \varepsilon \in \mathbb{R}\colon ((x-\varepsilon, x) \cup (x, x+\varepsilon)) \cap A \neq \varnothing \Rightarrow$
$\forall \varepsilon > 0, \varepsilon \in \mathbb{R}\colon \exists a' \in A | (x-\varepsilon < a' < x) \vee (x < a' < x+\varepsilon)$. (1)

Se $x \notin [a,b]$, então existe vizinhança perfurada de x sem intersecção com $[a,b]$ (o intervalo é fechado).

$x \notin [a,b] \Rightarrow$
$\exists \varepsilon > 0, \varepsilon \in \mathbb{R}\colon ((x-\varepsilon, x) \cup (x, x+\varepsilon)) \cap [a,b] = \varnothing \Rightarrow$
$\exists \varepsilon > 0, \varepsilon \in \mathbb{R}\colon \neg(\exists a' \in [a,b] | (x-\varepsilon < a' < x) \vee (x < a' < x+\varepsilon)) \Rightarrow$
$\exists \varepsilon > 0, \varepsilon \in \mathbb{R}\colon [\forall a' \in \mathbb{R} | (x-\varepsilon < a' < x) \vee (x < a' < x+\varepsilon)\colon a' \notin [a,b]]$ ou
$\exists \varepsilon > 0, \varepsilon \in \mathbb{R}\colon [\forall a' \in \mathbb{R}\colon (x-\varepsilon < a' < x) \vee (x < a' < x+\varepsilon) \Rightarrow a' \notin [a,b]]$.

Mas se $a' \notin [a,b]$, $a' \notin A$, contrariando (1).

Por exemplo,

$x = b + n, n > 0$
$\varepsilon = \frac{n}{2}$
$\dot{O}(x) = \left(x - \frac{n}{2}, x\right) \cup \left(x, x + \frac{n}{2}\right) = \left(b + n - \frac{n}{2}, x\right) \cup \left(x, x + \frac{n}{2}\right) = \left(b + \frac{n}{2}, x\right) \cup \left(x, x + \frac{n}{2}\right)$
$\forall n > 0\colon b + \frac{n}{2} > b \Rightarrow$
$\dot{O}(x) \cap [a,b] = \varnothing$.