

Aluno: Pedro Sobota

Exemplos

$$A = (0, 1).$$

$$x = 0 \Rightarrow x \in A'?$$

$$0 = \inf A \Rightarrow$$

$$\forall \varepsilon > 0, \varepsilon \in \mathbb{R}: \exists a \in A | 0 < a < \varepsilon \Leftrightarrow$$

$$\forall \dot{O}(0): \dot{O}(0) \cap A \neq \emptyset \Rightarrow$$

$$x \in A'.$$

$$x = 1 \Rightarrow x \in A'?$$

$$1 = \sup A \Rightarrow$$

$$\forall \varepsilon > 0, \varepsilon \in \mathbb{R}: \exists a \in A | 1 - \varepsilon < a < 1 \Leftrightarrow$$

$$\forall \dot{O}(1): \dot{O}(1) \cap A \neq \emptyset \Rightarrow$$

$$x \in A'.$$

$$x \in A \Rightarrow x \in A'?$$

$$x \in A \Rightarrow$$

$$\forall \dot{O}(x): \dot{O}(x) \cap A \neq \emptyset \Rightarrow$$

$$x \in A'.$$

Exercícios

Ex 1. $A = \mathbb{R} \Rightarrow A' = ?$

Suponha $A' \neq A$. Então $\exists x \notin A = \mathbb{R}$, absurdo.

$$A' = \mathbb{R}.$$

Ex 2. $A = \mathbb{Q} \Rightarrow A' = ?$

$$A' = \mathbb{Q}.$$

Ex 3. $A = \mathbb{N} \Rightarrow A' = ?$

$$\forall a, b \in \mathbb{N}: \exists c \in [a, b] | \dot{O}(c) = \emptyset. \text{ Então } A' = \emptyset.$$

Ex 4. $A = \left\{ \frac{1}{n}, n \in \mathbb{N} \right\} \Rightarrow A' = ?$

$$A = \left\{ \frac{1}{1}, \frac{1}{2}, \dots \right\}, \lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \right\} = 0.$$

$$A = (0, 1] \Rightarrow A' = [0, 1].$$

Ex 5. $A \subset [a, b]$, A é conj. infinito. Provar que \exists ao menos um ponto limite de A que $\in [a, b]$.

$$\begin{aligned}
& x \in A' \Rightarrow \\
& \forall \varepsilon > 0, \varepsilon \in \mathbb{R}: ((x - \varepsilon, x) \cup (x, x + \varepsilon)) \cap A \neq \emptyset \Rightarrow \\
& \forall \varepsilon > 0, \varepsilon \in \mathbb{R}: \exists a' \in A | (x - \varepsilon < a' < x) \vee (x < a' < x + \varepsilon). \quad (1)
\end{aligned}$$

Se $x \notin [a, b]$, então nenhuma vizinhança perfurada de x tem intersecção com $[a, b]$ (o intervalo é fechado).

$$\begin{aligned}
& x \notin [a, b] \Rightarrow \\
& \forall \varepsilon > 0, \varepsilon \in \mathbb{R}: ((x - \varepsilon, x) \cup (x, x + \varepsilon)) \cap [a, b] = \emptyset \Rightarrow \\
& \forall \varepsilon > 0, \varepsilon \in \mathbb{R}: \neg(\exists a' \in [a, b] | (x - \varepsilon < a' < x) \vee (x < a' < x + \varepsilon)) \Rightarrow \\
& \forall \varepsilon > 0, \varepsilon \in \mathbb{R}: [\forall a' \in \mathbb{R} | (x - \varepsilon < a' < x) \vee (x < a' < x + \varepsilon): a' \notin [a, b]].
\end{aligned}$$

Mas se $a' \notin [a, b]$, $a' \notin A$, contrariando (1).