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Exemplos

$A=(0,1)$.

$x=0\Rightarrow x\in A'$?
$0=\inf A\Rightarrow$
$\forall\varepsilon>0,\varepsilon\in\mathbb{R}\colon\exists a\in A|0<a<\varepsilon\Leftrightarrow$
$\forall\dot{O}(0)\colon\dot{O}(0)\cap A\neq\varnothing\Rightarrow$
$x\in A'$.

$x=1\Rightarrow x\in A'$?
$1=\sup A\Rightarrow$
$\forall\varepsilon>0,\varepsilon\in\mathbb{R}\colon\exists a\in A|1-\varepsilon<a<1\Leftrightarrow$
$\forall\dot{O}(1)\colon\dot{O}(1)\cap A\neq\varnothing\Rightarrow$
$x\in A'$.

$x\in A\Rightarrow x\in A'$?
$x\in A\Rightarrow$
$\forall\dot{O}(x)\colon\dot{O}(x)\cap A\neq\varnothing\Rightarrow$
$x\in A'$.

Exercícios

Ex 1. $A=\mathbb{R}\Rightarrow A'=?$

Suponha $A'\neq A$. Então $\exists x\notin A=\mathbb{R}$, absurdo.
$A'=\mathbb{R}$.

Ex 2. $A=\mathbb{Q}\Rightarrow A'=?$

$A'=\mathbb{Q}$.

Ex 3. $A=\mathbb{N}\Rightarrow A'=?$

$\forall a,b\in\mathbb{N}\colon\exists c\in[a,b]|\dot{O}(c)=\varnothing$. Então $A'=\varnothing$.

Ex 4. $A=\left\{\frac{1}{n},n\in\mathbb{N}\right\}\Rightarrow A'=?$

$A=\left\{\frac{1}{1},\frac{1}{2},....\right\},\ \lim_{n\to\infty}\left\{\frac{1}{n}\right\}=0.$
$A=(0,1]\Rightarrow A'=[0,1]$.

Ex 5. $A\subset[a,b]$, A é conj. infinito. Provar que $\exists$ ao menos um ponto limite de A que $\in[a,b]$.

$A'=A$ e $A\subset[a,b]$. Então $x\in A\Rightarrow x\in[a,b]$.

Ou:

Contradição: $\neg(\exists x \in A' | x \in [a,b]) = \forall x \in A' \colon x \notin [a,b]$.
$x \in A' \Rightarrow \forall \varepsilon > 0, \varepsilon \in \mathbb{R} \colon \exists x' \in A | x < x' < x + \varepsilon \vee x > x' > x - \varepsilon$.

Então,
$\forall \varepsilon > 0, \varepsilon \in \mathbb{R} \colon \exists x' \in A | (x < x' < x + \varepsilon \wedge x < a) \vee (x > x' > x - \varepsilon \wedge x > b)$.

Para $x = a - n$, tome $\varepsilon = \left|\frac{a-x}{2}\right|$.

$\neg(\forall \varepsilon \colon \exists x' | P(x')) = \exists \varepsilon | \forall x' \colon \neg P(x')$.

Então,
$\forall x' \in \mathbb{R} | x < x' < x + \varepsilon \colon x' < a \Rightarrow x' \notin [a,b]$.