In trigonometry, the basic relationship between the sine and the cosine is given by the Pythagorean identity:[1]

$$\sin^2\theta + \cos^2\theta = 1,$$

where $\sin^2\theta$ means $(\sin\theta)^2$ and $\cos^2\theta$ means $(\cos\theta)^2$.

This can be viewed as a version of the Pythagorean theorem, and follows from the equation $x^2 + y^2 = 1$ for the unit circle. This equation can be solved for either the sine or the cosine:

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}, \cos\theta = \pm\sqrt{1-\sin^2\theta},$$

where the sign depends on the quadrant of θ.

Dividing this identity by either $\sin^2\theta$ or $\cos^2\theta$ yields the other two Pythagorean identities:

$1 + \tan^2\theta = \sec^2\theta$ and

$1 + \cos^2\theta = \csc^2\theta.$

Using these identities together with the *ratio identities*, it is possible to express any trigonometric function in terms of any other (up to a plus or minus sign):

↓ in terms of →	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta$		$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta$	$\pm\sqrt{1-\sin^2\theta}$		$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$		$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$		$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$		$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	

(Some are symmetric — in orange —, some are almost — in cyan —, some are not.)

$$(\sin\theta)^2 = \sin^2\theta \text{ e } \sqrt{\sin^2\theta} = \sin\theta.$$

Expressar $\cos^4\theta$ como cossenos e senos de primeiro grau.

$\cos^4\theta =$

$\quad \cos^2\theta\cos^2\theta =$

...

$\sec\theta = \frac{1}{\cos\theta}, \csc\theta = \frac{1}{\sin\theta}, \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ são *recíprocos* ou *identidades de razão* (ratio).

$$\arcsin = \sin^{-1}, \arccos = \cos^{-1}, \arctan = \tan^{-1}, \operatorname{arcsec} = \sec^{-1}, \operatorname{arccsc} = \csc^{-1},$$

$\text{arccot} = \cot^{-1}$ são *funções inversas* das funções trigonométricas.

Power-reduction formulas are obtained by solving the second and third versions of the cosine double angle formula. “The trigonometric power reduction identities allow us to rewrite expressions involving trigonometric terms with trigonometric terms of smaller powers. This becomes important in several applications such as integrating powers of trigonometric expressions in calculus.”[2]

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2} \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^3\theta = \frac{3\sin\theta-\sin(3\theta)}{4} \qquad \cos^3\theta = \frac{3\cos\theta+\cos(3\theta)}{4}$$

$$\sin^4\theta = \frac{3-4\cos(2\theta)+\cos(4\theta)}{8} \qquad \cos^4\theta = \frac{3+4\cos(2\theta)+\cos(4\theta)}{8}$$

$$\sin^5\theta = \frac{10\sin\theta-5\sin(3\theta)+\sin(5\theta)}{16} \qquad \cos^5\theta = \frac{10\cos\theta+5\cos(3\theta)+\cos(5\theta)}{16}$$

Half-angle formulas:

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \quad \sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Dá para fatorar: exemplo: $2\cos^2\frac{\theta}{2} = 1 + \cos\theta$.

Product-to-sum:

$$2\cos\theta\cos\phi = \cos(\theta-\phi) + \cos(\theta+\phi) \quad 2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$$

$$2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi) \quad 2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$$

$$\tan\theta\tan\phi = \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)}$$

Sum-to-product:

$$\sin\theta \pm \sin\phi = 2\sin\left(\frac{\theta\pm\phi}{2}\right)\cos\left(\frac{\theta\pm\phi}{2}\right) \quad \cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

Valores comuns:

$$\sin\pi = 0 \quad \sin\frac{\pi}{2} = 1 \quad \cos\pi = -1 \quad \cos\frac{\pi}{2} = 0 \quad \cos 2\pi = 1 \quad \sin 2\pi = 0$$

Polar. To convert (a point) from rectangular to polar coordinates, use

$x = r\cos\theta$,

$y = r\sin\theta$.

To convert from polar to rectangular coordinates, use

$r = \sqrt{x^2 + y^2}$,

$\theta = \arctan\frac{y}{x}$.

1 https://en.wikipedia.org/wiki/List_of_trigonometric_identities

2 https://brilliant.org/wiki/trigonometric-power-reduction-identities/