

# Explicação dos Exercícios de Grafo

**Exercícios B, D, E e F**

# B - Roads of NITT

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- Existem **N hotels conectados** entre si por  **$N - 1$  arestas**.
- No início, sempre existe um **caminho entre dois pares de hotéis**, mas, com o tempo, algumas rotas se tornam **inutilizáveis**.
- Dadas **Q consultas**, determinar quantos **pares de hotéis** (X, Y) **não tem uma rota entre si**.

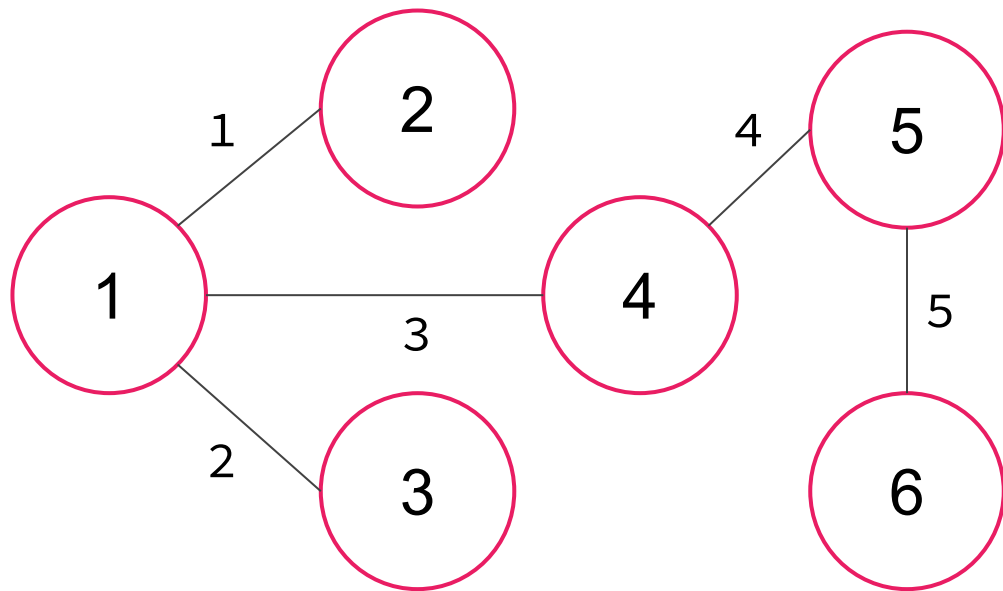
# B - Roads of NITT

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- As consultas podem ser de **2 tipos**:
  - **R X** -> **remove** a **estrada X**. É garantido que ela **não foi removida anteriormente**.
  - **Q** -> retorna o **resultado** do **número de pares** de hotéis (X, Y) que **não possuem rota entre si**.

# B - Roads of NITT

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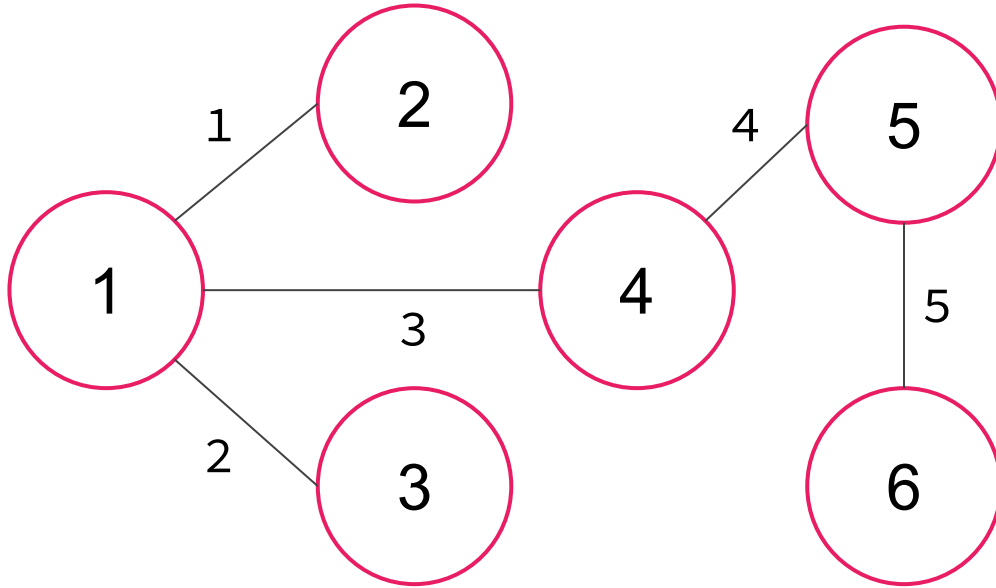
## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

Vamos passar por nossas queries e montar o grafo, ignorando as inserções das arestas removidas.

# B - Roads of NITT

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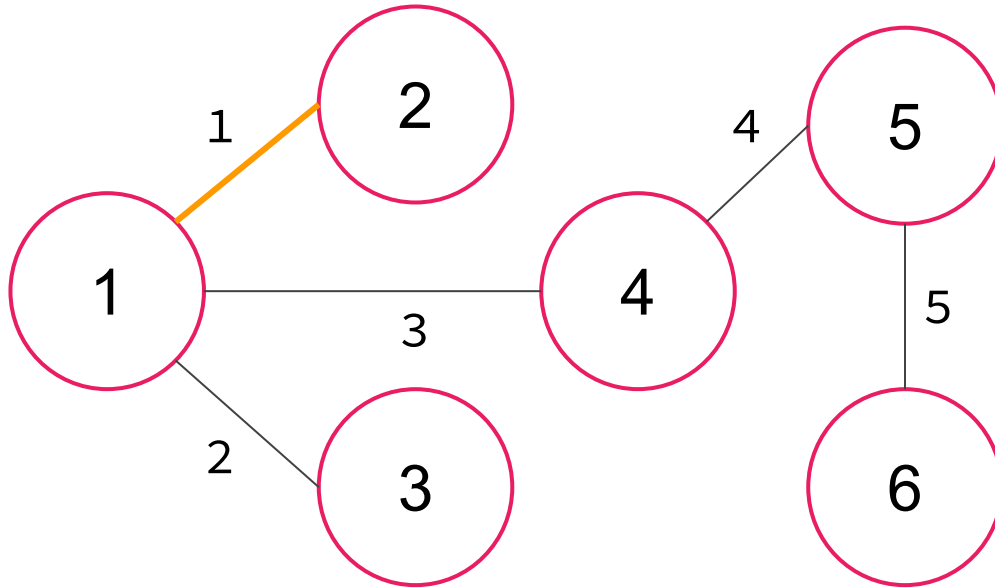
## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



# B - Roads of NITT

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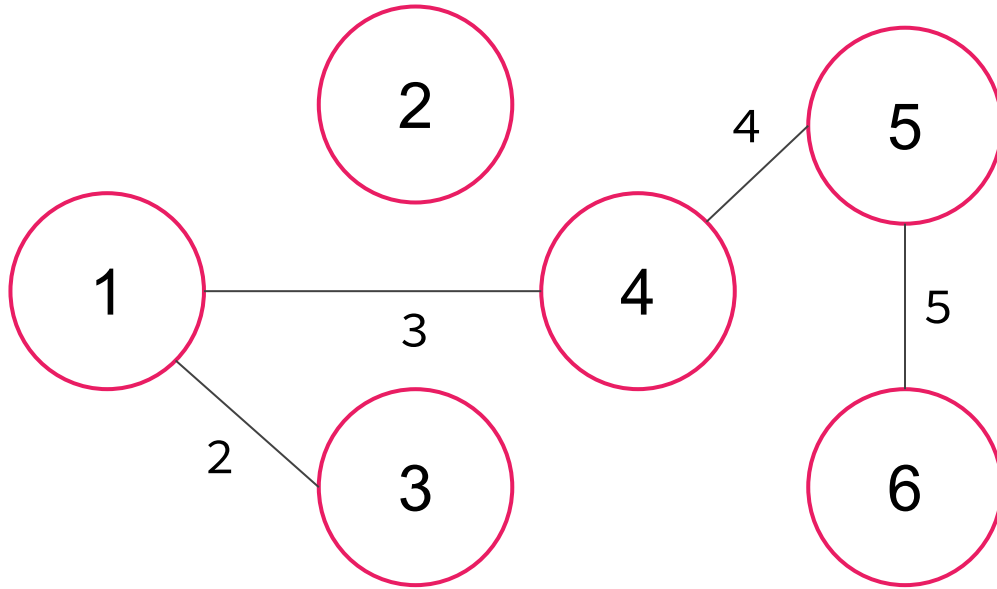
## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



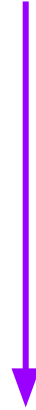
# B - Roads of NITT

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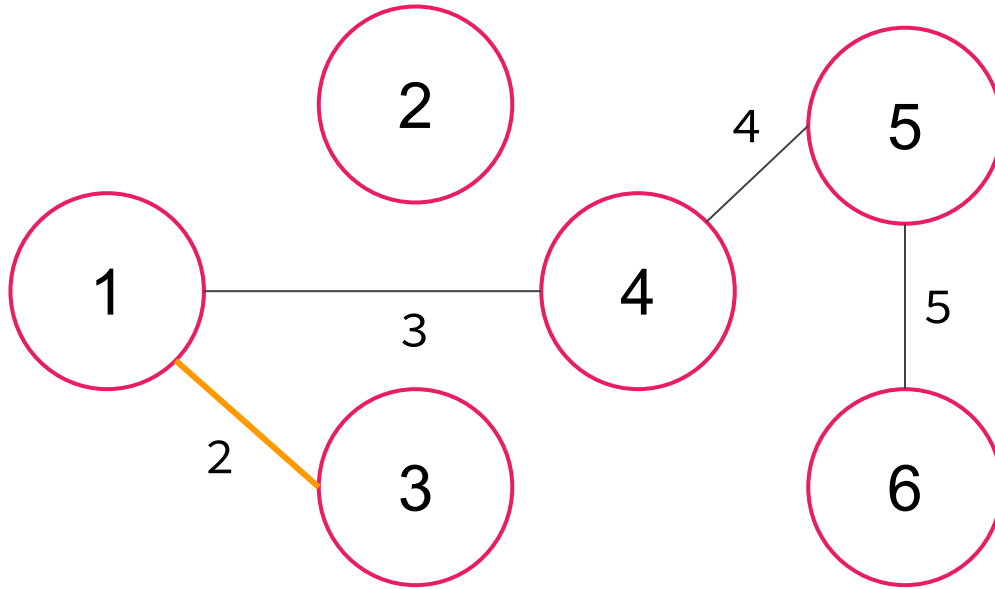
## Queries:

⇒ Q  
⇒ R 1  
⇒ **Q**  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



# B - Roads of NITT

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## Queries:

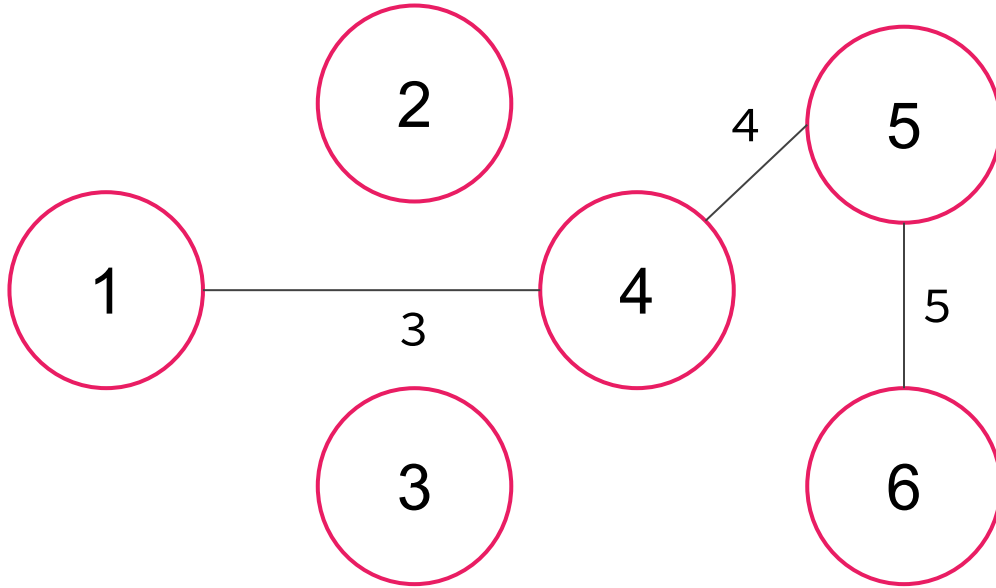
- ⇒ Q
- ⇒ R 1
- ⇒ Q
- ⇒ **R 2**
- ⇒ Q
- ⇒ R 3
- ⇒ Q





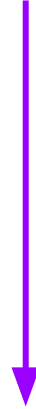
# B - Roads of NITT

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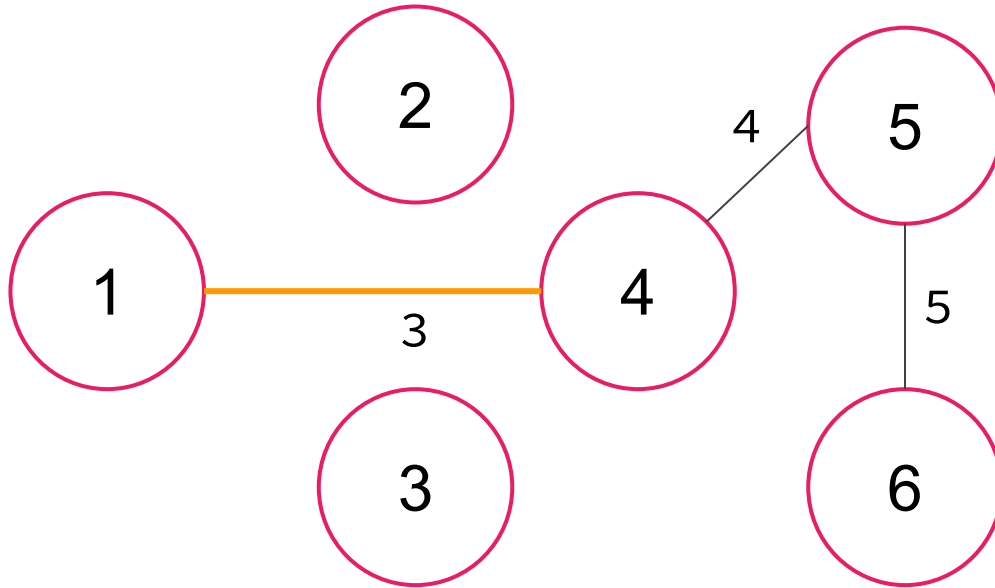
## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ **Q**  
⇒ R 3  
⇒ Q



# B - Roads of NITT

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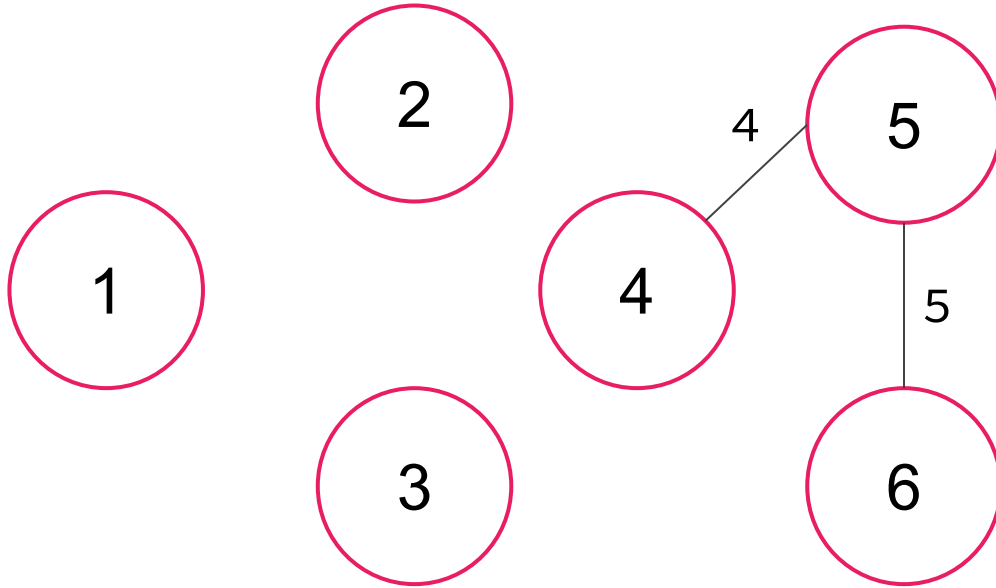
## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ **R 3**  
⇒ Q



# B - Roads of NITT

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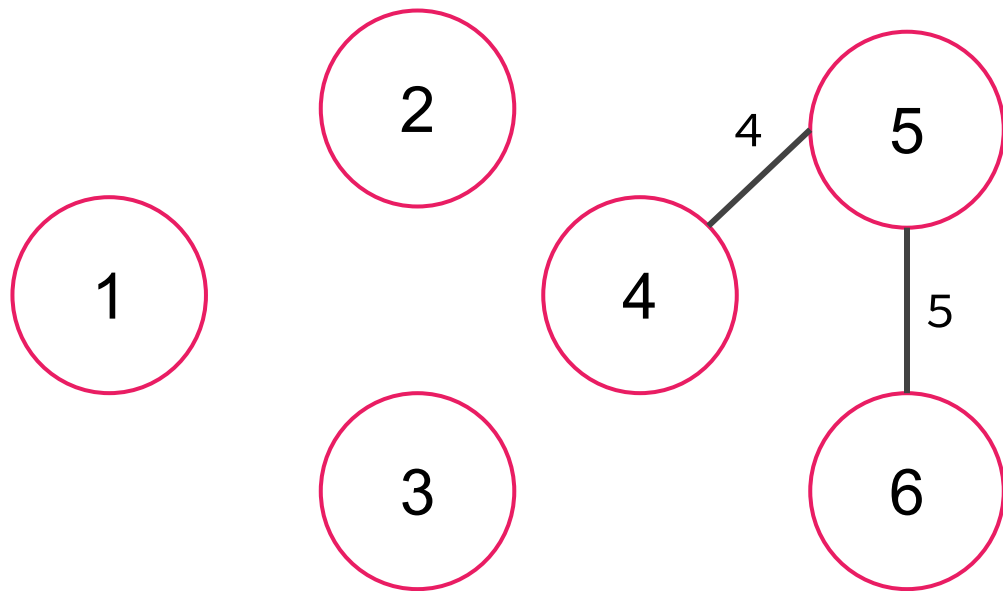
## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ **Q**



# B - Roads of NITT

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## Queries:

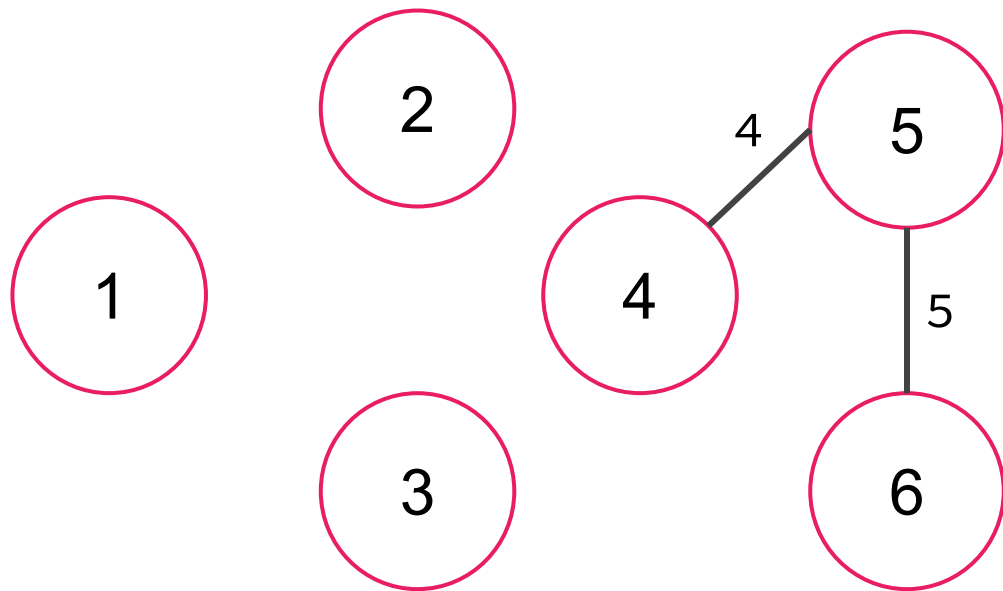
⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



As arestas que não foram removidas durante as queries permanecem e compõem nosso grafo inicial.

# B - Roads of NITT

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## Offline Queries:

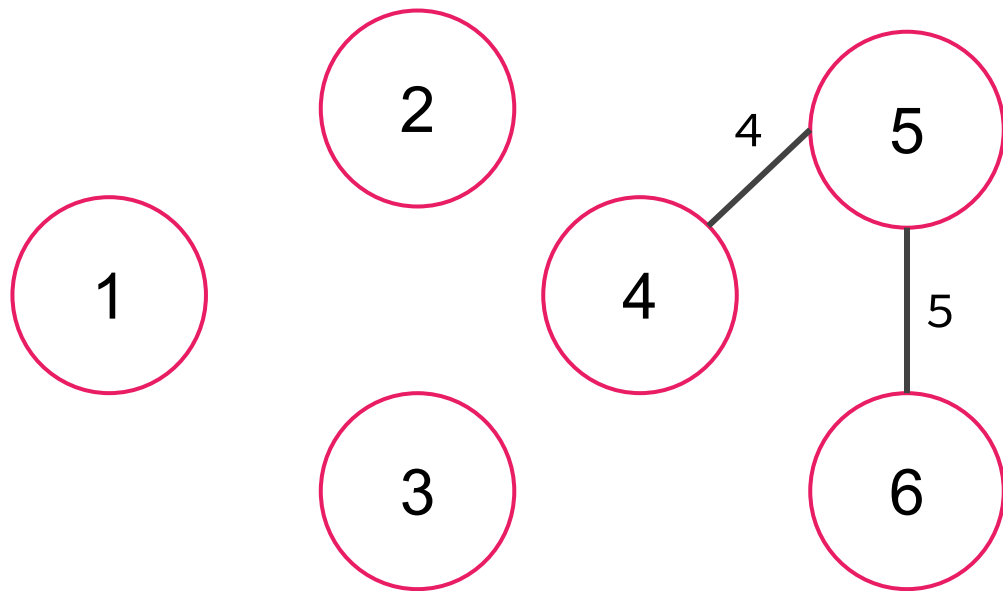
⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



Invertemos a ordem de nossas queries e, quando a consulta for de remoção, adicionamos a aresta X no grafo.

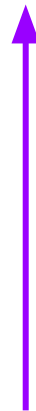
# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

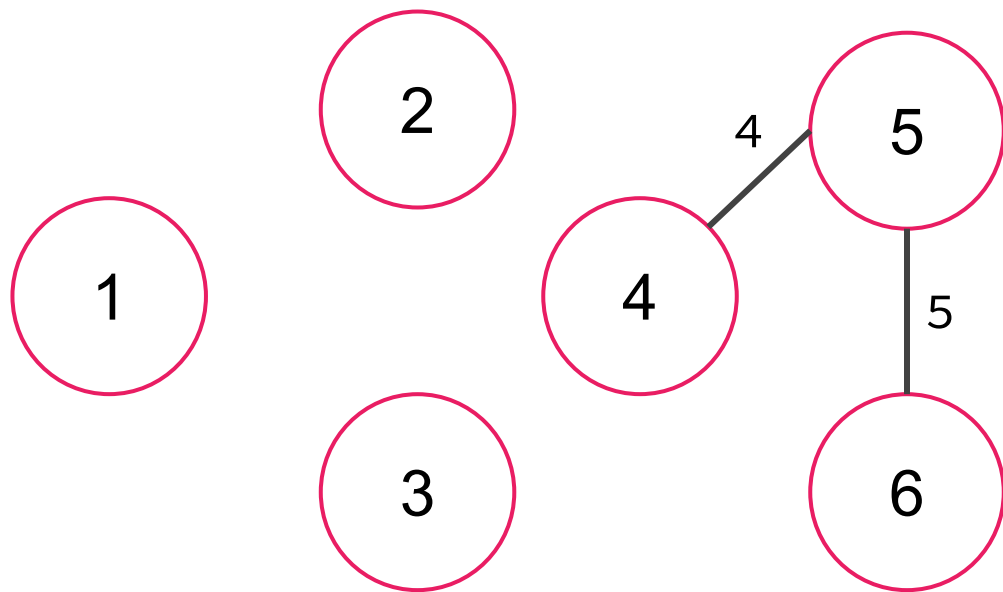


## Total de pares desconexos:

$$P = C_{N,2} - \sum_{\text{quantidade de pares conexos}}$$

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



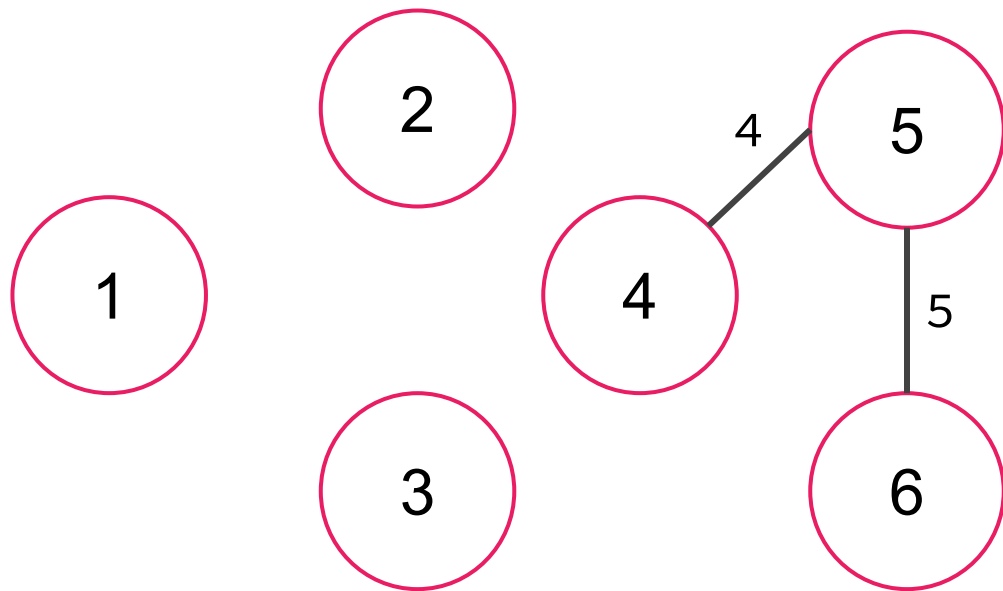
## Total de pares desconexos:

$$P = C_{N,2} - \sum (\text{tam}[U] - \text{tam}[V]),$$

sendo U e V componentes desconexas do grafo

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



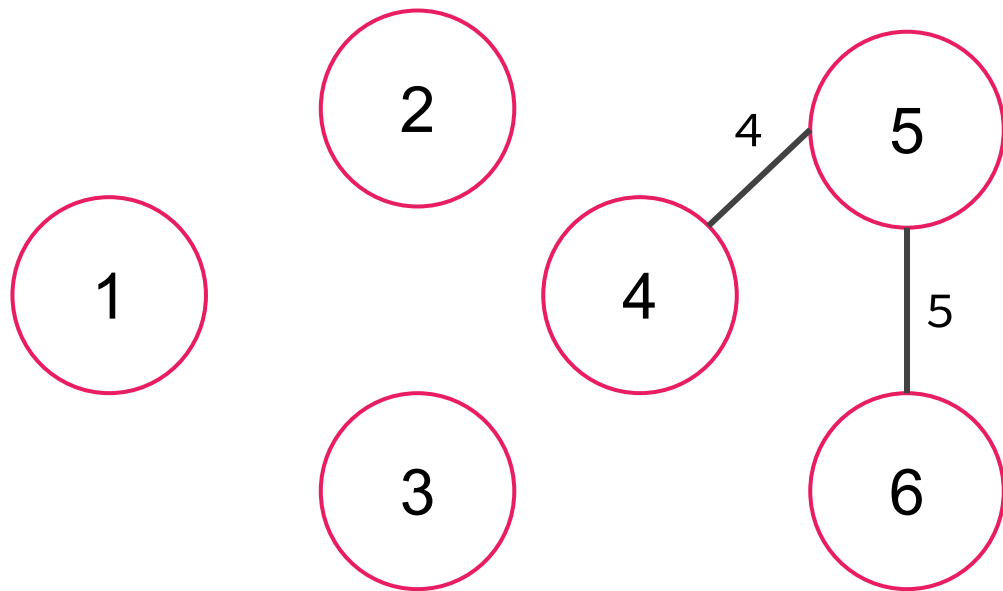
**Total de pares desconexos:**

$$P = C_{6,2} - (1 + 2)$$



# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

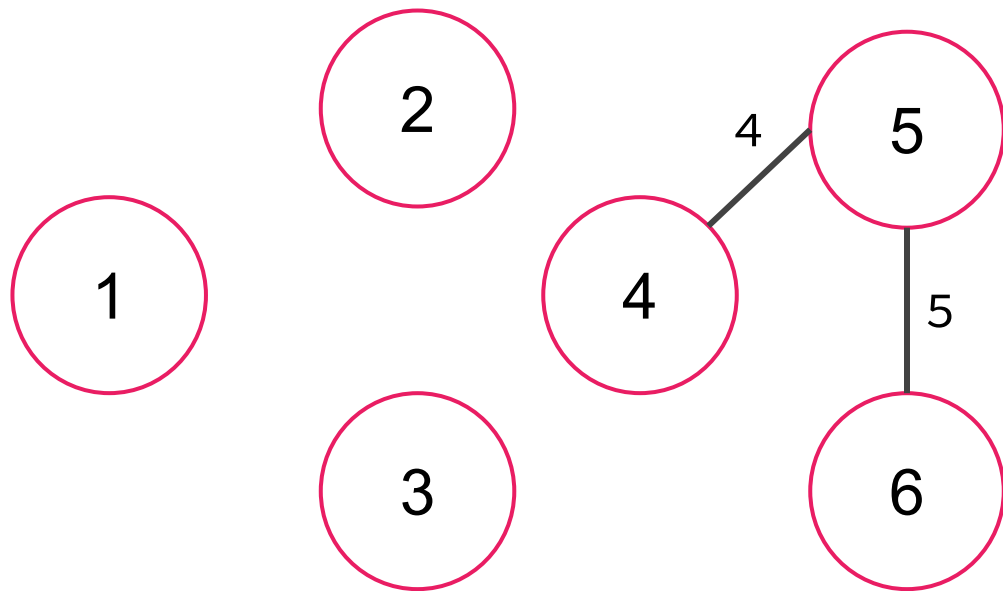


**Total de pares desconexos:**

$$P = 15 - 3 = 12$$

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

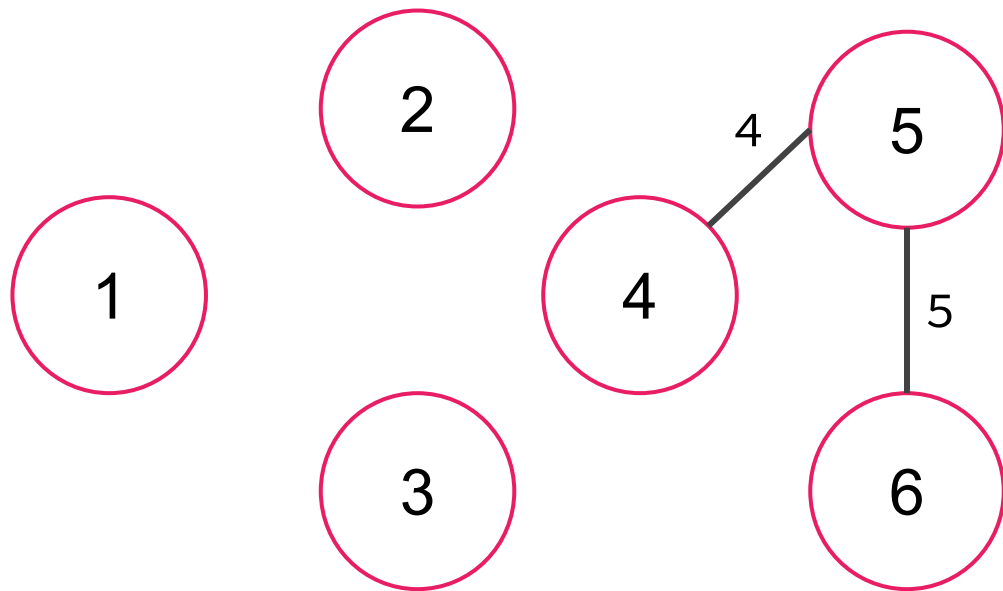


**Total de pares desconexos:**

P = 12

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

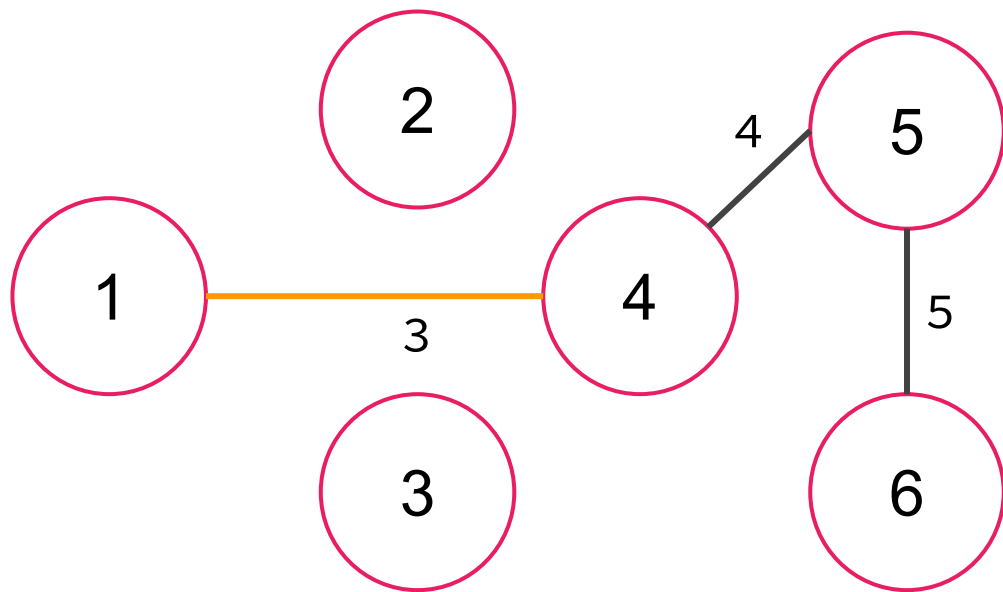


Total de pares desconexos:

P = 12

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ **R 3**  
⇒ Q

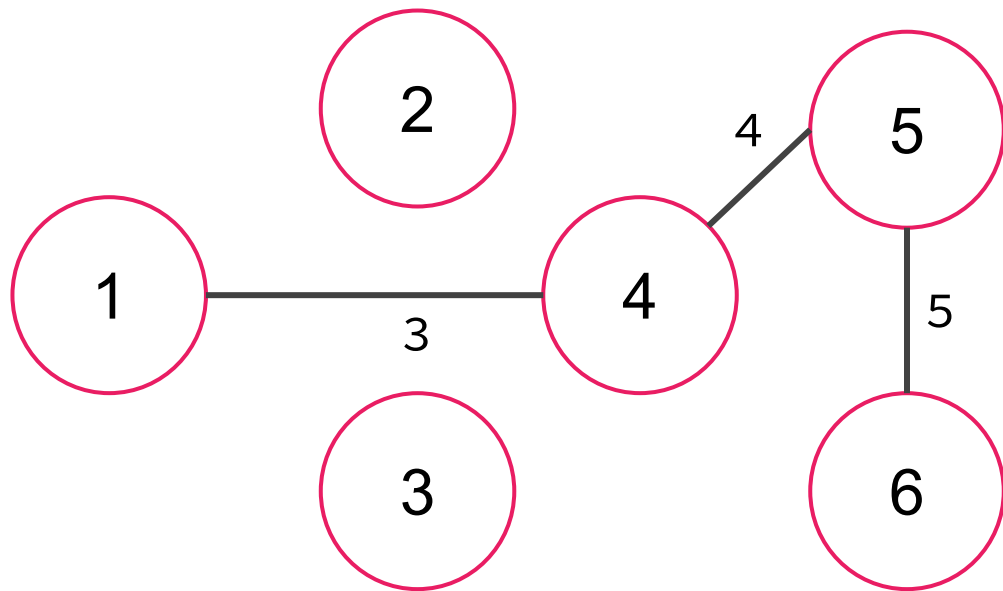


**Total de pares desconexos:**

$$P = 12 - (1 * 3) = 9$$

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ **Q**  
⇒ R 3  
⇒ Q

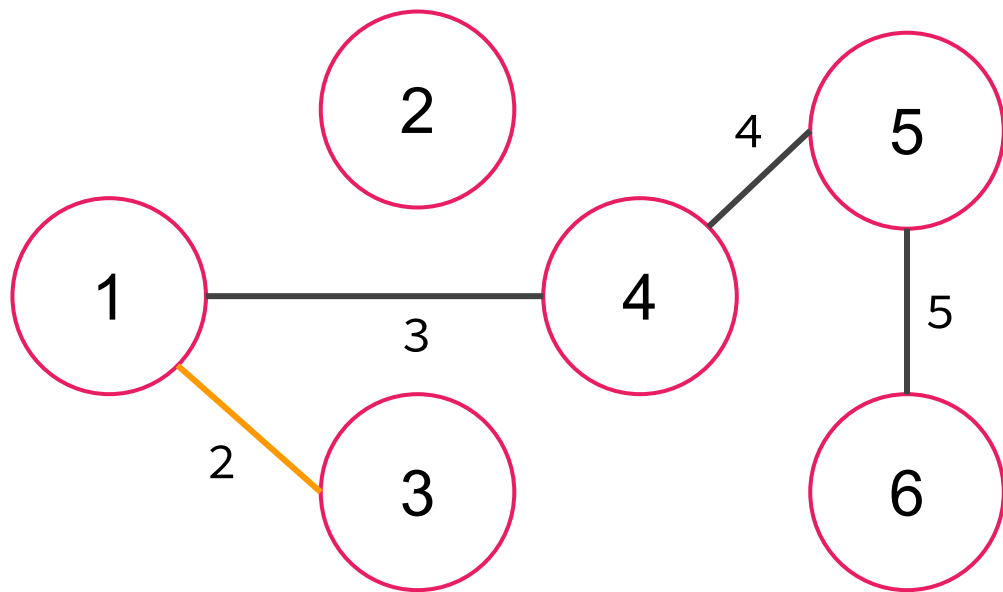


Total de pares desconexos:

**P = 9**

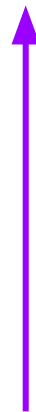
# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ **R 2**  
⇒ Q  
⇒ R 3  
⇒ Q

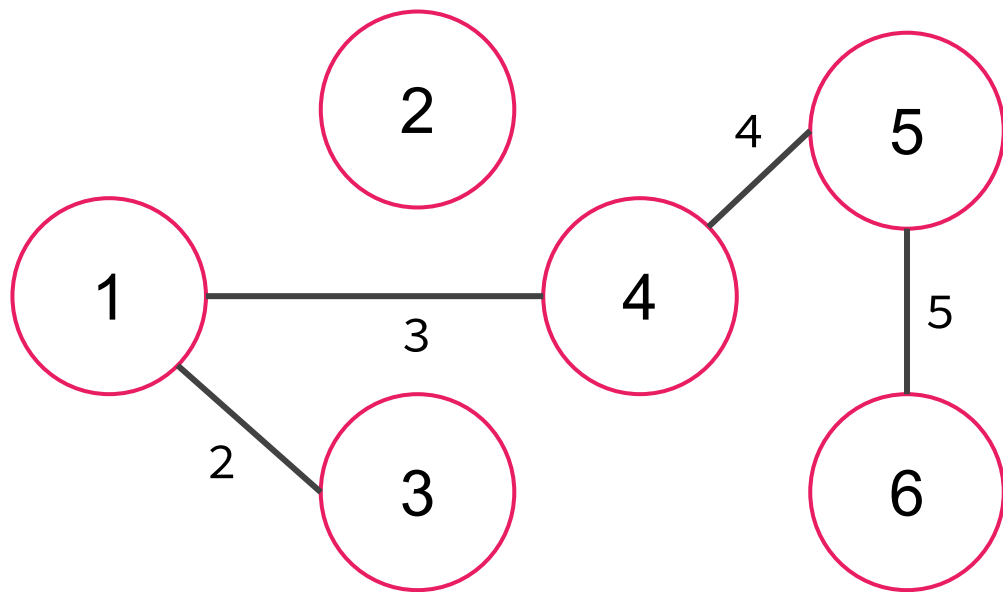


**Total de pares desconexos:**

$$P = 9 - (1 * 4) = 5$$

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ R 1  
⇒ **Q**  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

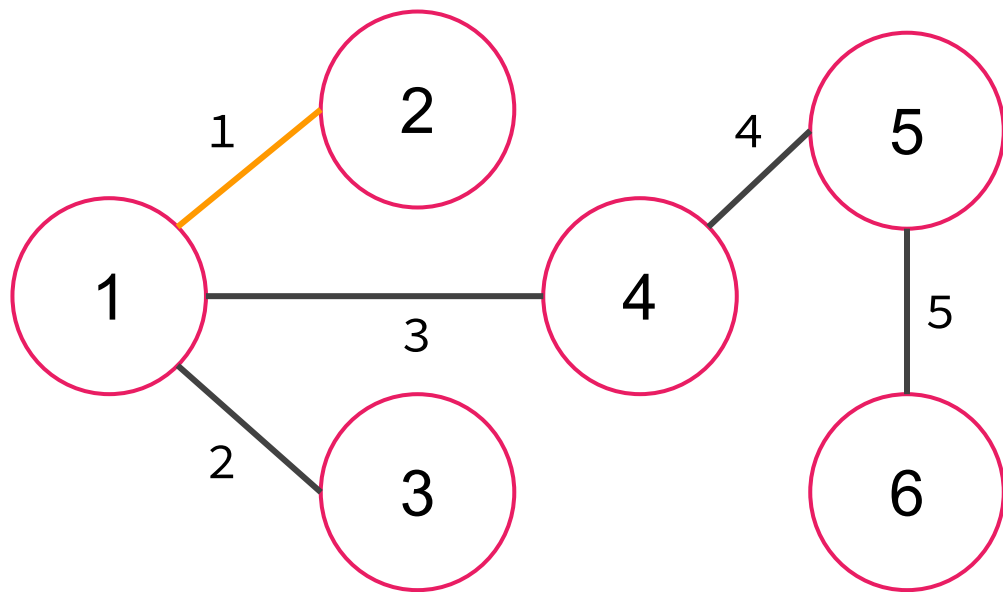


Total de pares desconexos:

**P = 5**

# B - Roads of NITT

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## Offline Queries:

⇒ Q  
⇒ **R 1**  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q



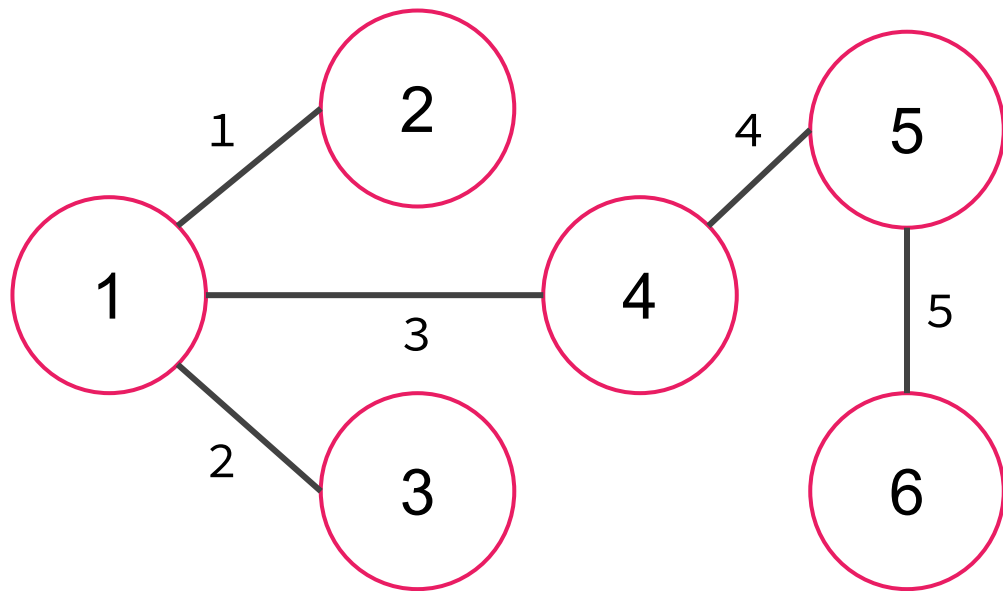
**Total de pares desconexos:**

$$P = 5 - (1 * 5) = 0$$



# B - Roads of NITT

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## Offline Queries:

⇒ Q

⇒ R 1

⇒ Q

⇒ R 2

⇒ Q

⇒ R 3

⇒ Q

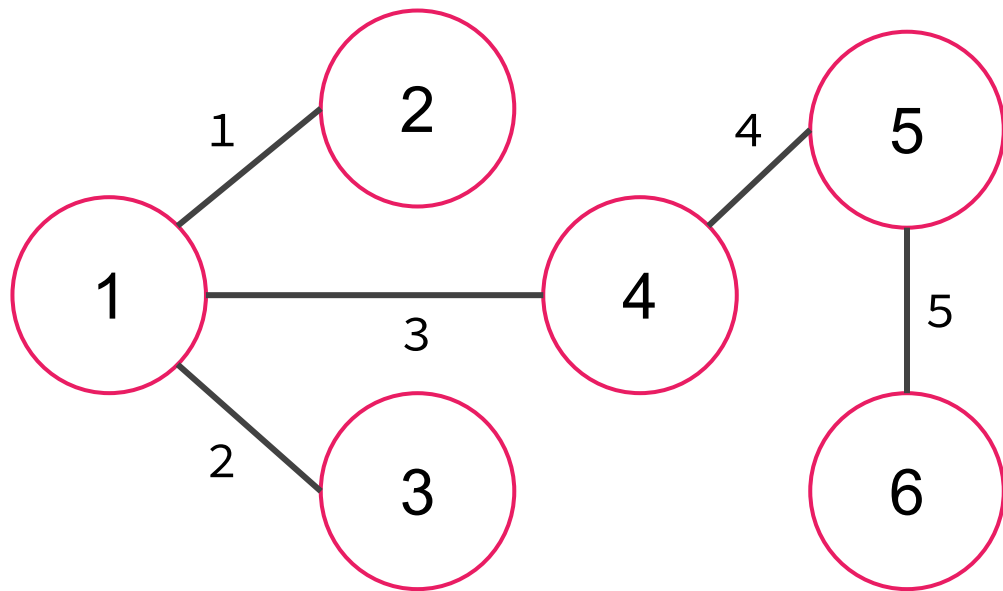


Total de pares desconexos:

P = 0

# B - Roads of NITT

---



## Queries:

⇒ Q  
⇒ R 1  
⇒ Q  
⇒ R 2  
⇒ Q  
⇒ R 3  
⇒ Q

**Total de pares desconexos:**

P = 0

# B - Roads of NITT

— — —

```
ll q;
cin >> q;

vector<bool> used(n);
vi queries(q);

for (ll i = 0; i < q; i++) {
    char op;
    ll id = -1;
    cin >> op;

    if (op == 'R') {
        cin >> id;
        used[id] = true;
    }

    queries[i] = id;
}
```

```
ll conn = 0;

for (ll i = 1; i < n; i++) {
    if (!used[i]) {
        int u, v;
        tie(u, v) = edges[i];
        u = find(u);
        v = find(v);

        if (u != v) {
            conn += len[u] * len[v];
            unite(u, v);
        }
    }
}

for (int i = 1; i <= n; i++) {
    find(i);
}
```

# B - Roads of NITT

— — —

```
ll tot = n * (n - 1) / 2 - conn;
stack<ll> ans;

for (ll i = q - 1; i >= 0; i--) {
    ll id = queries[i];

    if (id != -1) {
        ll u, v;
        tie(u, v) = edges[id];
        u = find(u);
        v = find(v);

        if (u != v) {
            tot -= len[u] * len[v];
            unite(u, v);
        }
    } else {
        ans.push(tot);
    }
}
```

```
while (!ans.empty()) {
    cout << ans.top() << "\n";
    ans.pop();
}

cout << "\n";
```

# B - Roads of NITT

— — —

```
ll tot = n * (n - 1) / 2 - conn;
stack<ll> ans;

for (ll i = q - 1; i >= 0; i--) {
    ll id = queries[i];

    if (id != -1) {
        ll u, v;
        tie(u, v) = edges[id];
        u = find(u);
        v = find(v);

        if (u != v) {
            tot -= len[u] * len[v];
            unite(u, v);
        }
    } else {
        ans.push(tot);
    }
}
```

whi

}

cc





Programação Competitiva Unesp Bauru  
apresenta

## Arissa “Tokidebug” Yoshida

Professora, programadora, musicista, futeboleira, 42

### D – Gifts

*“Os melhores presentes não são os mais caros,  
nem aqueles que combinam com nossa  
preferência, mas sim aqueles que dão  
balõezinhos!”*

*- Xilsu personificando a TokiDeBalloon*

**11/Julho** ◦ **Local:** aqui ◦ **Horário:** agora

**Valor:** R\$ 100.000.000,00  
(Ingressos a venda com o **Paiola**)



# E - Spanning Tree Fraction

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- Temos um **grafo conexo  $G$**  com  **$N$  vértices**, indexados de  $0$  até  $N - 1$ , e  **$M$  arestas**.
- Cada aresta desse grafo é uma **quádrupla  $(U, V, a, b)$** , sendo  **$U$  o vértice de origem**,  **$V$  o vértice de destino**,  **$a$  e  $b$  pesos** da nossa aresta.

# E - Spanning Tree Fraction

— — —

- Achar uma **árvore geradora**  $T$  que contém um **subconjunto das**  $M$  **arestas** e **maximize** a expressão  $\sum_{i \in T} a_i / \sum_{i \in T} b_i$ , sendo  $i$  o **índice** da **aresta inserida** em  $T$ .
- Escrever o termo resultante no formato de uma fração irredutível  $p / q$ .



# E - Spanning Tree Fraction

— — —

```
bool cmp(tuple<ii, ll, ll> a, tuple<ii, ll, ll> b) {
    ii p, q;
    p = get<0>(a);
    q = get<0>(b);

    return (p.first - p.second * x) > (q.first - q.second * x);
}

int main() {
    ll n, m;
    cin >> n >> m;
    parent = len = vi(n + 1);

    for (ll i = 0; i < m; i++) {
        ll u, v, a, b;
        cin >> u >> v >> a >> b;
        add_edge(u, v, a, b);
    }
```

```
ll p, q, gcd;
double left = 0., right = 1e7;

while (fabs(right - left) >= EPSILON) {
    x = (left + right) / 2;

    for (ll i = 0; i <= n; i++) {
        parent[i] = i;
        len[i] = 1;
    }

    p = q = 0;

    sort(begin(edges), end(edges), cmp);
    kruskal(p, q);

    if (p >= q * x) {
        left = x;
    } else {
        right = x;
    }
}
```

# E - Spanning Tree Fraction

— — —

```
gcd = __gcd(p, q);  
p /= gcd;  
q /= gcd;  
  
cout << p << "/" << q << "\n";  
  
return 0;  
}
```

# E - Spanning Tree Fraction

— — —

```
gcd = __gcd(p, q);  
p /= gcd;  
q /= gcd;  
  
cout << p << "/" << q << "\n";  
  
return 0;  
}
```



# F - Airports

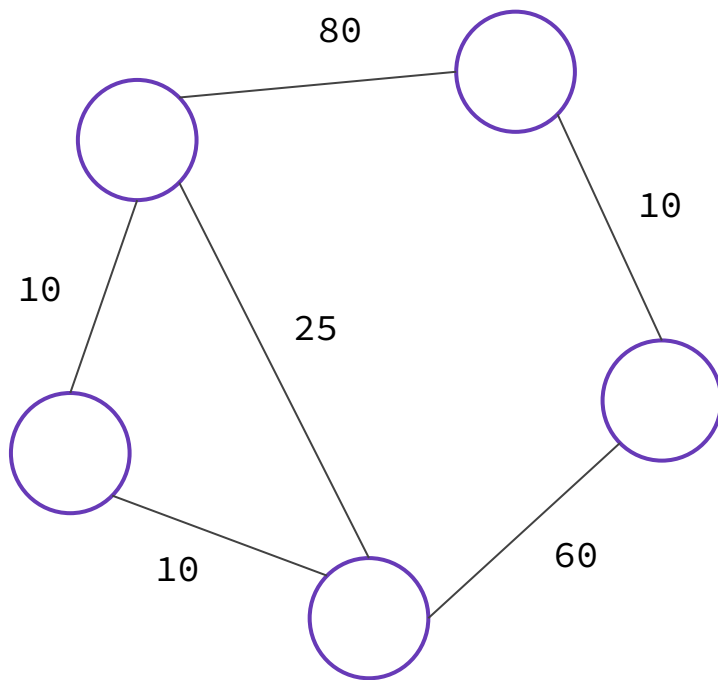
---

- Você pode colocar aeroportos em qualquer cidade.
- São dadas as possíveis pistas que ligam as cidades.
- Objetivo: Garantir que cada cidade tenha acesso a um aeroporto.

# F - Airports

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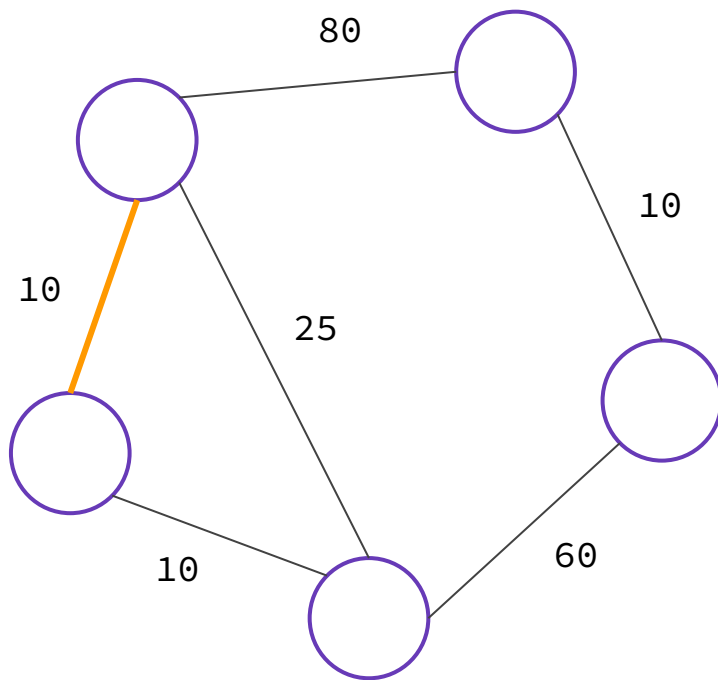
Custo Aeroporto: 50



# F - Airports

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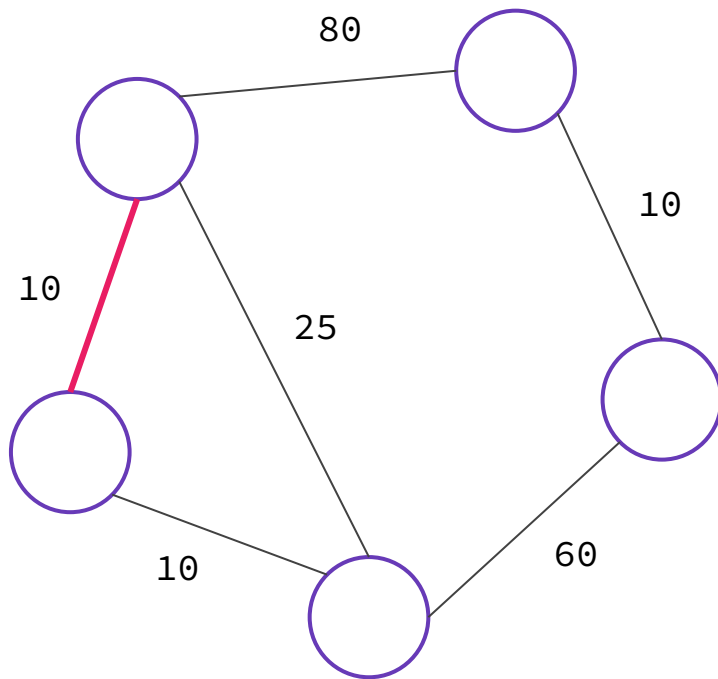
Custo Aeroporto: 50



# F - Airports

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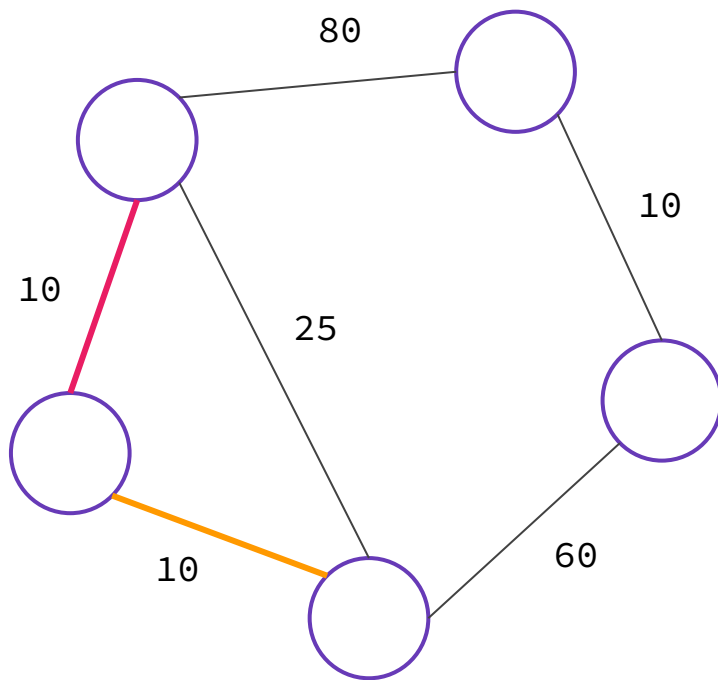
Custo Aeroporto: 50



# F - Airports

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Custo Aeroporto: 50

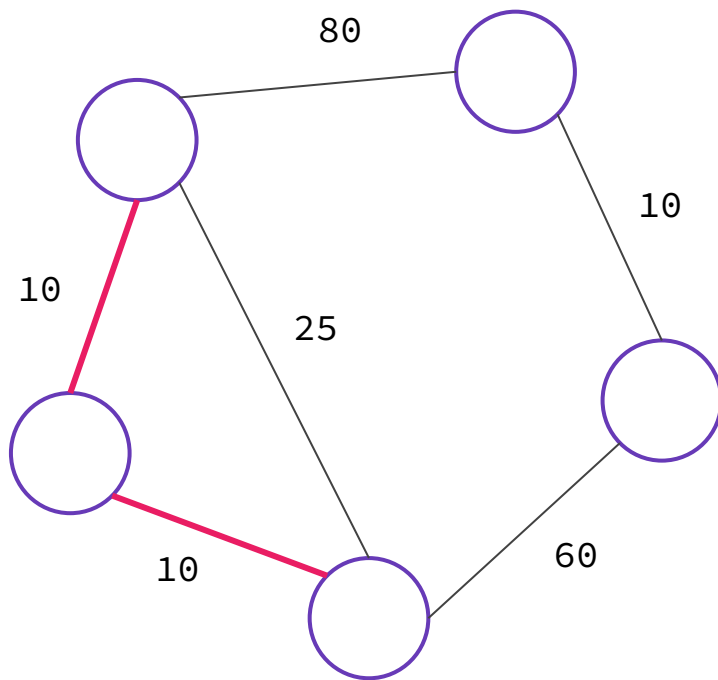




# F - Airports

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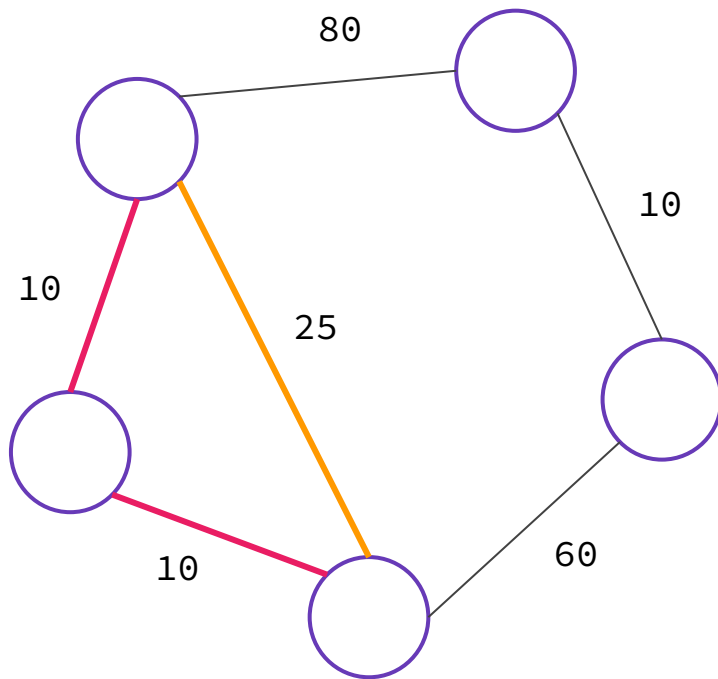
Custo Aeroporto: 50



# F - Airports

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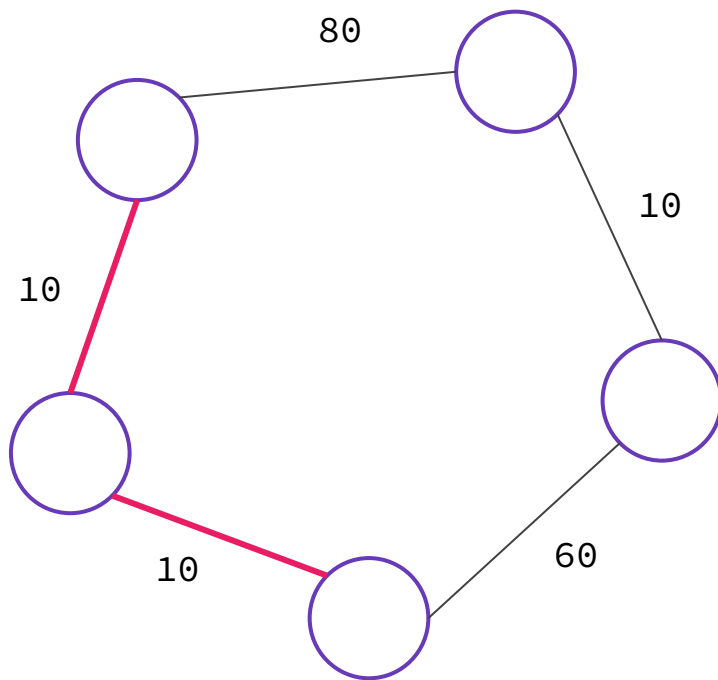
Custo Aeroporto: 50



# F - Airports

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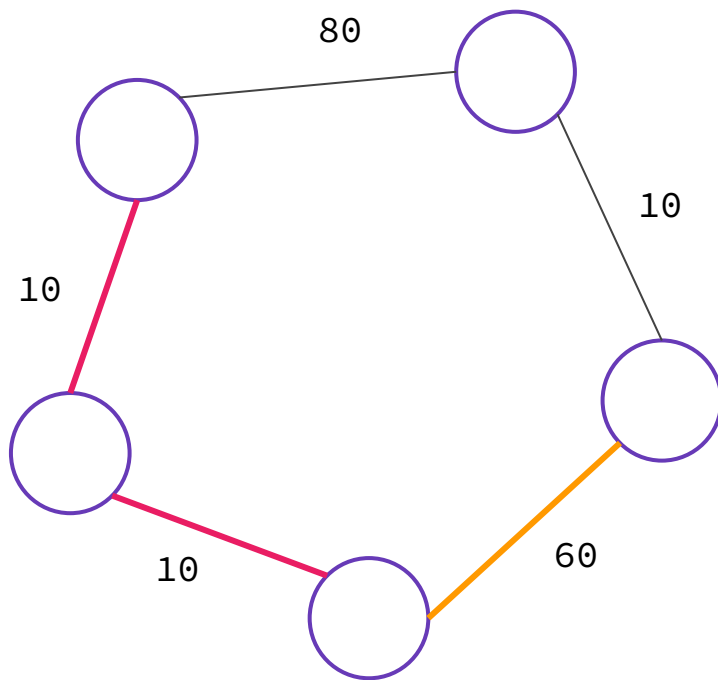
Custo Aeroporto: 50



# F - Airports

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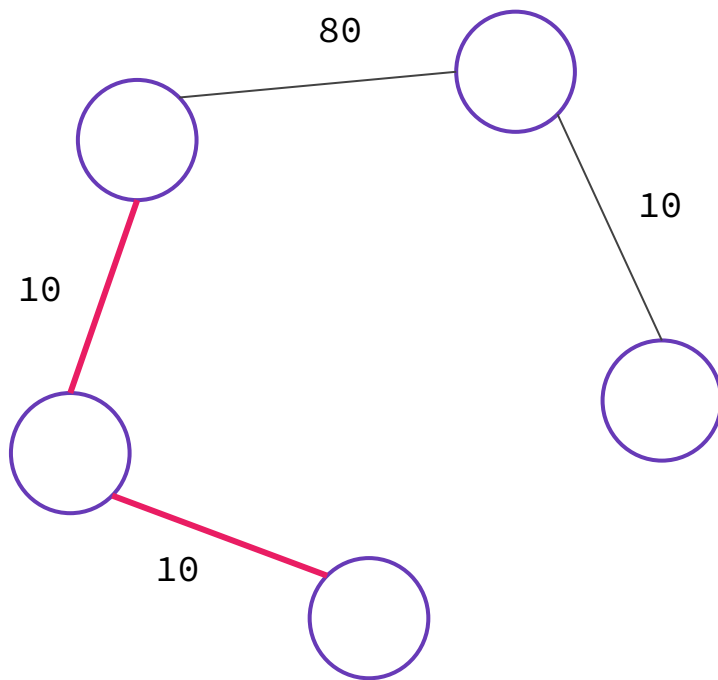
Custo Aeroporto: 50



# F - Airports

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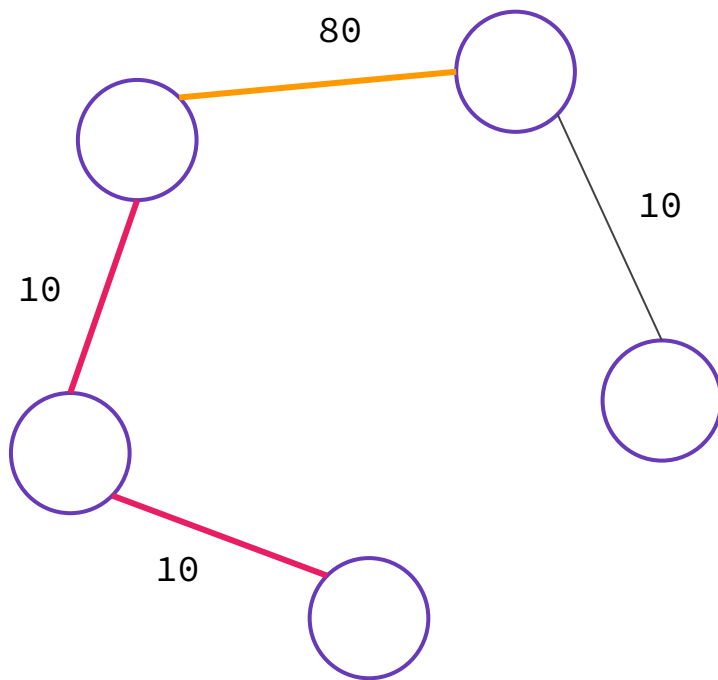
Custo Aeroporto: 50



# F - Airports

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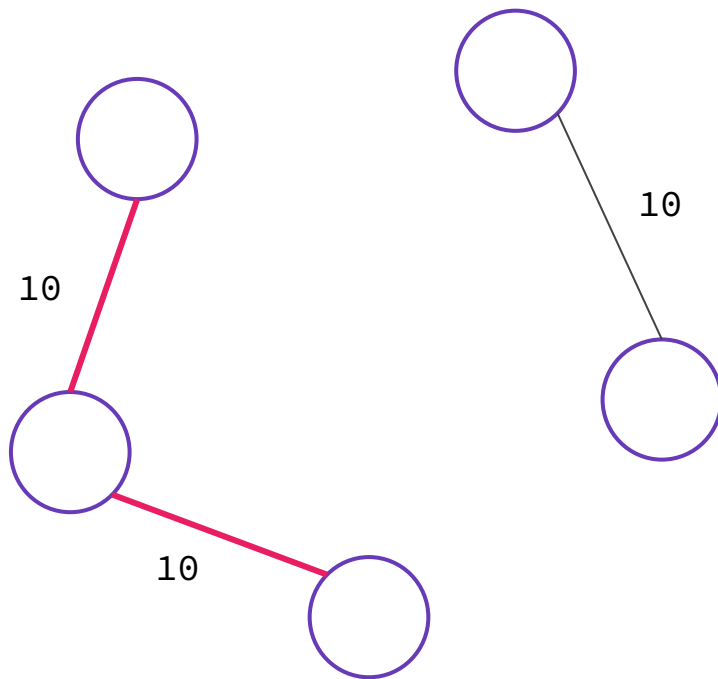
Custo Aeroporto: 50



# F - Airports

— — —

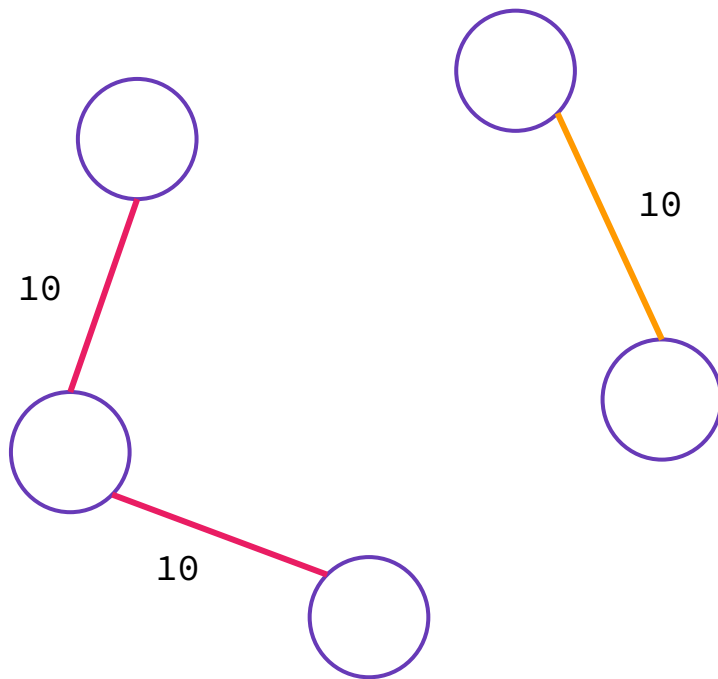
Custo Aeroporto: 50



# F - Airports

— — —

Custo Aeroporto: 50

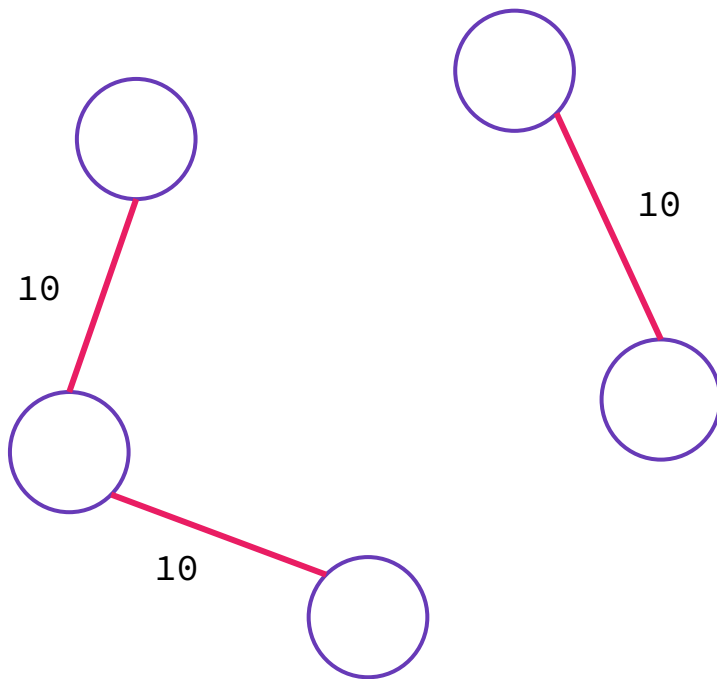




# F - Airports

— — —

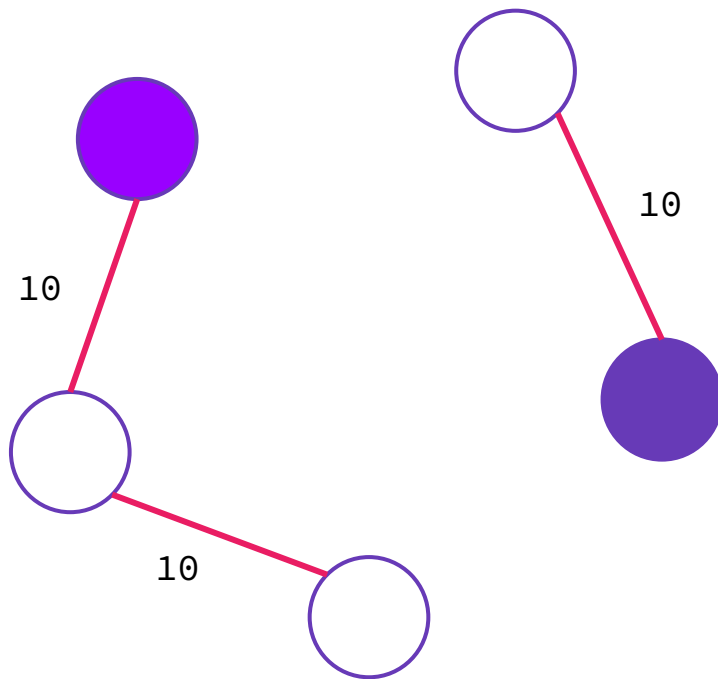
Custo Aeroporto: 50



# F - Airports

— — —

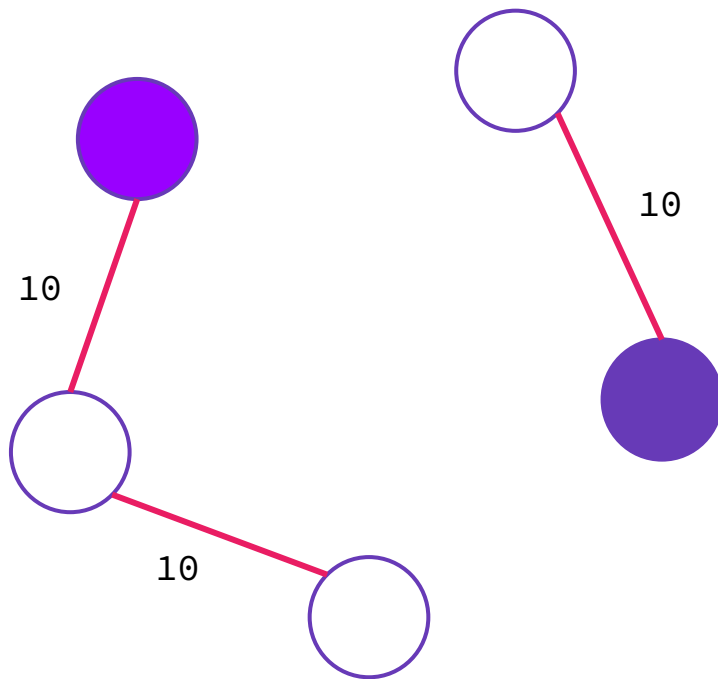
Custo Aeroporto: 50



# F - Airports

— — —

Custo Aeroporto: 50



Custo Total:  $10 + 10 + 10 + 50 + 50 = 130$

```
for(int i = 0; i < m; i++){
    cin >> a >> b >> c;
    arestas.push_back({c, a, b});
}

sort(arestas.begin(), arestas.end());

ll custo = 0;

ll ind = lower_bound(arestas.begin(), arestas.end(), make_tuple(aer, 1e17, 1e17))
- arestas.begin();

for(int j = 0; j < ind; j++){
    tie(c, a, b) = arestas[j];
    if(c == aer)continue;
    if(find(a) != find(b)){
        merge(a, b);
        custo += c;
    }
}
```

```
for(int i = 0; i < m; i++){
    cin >> a >> b >> c;
    arestas.push_back({c, a, b});
}

sort(arestas.begin(), arestas.end());

ll custo = 0;
ll ind = lower_bound(arestas.begin(), arestas.end()
- arestas.begin());

for(int j = 0; j < ind; j++){
    tie(c, a, b) = arestas[j];
    if(c == aer)continue;
    if(find(a) != find(b)){
        merge(a, b);
        custo += c;
    }
}
```

