# Linear Algebra (MAT-2233)
## Gaussian Elimination Program By Pedro Palacios

① consider the matrix $\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$

② Find the first nonzero for every row & save location

column: 0 1 2 3

Row 0: 0 0 **6** 2
Row 1: 0 **1** 4 -7
Row 2: **1** **1** 10 -5

→ [(0,2), (1,1), (2,0)]  (row, column)

③ Sort saved locations by column

[(2,0), (1,1), (0,2)] } saved pivots

④ Save the first location as first pivot location → [(2,0), ~~(1,1), (0,2)~~]

⑤ Save pivot

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 6 | 2 |
| 1 | 0 | 1 | 4 | -7 |
| 2 | **1** | 1 | 10 | -5 |

⑥ Look below pivot to the right for nonzero pivots →

⑦ Generate expected pivots from $\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$ ⇒ [(0,0), (1,1), (2,2)]

⑧ If expected pivots ≠ saved pivots, then row exchange at expected pivots

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 6 | 2 |
| 1 | 0 | 1 | 4 | -7 |
| 2 | 1 | 1 | 10 | -5 |

$R_0 = R_2$

$R_2 = R_{0\,previous}$

→

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | 1 | 10 | -5 |
| 1 | 0 | 1 | 4 | -7 |
| 2 | 0 | 0 | 6 | 2 |

(9) Find the first nonzero for every row & save location

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

→ [(0,0), (1,1), (2,2)]

(10) Sort locations by column

[(0,0), (1,1), (2,2)]

(11) Save the first location as the first pivot location

[(0,0), ~~(1,1), (2,2)~~]

(12) Save pivot location & look below first pivot to the right for nonzero pivots

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

(13) Save nonzero pivot locations

[(0,0), (1,1), (2,2)]

(14) Check RREF by checking if # of numbers in each pivot column ≤ 1

(15) If not RREF, continue

(16) For every pivot location, scale the value by $\frac{1}{\text{pivot}}$ AND look for nonzeros below and above pivot for row replacement

First Pivot

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array} \rightarrow \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

second pivot

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array} \xrightarrow{R_0 - R_1} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

Third Pivot

SCALE

$$\begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 6 & 2 \\ 1 & 0 & 1 & 4 & -7 \\ 2 & 0 & 0 & 6 & 2 \end{matrix} \xrightarrow{R_2 \cdot \frac{1}{6}} \begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 6 & 2 \\ 1 & 0 & 1 & 4 & -7 \\ 2 & 0 & 0 & 1 & 1/3 \end{matrix}$$

REPLACE

$$\begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 6 & 2 \\ 1 & 0 & 1 & 4 & -7 \\ 2 & 0 & 0 & 1 & 1/3 \end{matrix} \begin{matrix} R_0 - 6R_2 \\ R_1 - 4R_2 \end{matrix} \rightarrow \begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -8.33... \\ 2 & 0 & 0 & 1 & 1/3 \end{matrix}$$

(17) Update new pivot locations & verify consistency

(18) Check if matrix is in RREF

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -8.33... \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$ ✓

(19) Check if there's an infinite # of solutions

↓ No infinite # of solutions

(20) Output final matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -8.33... \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$