Linear Algebra (MAT-2233)
Gaussian Elimination Program By Pedro Palacios

① consider the matrix $\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$

② Find the first nonzero for every row & save location

column: 0 1 2 3 — Row: 0, 1, 2

$\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$

$[(0,2),(1,1),(2,0)]$ (row, column)

③ Sort saved locations by column

$[(2,0),(1,1),(0,2)]$

④ Save the first location as first pivot location

$[(2,0),(1,1),(0,2)]$

⑤ Save pivot

$\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$

⑥ Look below pivot to the right for nonzero pivots

⑦ Generate expected pivots from $\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$

⑧ If expected pivots ≠ saved pivots, then row exchange at expected pivots

$\begin{bmatrix} 0 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 1 & 1 & 10 & -5 \end{bmatrix}$ $R_0 = R_2$, $R_2 = R_{0\,previous}$ $\longrightarrow$ $\begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix}$

(9) Find the first nonzero for every row & save location

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

→ [(0,0), (1,1), (2,2)]

(10) Sort locations by column

[(0,0), (1,1), (2,2)]

(11) Save the first location as the first pivot location

[(0,0), ~~(1,1), (2,2)~~]

(12) Save pivot location & look below first pivot to the right for nonzero pivots

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

(13) Save nonzero pivot locations

[(0,0), (1,1), (2,2)]

(14) Check RREF by checking if # of numbers in each pivot column ≤ 1

(15) If not RREF, continue

(16) For every pivot location, scale the value by $\frac{1}{\text{pivot}}$ AND look for nonzeros below and above pivot for row replacement

First Pivot

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array} \rightarrow \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

second pivot

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 10 & -5 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array} \xrightarrow{R_0 - R_1} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 6 & 2 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 6 & 2 \end{bmatrix} \end{array}$$

Third Pivot

SCALE

$$\begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 6 & 2 \\ 1 & 0 & 1 & 4 & -7 \\ 2 & 0 & 0 & 6 & 2 \end{matrix} \xrightarrow{R_2 \cdot \frac{1}{6}} \begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 6 & 2 \\ 1 & 0 & 1 & 4 & -7 \\ 2 & 0 & 0 & 1 & 1/3 \end{matrix}$$

REPLACE

$$\begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 6 & 2 \\ 1 & 0 & 1 & 4 & -7 \\ 2 & 0 & 0 & 1 & 1/3 \end{matrix} \begin{matrix} R_0 - 6R_2 \\ R_1 - 4R_2 \end{matrix} \rightarrow \begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -8.33... \\ 2 & 0 & 0 & 1 & 1/3 \end{matrix}$$

(17) Update new pivot locations & verify consistency

(18) Check if matrix is in RREF

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -8.33... \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$ ✓

(19) Check if there's an infinite # of solutions

↓ No infinite # of solutions

(20) Output final matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -8.33... \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$