Métodos Numéricos

Clase muestra:
Interpolación

Sistemas computacionales

Pedro Fernando
Flores Palmeros

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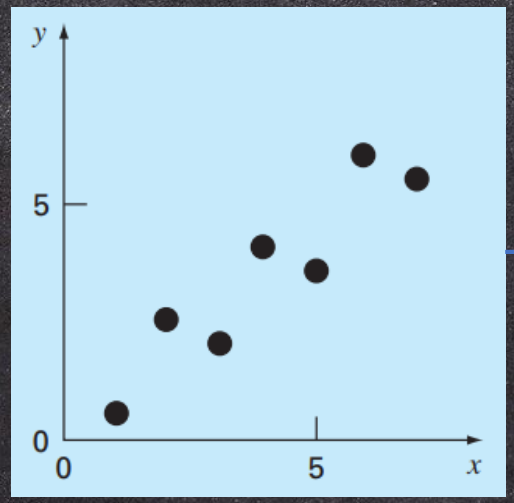
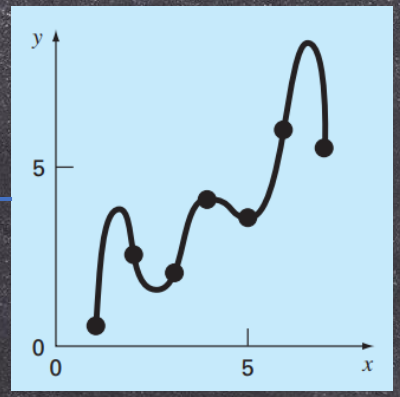
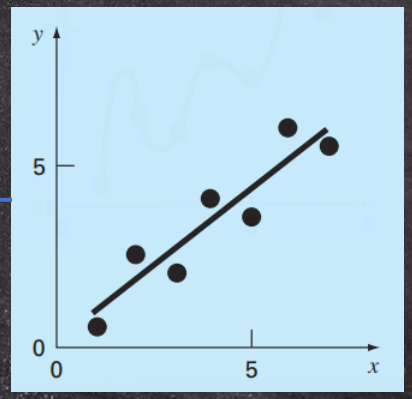
Introducción







Interpolación Polinomial

Puntos	0	1	2	3
T [C]	56.5	113	181.0	214.5
P[atm]	1	5	20	40

¿Qué temperatura se necesita cuando se tiene una presión de 10 atm?

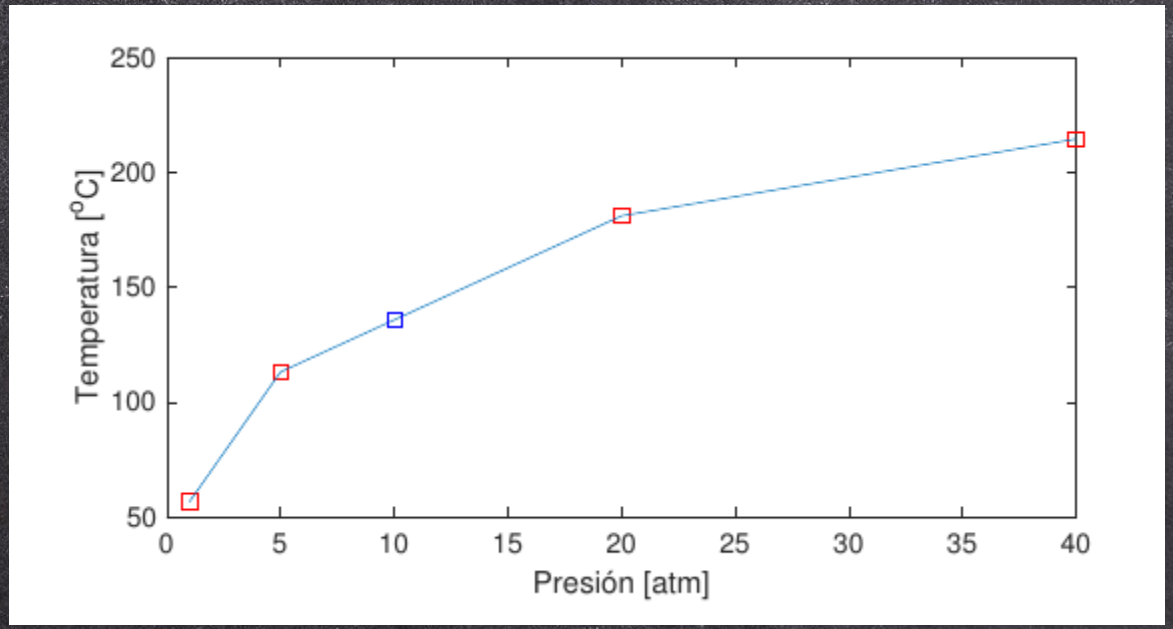
$$\hat{y} = a_0 + a_1 x$$

$$113 = a_0 + 5a_1$$
$$181 = a_0 + 20a_1$$

$$a_0 = 90.33$$
$$a_1 = 4.53$$

$$y = 90.33 + 4.53x$$

$$y = 90.33 + 4.53x$$
$$= 90.33 + 4.53(10)$$
$$= 135.63$$



Interpolación de Segundo Orden

¿Qué temperatura se necesita cuando se tiene una presión de 10 atm?

$$\hat{y} = a_0 + a_1 x + a_2 x^2$$

$$\begin{aligned} 56.5 &= a_0 + a_1(1) + a_2(1)^2 \\ 113 &= a_0 + a_1(5) + a_2(5)(5)^2 \\ 181 &= a_0 + a_1(20) + a_2(20)^2 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 25 \\ 1 & 20 & 400 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 56.5 \\ 113 \\ 181 \end{bmatrix}$$

$$\begin{aligned} a_0 &= 39.8509 \\ a_1 &= 17.1539 \\ a_2 &= -0.5048 \end{aligned}$$

$$\hat{y} = 39.8509 + 17.1539x - 0.5048x^2$$

$$\begin{aligned} \hat{y} &= 39.8509 + 17.1539x - 0.5048x^2 \\ &= 39.8509 + 1715.39 - 50.48 \\ &= 156.5899 \end{aligned}$$

Interpolación de Lagrange

Polinomio de Lagrange de primer orden

Puntos	0	1	2	3
T [C]	56.5	113	181.0	214.5
P[atm]	1	5	20	40

$$P(x) = a_0(x - x_1) + a_1(x - x_0)$$

$$\begin{aligned} P(x_0) &= a_0(x_0 - x_1) + a_1(x_0 - x_0) \\ &= a_0(x_0 - x_1) \end{aligned}$$

$$a_0 = \frac{P(x_0)}{x_0 - x_1} = \frac{f(x_0)}{x_0 - x_1}$$

$$P(x_1) = a_0(x_1 - x_1) + a_1(x_1 - x_0)$$
$$= a_1(x_1 - x_0)$$

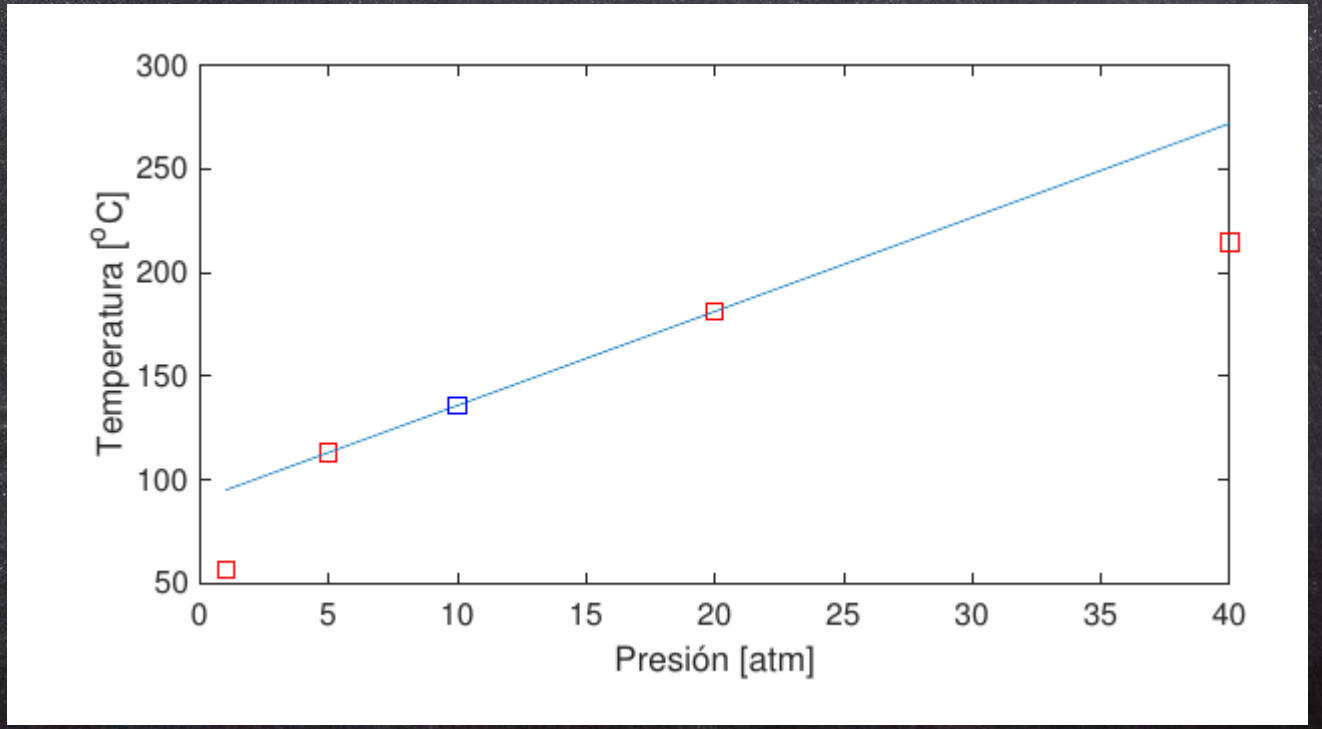
$$a_1 = \frac{f(x_1)}{x_1 - x_0}$$

$$P(x) = \frac{f(x_0)}{x_0 - x_1}(x - x_1) + \frac{f(x_1)}{x_1 - x_0}(x - x_0)$$
$$= \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$
$$= L_0(x) f(x_0) + L_1(x) f(x_1)$$

$$x_0 = 5 \qquad x_1 = 20$$
$$f(x_0) = 113 \qquad f(x_1) = 181$$

$$P(x) = \frac{113}{5-20}(x-2) + \frac{181}{20-5}(x-5)$$
$$= -7.533(x-20) + 12.066(x-5)$$
$$= 90.333 + 4.5333x$$

$$y = 90.333 + 4.5333x$$
$$= 90.3333 + 45.333$$
$$= 135.6663$$



Generalización

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

$$L_i(x) = \sum_{j=0, j\neq i}^{n} \left(\frac{x - x_j}{x_i - x_j} \right)$$