TUTORIAL 3 – MODELING OF TMS ROBOT

Q

1- FKM $\xi = f(q)$ with $q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$ and $\xi = O_4$ position

$\nearrow$ computed first in $\mathcal{F}_0$: ${}^{0}\xi$

$\searrow$ change of base to find ${}^{b}\xi$

2-

$$T_{1,2}\, T_{2,3} = \left(\begin{array}{ccc|c} C_2 & -S_2 & 0 & 0 \\ Cs_1 S_2 & Cs_1 C_2 & Ss_1 & 0 \\ -Ss_1 S_2 & -Ss_1 C_2 & Cs_1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & * & 0 & 0 \\ 0 & * & Ss_2 & q_3 Ss_2 \\ 0 & * & Cs_2 & q_3 Cs_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} C_2 & * & D_1 & D_2 \\ D_3 & * & D_4 & D_5 \\ D_6 & * & D_7 & D_8 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \quad \text{with} \quad \left|\begin{array}{l} D_1 = -S_2 Ss_2 \\ D_2 = q_3 D_1 \\ D_3 = Cs_1 S_2 \\ D_4 = Cs_1 C_2 Ss_2 + Ss_1 Cs_2 \\ D_5 = q_3 D_4 \\ D_6 = -Ss_1 S_2 \\ D_7 = -Ss_2\, Ss_1 C_2 + Cs_1 Cs_2 \\ D_8 = q_3 D_7 \end{array}\right.$$

$$T_{0,1}\, T_{1,3} = \left(\begin{array}{ccc|c} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} C_2 & * & D_1 & D_2 \\ D_3 & * & D_4 & D_5 \\ D_6 & * & D_7 & D_8 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} D_9 & * & D_{10} & D_{11} \\ D_{12} & * & D_{13} & D_{14} \\ D_6 & * & D_7 & D_8 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \quad \text{with} \quad \left|\begin{array}{l} D_9 = C_1 C_2 - S_1 D_3 \\ D_{10} = C_1 D_1 - S_1 D_4 \\ D_{11} = C_1 D_2 - S_1 D_5 \\ D_{12} = S_1 C_2 + C_1 D_3 \\ D_{13} = S_1 D_1 + C_1 D_4 \\ D_{14} = S_1 D_2 + C_1 D_5 \end{array}\right.$$

and then $\boxed{\begin{cases} x = D_{11} + r_4 D_{10} \\ y = D_{14} + r_4 D_{13} \\ z = D_8 + r_4 D_7 \end{cases}}$

3- $\quad {}^{b}\xi = T_{b0}\,{}^{0}\xi$ $\qquad$ see Tutorial 3 ...

with $T_{b0} = \left(\begin{array}{c|c} R_{b0} & p_{b0} \\ \hline 0 & 1 \end{array}\right)$.

$$R_{b0} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_0 & -S_0 \\ 0 & S_0 & C_0 \end{pmatrix}$$

with $C_0 = \cos s_0$
$S_0 = \sin s_0$
upper case $\quad$ lower case

4- To solve the IKM you should solve in $\mathcal{F}_0$ as it is the frame in which the FKM was written.

So if the IKM problem writes:

Find $\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$ that correspond to ${}^{b}\xi$

Then first compute ${}^{0}\xi = (T_{b0})^{-1}\,{}^{b}\xi$ and then solve with the equation of ${}^{0}\xi$

Let us assume that we have determined ${}^{0}\xi = (T_{b0})^{-1}\,{}^{b}\xi$ we should solve the system of equations:

$D_1 = -S_2\, S_{s2}$
$D_2 = q_3 D_1$
$D_3 = C_{s1} S_2$
$D_4 = C_{s1} C_2 S_{s2} + S_{s1} C_{s2}$
$D_5 = q_3 D_4$
$D_6 = -S_{s1} S_2$
$D_7 = -S_{s2} S_{s1} C_2 + C_{s1} C_{s2}$
$D_8 = q_3 D_7$
$D_9 = C_1 C_2 - S_1 D_3$
$D_{10} = C_1 D_1 - S_1 D_4$

$D_{11} = C_1 D_2 - S_1 D_5$
$D_{12} = S_1 C_2 + C_1 D_3$
$D_{13} = S_1 D_1 + C_1 D_4$
$D_{14} = S_1 D_2 + C_1 D_5$

$x = D_{11} + r_4 D_{10}$
$y = D_{14} + r_4 D_{13}$
$z = D_8 + r_4 D_7$

... that looks not very easy...

First have a look at the equations and observe that:

$$x = D_{11} + r_4 D_{10} = (q_3 + r_4) D_{10}$$
$$y = D_{14} + r_4 D_{13} = (q_3 + r_4) D_{13} \qquad (E)$$
$$z = D_8 + r_4 D_7 = (q_3 + r_4) D_7$$

Note that $q_3 + r_4$ is the distance between O_3 and O_4, which suggests to compute

$$x^2 + y^2 + z^2 = (q_3 + r_4)^2 (D_{10}^2 + D_{13}^2 + D_7^2)$$

Computation of $D_{10}^2 + D_{13}^2 + D_7^2$

$$D_{10}^2 + D_{13}^2 = (C_1 D_1 - S_1 D_4)^2 + (S_1 D_1 + C_1 D_4)^2 = D_1^2 + D_4^2$$

$$\Rightarrow D_1^2 + D_4^2 + D_7^2 = (-S_2\, Ss_2)^2 + (Cs_1\, C_2\, Ss_2 + Ss_1\, Cs_2)^2 + (-Ss_2\, Ss_1\, C_2 + Cs_1\, Cs_2)^2$$

$$= Ss_2^2 (S_2^2 + (C_2 Cs_1)^2 + (-C_2 Ss_1)^2) + Cs_2^2 (Ss_1^2 + Cs_1^2) + 2\, Cs_1\, Cs_2\, Ss_1\, Ss_2\, C_2 - 2\, Cs_1\, Cs_2\, Ss_1\, Ss_2\, C_2$$

$$= 1 \,!$$

$$\Rightarrow x^2 + y^2 + z^2 = (q_3 + r_4)^2 \quad \text{with } r_4 = -rh$$

$$\Rightarrow \boxed{q_3 = rh - \sqrt{x^2 + y^2 + z^2}}$$

we use the $-$ solution since ① q_3 should be positive or negative

$O_3O_4 < rh$ $\qquad$ $O_3O_4 > rh$

② q_3 is assumed $< rh$ which corresponds to O_4 in O_T

As a result $q_3 < rh$

$(\Leftrightarrow q_3 < -r_4)$

$\Rightarrow q_3 + r_4 < 0$

From (E) D_7, D_{10} and D_{13} are now known

$D_7 = Cs_1 Cs_2 - Ss_1 Ss_2 C_2 \Rightarrow C_2$ known

As it is clear that there are two symmetrical solutions for q_2 (as in the SCARA case)

$$C_2 = \frac{Cs_1 Cs_2 - D_7}{Ss_1 Ss_2} = \frac{Cs_1 Cs_2 - z/(q_3+r_4)}{Ss_1 \; Ss_2}$$ (always $\neq 0$ since $q_3 + r_4 < 0$)

C_2 not defined iff $Ss_1 Ss_2 = 0$ which does not correspond to a smart design (0 or 180 deg segments)

Then $S_2 = \varepsilon_2 \sqrt{1 - C_2^2}$ with $\varepsilon_2 = \pm 1$

$$\boxed{q_2 = \operatorname{atan2}(S_2, C_2)}$$

Finally we use D_{10} and D_{13}:

$$\begin{cases} D_{10} = C_1 D_1 - S_1 D_4 = x/(q_3 + r_4) \\ D_{13} = S_1 D_1 + C_1 D_4 = y/(q_3 + r_4) \end{cases}$$ with $r_4 = -r_h$

$$\begin{pmatrix} -D_4 & D_1 \\ D_1 & D_4 \end{pmatrix} \begin{pmatrix} S_1 \\ C_1 \end{pmatrix} = \frac{1}{q_3 - r_h} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow \text{if } D_1^2 - D_4^2 \neq 0, \quad \begin{pmatrix} S_1 \\ C_1 \end{pmatrix} = \frac{-1}{(q_3 - r_h)(D_1^2 + D_4^2)} \begin{pmatrix} D_4 & -D_1 \\ -D_1 & -D_4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\boxed{q_1 = \operatorname{atan2}(S_1, C_1)}$$

Note: singularities if $q_3 = r_h$, avoided by assumption

or $D_1^2 = D_4^2 \rightarrow S_2^2 Ss_2^2 = (Cs_1 C_2 Ss_2 + Ss_1 Cs_2)^2$