Case Study on Transcranial Magnetic Stimulation

Tutorial 1 – Mathematical tools for robotics

This tutorial deals with some mathematical modeling tools used in robotics, in particular the concept of homogeneous transformation.

The considered robot architecture is composed of two subsystems in series. The first subsystem is called *supporting structure* in the following. The second one is a spherical wrist that allows to rotate the coil. Its study is not considered here.

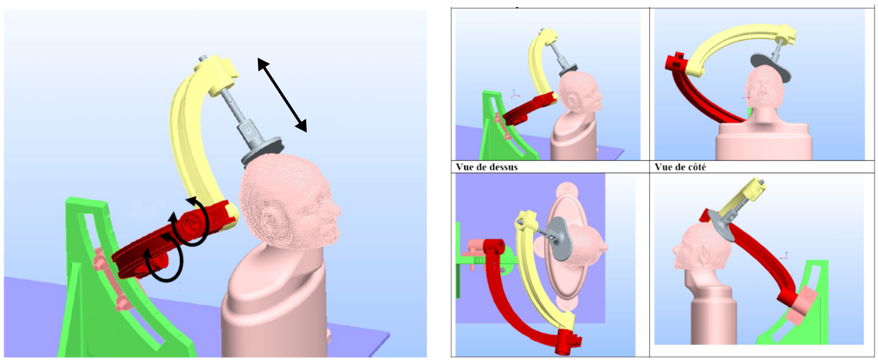

Figure 1: Supporting structure with two 90 deg. circular segments [Lebossé 2008].

The supporting structure can be placed in different configurations, as it is mounted on a carriage that can translate on a circular rail. Figure 2 represents this system in a planar sagittal view.

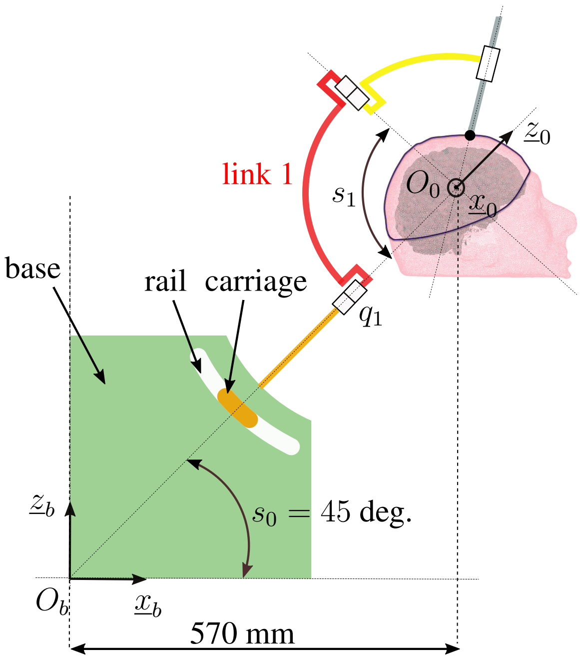

Figure 2: Supporting structure in a planar (sagittal) configuration.

To get used to homogeneous transformations, very frequent in robotics, we propose to study the display of the robot base and first link (in red) in different configurations, as MATLAB 3D plots.

This same (type of) development will be later used in the next tutorials to display the whole robot in different configurations.

Let $\mathcal{F}_0 = (O_0, \underline{x}_0, \underline{y}_0, \underline{z}_0)$ and $\mathcal{F}_b = (O_b, \underline{x}_b, \underline{y}_b, \underline{z}_b)$ denote the head and robot base reference frames, respectively. The other notations, in particular the problem parameters or numerical values, are defined in Figure 2. Note that $\underline{y}$ axes are not represented, as usual in robotics modeling.

Questions

1 Consider the robot links and joints. Note that all the joints axes intersect in O_0. Characterize the links motions, with respect to the head center. As a result, what type of transformations are involved here?

2 An example of $2\pi/3$ circular arc of the (x, y) plane (of MATLAB 3D plots) centered on the reference frame origin can be defined by the following MATLAB sequence:

```
arc_length=2*pi/3;
radius=0.5;
n=100;
direction=1;
angle=0;
for i = 1 : n+1
xc(:,i) = radius*cos(angle);
yc(:,i) = radius*sin(angle);
zc(:,i) = 0;
angle=angle+direction*(arc_length)/n;
end
plot3(xc,yc,zc,'b-')
```

Analyse this code. What is in particular the use of the different parameters?

3 Looking now at Figure 2, propose a similar parameterization to define link 1, in $\mathcal{F}_0$.

4 Link 1 rotates about axes $\underline{z}_0$ by an angle q_1 (positive in the counter-clockwise direction if nothing more is specified). Calculate the corresponding homogeneous transformation $T_{0,1}$.

5 Now calculate the position of the points of link 1 in the base frame $\mathcal{F}_b$.

6 Using MATLAB, implement the previous elements with the two proposed .m files. Program and check :

- that the rotation of link 1 is correct, with the provided MATLAB function `circular_arc.m` of course with several values of q_1 and s_1 (just in case ...);
- that once projected into base frame $\mathcal{F}_b$ the display of link 1 is still correct. Plot link 1 with the provided `test.m` MATLAB script, and check with several values for s_0 and q_1.