# Folha 6

$\boxed{1}$

a) $x^2 + y^2 - 2xy + 2x + 4y + 5 = 0$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 5 = 0$$

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad A - \lambda I = \begin{bmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix}$$

$$P_A(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 1 = \cancel{1} - 2\lambda + \lambda^2 - \cancel{1} = \lambda(\lambda - 2)$$

$P_A(\lambda) = 0 \Leftrightarrow \lambda \in \{0, 2\}$

$\boxed{\lambda = 2}$

$$(A - 2I)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \left[\begin{array}{cc|c} -1 & -1 & 0 \\ -1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightsquigarrow x_1 = -x_2$$

$$U_2 = \left\langle \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\rangle \rightsquigarrow v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = 0}$

$$(A - 0I)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Leftrightarrow AX = 0 \Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightsquigarrow x_1 = x_2$$

$$U_2 = \left\langle \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\rangle \rightsquigarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Como $A$ é simétrica $\Rightarrow$ A é ortogonalmente diagonalizável, os vetores próprios associados a valores próprios distintos são ortogonais, logo:

$$v_1 \cdot v_2 = 0$$

$$v_1' = \frac{v_1}{\|v_1\|} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \qquad v_2' = \frac{v_2}{\|v_2\|} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

Seja $P = [v_1 | v_2]$ uma matriz diagonalizante ortogonal

$$P = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}, \text{ tal que: } P^T A P = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Voltando à eq.

Fazer $X = P\bar{X}$ onde $\bar{X} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}$ e substituir em:

$$X^T A X + BX + 5 = 0$$

$$\Leftrightarrow (P\bar{X})^T A (P\bar{X}) + B(P\bar{X}) + 5 = 0$$

$$\Leftrightarrow \bar{X}^T \underbrace{P^T A P}_{D} \bar{X} + (BP)\bar{X} + 5 = 0$$

$$BP = \begin{bmatrix} 2 & 4 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & 3\sqrt{2} \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \bar{x} & \bar{y} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \sqrt{2} & 3\sqrt{2} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + 5 = 0$$

$$\Leftrightarrow 2\bar{x}^2 + \sqrt{2}\bar{x} + 3\sqrt{2}\,\bar{y} + 5 = 0$$

$$2a = \frac{\sqrt{2}}{2} \Rightarrow a = \frac{\sqrt{2}}{4}$$

$$\Leftrightarrow 2\left(\bar{x}^2 + \frac{\sqrt{2}}{2}\bar{x} + \frac{1}{8} - \frac{1}{8}\right) + 3\sqrt{2}\,\bar{y} + 5 = 0$$

$$\Leftrightarrow 2\left(\bar{x} + \frac{\sqrt{2}}{4}\right)^2 + 3\sqrt{2}\,\bar{y} + \frac{19}{4}$$

$$\Leftrightarrow 2\left(\bar{x} + \frac{\sqrt{2}}{4}\right)^2 + 3\sqrt{2}\left(\bar{y} + \frac{19}{4 \times 3\sqrt{2}}\right) = 0$$

$$\Leftrightarrow 2\left(\bar{x} + \frac{\sqrt{2}}{4}\right)^2 + 3\sqrt{2}\left(\bar{y} + \frac{19}{24}\sqrt{2}\right) = 0$$

$$\tilde{x} = \bar{x} + \frac{\sqrt{2}}{4}$$

$$\tilde{y} = \bar{y} + \frac{19}{24}\sqrt{2}$$

$$\Leftrightarrow 2\tilde{x}^2 + 3\sqrt{2}\,\tilde{y} = 0 \Leftrightarrow \tilde{y} = -\frac{2}{3\sqrt{2}}\tilde{x}^2$$

$$\Leftrightarrow \boxed{\tilde{y} = -\frac{\sqrt{2}}{3}\tilde{x}^2}$$

Parábola, c.v.p.b

b)

$$4xy - 2x + 6y + 3 = 0$$

$$(*) \quad \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 3 = 0$$

$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ é uma matriz simétrica, logo é ortogonalmente diagonalizável

$$A - \lambda I = \begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix} \qquad P_A(\lambda) = \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = \lambda^2 - 4$$

$$P_A(\lambda) = 0 \Leftrightarrow \lambda^2 - 4 = 0 \Leftrightarrow \lambda = -2 \vee \lambda = 2$$

$\boxed{\lambda = -2}$

$$U_{\lambda=-2} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \right\}$$

$$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 2 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \leadsto x + y = 0 \Leftrightarrow x = -y$$

$$U_{\lambda=-2} = \left\{ \begin{bmatrix} y \\ -y \end{bmatrix}, y \in \mathbb{R} \right\} = \left\langle \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\rangle$$

$\hookrightarrow v_1$, vetor próprio associado ao valor próprio $\lambda = -2$

$\boxed{\lambda = 2}$

$$U_{\lambda=2} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \right\}$$

$$\left[\begin{array}{cc|c} -2 & 2 & 0 \\ 2 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \leadsto -x + y = 0 \Leftrightarrow y = x$$

$$U_{\lambda=2} = \left\{ \begin{bmatrix} x \\ x \end{bmatrix}, x \in \mathbb{R} \right\} = \left\langle \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\rangle$$

$\hookrightarrow v_2$

$$v_1 \cdot v_2 = 0$$

$$v_1' = \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix} \qquad v_2' = \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \qquad \Rightarrow v_1' \cdot v_2' = 0$$

$\|v_1'\| = 1$ $\qquad$ $\|v_2'\| = 1$

Seja $P = [v_1' \mid v_2']$ a matriz diagonalizante ortogonal de $A$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \qquad P^T A P = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 6 \end{bmatrix}$$

$$(*) \Rightarrow X^T A X + B X + 3 = 0$$

$X = P\bar{X}$ , onde $\bar{X} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}$

C.A.

$$BP = \begin{bmatrix} -2 & 6 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} -4\sqrt{2} & 2\sqrt{2} \end{bmatrix}$$

$$\circledast \Rightarrow (P\bar{X})^T A (P\bar{X}) + B(P\bar{X}) + 3 = 0$$

$$\Leftrightarrow \bar{X}^T (P^T A P)\bar{X} + (BP)\bar{X} + 3 = 0$$

$$\Leftrightarrow \begin{bmatrix} \bar{x} & \bar{y} \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} -4\sqrt{2} & 2\sqrt{2} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + 3 = 0$$

$$\Leftrightarrow -2\bar{x}^2 + 2\bar{y}^2 - 4\sqrt{2}\bar{x} + 2\sqrt{2}\bar{y} + 3 = 0$$

$$\Leftrightarrow -2(\bar{x}^2 + 2\sqrt{2}\bar{x}) + 2(\bar{y}^2 + \sqrt{2}\bar{y}) + 3 = 0$$

$2a = 2\sqrt{2}$ $\Leftrightarrow a = \sqrt{2}$ $\Leftrightarrow a^2 = 2$

$2b = \sqrt{2}$ $\Leftrightarrow b = \frac{\sqrt{2}}{2}$ $\Leftrightarrow b^2 = \frac{1}{2}$

$$\Leftrightarrow -2(\bar{x}^2 + 2\sqrt{2}x + 2 - 2) + 2(\bar{y}^2 + \sqrt{2}\bar{y} + \frac{1}{2} - \frac{1}{2}) + 3 = 0$$

$$\Leftrightarrow -2(\bar{x} + \sqrt{2})^2 + 4 + 2(\bar{y} + \frac{\sqrt{2}}{2})^2 - 1 + 3 = 0$$

$$\Leftrightarrow -2(\bar{x} + \sqrt{2})^2 + 2(\bar{y} + \frac{\sqrt{2}}{2})^2 = -6$$

$\tilde{x} = \bar{x} + \sqrt{2}$

$\tilde{y} = \bar{y} + \frac{\sqrt{2}}{2}$

$$\Leftrightarrow -2\tilde{x}^2 + 2\tilde{y}^2 = -6$$

$$\Leftrightarrow \boxed{\frac{\tilde{x}^2}{3} - \frac{\tilde{y}^2}{3} = 1} \longrightarrow \text{Hipérbole!}$$

c)

$$x^2 + 2x + y^2 - 4y = 0$$

$2a = 2$ $\Leftrightarrow a = 1$ $\Leftrightarrow a^2 = 1$

$2b = 4$ $\Leftrightarrow b = 2$ $b^2 = 4$

$$\Leftrightarrow x^2 + 2x + 1 - 1 + y^2 - 4y + 4 - 4 = 0$$

$$\Leftrightarrow (x+1)^2 - 1 + (y-2)^2 - 4 = 0$$

$\tilde{x} = x + 1$

$\tilde{y} = y - 2$

$$\Leftrightarrow \tilde{x}^2 + \tilde{y}^2 = 5$$

$$\Leftrightarrow \boxed{\frac{\tilde{x}^2}{5} + \frac{\tilde{y}}{5} = 1} \longrightarrow \text{Elipse!}$$

$\boxed{2}$

a)

$$x^2 - y^2 - 6z^2 + 4x - 6y - 9 = 0$$
$$\Leftrightarrow x^2 + 4x + 4 - 4 \quad - (y^2 + 6y + 9) - 6z^2 = 0$$
$$\Leftrightarrow (x+2)^2 - 4 - (y+3)^2 - 6z^2 = 0$$

$2a = 6$
$\Leftrightarrow a = 3$
$a^2 = 9$

$\tilde{x} = x + 2$
$\tilde{y} = y + 3$
$\tilde{z} = z$

$$\Leftrightarrow \tilde{x}^2 - \tilde{y}^2 - 6\tilde{z}^2 = 4$$
$$\Leftrightarrow \boxed{\frac{\tilde{x}^2}{4} - \frac{\tilde{y}^2}{4} - \frac{\tilde{z}^2}{\frac{2}{3}} = 1} \Rightarrow \text{Hiperbolóide de duas folhas}$$

b)

$$x^2 + 2y^2 + z^2 - 2x + 4y = 0$$
$$\Leftrightarrow x^2 - 2x + 2y^2 + 4y + z^2 = 0$$
$$\Leftrightarrow x^2 - 2x + 1 - 1 \quad + 2(y^2 + 2y + 1 - 1) + z^2 = 0$$
$$\Leftrightarrow (x-1)^2 - 1 + 2(y+1)^2 - 2 + z^2 = 0$$

$\tilde{x} = x - 1$
$\tilde{y} = y + 1$
$\tilde{z} = z$

$$\Leftrightarrow \tilde{x}^2 + 2\tilde{y}^2 + \tilde{z}^2 = 3$$
$$\Leftrightarrow \boxed{\frac{\tilde{x}^2}{3} + \frac{\tilde{y}^2}{\frac{3}{2}} + \frac{\tilde{z}^2}{3} = 1} \Rightarrow \text{Elipsóide}$$

c)

$$x^2 + y^2 + 4x - 6y - z = 0$$
$$\Leftrightarrow x^2 + 4x + 4 - 4 + y^2 - 6y + 9 - 9 = z$$
$$\Leftrightarrow (x+2)^2 - 4 + (y-3)^2 - 9 = z$$

$2a = 6$
$\Leftrightarrow a = 3$
$a^2 = 9$

$\tilde{x} = x + 2$
$\tilde{y} = y - 3$
$\tilde{z} = z + 13$

$$\Leftrightarrow \tilde{x}^2 + \tilde{y}^2 = \tilde{z}$$

$\tilde{x} = 0 \Rightarrow \tilde{z} = \tilde{y}^2 \Rightarrow$ parábola
$\tilde{y} = 0 \Rightarrow \tilde{z} = \tilde{x}^2 \Rightarrow$ parábola
$\tilde{z} = 0 \Rightarrow \tilde{x}^2 + \tilde{y}^2 = 0 \Rightarrow x = 0 \wedge y = 0 \Rightarrow (0,0)$
$\tilde{z} = k,\ k > 0 \Rightarrow \tilde{x}^2 + \tilde{y}^2 = k \Leftrightarrow \frac{\tilde{x}^2}{k} + \frac{\tilde{y}^2}{k} = 1 \Rightarrow$ elipse
$\tilde{z} = k,\ k < 0 \Rightarrow \tilde{x}^2 + \tilde{y}^2 = k \Rightarrow$ Impossível $\Rightarrow \emptyset$

Parabolóide elítica

d)

$$x^2 + 4y^2 + 4xy - 2x - 4y + 2z + 1 = 0$$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} -2 & -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + 1 = 0$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad B = \begin{bmatrix} -2 & -4 & 2 \end{bmatrix}$$

↳ A é ortogonalmente diagonalizável pois é uma matriz simétrica

$$p_A(\lambda) = \begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & 4-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} \underset{\substack{T.L. \\ 3^a \text{ coluna}}}{=} -\lambda \times \begin{vmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = -\lambda \times \left((1-\lambda)(4-\lambda) - 4\right) = -\lambda \times (\cancel{4} - \lambda - 4\lambda + \lambda^2 - \cancel{4})$$

$$= 5\lambda^2 - \lambda^3 = \lambda^2(5-\lambda)$$

$$p_A(\lambda) = 0 \Leftrightarrow \lambda = 0 \lor \lambda = 5$$

$\boxed{\lambda = 0}$

$$U_{\lambda=0} = \left\{ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R} : \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0 \right\}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightsquigarrow x + 2y = 0 \Leftrightarrow x = -2y$$

$$U_{\lambda=0} = \left\{ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R}^3 : x = -2y \right\} = \left\{ \begin{bmatrix} -2y \\ Y \\ Z \end{bmatrix}, y, z \in \mathbb{R} \right\}$$

$$= \left\langle \underbrace{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}}_{V_1}, \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{V_2} \right\rangle$$

$\boxed{\lambda = 5}$

$$U_{\lambda=5} = \left\{ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R}^3 : \begin{bmatrix} -4 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 0 \right\}$$

$$\left[\begin{array}{ccc|c} -4 & 2 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & -5 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightsquigarrow \begin{cases} 2x - y = 0 \\ z = 0 \end{cases} \begin{cases} y = 2x \\ z = 0 \end{cases}$$

$$U_{\lambda=5} = \left\{ \begin{bmatrix} X \\ 2x \\ 0 \end{bmatrix}, x, z \in \mathbb{R} \right\} = \left\langle \underbrace{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}_{V_3} \right\rangle$$

$$v'_1 = \frac{1}{\sqrt{5}} \times \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \\ 0 \end{bmatrix}$$

$\|v'_1\| = 1$

$$v'_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad v'_3 = \frac{1}{\sqrt{5}} \times \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} \\ 0 \end{bmatrix}$$

$\|v'_2\| = 1 \qquad \|v'_3\| = 1$

• Como A é uma matriz simétrica os vetores próprios são ortogonais entre eles, logo $v'_1$, $v'_2$ e $v'_3$ também são ortogonais entre si

Seja $P = [v_3' \mid v_2' \mid v_1']$ uma matriz diagonalizante ortogonal de $A$.

↕ Quando à termos nulos o maior é o primeiro!

$$= \begin{bmatrix} \frac{\sqrt{5}}{5} & 0 & -\frac{2\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} & 0 & \frac{\sqrt{5}}{5} \\ 0 & 1 & 0 \end{bmatrix} \qquad P^T A P = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[x\ y\ z] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + [-2\ -4\ 2] \begin{bmatrix} x \\ y \\ z \end{bmatrix} + 1 = 0 \Rightarrow X^T A X + B X + 1 = 0 \quad ⊛$$

Seja $X = P\bar{X}$, onde $\bar{X} = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}$

$$BP = [-2\ -4\ 2] \begin{bmatrix} \frac{\sqrt{5}}{5} & 0 & -\frac{2\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} & 0 & \frac{\sqrt{5}}{5} \\ 0 & 1 & 0 \end{bmatrix} = [-2\sqrt{5}\ \ 2\ \ 0]$$

$$⊛ \Leftrightarrow (P\bar{X})^T A (P\bar{X}) + B(P\bar{X}) + 1 = 0$$

$$\Leftrightarrow \bar{X}^T (P^T A P)\bar{X} + (BP)\bar{X} + 1 = 0$$

$$\Leftrightarrow [\bar{x}\ \bar{y}\ \bar{z}] \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} + [-2\sqrt{5}\ 2\ 0] \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} + 1 = 0$$

$$\Leftrightarrow 5\bar{x}^2 - 2\sqrt{5}\bar{x} + 2\bar{y} + 1 = 0$$

$$\Leftrightarrow 5\left(\bar{x}^2 - \frac{2\sqrt{5}}{5}\bar{x}\right) + 2\bar{y} + 1 = 0$$

$$2a = \frac{2\sqrt{5}}{5} \Leftrightarrow a = \frac{\sqrt{5}}{5} \Leftrightarrow a = \frac{5}{25} \Leftrightarrow a = \frac{1}{5}$$

$$\Leftrightarrow 5\left(\bar{x}^2 - \frac{2\sqrt{5}}{5}\bar{x} + \frac{1}{5} - \cancel{\frac{1}{5}}\right) + 2\bar{y} + \cancel{1} = 0$$

$$\Leftrightarrow 5\left(\bar{x} - \frac{\sqrt{5}}{5}\right)^2 + 2\bar{y} = 0$$

$\tilde{x} = \bar{x} - \frac{\sqrt{5}}{5}$

$\tilde{y} = \bar{y}$

$\tilde{z} = \bar{z}$

$$\Leftrightarrow 5\tilde{x}^2 + 2\tilde{y} = 0$$

$$\Leftrightarrow \boxed{\tilde{y} = -\frac{5}{2}\tilde{x}^2} \rightarrow \text{Cilindro parabólico!}$$

e)

$$3y^2 + 4xz + 6y + 1 = 0$$

$$\begin{bmatrix} x & y & z \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 & 6 & 0 \end{bmatrix}}_{B} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + 1 = 0$$

$\Leftrightarrow \quad X^T A X + B X + 1 = 0$

(*)

A é uma matriz ortogonalmente diagonalizável pois é uma matriz simétrica

$$p_A(\lambda) = \begin{vmatrix} -\lambda & 0 & 2 \\ 0 & 3-\lambda & 0 \\ 2 & 0 & -\lambda \end{vmatrix} \underset{\substack{\text{T.L.} \\ \text{2ª coluna}}}{=} (3-\lambda) \times \begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} = (3-\lambda) \times (\lambda^2 - 4)$$

$p_A(\lambda) = 0 \Leftrightarrow \lambda = 3 \vee \lambda = 2 \vee \lambda = -2$

$\boxed{\lambda = -2}$

$(A + 2I)X = 0$

$$\left[\begin{array}{ccc|c} 2 & 0 & 2 & 0 \\ 0 & 5 & 0 & 0 \\ 2 & 0 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightsquigarrow \begin{cases} x = -z \\ y = 0 \end{cases} \Rightarrow \quad v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \hat{v}_1 = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$\boxed{\lambda = 2}$

$(A - 2I)X = 0$

$$\left[\begin{array}{ccc|c} -2 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightsquigarrow \begin{cases} x = z \\ y = 0 \end{cases} \Rightarrow \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ e } \hat{v}_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$\boxed{\lambda = 3}$

$(A - 3I)X = 0$

$$\left[\begin{array}{ccc|c} -3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & -3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} -3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightsquigarrow \begin{cases} x = 0 \\ z = 0 \end{cases} \Rightarrow \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ e } \hat{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\|\hat{v}_1\| = \|\hat{v}_2\| = \|\hat{v}_3\| = 1$

$\hat{v}_1 \cdot \hat{v}_2 = 0$

$\hat{v}_1 \cdot \hat{v}_3 = 0$

$\hat{v}_2 \cdot \hat{v}_3 = 0$

Seja $P = [\hat{v}_3 \mid \hat{v}_2 \mid \hat{v}_1]$ uma matriz diagonalizante ortogonal de $A$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \qquad P^T A P = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$BP = [6 \ 0 \ 0]$$

$$X = P\overline{X}, \text{ onde } \overline{X} = \begin{bmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{bmatrix}$$

$$(*) \Leftrightarrow (P\overline{X})^T A (P\overline{X}) + B(P\overline{X}) + 1 = 0$$

$$\Leftrightarrow \overline{X}^T (P^T A P)\overline{X} + (BP)\overline{X} + 1 = 0$$

$$\Leftrightarrow [\overline{x} \ \overline{y} \ \overline{z}] \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{bmatrix} + [6 \ 0 \ 0] \begin{bmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{bmatrix} + 1 = 0$$

$$\Leftrightarrow 3\overline{x}^2 + 2\overline{y}^2 - 2\overline{z}^2 + 6\overline{x} + 1 = 0$$

$$\Leftrightarrow 3(\overline{x}^2 + 2\overline{x} + 1 - 1) + 2\overline{y}^2 - 2\overline{z}^2 = -1$$

$$\Leftrightarrow 3(\overline{x} + 1)^2 + 2\overline{y}^2 - 2\overline{z}^2 = 2$$

$\tilde{x} = \overline{x} + 1$
$\tilde{y} = \overline{y}$
$\tilde{z} = \overline{z}$

$$\Leftrightarrow 3\tilde{x}^2 + 2\tilde{y}^2 - 2\tilde{z}^2 = 2$$

$$\Leftrightarrow \boxed{\frac{\tilde{x}^2}{\frac{2}{3}} + \frac{\tilde{y}^2}{1} - \frac{\tilde{z}^2}{1} = 1} \longrightarrow \text{Hipérbole de uma folha}$$

f) mais do mesmo !

g)

$$-x^2 + y^2 - 2x - 4y + z = 0$$

$$\Leftrightarrow -(x^2 + 2x + 1 - 1) + (y^2 - 4x + 4 - 4) = -2$$

$$\Leftrightarrow -(x+1)^2 + 1 + (y-2)^2 - 4 = -2$$

$$\Leftrightarrow -(x+1)^2 + (y-2)^2 = 1$$

$\tilde{x} = x + 1$
$\tilde{y} = y - 2$
$\tilde{z} = z$

$$\Leftrightarrow -\tilde{x}^2 + \tilde{y}^2 = 1$$

$$\Leftrightarrow \boxed{\tilde{y}^2 - \tilde{x}^2 = 1} \longrightarrow \text{Cilindro Hiperbólico}$$

$\boxed{3}$

$$5x^2 + 5y^2 + 2xy + 2x - 2y + \alpha = 0$$

$$\underbrace{\begin{bmatrix} x & y \end{bmatrix}}\underbrace{\begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 2 & -2 \end{bmatrix}}_{B}\begin{bmatrix} x \\ y \end{bmatrix} + \alpha = 0$$

A é uma matriz ortogonalmente diagonalizável pois é uma matriz simétrica

$$p_A(\lambda) = \begin{vmatrix} 5-\lambda & 1 \\ 1 & 5-\lambda \end{vmatrix} = (5-\lambda)^2 - 1 = 25 - 10\lambda + \lambda^2 - 1$$
$$= \lambda^2 - 10\lambda + 24$$

$$p_A(\lambda) = 0 \Leftrightarrow \lambda = \frac{10 \pm \sqrt{100 - 4 \times 1 \times 24}}{2}$$

$$\Leftrightarrow \lambda = \frac{10-2}{2} \vee \lambda = \frac{10+2}{2}$$

$$\Leftrightarrow \lambda = 4 \vee \lambda = 6$$

$\boxed{\lambda = 4}$

$(A - 4I)X = 0$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightsquigarrow x = -y \Rightarrow v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ e } \hat{v}_1 = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$\boxed{\lambda = 6}$

$(A - 6I)X = 0$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightsquigarrow x = y \Rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ e } \hat{v}_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

Seja $P = [\hat{v}_2 \mid \hat{v}_1]$ uma matriz diagonalizante ortogonal de A

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \quad \text{e} \quad P^T A P = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$$

$X = P\bar{X}$, onde $\bar{X} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}$ $\qquad BP = \begin{bmatrix} 0 & 2\sqrt{2} \end{bmatrix}$

Eq. da cónica $\rightarrow (P\bar{X})^T A (P\bar{X}) + B(P\bar{X}) + \alpha = 0$

$$\Leftrightarrow \bar{X}^T (P^T A P)\bar{X} + (BP)\bar{X} + \alpha = 0$$

$$\Leftrightarrow \begin{bmatrix} \bar{x} & \bar{y} \end{bmatrix}\begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} 0 & 2\sqrt{2} \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \alpha = 0$$

$$\Leftrightarrow 6\bar{x}^2 + 4\bar{y}^2 + 2\sqrt{2}\,\bar{y} + \alpha = 0$$

$$\Leftrightarrow 6\bar{x}^2 + 4\left(\bar{y}^2 + \frac{\sqrt{2}}{2}\bar{y} + \frac{1}{8} - \frac{1}{8}\right) + \alpha = 0$$

$$2a = \frac{\sqrt{2}}{2}$$
$$\Leftrightarrow a = \frac{\sqrt{2}}{4}$$
$$\Leftrightarrow a^2 = \frac{2}{16} \Leftrightarrow a^2 = \frac{1}{8}$$

$$\Leftrightarrow 6\bar{x}^2 + 4\left(\bar{y} + \frac{\sqrt{2}}{4}\right)^2 - \frac{1}{2} + \alpha = 0$$

$\tilde{x} = \bar{x}$

$\tilde{y} = \bar{y} + \frac{\sqrt{2}}{4}$

$$\Leftrightarrow 6\tilde{x}^2 + 4\tilde{y}^2 = \frac{1}{2} - \alpha$$

$$\Leftrightarrow 12\tilde{x}^2 + 8\tilde{y}^2 = 1 - 2\alpha$$

$$\Leftrightarrow \frac{12\tilde{x}^2}{1-2\alpha} + \frac{8\tilde{y}^2}{1-2\alpha} = 1 \quad , \; 1 - 2\alpha \neq 0 \Rightarrow \alpha \neq \frac{1}{2}$$

$$\Leftrightarrow \boxed{\frac{\tilde{x}^2}{\frac{1-2\alpha}{12}} + \frac{\tilde{y}^2}{\frac{1-2\alpha}{8}} = 1} \rightarrow \text{Para ser uma elipse } 1 - 2\alpha > 0$$

$\Leftrightarrow 2\alpha < 1$

$\Leftrightarrow \boxed{\alpha < \frac{1}{2}}$

**4**

a)

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \quad e \quad P = \begin{array}{cc} v_1 & v_2 \\ \left[\begin{array}{cc} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{array}\right] \end{array}$$

$\rightarrow$ P é uma matriz ortogonal pois as suas colunas são uma base ortonormada de $\mathbb{R}^2$ onde

$P = [v_1 \mid v_2] \Rightarrow v_1 \cdot v_2 = 0 \;\wedge \|v_1\| = \|v_2\| = 1$

$$P^T A P = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = D$$

b)

$$4xy + x + y = 0$$

$$\Leftrightarrow [x \; y] \underbrace{\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} + \underbrace{[1 \; 1]}_{B} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\Leftrightarrow X^T A X + BX = 0$$

alínea anterior

Seja $X = P\bar{X}$, onde $\bar{X} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}$

$$\Leftrightarrow (P\bar{X})^T A (P\bar{X}) + B(P\bar{X}) = 0$$

$$\Leftrightarrow \bar{X}^T \underbrace{(P^T A P)}_{\text{alínea anterior}} \bar{X} + (BP)\bar{X} = 0$$

$$BP = [1 \; 1] \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = [\sqrt{2} \; 0]$$

$$\Leftrightarrow [\bar{x} \; \bar{y}] \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + [\sqrt{2} \; 0] \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = 0$$

$$\Leftrightarrow 2\bar{x}^2 - 2\bar{y}^2 + \sqrt{2}\bar{x} = 0$$

$$\Leftrightarrow 2\left(\bar{x}^2 + \frac{\sqrt{2}}{2}\bar{x} + \frac{1}{8} - \frac{1}{8}\right) - 2\bar{y} = 0$$

$2a = \frac{\sqrt{2}}{2}$

$\Leftrightarrow a = \frac{\sqrt{2}}{4}$

$\Leftrightarrow a^2 = \frac{1}{8}$

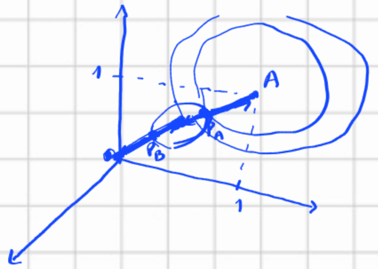
$\tilde{x} = \bar{x} + \frac{\sqrt{2}}{4}$

$\tilde{y} = \bar{y}$

$$\Leftrightarrow 2\tilde{x} - \frac{1}{4} - 2\tilde{y} = 0$$

$$\Leftrightarrow \boxed{\frac{\tilde{x}}{\frac{1}{8}} - \frac{\tilde{y}}{\frac{1}{8}} = 1} \rightarrow \text{Hipérbole}$$

$\boxed{5}$




$A = (0,1,1) \qquad P(x,y,z)$

$$\vec{AP} = P - A = (x, y-1, z-1)$$
$$\vec{OP} = P - O = (x, y, z)$$

$$\|\vec{AP}\| = \|\vec{OP}\| + 1$$

$$\Leftrightarrow \sqrt{x^2 + (y-1)^2 + (z-1)^2} = \sqrt{x^2 + y^2 + z^2} + 1$$

$\Leftrightarrow (\ldots)$

$\Leftrightarrow 4x^2 - 8yz + 4y + 4z - 1 = 0$

Classificavamos ...
e dava Hiperbole de uma folha