$\boxed{1}$

$X = (1, -2, 1) \qquad Y = (-1, 1, 0)$

(a)

$\cos\theta = \dfrac{x \cdot Y}{\|x\| \cdot \|Y\|}$

Para $X \perp Y$: $X \cdot Y = 0$

$$\Leftrightarrow \begin{bmatrix} 1, -2, 1 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\Leftrightarrow -1 - 2 = 0$$

$$\Leftrightarrow \underbrace{-3 = 0}_{}$$

C.I. $\Rightarrow$ logo X não é $\perp$ a Y

Como: $\dfrac{1}{-1} \neq \dfrac{-2}{1} \neq \dfrac{1}{0}$, logo X e Y não são colineares

b)

(i)

$$\cos\theta = \frac{X \cdot Y}{\|X\| \cdot \|Y\|}$$

$$\|X\| = \sqrt{1+4+1} = \sqrt{6} = \sqrt{2} \times \sqrt{3}$$

$$\|Y\| = \sqrt{1+1} = \sqrt{2}$$

$$\cos\theta = \frac{-3}{\sqrt{2} \times \sqrt{2} \times \sqrt{3}} = \frac{-3 \times \sqrt{3}}{2 \times 3} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

(ii)

$-Y = (1, -1, 0)$

$X \cdot (-Y) = (1, -2, 1) \cdot (1, -1, 0)$
$= 1 + 2 = 3$

$\|-Y\| = \|Y\| = \sqrt{2}$

$\|X\| = \sqrt{6}$

$\cos\theta = \dfrac{3}{\sqrt{2} \times \sqrt{2} \times \sqrt{3}} = \dfrac{\sqrt{3}}{2}$, logo $\theta = \dfrac{\pi}{6}$

(iii)

$$X+Y=(1,-2,1)+(-1,1,0)=(0,-1,1)$$

$$X-Y=(1,-2,1)-(-1,1,0)=(2,-3,1)$$

$$\|X+Y\|=\sqrt{1+1}=\sqrt{2}$$

$$\|X-Y\|=\sqrt{4+9+1}=\sqrt{2}\times\sqrt{7}$$

$$(X+Y)\cdot(X-Y)=(0,-1,1)\cdot(2,-3,1)=0+3+1=4$$

$$\cos\theta=\frac{4}{\sqrt{2}\times\sqrt{2}\times\sqrt{7}}=\frac{2\sqrt{7}}{7}\text{ , logo }\theta=\arccos\left(\frac{2\sqrt{7}}{7}\right)$$

c) Se u o vetor unitário com a direção do vetor X

$$u=\frac{1}{\|X\|}\times X=\frac{1}{\sqrt{6}}\times(1,-2,1)=\left(\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6}\right)$$

d)

$$2u=\left(\frac{\sqrt{6}}{3},-\frac{2\sqrt{6}}{3},\frac{\sqrt{6}}{3}\right)\quad\text{e}\quad -2u=\left(-\frac{\sqrt{6}}{3},\frac{2\sqrt{6}}{3},-\frac{\sqrt{6}}{3}\right)$$

$\downarrow$ tem o sentido de X

$\downarrow$ tem o sentido oposto a X

e)

Seja v um vetor ortogonal a Y, $v=(a,b,c)$, $a,b,c\in\mathbb{R}$

$$Y\cdot v=0$$

$$(-1,1,0)\cdot(a,b,c)=0$$

$$\Leftrightarrow -a+b=0$$
$$\Leftrightarrow a=b$$

$$v=(a,a,c)\text{ , } a,c\in\mathbb{R}$$

Se w um vetor com a direção de Y.

$$w=\alpha(-1,1,0)$$

$(v+w) = (1,-2,1)$

$(a,a,c) + \alpha(-1,1,0) = (1,-2,1)$

$\begin{cases} a - \alpha = 1 \\ a + \alpha = -2 \\ c = 1 \end{cases}$ $\begin{cases} a - \alpha = 1 \\ (a-\alpha)+\alpha+\alpha = -2 \\ c = 1 \end{cases}$ $\begin{cases} a - \alpha = 1 \\ 1 + 2\alpha = -2 \\ c = 1 \end{cases}$ $\begin{cases} a = 1 - \frac{3}{2} \\ \alpha = -\frac{3}{2} \\ c = 1 \end{cases}$ $\begin{cases} a = \frac{1}{2} \\ \alpha = -\frac{3}{2} \\ c = 1 \end{cases}$

$X = \left(\frac{1}{2}, \frac{1}{2}, 1\right) - \frac{3}{2}(-1,1,0)$

f)

Seja z um vetor perpendicular a X e a Y

$z = (a,b,c)$

$z \cdot X = 0 \quad \wedge \quad z \cdot Y = 0$

$(a,b,c)\cdot(1,-2,1) = 0 \quad \wedge \quad (a,b,c)\cdot(-1,1,0) = 0$

$\Leftrightarrow \begin{cases} a - 2b + c = 0 \\ -a + b = 0 \end{cases}$ $\begin{cases} -a + c = 0 \\ a = b \end{cases}$ $\begin{cases} a = c \\ a = b \end{cases}$ $\begin{cases} a = a \\ b = a \\ c = a \end{cases}$

$z = (a,a,a) = a(1,1,1), \quad a \in \mathbb{R}$

g)

Seja $\beta(a,b,c)$, vetor ortogonal a X

$(a,b,c)\cdot(1,-2,1) = 0$

$\Leftrightarrow a - 2b + c = 0 \Leftrightarrow a = 2b - c$

$\beta = (2b-c, b, c) = (2b, b, 0) - (c, 0, -c)$

$= b(2,1,0) - c(1,0,-1)$

Com $a = x$, $b = y$, $c = z$:

$\beta = y(2,1,0) - z(1,0,-1), \quad y,z \in \mathbb{R}$

$\boxed{2}$

$$u = \overrightarrow{P_1P_2} = P_2 - P_1 = (3-2, 1-3, 2+4) = (1,-2,6)$$

$$\|u\| = \sqrt{1+4+36} = \sqrt{41}$$

$$v = \overrightarrow{P_2P_3} = P_3 - P_2 = (-3-3, 0-1, 4-2) = (-6,-1,-2)$$

$$\|v\| = \sqrt{36+1+4} = \sqrt{41}$$

$$w = \overrightarrow{P_3P_1} = P_1 - P_3 = (2+3, 3-0, -4-4) = (5,3,-8)$$

$$\|w\| = \sqrt{25+9+64} = \sqrt{98}$$

Como $\left[\|u\| = \|v\|\right] \neq \|w\|$, logo o triangulo de vetores u, v e w é isósceles

$\boxed{3}$

$$u = (1,0,0)$$

Seja w todos os vetores que fazem um ângulo de $\frac{\pi}{3}$ com $(1,0,0)$ e seja θ esse ângulo. $w = (a, b, c)$, $a > 0$, $b, c \in \mathbb{R}$

$$\theta = \frac{\pi}{3} \qquad \cos\theta = \frac{u \cdot w}{\|u\| \cdot \|w\|}$$

$$\cos\frac{\pi}{3} = \frac{1}{2} \qquad \|u\| = 1$$

$$\frac{1}{2} = \frac{u \cdot w}{\|w\|} \Leftrightarrow 2(u \cdot w) = \|w\|$$

$$\Leftrightarrow 2\left((1,0,0)\cdot(a,b,c)\right) = \sqrt{a^2+b^2+c^2}$$

$$\Leftrightarrow 2a = \sqrt{a^2+b^2+c^2}$$

$$\Leftrightarrow 4a^2 - a^2 - b^2 - c^2 = 0$$

$$\Leftrightarrow 3a^2 = b^2 + c^2$$

$$\Leftrightarrow a = \underset{a>0}{\underbrace{+}}\sqrt{\frac{b^2+c^2}{3}} \quad \wedge \quad b^2+c^2 \neq 0$$

$$w = \left(\sqrt{\frac{b^2+c^2}{3}}, b, c\right), \quad b^2+c^2 \neq 0$$

$\boxed{4}$

a) $\|X+Y\|^2 + \|X-Y\|^2 = (X+Y)\cdot(X+Y) + (X-Y)\cdot(X-Y)$

$= X\cdot(X+Y) + Y\cdot(X+Y) + X\cdot(X-Y) + (-Y)\cdot(X-Y)$

$= X\cdot X + \cancel{X\cdot Y} + \cancel{Y\cdot X} + Y\cdot Y + X\cdot X - \cancel{X\cdot Y} - \cancel{Y\cdot X} + Y\cdot Y$

$= 2\times(X\cdot X) + 2\times(Y\cdot Y)$

$= 2\|X\|^2 + 2\|Y\|^2$

$= 2\left(\|X\|^2 + \|Y\|^2\right)$

b) $\|X+Y\|^2 = (X+Y)\cdot(X+Y)$

$= (X+Y)\cdot X + (X+Y)\cdot Y$

$= X\cdot X + Y\cdot X + X\cdot Y + Y\cdot Y$

$= \|X^2\| + \underbrace{X\cdot Y}_{0} + \underbrace{X\cdot Y}_{0} + \|Y\|^2$

$\underset{X\perp Y}{=} \|X\|^2 + \|Y\|^2$

$\boxed{5}$

a) $X\times Y = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 0 & 2 & -1 \end{vmatrix} \underset{\substack{\text{T.L.}\\ 1^{\text{a}}\text{ linha}}}{=} \hat{\imath}\times\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} - \hat{\jmath}\times\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} + \hat{k}\times\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix}$

$= \hat{\imath}\times(-1) - \hat{\jmath}\times(-2) + \hat{k}\times(4)$

$= -\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

$= (-1, 2, 4)$

b) $(-1,2,4)\cdot(2,-1,1) = 0$

$\Leftrightarrow -2-2+4=0$

$\Leftrightarrow 0=0$, logo $(X\times Y)\perp X$

$(-1,2,4)\cdot(0,2,-1)=0$

$\Leftrightarrow 0+4-4=0$

$\Leftrightarrow 0=0$, logo $(X\times Y)\perp Y$

$\boxed{6}$

$X \neq 0$, $Y \neq 0$, $X, Y \in \mathbb{R}^3$

a) $X(x_1, x_2, x_3)$

Para X e Y serem colineares: $Y = KX$, $K \in \mathbb{R}$

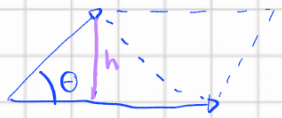
Logo, $X \times Y = X \times KX = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & x_2 & x_3 \\ Kx_1 & Kx_2 & Kx_3 \end{vmatrix} = K \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \end{vmatrix} = K \times 0 = (0,0,0)$

Logo, X e Y são colineares sse $X \times Y = (0,0,0)$

$\boxed{7}$

a)

(i)



$\sin\theta = \frac{\text{c. opst.}}{\text{hip.}} \Leftrightarrow \sin\theta = \frac{h}{\|Y\|} \Leftrightarrow h = \|Y\| \times \sin\theta$

(ii)

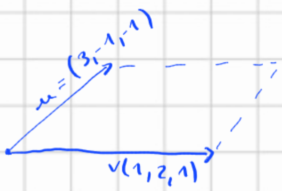
$A_{▱} = \|X\| \times h = \|X\| \times \|Y\| \times \sin\theta = \|X \times Y\|$

(iii)

$A_{\Delta} = \frac{1}{2} \times A_{▱} = \frac{1}{2} \times \|X \times Y\|$

b)

(i)



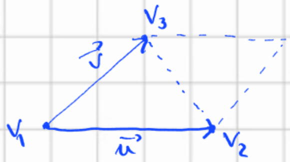
$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i} \times \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix}$

$= 2\hat{i} - 4\hat{j} + 7\hat{k}$

$= (1, -4, 7)$

$A_{▱} = \|u \times v\| = \sqrt{1 + 16 + 49} = \sqrt{66}$

(ii)



$V_1 = (1,0,1)$
$V_2 = (0,1,1)$
$V_3 = (1,1,2)$

$\vec{u} = \overrightarrow{V_1V_2} = V_2 - V_1 = (-1,1,0)$
$\vec{v} = \overrightarrow{V_1V_3} = V_3 - V_1 = (0,1,1)$

$$A_\Delta = \frac{\|\vec{u} \times \vec{v}\|}{2}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{\imath} \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - \hat{\jmath} \times \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} + \hat{k} \times \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = \hat{\imath} + \hat{\jmath} - \hat{k} = (1,1,-1)$$

$$\|\vec{u} \times \vec{v}\| = \|(1,1,-1)\| = \sqrt{3}$$

$$A_\Delta = \frac{\sqrt{3}}{2}$$

$\boxed{8}$

a)

Seja Z, vetor ortogonal a X e a Y

$$Z = X \times Y = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \hat{\imath} \times \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$$

$$= (2,-1,-3)$$

Todos os vetores ortogonais a X e a Y: $\alpha Z = \alpha(2,-1,-3)$, $\alpha \in \mathbb{R}$

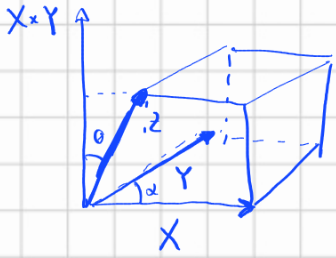
b)

$$A_{\square} = \|(2,-1,3)\| = \sqrt{4+1+9} = \sqrt{14}$$

$\boxed{9}$

a)

(i)



$$\cos\theta = \frac{h}{\|Z\|} \iff h = \|Z\| \times |\cos\theta|$$

(ii)

$$V = \underbrace{\|X \times Y\|}_{u} \times \underbrace{\|Z\|}_{v} \times |\cos(\theta)|$$

$$|\cos\theta| = \left|\frac{u \cdot v}{\|u\|\,\|v\|}\right| = \frac{|u \cdot v|}{\|u\|\,\|v\|}$$

$$|u \cdot v| = \|u\|\,\|v\|\,|\cos\theta|$$

$$= \|u\|\,\|v\|\cos(\theta)$$

$$= |u \cdot v|$$

$$= |(X \times Y) \cdot Z|$$

b)

$$V = |(X \times Y) \cdot Z| = |X \cdot (Y \times Z)|$$

$X = (3, -2, 1)$

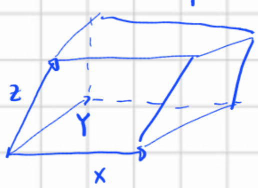
$Y = (1, 2, 3)$

$Z = (2, -1, 2)$

$$V = \left|\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 2 \end{vmatrix}\right| \underset{\substack{L_1' := L_1 - 3L_2 \\ L_3' := L_3 - 2L_2}}{\sim} \left|\begin{vmatrix} 0 & -8 & -8 \\ 1 & 2 & 3 \\ 0 & -5 & -4 \end{vmatrix}\right| \underset{\substack{\text{T.L.} \\ 1^{a}\text{ coluna}}}{=} \left| -1 \times \begin{vmatrix} -8 & -8 \\ -5 & -4 \end{vmatrix} \right| = \left| -1 \times (32 - 40) \right|$$

$$= 8$$

(ii)



$$V = |(X \times Y) \cdot Z|$$

$$= |X \cdot (Y \times Z)|$$

$X = (2, 1, 1)$

$Y = (2, 3, 4)$

$Z = (1, 0, -1)$

$$= \left|\begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{vmatrix}\right| \underset{L_3' := L_3 + L_1}{\sim} \left|\begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 0 & 0 \end{vmatrix}\right| \underset{\substack{\text{T.L.} \\ 3^{a}\text{ linha}}}{=} \left| 3 \times \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \right| = |3 \times (4 - 3)| = |3| = 3$$

$\boxed{10}$

$$\begin{cases} x + y - z = 2 \\ x - y + z = 0 \end{cases}$$

Se $z = 0 \Rightarrow x + y = 2 \wedge x - y = 0$
$\Leftrightarrow x = 1 \wedge y = 1$

Se $y = 0 \Rightarrow x - z = 2 \wedge x + z = 0$
$\Leftrightarrow z = -1 \wedge x = 1$

$A = (1,1,0) \in \mathcal{R}$

$A' = (1,0,-1) \in \mathcal{R}$

Seja $\overrightarrow{AA'}$ um vetor diretor de $\mathcal{R}$

$\overrightarrow{AA'} = A' - A = (0,-1,-1)$

Logo, $\mathcal{R}: (x,y,z) = (1,1,0) + k(0,-1,-1), k \in \mathbb{R}$

$P = (2,2,1)$

Seja $\vec{w}$ um vetor diretor do plano β

$\vec{w} \cdot \overrightarrow{AA'} = 0$ $\qquad \overrightarrow{AA'} = (0,-1,-1)$

$\Leftrightarrow x_w \times 0 + y_w \times (-1) + z_w \times (-1) = 0$
$\Leftrightarrow -y_w - z_w = 0$
$\Leftrightarrow y_w = -z_w$

$\vec{w} = (0,1,-1)$

$\beta: y - z + d = 0$

Como $P \in \beta \Rightarrow 2 - 1 + d = 0 \Leftrightarrow d = -1$

$\beta: y - z = 1$

Outro vetor diretor de β é $\overrightarrow{AP} = P - A = (2,2,1) - (1,1,0)$
$= (1,1,1)$

Logo $\beta: (x,y,z) = (2,2,1) + \alpha(0,1,-1) + \beta(1,1,1), \alpha, \beta \in \mathbb{R}$

$\boxed{11}$

A(-1,0,2)
B(1,-1,1)

$$d(A,\beta) = d(B,\beta)$$

Seja $M \in \beta$, o ponto médio de $\overline{AB}$

$$M = \left(\frac{-1+1}{2}, \frac{0-1}{2}, \frac{2+1}{2}\right) = \left(0, -\frac{1}{2}, \frac{3}{2}\right)$$

$$\overrightarrow{MB} = B - M = \left(1, -1+\frac{1}{2}, 1-\frac{3}{2}\right) = \left(1, -\frac{1}{2}, -\frac{1}{2}\right)$$

$$\beta: \ x - \frac{1}{2}y - \frac{1}{2}z + d = 0$$

$$\frac{1}{4} - \frac{3}{4} + d = 0 \Leftrightarrow d = \frac{1}{2}$$

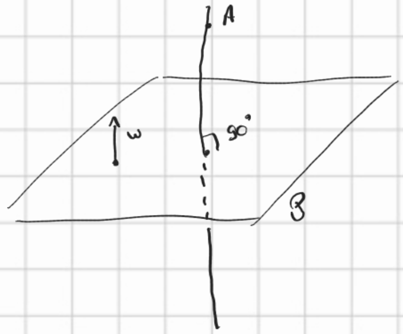
$$\Rightarrow \beta: x - \frac{1}{2}y - \frac{1}{2}z + \frac{1}{2} = 0$$

R: Todos os pontos que satisfaçam esta condição : $x - \frac{1}{2}y - \frac{1}{2}z + \frac{1}{2} = 0 \Leftrightarrow 2x - y - z + 1 = 0$

$\boxed{12}$

a) $A = \left(3, \frac{1}{2}, -\frac{7}{2}\right)$

$\beta : y + z = -1$



$0x + 1y + 1z = -1$

$w = (0,1,1)$

$(x,y,z) = \left(3, \frac{1}{2}, -\frac{7}{2}\right) + \alpha(0,1,1), \alpha \in \mathbb{R}$

<u>A mais</u>! (dá jeito na pergunta seguinte)

Eq. paramétricas:

$$\begin{cases} x = 3 \\ y = \frac{1}{2} + \alpha \\ z = -\frac{7}{2} + \alpha \end{cases}, \alpha \in \mathbb{R}$$

Obtendo α no sistema anterior:

$$\begin{cases} x = 3 \\ y = \frac{1}{2} + z + \frac{7}{2} \\ \alpha = z + \frac{7}{2} \end{cases} \Leftrightarrow \begin{cases} x = 3 \\ y = 4 + z \\ \alpha = z + \frac{7}{2} \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} x = 3 \\ y - z = 4 \end{cases}$$

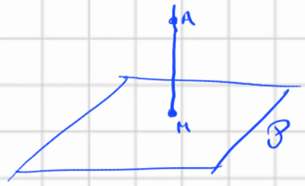
Eq. cartesianas de $\mathbb{R}$

b)

1.º Processo

$A\left(3, \frac{1}{2}, -\frac{7}{2}\right)$

$\beta : y + z = -1$



$d(A, \beta) = d(A, M)$
$= \|\overrightarrow{AM}\|$

$(x,y,z) = \left(3, \frac{1}{2}, -\frac{7}{2}\right) + \alpha(0,1,1), \alpha \in \mathbb{R}$

$$R_0 \cap \beta : \begin{cases} x = 3 \\ y - z = 4 \\ y + z = -1 \end{cases} \quad \begin{cases} x = 3 \\ y = 4 + z \\ z = -1 - y \end{cases} \quad \begin{cases} x = 3 \\ 2y = 3 \\ z = -1 - y \end{cases} \quad \begin{cases} x = 3 \\ y = 3/2 \\ z = -5/2 \end{cases}$$

$\overrightarrow{AM} = M - A = \left(3 - 3, \frac{3}{2} - \frac{1}{2}, -\frac{5}{2} + \frac{7}{2}\right) = (0, 1, 1)$

$d(A, M) = \|\overrightarrow{AM}\| = \sqrt{2}$

—— // ——

2.º Processo M

$$d(A, \beta) = \frac{|a x_0 + b x_0 + c x_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

$(a,b,c) = (0,1,1)$, $y + z = -1 \Rightarrow d = 1$

$(x_0, y_0, z_0) = \left(3, \frac{1}{2}, -\frac{7}{2}\right)$

$$d(A, \beta) = \frac{\left|\frac{1}{2} - \frac{7}{2} + 1\right|}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$$

—— // ——

3.º Processo

$$d(A, \beta) = \frac{|\vec{PA} \cdot w|}{\|w\|}$$

$A\left(3, \frac{1}{2}, -\frac{7}{2}\right)$

$P \in \beta$ ou seja P que verifica $y+z+1=0$

Tomando $y=-1 \Rightarrow z=0$, P pode ser $P=(0,-1,0)$

Poderia ser um qualquer

$w = (0,1,1)$

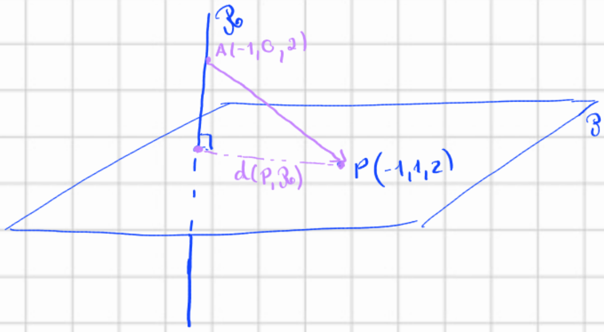
$\|w\| = \sqrt{2}$

$$\vec{PA} = A - P = \left(3, \frac{3}{2}, -\frac{7}{2}\right)$$

$$d(A, \beta) = \frac{\left|\left(3, \frac{3}{2}, -\frac{7}{2}\right) \cdot (0,1,1)\right|}{\sqrt{2}} = \frac{\left|\frac{3}{2} - \frac{7}{2}\right|}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$\boxed{13}$

a)



$$\overrightarrow{AB} = B - A = (-1, 0, 1)$$

$$\mathcal{R} : (-1, 0, 2) + \alpha(-1, 0, 1) \quad , \alpha \in \mathbb{R}$$

Seja $\vec{v}$ o vetor diretor de β.

$$\vec{v} = \overrightarrow{AB} = (-1, 0, 1)$$

$$\beta : -x + z + d = 0$$

$P \in \beta$, $P(-1, 1, 2)$

$$1 + 2 + d = 0 \Leftrightarrow d = -3 \implies \beta : -x + z - 3 = 0$$

b)

$$d(P, \mathcal{R}_0) = \frac{\| u \times \overrightarrow{AP} \|}{\|u\|}$$

Seja u, o vetor diretor de $\mathcal{R}_0$: $u = (-1, 0, 1)$

$$\overrightarrow{AP} = P - A = (-2, 1, 2)$$

$$\|u\| = \sqrt{2}$$

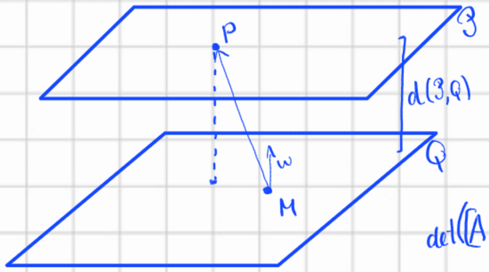
$$\| u \times \overrightarrow{AP} \| = \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ -2 & 1 & 2 \end{vmatrix} \right\| = \left\| \hat{i} \times \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} - \hat{j} \begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} \right\|$$

$$= \| (-1, 0, -1) \|$$

$$= \sqrt{2}$$

$$d(P, \mathcal{R}_0) = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$\boxed{14}$



$P \cap Q = ?$

$$\begin{cases} x + y + 2z = 3 \\ 2x + 2y + 4z = 2 \end{cases}$$

Seja $[A|B]$ a matriz ampliada do sistema

$$\det([A|B]) = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & 2 & 4 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right],$$

logo como $\text{car}(A) < \text{car}([A|B])$ o sistema é impossível, logo $P \cap Q = \emptyset$, conclui-se que P e Q são estritamente paralelos.

Seja $P \in \mathcal{P}$ e $M \in Q$, e w o vetor diretor de Q.

$$d(\mathcal{P},Q) = d(P,Q) = \frac{|\overrightarrow{MP} \cdot w|}{\|w\|}$$

$$Q: 2x + 2y + 4z = 2 \Leftrightarrow x + y + 2z = 1 \Rightarrow w = (1,1,2)$$

$$\overrightarrow{MP} = P - M$$

Seja $P = (3,0,0)$ e $M(1,0,0)$

$$\overrightarrow{MP} = (2,0,0)$$

$$|\overrightarrow{MP} \cdot w| = |(2,0,0) \cdot (1,1,2)| = 2$$

$$\|w\| = \sqrt{6}$$

$$d(P,Q) = \frac{2}{\sqrt{6}} = d(\mathcal{P},Q)$$

$\boxed{15}$

$$\mathcal{P}: x - y + z = 1$$

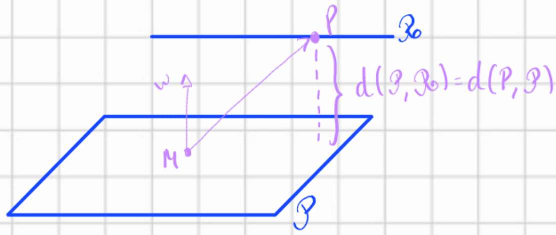
$$\mathcal{R}_o: \begin{cases} x - 2y = -1 \\ y + z = 3 \end{cases}$$

Se $x = -1 \Rightarrow y = 0 \wedge z = 3$ Se $z = 0 \Rightarrow y = 3 \wedge x = 5$

Seja $\vec{v}$ o vetor diretor da reta $\mathcal{R}_o$.

$$\vec{w} = (-1,0,3) - (5,3,0) = (-6,-3,3)$$

Logo, como $\vec{v}$ é colinear com $\vec{w}$, $\vec{v} = (-6,-3,3) \times (-\frac{1}{3}) = (2,1,-1)$



Seja P um ponto da reta $\mathcal{R}$ e γ o vetor diretor do plano $\mathcal{P}$ e M um ponto de $\mathcal{P}$

$P=(-1,0,3)$

$\gamma=(1,-1,1)$ $\quad M=(0,0,1)$

$$d(P,\mathcal{P})=\frac{|\overrightarrow{MP}\cdot\gamma|}{\|\gamma\|}$$

$\|\gamma\|=\sqrt{3}$

$\overrightarrow{MP}=P-M=(-1,0,2)$

$|\overrightarrow{MP}\cdot\gamma|=|(-1,0,2)\cdot(1,-1,1)|=|-1-0+2|=1$

$$d(P,\mathcal{P})=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$P\cap\mathcal{R}=?$

$$\begin{cases} x-y+z=1\\ x-2y=-1\\ y+z=3\end{cases}$$

Seja $[A|B]$ a matriz ampliada do sistema

$$[A|B]=\left[\begin{array}{ccc|c}1&-1&1&1\\1&-2&0&-1\\0&1&1&3\end{array}\right]\sim\left[\begin{array}{ccc|c}1&-1&1&1\\0&-1&-1&-2\\0&1&1&3\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&-1&1&1\\0&1&1&2\\0&0&0&1\end{array}\right]$$

, logo como $car(A) < car([A|B])$, conclui-se que o sistema é impossível, logo $\mathcal{P}$ e $\mathcal{R}$ são estritamente paralelos

$\boxed{16}$

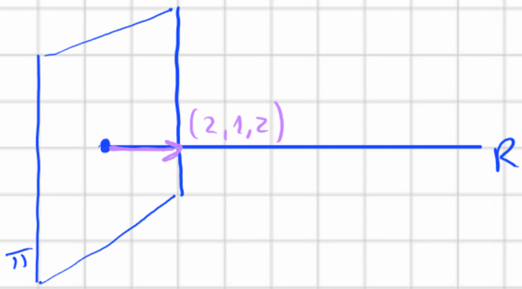
$R: \ x = 2y = z - 1$

$$\frac{x-0}{1} = \frac{y-0}{\frac{1}{2}} = \frac{z-1}{1} \Rightarrow R: (0,0,1) + \alpha(2,1,2), \alpha \in \mathbb{R}$$

Uma direção da reta é $u = (1, \frac{1}{2}, 1)$

ou

$u^* = (2,1,2)$



Seja $\pi: 2x + y + 2z + d = 0$ o plano perpendicular a R e a $d(O, \pi) = 1$, onde $O(0,0,0)$

$d(O, \pi) = 1$

$$\Leftrightarrow \frac{|2 \times 0 + 1 \times 0 + 2 \times 0 + d|}{\sqrt{9}} = 1$$

$\Leftrightarrow |d| = 3 \Leftrightarrow d = -3 \vee d = 3$

Assim:

$\pi: 2x + y + 2z + 3 = 0$

ou

$\pi: 2x + y + 2z - 3 = 0$

$R_1 : (1,1,-1) + \alpha(-1,2,-1), \alpha \in \mathbb{R}$

$v = (0,1,-1) - (1,-1,0)$

$= (-1, 2, -1)$

$R_2 : (1,-1,0) + \beta(-1,2,-1), \beta \in \mathbb{R}$

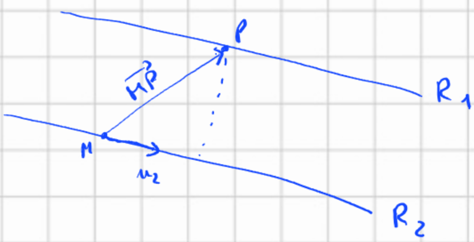
R_1 e R_2 são paralelos, visto que, os vetores diretores são colineares. São estritamente paralelos?

Seja $P = (1,1,-1) \in R_1$

$P \in R_2$?

$(1,1,-1) = (1,-1,0) + \beta(-1,2,-1), \beta \in \mathbb{R}$

$\begin{cases} 1 = 1 - \beta \\ 1 = -1 + 2\beta \\ -1 = -\beta \end{cases} \quad \begin{cases} \beta = 0 \\ \beta = 1 \\ \beta = 1 \end{cases}$ $\not\exists$, logo R_1 e R_2 são estritamente paralelos



$d(R_1, R_2) = d(P, R_2), P \in R_1$

$\Leftrightarrow d(P, R_2) = \dfrac{\| u_2 \times \vec{MP} \|}{\|u_2\|}$

Seja $M = (1,-1,0)$
$P = (1,1,-1)$

$\vec{MP} = P - M = (0, 2, -1)$

$u_2 = (-1,2,-1)$

$\|u_2\| = \sqrt{1+4+1} = \sqrt{6}$

$$u_2 \times \vec{MP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 0 & 2 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} -1 & 2 \\ 0 & 2 \end{vmatrix}$$

$$= 0\hat{i} - \hat{j} - 2\hat{k} = (0,-1,-2)$$

$$\|u_2 \times \vec{MP}\| = \sqrt{5}$$

$$d(P, R_2) = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{5}\times\sqrt{6}}{6} = \frac{\sqrt{30}}{6}$$

$\boxed{1^{\circ}}$

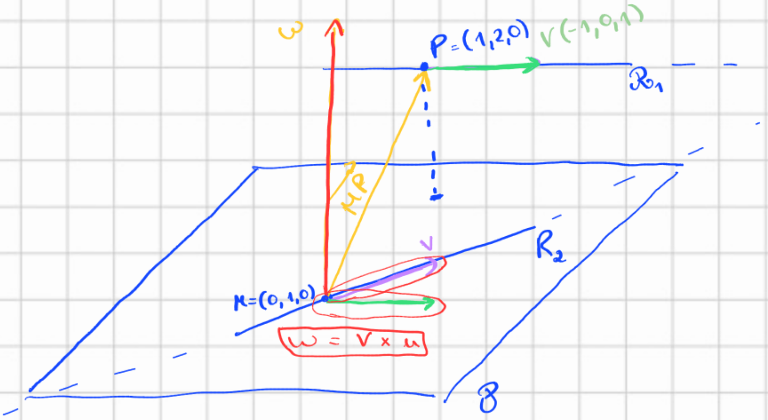
a)

Comos os vetores diretores de R_1 e R_2 não são colineares logo não são estritamente paralelos

$$R_1 \cap R_2 \Rightarrow (1,2,0) + \alpha(-1,0,1) = (0,1,0) + \alpha(0,-1,1)$$

$$\Leftrightarrow \begin{cases} 1-\alpha = 0 + 0\alpha \\ 2 = 1-\alpha \\ 0+\alpha = 0+\alpha \end{cases} \quad \begin{cases} \alpha = 1 \\ \alpha = -1 \\ 0 = 0 \end{cases} \quad \neq \quad \text{, logo } R_1 \cap R_2 = \emptyset$$

$R \cap R'' = \emptyset$ e os seu vetores diretores são não colineares, logo são enviesados



• Considerando o plano β que contem R'' e é paralelo a R

$$\beta : (x,y,z) = \underbrace{(0,1,0) + \alpha(-1,0,1)}_{R_2} + \underbrace{\beta(0,-1,1)}_{\text{vetor diretor de } R_1}, \quad \alpha,\beta \in \mathbb{R}$$

$$\begin{cases} x = -\alpha \\ y = 1-\beta \\ z = \alpha+\beta \end{cases} \quad \begin{cases} \alpha = -x \\ \beta = 1-y \\ z = -x+1-y \end{cases} \quad \begin{cases} \alpha = -x \\ \beta = 1-y \\ \boxed{x+y+z=1} \end{cases}$$

$$\beta : x+y+z = 1$$

b)

$$d(\mathcal{R}_1, \mathcal{R}_2) = d(\mathcal{R}_1, \mathcal{P}) = d(P, \mathcal{P}) = \frac{|\vec{MP} \cdot w|}{\|w\|}$$

Seja w um vetor diretor do plano $\mathcal{P}$

Seja $P \in \mathcal{R}_1$ e $M \in \mathcal{R}_2$

$P = (1,2,0)$

$M = (0,1,0)$

$w = (1,1,1)$ $\|w\| = \sqrt{3}$

$\vec{MP} = P - M = (1,1,0)$

$\vec{MP} \cdot w = 1 + 1 + 0 = 2$

$$d(P, \mathcal{P}) = \frac{|2|}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos\theta = \frac{|u \cdot v|}{\|u\|\,\|v\|} \Rightarrow \theta = \arccos\left[\frac{|u \cdot v|}{\|u\|\,\|v\|}\right]$$

Sendo $u = (-1,0,1)$ o vetor diretor de $\mathcal{R}_1$ e $v = (0,-1,1)$ vetor diretor de $\mathcal{R}_2$

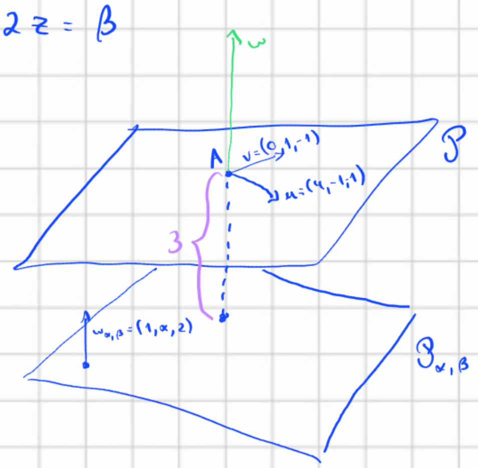
$$\theta = \arccos\left[\frac{1}{2}\right] = \frac{\pi}{3}$$

$\boxed{19}$

$\mathcal{P} = (x,y,z) = (1,1,-1) + s(0,1,-1) + t(4,-1,-1)$, $s, t \in \mathbb{R}$

e

$\mathcal{P}_{\alpha,\beta} = x + \alpha y + 2z = \beta$



$w = (0,1,-1) \times (4,-1,-1)$

$$w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 4 & -1 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & -1 \\ 4 & -1 \end{vmatrix} + k\begin{vmatrix} 0 & 1 \\ 4 & -1 \end{vmatrix}$$

$$= -2\hat{i} - 4\hat{j} - 4\hat{k}$$

$$= (-2,-4,-4)$$

Usemos $w' = (1,2,2)$, colinear com o anterior

A equação de $\mathcal{P}$ é:

$$1x + 2y + 2z = d \underset{\substack{(1,1,-1) \\ \in \mathcal{P}}}{\Rightarrow} 1 + 2 - 2 = d \Leftrightarrow d = 1$$

Logo, $\boxed{\mathcal{P}: x + 2y + 2z = 1}$

Como $\mathcal{P} \parallel \mathcal{P}_{\alpha,\beta}$

$$\Downarrow$$

$$\omega_\alpha = \omega' \Rightarrow \alpha = 2$$

<u>Resta calcular β</u>

$$d(\mathcal{P}, \mathcal{P}_{\alpha,\beta}) = 3$$

$$\Downarrow \; A \in \mathcal{P}$$

$$d(A, \mathcal{P}_{\alpha,\beta}) = 3$$

Formula geral:

$$d(P, \mathcal{P}) = \frac{|a x_0 + b y_0 + c z_0 - d|}{\sqrt{a^2+b^2+c^2}},$$

Poderia ser $+d$, dependendo de como escrevemos a equação do plano

onde $ax + by + cz - d = 0$

$P = (x_0, y_0, z_0), \notin \mathcal{P}$

—— // ——

$$3 = d(A, \mathcal{P}_{\alpha,\beta}) = \frac{|1 \times 1 + 1 \times 2 - 1 \times 2 - \beta|}{\sqrt{9}}$$

$A = (1, 1, -1)$

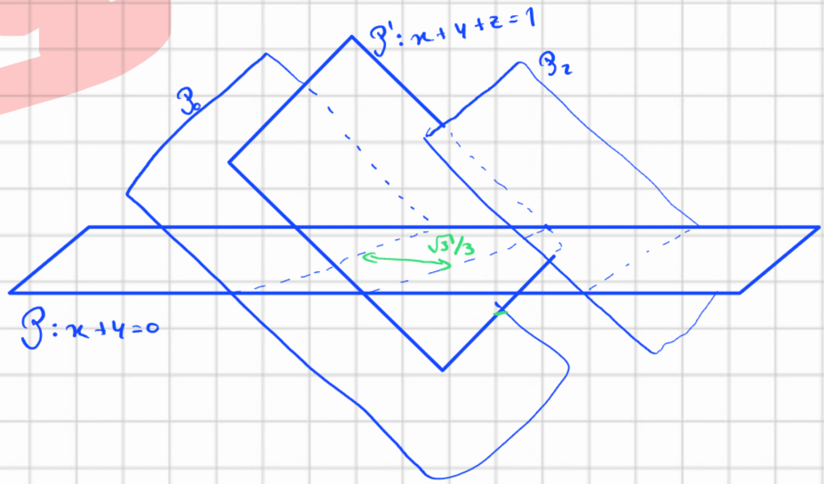
$\omega' = (1, 2, 2)$

$$\Leftrightarrow 3 = \frac{|1 - \beta|}{3} \Leftrightarrow |1-\beta| = 9 \Leftrightarrow 1 - \beta = -9 \vee 1 - \beta = 9$$

$$\Leftrightarrow \beta = 10 \vee \beta = -8$$

R: $\beta \in \{-8, 10\} \wedge \alpha = 2$

$\boxed{20}$



- Consideramos os planos // a β',

β_K cuja distância a β' é $\frac{\sqrt{3}}{3}$

Vamos ver que planos são esses β_K
vão ser na forma $x+y+z=k$

Mais,

$$d(\beta', \beta_K) = \frac{\sqrt{3}}{3}$$

$$\underset{\substack{A \in \beta' \\ A=(0,0,1)}}{\Leftrightarrow} \frac{\sqrt{3}}{3} = \frac{|1\cdot 0 + 1\cdot 0 + 1\cdot 1 - k|}{\sqrt{1+1+1}}$$

$ax+by+cz=d$

$$\to d(P,\beta) = \frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}$$

$$\Leftrightarrow \frac{|1-k|}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Leftrightarrow |1-k| = 1$$

$$\Leftrightarrow k=0 \vee k=2$$

Assim, os planos β_K são da forma $x+y+z=0$
$x+y+z=2$

$$R_1: \begin{cases} x+y=0 \\ x+y+z=0 \end{cases} \begin{cases} x+y=0 \\ z=0 \end{cases} \qquad R_2: \begin{cases} x+y=0 \\ x+y+z=2 \end{cases} \begin{cases} x+y=0 \\ z=2 \end{cases}$$

$\boxed{21}$

Falta!