1. a) $f(x) = \frac{2}{\sqrt{x}}$, $D_f = \mathbb{R}^+$

$g(x) = \sqrt{5-x}$, $D_g =]-\infty, 5]$

Os gráficos de f e g só se podem interceptar em $I = D_f \cap D_g =]0, 5]$. Para $x \in I$:

$$\frac{2}{\sqrt{x}} = \sqrt{5-x} \Leftrightarrow \sqrt{x}\,\sqrt{5-x} = 2 \underset{x \in I}{\Leftrightarrow}$$

$$x(5-x) = 4 \Leftrightarrow 5x - x^2 = 4 \Leftrightarrow$$

$$x^2 - 5x + 4 = 0 \Leftrightarrow x = \frac{5 \pm \sqrt{25 - 4(4)}}{2} \Leftrightarrow$$

$$x = \frac{5 \pm 3}{2} \Leftrightarrow x = 1 \ \vee \ x = 4$$

Os pontos de intersecção são $P = (1, \frac{2}{\sqrt{1}})$ e $Q = (4, \frac{2}{\sqrt{4}})$. $P = (1,2)$ e $Q = (4,1)$

b) f e g são de classe C^∞ em $]0,5[$ e

$$f'(x) = \left[2\,x^{-1/2}\right]' = 2\left(-\tfrac{1}{2}\right)x^{-3/2} = -\,x^{-3/2} < 0$$

$$f''(x) = -\left(-\tfrac{3}{2}\right)x^{-5/2} = \tfrac{3}{2}\,x^{-5/2} > 0$$

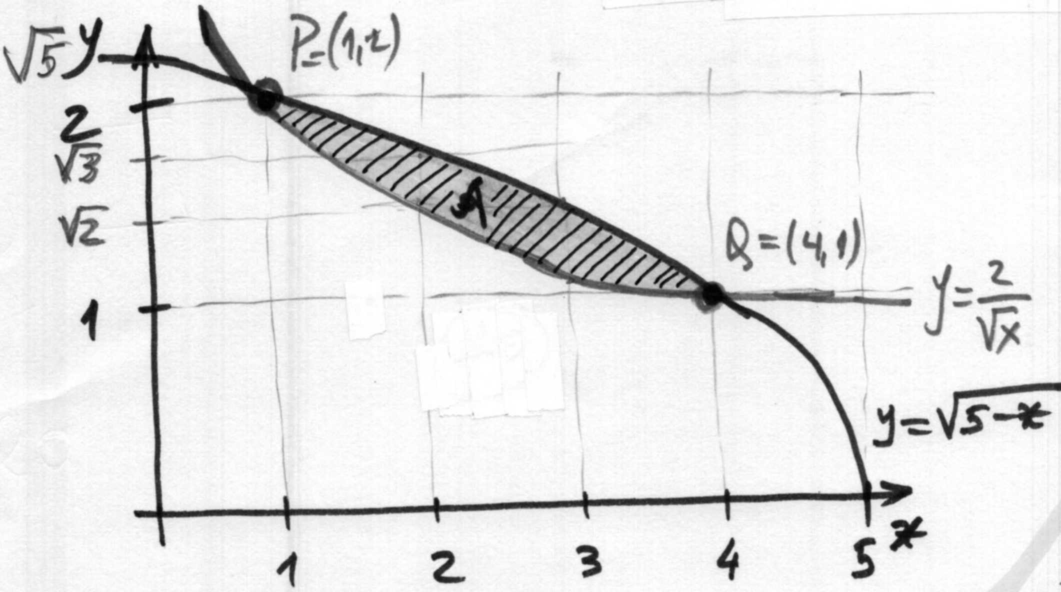
f é estritamente decrescente e convexa em I

$$g'(x) = \left[(5-x)^{1/2}\right]' = \tfrac{1}{2}(5-x)^{-1/2}(-1) = -\tfrac{1}{2}(5-x)^{-1/2} < 0$$

$$g''(x) = -\tfrac{1}{2}\left(-\tfrac{1}{2}\right)(5-x)^{-3/2}(-1) = -\tfrac{1}{4}(5-x)^{3/2} > 0$$

g é estrit. decrescente e côncava em I

…/…



$\sqrt{5} \simeq 2{,}2$

$\sqrt{2} \simeq 1{,}414$

$\sqrt{3} \simeq 1{,}7$

x	$\frac{2}{\sqrt{x}}$	$\sqrt{5-x}$
0	$+\infty$	$\sqrt{5}$
1	2	2
2	$\frac{2}{\sqrt{2}}=\sqrt{2}$	$\sqrt{3}$
3	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$
4	1	1

c)

$$A = \int_1^4 \left| \frac{2}{\sqrt{x}} - \sqrt{5-x} \right| dx = \int_1^4 \sqrt{5-x} - \frac{2}{\sqrt{x}}\, dx$$

$$A = \int_1^4 (5-x)^{1/2} - 2\, x^{-1/2}\, dx$$

Regra Barrow
$\supset$ f e g contínuas

$$A = \left[\frac{-(5-x)^{+1/2+1}}{+1/2+1} - \frac{2\, x^{-1/2+1}}{-1/2+1} \right]_1^4 = \left[-\frac{2}{3}(5-x)^{3/2} - 4\, x^{1/2} \right]_1^4$$

$$A = \left[-\frac{2}{3}(5-4)^{3/2} - 4\,(4)^{1/2} \right] - \left[-\frac{2}{3}(5-1)^{3/2} - 4\,(1)^{1/2} \right]$$

$$A = -\frac{2}{3}\, 1^{3/2} - 8 + \frac{2}{3}(4)^{3/2} + 4 = -\frac{2}{3}(1) - 4 + \frac{2}{3}\, 4\sqrt{4}$$

$$A = -\frac{2}{3}(1) - 4 + \frac{8}{3}(2) = \frac{-2-12+16}{3} = \underline{\underline{\frac{2}{3}}} > 0$$