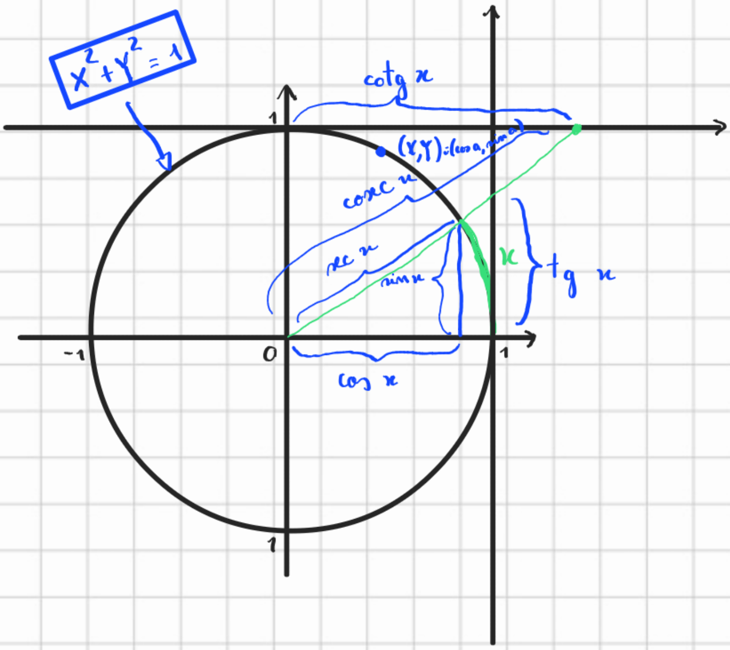
Aula 1

• Funções trigonométricas e as suas Inversas

Revisões:

$$\arcsin x = \sin^{-1} x$$



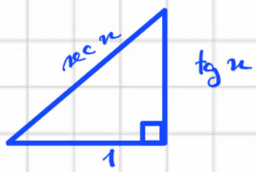
$$\sec x = \frac{1}{\cos x}$$

$$\cotg x = \frac{1}{tg\, x}$$

$$sen^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = sen^2 x$$

$$\sec x = \frac{1}{\cos x}, \quad x \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$



$$1 + \tan^2 x = sen^2 x$$

$$1 + \frac{sen^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 = \cos^2 x + \sin^2 x$$

Admito

$$\boxed{\sec x = \frac{1}{\cos x}}, \quad x \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

Utilizando

① $\operatorname{cosec} x = \dfrac{1}{\sin x}, \quad x \neq k\pi, \; k \in \mathbb{Z}$

② $\operatorname{cotg} x = \dfrac{1}{\operatorname{tg} x}$

$\operatorname{cosec}^2 x = 1 + \operatorname{cotg}^2 x, \; x \neq k\pi, k \in \mathbb{Z}$

"triangulo de circunferencia"

Mult. por $\sin^2 x$, $x \neq k\pi, k \in \mathbb{Z}$

$$\frac{1}{\operatorname{sen}^2 x} = 1 + \frac{1}{\operatorname{tg}^2 x} = 1 + \frac{\cos^2 x}{\sin^2 x}$$

$$\boxed{1 = \sin^2 x + \cos^2 x}$$

x	0 (0°)	$\pi/6$ (30°)	$\pi/4$ (45°)	$\pi/3$ (60°)	$\pi/2$ (90°)
$\sin x$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\cos x$	$\frac{\sqrt{4}}{2} = 2$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{0}}{2} = 0$
$\operatorname{tg} x = \frac{\sin x}{\cos x}$	$\frac{0}{2} = 0$	$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}$	$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$	"$+\infty$"
$\operatorname{cotg} x = \frac{1}{\tan x}$					
$\sec x = \frac{1}{\cos x}$	1	$\frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{6}$	$\frac{2}{\sqrt{2}} = \sqrt{2}$	2	"$+\infty$"
$\operatorname{cosec} x = \frac{1}{\sin x}$					

$\pi = 4 \times \operatorname{atan}(1)$

$\operatorname{tg} \frac{\pi}{4} = 1 \Leftrightarrow \frac{\pi}{4} = \arctan(1)$

• Funções Hiperbólicas (obtêm-se do gráfico da hipérbole padrão $X^2 - Y^2 = 0$)

Lê-se "seno hiperbólico" de x

$\boxed{\sinh x \text{ ou } \operatorname{senh} x}$

Na calculadora:

[hyp] [sin]

Sempre IR

$$\sinh : \underset{D_f}{\mathbb{R}} \longrightarrow \mathbb{R}$$

$$x \longmapsto y = \sinh x = \frac{e^x - e^{-x}}{2}$$

$\sin 0 = 0$

$\sinh 0 = \frac{e^0 - e^0}{2} = 0$

!

$D_f = \mathbb{R}$, $CD_f = \mathbb{R}$

$$(\sinh x)' = \left(\frac{e^x - e^{-x}}{2}\right)' = \frac{e^x + e^{-x}}{2}$$

$$= \cosh x$$

cosseno hiperbólico de x

$$(\cosh x)' = \left(\frac{e^x + e^{-x}}{2}\right)' = \frac{e^x - e^{-x}}{2}$$

$$= \sinh x$$

$\cosh x$

$$(\sinh x)'' = \sinh x$$

$$(\cosh x)'' = \cosh x$$

$$\cosh^2 x - \sinh^2 x = \left[\frac{e^x + e^{-x}}{2}\right]^2 - \left[\frac{e^x - e^{-x}}{2}\right]^2$$

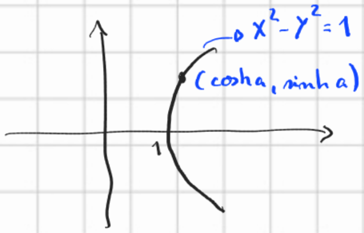
$$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4}$$

$$= 1$$

Relação Fundamental da geometria hiperbólica

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

Temos que todo o ponto de coordenadas $(\cosh a, \sinh a)$ satisfaz a equação da hipérbole



Mais:

$$\text{tgh}\, x = \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\text{cotgh}\, x = \frac{\cosh x}{\sinh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$$\text{sech}\, x = \frac{1}{\sinh x}$$

$$\text{cosech}\, x = \frac{1}{\cosh x}$$

$$(\text{tg}\, x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$(\text{tg}\, x)' = \sec^2 x$$

$$(\text{cotg}\, x)' = \left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

$$(\text{cotg}\, x)' = -\text{cosec}^2 x$$

$$(\text{tgh}\, x)' = \left(\frac{\sinh x}{\cosh x}\right)' = \frac{\cosh^2 x - \sinh^2 x}{\cos^2 h\, x} = \frac{1}{\cosh^2 x}$$

$$(\text{tgh}\, x)' = \text{sech}^2 x$$

$$(\text{cotgh}\, x)' = \left(\frac{\cosh x}{\sinh x}\right)' = \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{1}{\sinh^2 x}$$

$$(\text{cotg}\, x)' = -\text{cosech}^2 x$$

Regra da cadeia ("chain rule") → derivação da função composta

- $(\operatorname{sen} u)' = u' \cdot (\operatorname{sen} u)'$
  $= u' \cdot \cos u$

- $(\cos u)' = u' \cdot (\cos u)'$
  $= u' \cdot (-\operatorname{sen} u)$

! $\boxed{CD_h \subset D_g \wedge CD_g \subset D_f}$

Noutra notação:

$$[(f \circ g)(x)]' = [f(g(x))]' = g'(x) \cdot f'(g(x))$$

$$[f(g(h(x)))]' = h'(x) \cdot g'(h(x)) \cdot f'(g(h(x)))$$

Por exemplo:

$$f(x) = \sin(\cos(e^x))$$

$$f'(x) = [\sin(\cos(e^x))]' = (e^x)' \cdot \cos'(e^x) \cdot \sin'(\cos(e^x))$$

$$= e^x \cdot [-\sin(e^x)] \cdot \cos(\cos(e^x))$$

$$= -e^x \cdot \sin(e^x) \cdot \cos(\cos(e^x))$$

Caso interessante: $y(x) = \ln(-x^2)$ não está definido pois $D_y = \{\}$

$$CD_{-x^2} = \mathbb{R}_0^- \not\subset D_{\ln z} = \mathbb{R}^+$$

Pensar no domínio da função:

$$f(x) = (-e)^x$$

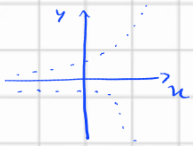
Pensar nos pontos onde $f(x) > 0$ $f(x) < 0$

$f(x)$ será contínua? Terá zeros?

Nota: $f(-1) = (-e)^{-1} = -\frac{1}{e} < 0$

$f(-2) = (-e)^{-2} = \frac{1}{(-e)^2} = \frac{1}{e^2} > 0$

$f(x) = (-e^x)$



$f(x) > 0$ quando x é par

$f(x) < 0$ quando x é ímpar

$f(x)$ não é contínua e não terá zeros