2. a) $\displaystyle\int \underbrace{(2x)}_{u'} \underbrace{\operatorname{arctg}(x^2)}_{v} = x^2\operatorname{arctg}(x^2) - \int x^2 \frac{2x}{1+x^4}\,dx$

$\displaystyle = x^2\operatorname{arctg}(x^2) - 2\frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx$

$\displaystyle = x^2\operatorname{arctg}(x^2) - \frac{1}{2}\ln(1+x^4) + C,$

$C \in \mathbb{R}$ em intervalos.

C.A.

$u' = 2x \quad v = \operatorname{arctg}(x^2)$

$u = x^2 \quad v' = \dfrac{2x}{1+x^4}$

$P u'v = uv - P uv'$

2. b) $\displaystyle\int \frac{1}{1+\cosh x}\,dx = \int \frac{1}{1+\frac{e^x+e^{-x}}{2}}\,dx$

$\displaystyle = \int \frac{2}{2+e^x+e^{-x}}\,dx$

$\displaystyle = \int \frac{2e^x}{2e^x+e^{2x}+1}\,dx$

$\displaystyle = \int \frac{2\cancel{t}}{2t+t^2+1}\,\frac{1}{\cancel{t}}\,dt$

$\displaystyle = 2\int \frac{1}{t^2+2t+1}\,dt$

$\displaystyle = 2\int \frac{1}{(t+1)^2}\,dt = 2\int (t+1)^{-2}\,dt$

$\displaystyle = 2\,\frac{(t+1)^{-2+1}}{-2+1} + C$

$\displaystyle = -\frac{2}{t+1} + C = -\frac{2}{e^x+1} + C,$

$C \in \mathbb{R}$ em intervalos.

Uma possível substituição:

C.A.

Subst.: $e^x = t$

$x = \varphi(t) = \ln t,\ t \in \mathbb{R}^+$

$\varphi'(t) = \dfrac{1}{t} > 0,\ \forall t \in \mathbb{R}^+$

φ é diferenciável e invertível em $\mathbb{R}^+$

$dx = \dfrac{1}{t}\,dt$

C.A.

$t^2+2t+1=0$

$t = \dfrac{-2 \pm \sqrt{4-4(1)}}{2}$

$t=-1 \ \vee\ t=-1$

$t^2+2t+1 = (t+1)(t+1)$