Aula 0

Wikidot Calculo

! site onde está todo o curso (CI/CII/CIII)

Pacotes Scorm → elearning

Determinar, em IR, o conjunto soluções das inequações:

1) $\frac{4x-1}{2x+3} > 1 \Leftrightarrow \frac{4x-1}{2x+3} - \frac{2x+3}{2x+3} > 0$

$\Leftrightarrow \frac{4x-2x-1-3}{2x+3} > 0$

$\Leftrightarrow \frac{2x-4}{2x+3} > 0$

Seja $f(x) = \frac{2x-4}{2x+3}$

$f(x) = 0 \Leftrightarrow 2x+3 \neq 0 \wedge 2x-4=0$

$\Leftrightarrow x \neq -\frac{3}{2} \wedge x = 2$

x	$-\infty$	$-\frac{3}{2}$		2	$+\infty$
$2x-4$	$-$	$-$	$-$	0	$+$
$2x+3$	$-$	0	$+$	$+$	$+$
$f(x)$	$+$	S.S.	$-$	0	$+$

→ C.S. $=]-\infty, -\frac{3}{2}[\cup [2, +\infty[$

2)

$$e^{\frac{1}{x}} + \cancel{2} < \cancel{2} + e \Leftrightarrow e^{\frac{1}{x}} < e$$

$$\Leftrightarrow \frac{1}{x} < 1$$

$$\Leftrightarrow \frac{1}{x} - 1 < 0$$

$$\Leftrightarrow \frac{1-x}{x} < 0$$

Seja $f(x) = \frac{1-x}{x}$

$$f(x) = 0 \Leftrightarrow \frac{1-x}{x} = 0 \Leftrightarrow 1 - x = 0 \land x \neq 0$$

$$\Leftrightarrow x = 1 \land x \neq 0$$

x	$-\infty$		0		1		$+\infty$
$1-x$	+	+	+	+	0	−	−
x	−	−	0	+	+	+	+
$f(x)$	−	−	S.S.	+	0	−	−

$$f(x) < 0 \Leftrightarrow x \in]-\infty, 0[\cup]1, +\infty[$$

$$C.S. =]-\infty, 0[\cup]1, +\infty[$$

3)

$$0 < \frac{x+1}{x^2+1} < 2 \Leftrightarrow \underbrace{0 < \frac{x+1}{x^2+1}}_{1} \land \underbrace{\frac{x+1}{x^2+1} < 2}_{2}$$

1

$$\frac{x+1}{x^2+1} > 0 \Leftrightarrow x + 1 > 0, \quad x^2+1 > 0 \ \forall x \in \mathbb{R}$$

$$\Leftrightarrow x > -1 \Leftrightarrow x \in]-1, +\infty[$$

2

$$\frac{x+1}{x^2+1} < 2 \Leftrightarrow \frac{x + 1 - 2x^2 - 2}{x^2+1} < 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{-2x^2 + x - 1}{x^2+1} < 0 \Leftrightarrow -2x^2 + x - 1 < 0, \quad x^2+1 > 0 \ \forall x \in \mathbb{R}$$

$\Leftrightarrow -2x^2 + x - 1 < 0$

Seja $g(x) = -2x^2 + x - 1$

$g(x) = 0 \Leftrightarrow x = \frac{-1 \pm \sqrt{1 - 4 \times (-2) \times (-1)}}{-4}$

$\Leftrightarrow x = \frac{-1 \pm \sqrt{-7}}{-4}$

$\Leftrightarrow x \notin \mathbb{R}$

Como $g(x) = 0 \Leftrightarrow x \notin \mathbb{R}$ logo $g(x)$ não tem zeros,
e como a função g é contínua em $\mathbb{R}$ e $g(1) = -2 + 1 - 1 = -2 < 0$,
logo $g(x) < 0 \ \forall x \in \mathbb{R}$

$0 < \frac{x+1}{x^2+1} < 2 \Leftrightarrow x > -1 \land -2x^2 + x - 1 < 0$

$\Leftrightarrow x > -1 \land g(x) < 0$

$\Leftrightarrow x > -1 \land x \in \mathbb{R}, \ g(x) < 0 \ \forall x \in \mathbb{R}$

$\Leftrightarrow x \in]-1, +\infty[$

C.S. $=]-1, +\infty[$

4)

$\frac{2x^2 + 1}{2\sqrt{2}x} \leq 1 \Leftrightarrow \frac{2x^2 + 1 - 2\sqrt{2}x}{2\sqrt{2}x} \leq 0$

Seja $f(x) = \frac{2x^2 + 1 - 2\sqrt{2}x}{2\sqrt{2}x}$

$f(x) = 0 \Leftrightarrow 2x^2 + 1 - 2\sqrt{2}x = 0 \land 2\sqrt{2}x \neq 0$

$\Leftrightarrow x = \frac{2\sqrt{2} \pm \sqrt{4 \times 2 - 4 \times 2 \times 1}}{4} \land x \neq 0$

$\Leftrightarrow x = \frac{2\sqrt{2} \pm \sqrt{0}}{4} \land x \neq 0$

$$\Leftrightarrow x = \frac{2\sqrt{2}}{4} \wedge x \neq 0$$

$$\Leftrightarrow x = \frac{\sqrt{2}}{2} \wedge x \neq 0$$

x	$-\infty$		0		$\frac{\sqrt{2}}{2}$	$+\infty$
$2x^2 - 2\sqrt{2}x + 1$		$+$	$+$	$+$	0	$+$
$2\sqrt{2}x$		$-$	0	$+$	$+$	$+$
$f(x)$		$-$	S.S.	$+$	0	$+$

$$f(x) \leq 0 \Leftrightarrow x \in \left]-\infty, 0\right[\cup \left\{\frac{\sqrt{2}}{2}\right\}$$

$$C.S = \left]-\infty, 0\right[\cup \left\{\frac{\sqrt{2}}{2}\right\}$$

5)

$$\frac{1}{\sqrt{x-1}} \leq 1 \Leftrightarrow \frac{1 - \sqrt{x-1}}{\sqrt{x-1}} \leq 0$$

$$\Leftrightarrow 1 - \sqrt{x-1} \leq 0 \wedge \sqrt{x-1} \neq 0 \quad , \quad \sqrt{x-1} \geq 0 \ \forall x \in \mathbb{R}$$

$$\Leftrightarrow \sqrt{x-1} \geq 1 \wedge x \neq 1$$

$$\Leftrightarrow x - 1 \geq 1 \wedge x \neq 1$$

$$\Leftrightarrow x \geq 2 \wedge x \neq 1$$

$$\Leftrightarrow x \in \left[2, +\infty\right[$$

6)

$$\frac{\ln(x+e) + x}{x+1} < 1$$

$$\Leftrightarrow \frac{\ln(x+e) + \cancel{x} - \cancel{x} - 1}{x+1} < 0 \Leftrightarrow \frac{\ln(x+e) - 1}{x+1} < 0$$

Seja $f(x) = \frac{\ln(x+e)-1}{x+1}$

$D_f = \{x \in \mathbb{R} : x+e>0 \wedge x+1 \neq 0\} =]-e,+\infty[\wedge x \neq -1$

$f(x)=0 \Leftrightarrow \ln(x+e)-1=0 \wedge x+1\neq 0$

$\Leftrightarrow \ln(x+e)=1 \wedge x\neq -1$

$\Leftrightarrow \cancel{\ln}(x+e) = \cancel{\ln}(e)$

$\Leftrightarrow x+e=e \wedge x\neq -1$

$\Leftrightarrow x=0 \wedge x\neq -1$

x	$-e$		-1		0		$+\infty$
$\ln(x+e)-1$		$-$	$-$	$-$	0	$+$	$+$
$x+1$		$-$	0	$+$	$+$	$+$	$+$
$f(x)$		$+$	S.S.	$-$	0	$+$	$+$

$f(x)<0 \Leftrightarrow x \in]-1,0[$

$C.S =]-1,0[$

7)

$\ln(x) > \frac{x-1}{\sqrt{x}}$

$\Leftrightarrow \ln(x) - \frac{x-1}{\sqrt{x}} > 0$

$f(x) = \ln(x) - \frac{x-1}{\sqrt{x}}$

$D_f =]0,+\infty[$

Sabemos que $f(x)=0 \Leftrightarrow x=1$

$$f'(x) = \frac{1}{x} - \frac{(x-1)' \times (\sqrt{x}) - (x-1) \times (\sqrt{x})'}{x}$$

$$= \frac{1}{x} - \frac{1 \times \sqrt{x} - (x-1) \times (x^{\frac{1}{2}})'}{x}$$

$$= \frac{1}{x} - \frac{\sqrt{x} - (x-1) \times \left(\frac{1}{2} \times x^{-\frac{1}{2}} \times 1\right)}{x}$$

$$= \frac{1}{x} - \frac{\sqrt{x} - (x-1) \times \frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{1}{x} - \frac{\sqrt{x} - \frac{x-1}{2\sqrt{x}}}{x}$$

$$= \frac{1 - \sqrt{x} + \frac{x-1}{2\sqrt{x}}}{x} = \frac{\frac{2\sqrt{x}}{2\sqrt{x}} - \frac{2\sqrt{x} \times \sqrt{x}}{2\sqrt{x}} + \frac{x-1}{2\sqrt{x}}}{x}$$

$$= \frac{2\sqrt{x} - 2x + x - 1}{2x\sqrt{x}} = \frac{2\sqrt{x} - x - 1}{2x\sqrt{x}}$$

$$= \frac{-x + 2\sqrt{x} - 1}{2x\sqrt{x}} = \frac{-(x - 2\sqrt{x} + 1)}{2x\sqrt{x}}$$

$$= \frac{-(\sqrt{x} - 1)^2}{2x\sqrt{x}}$$

Como $D_f =]0, +\infty[\implies 2x\sqrt{x} > 0 \quad \forall x \in D_f$

$-(\sqrt{x} - 1)^2 < 0, \forall x \in D_f \setminus \{1\}$

$f'(x) < 0, \forall x \in D_f \setminus \{1\}$

$\hookrightarrow$ logo f é estritamente crescente

x	0		1		$+\infty$
$f'(x)$		$-$	0	$-$	
$f(x)$		$\searrow$	0	$\searrow$	
$f(x)$		$+$	0	$-$	

$f(x) > 0 \iff x \in]0,1[\qquad C.S. =]0,1[$