Ficha 4

$\boxed{1}$

a)

$$\int_0^{+\infty} \frac{5}{4+x^2}\,dx = \lim_{t\to+\infty} \int_0^t \frac{5}{4+x^2}\,dx$$

$$\int \frac{5}{4+x^2}\,dx = \frac{5}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx = \frac{5}{2}\int \frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}\,dx = \frac{5}{2}\operatorname{arctg}\left(\frac{x}{2}\right)$$

Logo, $\displaystyle\lim_{t\to+\infty}\int_0^t \frac{5}{4+x^2}\,dx = \lim_{t\to+\infty}\left[\frac{5}{2}\operatorname{arctg}\left(\frac{x}{2}\right)\right]_0^t$

$$= \frac{5}{2}\left(\operatorname{arctg}(+\infty) - \operatorname{arctg}(0)\right)$$

$$= \frac{5}{2}\times\frac{\pi}{2} = \frac{5\pi}{4}$$

b)

$$\int_\pi^{+\infty} \cos(3x)\,dx = \frac{1}{3}\lim_{t\to+\infty}\int_\pi^t 3\cos(3x)\,dx = \frac{1}{3}\lim_{t\to+\infty}\left[\sin(3x)\right]_\pi^t$$

$$= \frac{1}{3}\lim_{t\to+\infty}\left(\sin(3t) - \underbrace{\cancel{\sin(3\pi)}}_{0}\right)$$

$$= \frac{1}{3}\sin(+\infty)$$

Não está definido, logo o integral dado diverge

c)

$$\int_{-\infty}^{2} \frac{1}{(4-x)^2}\,dx = -\lim_{b\to-\infty}\int_b^2 -(4-x)^{-2}\,dx = -\lim_{b\to-\infty}\left[-\frac{1}{4-x}\right]_b^2$$

$$= -\lim_{b\to-\infty}\left(-\frac{1}{2} + \frac{1}{4-b}\right]$$

$$= \frac{1}{2} - \frac{1}{4-(-\infty)} = \frac{1}{2} - \cancel{\frac{1}{+\infty}}^{\,0} = \frac{1}{2}$$

d)

$$\int_{-\infty}^{+\infty} x\,dx = \underbrace{\lim_{b\to-\infty}\int_b^0 x\,dx}_{\text{Divergente, logo o dado também é DIV}} + \lim_{t\to+\infty}\int_0^t x\,dx$$

e)

$$\int_{-\infty}^{+\infty} \frac{1}{1+x^2}\,dx = \lim_{b\to-\infty} \int_b^0 \frac{1}{1+x^2}\,dx + \lim_{t\to+\infty}\int_0^t \frac{1}{1+x^2}\,dx$$

$$= \cancel{\operatorname{arctg}(0)}^{0} - \operatorname{arctg}(-\infty) + \operatorname{arctg}(+\infty) - \cancel{\operatorname{arctg}(0)}^{0}$$

$$= -\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = \pi$$

f)

$$\int_{-\infty}^{0} x e^{-x^2}\,dx = -\frac{1}{2}\lim_{b\to-\infty}\int_b^0 -2x e^{-x^2}\,dx = -\frac{1}{2}\lim_{b\to-\infty}\left[e^{-x^2}\right]_b^0$$

$$= -\frac{1}{2}\left(e^0 - e^{-(-\infty)^2}\right) = -\frac{1}{2}\left(1 - e^{-\infty}\right) = -\frac{1}{2}(1-0) = -\frac{1}{2}$$

g)

$$\int_{-\infty}^{+\infty} e^{-|x|}\,dx = \lim_{b\to-\infty}\int_b^0 e^{x}\,dx - \lim_{t\to+\infty}\int_0^{+\infty} -e^{-x}\,dx$$

$$|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$$

$$= \lim_{b\to-\infty}\left[e^{x}\right]_{-\infty}^{0} - \lim_{t\to+\infty}\left[e^{-x}\right]_0^{+\infty}$$

$$= e^0 - e^{-\infty} - \left(e^{-\infty} - e^0\right)$$

$$= 1 - 0 - 0 + 1$$

$$= 2$$

h)

$$\int_0^{+\infty} \frac{1}{\sqrt[3]{x+1}}\,dx = \lim_{t\to+\infty}\int_0^t (x+1)^{-1/3}\,dx = \lim_{t\to+\infty}\left[(x+1)^{2/3}\times 3/2\right]_0^t$$

$$= (+\infty)^{2/3}\times\frac{3}{2} - 1\times\frac{3}{2}$$

$= +\infty - \frac{3}{2} = +\infty$, logo o integral dado é divergente

i)

$$\int_{-\infty}^{+\infty} \frac{x}{(1+x^2)^2}\,dx = \frac{1}{2}\lim_{b\to-\infty}\int_b^0 (2x)(1+x^2)^{-2}\,dx + \frac{1}{2}\lim_{t\to+\infty}\int_0^t (2x)(1+x^2)^{-2}\,dx$$

$$= \frac{1}{2}\lim_{b\to-\infty}\left[-\frac{1}{1+x^2}\right]_b^0 + \frac{1}{2}\lim_{t\to+\infty}\left[-\frac{1}{1+x^2}\right]_0^t$$

$$= \frac{1}{2}\times(-1) - \frac{1}{2}\times\left(-\frac{1}{+\infty}\right) + \frac{1}{2}\times\left(-\frac{1}{+\infty}\right) - \frac{1}{2}\times(-1)$$

$$= -\frac{1}{2} + 0 - 0 + \frac{1}{2} = 0$$

j)

$$\int_e^{+\infty} \ln x \, dx = \lim_{t\to+\infty} \int_e^t \ln x \, dx = \lim_{t\to+\infty} \left[x(\ln(x)-1)\right]_e^t$$

$$= +\infty(+\infty - 1) = +\infty \quad \text{DIV}$$

$$\int 1 \times \ln x \, dx = x\ln(x) - x$$
$$= x(\ln(x) - 1)$$

$u = x \quad v = \ln(x)$

$u' = 1 \quad v' = \frac{1}{x}$

K)

$$\int_1^{+\infty} \frac{1}{x}(\ln x)^3 \, dx = \lim_{t\to+\infty} \left[[\ln(x)]^2 \times \frac{1}{2}\right]_1^t = +\infty - 0 = +\infty \quad \text{DIV}$$

l)

$$\int_{-\infty}^0 \frac{4}{1+(x+1)^2} \, dx = 4\lim_{b\to-\infty} \int_b^0 \frac{1}{1+(x+1)^2} \, dx = 4\lim_{b\to-\infty} \left[\operatorname{arctg}(x+1)\right]_b^0$$

$$= 4\operatorname{arctg}(1) - 4\operatorname{arctg}(-\infty)$$
$$= \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) - 4\left(-\frac{\pi}{2}\right)$$
$$= \pi + 2\pi = 3\pi$$

m)

$$\int_{-\infty}^0 \frac{e^{\operatorname{arctg} x}}{1+x^2} \, dx = \lim_{b\to-\infty} \int_b^0 (\operatorname{arctg} x)' e^{\operatorname{arctg} x} \, dx$$

$$= \lim_{b\to-\infty} \left[e^{\operatorname{arctg} x}\right]_b^0 = e^{\operatorname{arctg}(0)} - e^{\operatorname{arctg}(-\infty)} = e^0 - e^{-\frac{\pi}{2}} = 1 - e^{-\pi/2}$$

$\boxed{2}$

a)

$$\int_1^{+\infty} \frac{\operatorname{sen}^2 x}{x^{5/2}} \, dx$$

$0 \leq \frac{\operatorname{sen}^2 x}{x^{5/2}} \leq \frac{1}{x^{5/2}}$, logo como $\int_1^{+\infty} \frac{1}{x^{5/2}} dx$ converge (Dirichlet $\alpha = 5/2 > 1$) então o integral dado também converge

b)

$$\int_1^{+\infty} \frac{2x}{e^{2x}-1}$$

$$L = \lim_{x\to+\infty} \frac{\frac{2x}{e^{2x}-1}}{\frac{2x}{e^{2x}}} = \lim_{x\to+\infty} \frac{\cancel{e^{2x}} \times \cancel{2x}}{\cancel{2x}\,\cancel{e^{2x}}\left(1-\frac{1}{e^{2x}}\right)} = 1 \in \mathbb{R}^+ \text{ logo } \int_1^{+\infty} \frac{2x}{e^{2x}} dx \text{ tem a mesma natureza do integral dado}$$

$$\int_1^{+\infty} 2x\, e^{-2x} \, dx = -\lim_{t\to+\infty} \int_1^t (-2x)\, e^{-2x} \, dx = -\lim_{t\to+\infty} \left[e^{-2x}\right]_1^t$$

$$= -e^{-\infty} + e^{-2} = -0 + e^{-2} = \frac{1}{e^2}$$ CONV

$\hookrightarrow$ logo o dado também converge

c)

$$\int_1^{+\infty} \frac{5x^2-3}{x^8+x-1}\,dx = \int_1^{+\infty} \frac{\cancel{x^2}(5-\frac{3}{x^2})}{\cancel{x^2}(x^6+\frac{1}{x}-\frac{1}{x^2})}$$

$$L = \lim_{x\to+\infty} \frac{\frac{5-3/x^2}{x^6+1/x-1/x^2}}{\frac{1}{x^6}} = \lim_{x\to+\infty} \frac{\cancel{x^6}(5-3/x^2)}{\cancel{x^6}(1+1/x^7-1/x^8)} = \frac{5-0}{1+0-0} = 5 \in \mathbb{R}^+,\text{ logo}$$

$\int_1^{+\infty} \frac{1}{x^6}\,dx$ que converge (Dirichlet $\alpha = 6 > 1$) tem a mesma natureza do integral dado, logo o dado converge

d)

$$\int_0^{+\infty} e^{x^2}\,dx$$

$$0 \leq e^x \leq e^{x^2}$$

Logo, como $\int_0^{+\infty} e^x\,dx$ diverge (Dirichlet $\beta = 1 \geq 0$), logo o dado também diverge

$\boxed{3}$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{0} \frac{1}{x-1}\,dx + \int_{0}^{+\infty} \operatorname{arctg}(x)\,dx$$

não está def. $\longrightarrow$ $\operatorname{arctg}(0) = 0$

Para o integral dado convergir ambos os limites têm de existir e ser finitos:

$$\lim_{b\to-\infty} \int_b^0 \frac{1}{x-1}\,dx \quad \text{e} \quad \lim_{t\to+\infty} \int_0^t \operatorname{arctg}(x)\,dx$$

$\int_0^{+\infty} \operatorname{arctg}(x)\,dx$ tem a mesma natureza de $\int_1^{+\infty} \operatorname{arctg}(x)\,dx$

Como $0 \leq \frac{\pi}{4} \leq \operatorname{arctg}(x)$, $x \in [1, +\infty[$ e $\int_1^{+\infty} \frac{\pi}{4}\,dx$ diverge então o o integral $\int_0^{+\infty} \operatorname{arctg}(x)\,dx$ diverge e consequentemente $\lim_{t\to+\infty} \int_0^t \operatorname{arctg}(x)\,dx$ não é finito, logo o integral dado Diverge

$\boxed{4}$

$$\int_1^{+\infty} \frac{\cos x}{x^3}\, dx$$

Estudando o integral dos módulos

$$\int_1^{+\infty} \frac{|\cos x|}{|x^3|}\, dx = \int_1^{+\infty} \frac{|\cos x|}{x^3}$$

Como $0 \leq \frac{|\cos x|}{x^3} \leq \frac{1}{x^3}$

Como $\int_1^{+\infty} \frac{1}{x^3}\, dx$ converge (Dirichlet $\alpha = 3 > 1$) logo o integral dado é absolutamente convergente

$\boxed{5}$

a) $\int_{-1}^{0} \frac{x}{\sqrt{1-x^2}}\, dx$, integral impróprio de 2ª espécie!

• a função integrada não está definida para $x = -1$

$$= \lim_{b \to -1^+} \int_b^0 x \times (1-x^2)^{-1/2}\, dx = \frac{-1}{2} \lim_{b \to -1^+} \int_b^0 (-2x)(1-x^2)^{-1/2} dx$$

$$= -\frac{1}{\cancel{2}} \lim_{b \to -1^+} \left[\cancel{2}(1-x^2)^{1/2}\right]_b^0 = -\left(1 - (1-(-1^+))^{1/2}\right)$$

$$= -1 + 0 = -1$$

b) $\cot g\, x = \frac{\cos x}{\sin x}$

$\int_{\pi/2}^{\pi} \cot g\, x\, dx$, integral impróprio de 2ª espécie

a função integrada não está definida para $x = \pi$

$$= \lim_{t \to \pi^-} \int_{\pi/2}^{t} \cot g\, x\, dx = \lim_{t \to \pi^-} \int_{\pi/2}^{t} \frac{(\sin x)'}{\sin x}\, dx$$

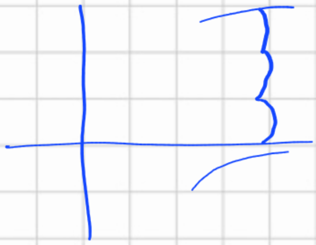
$$= \lim_{t \to \pi^-} \left[\ln|\sin x|\right]_{\pi/2}^{t} = \ln|\sin(\pi^-)| - \ln\left|\sin \frac{\pi}{2}\right|$$

$= \ln(0^+) - \ln(1) = -\infty - 0 = -\infty$, logo o integral dado diverge

c) $$\int_{-1}^{3} \frac{1}{9-x^2}\,dx = \lim_{t\to 3^-} \int_{-1}^{t} \frac{1}{9-x^2}\,dx = -\lim_{t\to 3^-} \int_{-1}^{t} \frac{1}{(x-3)(x+3)}\,dx$$

$$\frac{1}{(x-3)(x+3)} = \frac{A}{(x-3)} + \frac{B}{(x+3)} \Leftrightarrow 1 = A(x+3) + B(x-3)$$

$$\begin{cases} A+B=0 \\ 3A-3B=1 \end{cases} \begin{cases} A=-B \\ B=-1/6 \end{cases} \begin{cases} A=1/6 \\ B=-1/6 \end{cases}$$



$$= -\lim_{t\to 3^-}\left(\left[\frac{1}{6}\ln|x-3|\right]_{-1}^{t} - \left[\frac{1}{6}\ln|x+3|\right]_{-1}^{t}\right)$$

$$= -\left(\left(\frac{1}{6}\ln|0^-| - \frac{1}{6}\ln|-4|\right) - \left(\frac{1}{6}\ln(6) - \frac{1}{6}\ln(2)\right)\right)$$

$$= -\frac{1}{6}\left(-\infty - \ln(4) - \ln(6) + \ln(2)\right) = -\frac{1}{6}\times(-\infty) = +\infty$$, logo o integral dado diverge

d) $$\int_0^1 \ln x\,dx = \lim_{b\to 0^+}\int_b^1 \underbrace{1}_{u'}\times\underbrace{\ln(x)}_{v}\,dx = \lim_{b\to 0^+}\left(\left[x\ln(x)\right]_b^1 - \int_b^1 x\times\frac{1}{x}\,dx\right)$$

$u = x \quad v = \ln(x)$

$u' = 1 \quad v' = \frac{1}{x}$

$$= \lim_{b\to 0^+}\left(-b\ln(b) - \left[x\right]_b^1\right)$$

$$= -\lim_{b\to 0^+}\left(b\ln(b)\right) - 1 + 0^+$$

$$= -\lim_{b\to 0^+}\left(b\ln(b)\right) - 1$$

$$\lim_{b\to 0^+}\left(b\ln(b)\right) = \lim_{b\to 0^+}\left(\frac{\ln(b)}{\frac{1}{b}}\right)$$

$\left(\frac{\infty}{\infty}\right)$ R.C.

$$= \lim_{b\to 0^+}\left(\frac{\frac{1}{b}}{-\frac{1}{b^2}}\right) = \lim_{b\to 0^+}(-b) = 0 \qquad = 0 - 1 = -1$$

e) $$\int_{-2}^{1}\frac{1}{|x|}\,dx = \int_{-2}^{0}\frac{1}{-x}\,dx + \int_0^1 \frac{1}{x}\,dx$$

$$= -\lim_{t\to 0^-}\left[\ln|x|\right]_{-2}^{b} + \lim_{b\to 0^+}\left[\ln|x|\right]_b^1$$

$$= -\left(\ln(0^+) - \ln(2)\right) + \ln(1) - \ln(0^+)$$

$$= -2\ln(0^+) + \ln(2) + \ln(1) = -(-\infty) = +\infty$$, logo DIV

$\boxed{6}$

a) $$\int_0^1 \frac{\pi}{1-\sqrt{x}}\,dx = \lim_{b\to 1^-} \int_0^b \frac{\pi}{1-\sqrt{x}}\,dx$$

$$L = \lim_{b\to 1^-} \frac{\frac{\cancel{\pi}}{1-\sqrt{x}}}{\frac{\cancel{\pi}}{1-x}} = \lim_{b\to 1^-} \frac{1-x}{1-\sqrt{x}} \overset{\left(\frac{0}{0}\right)}{=} \lim_{b\to 1^-} \frac{-1}{-\frac{1}{2}\times\frac{1}{\sqrt{x}}} = \frac{-1}{-\frac{1}{2}\times\frac{1}{1}} = 2$$

$L \in \mathbb{R}^+$, logo tem a mesma natureza de $\int_0^1 \frac{\pi}{1-x}\,dx$

$$= -\pi \lim_{t\to 1^-} \Big[\ln|1-x|\Big]_0^t = -\pi(-\infty) = +\infty$$

logo DIV