Ficha 5

$\boxed{1}$

a)

$$\sum_{n=1}^{+\infty} 2^n \rightarrow a_n = 2^n \text{ sucessão geométrica} \quad r = 2 \qquad S_n = a_1 \times \frac{1-r^n}{1-r}$$

Notação perigosa $\neq$

$$S_n = 2 \times \frac{1-2^n}{1-2} = -2 + 2\times 2^n = 2^{n+1} - 1$$

$$\lim_{n\to+\infty} (S_n) = 2^{+\infty} = +\infty \text{, logo DIV}$$

b)

$$\sum_{n=1}^{+\infty} 2n \rightarrow a_n = 2n \quad \text{sucessão aritmética} \qquad S_n = \left(\frac{a_1 + a_n}{2}\right) \times n$$

$$S_n = \frac{2 + 2n}{2} \times n = n + n^2$$

$$\lim_{n\to+\infty} (S_n) = +\infty \text{, logo DIV}$$

c)

$$\sum_{n=1}^{+\infty} 3^{-2n+3} = \sum_{n=1}^{+\infty} \left(\frac{1}{9}\right)^n \times 27$$

$$S_n = 3 \times \frac{1-\left(\frac{1}{9}\right)^n}{1-\frac{1}{9}} = \frac{27}{8}\left(1-\left(\frac{1}{9}\right)^n\right)$$

$$\lim_{n\to+\infty} S_n = \frac{27}{8} \times \left(1 - \underbrace{\left(\frac{1}{9}\right)^{+\infty}}_{<1}\right) = \frac{27}{8}$$

d)

$$\sum_{n=2}^{+\infty} \frac{2}{n^2-1} = \sum_{n=2}^{+\infty} \frac{2}{(n-1)(n+1)} = \sum_{n=2}^{+\infty} \left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

$$\frac{2}{(n-1)(n+1)} = \frac{A}{(n-1)} + \frac{B}{(n+1)} \Leftrightarrow 2 = A(n+1) + B(n-1)$$

$$\begin{cases} A + B = 0 \\ A - B = 2 \end{cases} \quad \begin{cases} A = -B \\ B = -1 \end{cases} \quad \begin{cases} A = 1 \\ B = -1 \end{cases}$$

$$S_n = \frac{1}{1} + \frac{1}{2} + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \dots + \cancel{\frac{1}{n-2}} + \cancel{\frac{1}{n-1}}$$

$$- \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} - \dots - \cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n-1}} - \frac{1}{n} - \frac{1}{n+1}$$

$$= \frac{3}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$\lim_{n\to+\infty} (S_n) = \frac{3}{2} - 0 - 0 = \frac{3}{2}$$

e) $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{1}{n+2} - \frac{1}{n}\right)$

$$S_n = \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{5}} + \dots + \cancel{\frac{1}{n}} + \frac{1}{n+1} + \frac{1}{n+2}$$
$$-\frac{1}{1} - \frac{1}{2} - \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} - \dots - \cancel{\frac{1}{n}} = -\frac{3}{2} + \frac{1}{n+1} + \frac{1}{n+2}$$

$$\lim_{n\to+\infty} S_n = -\frac{3}{2}$$

f) $\displaystyle\sum_{n=1}^{+\infty} \frac{2n+1}{n^2(n+1)^2}$

$$\frac{2n+1}{n^2(n+1)^2} = \frac{A}{n} + \frac{B}{n^2} + \frac{C}{n+1} + \frac{D}{(n+1)^2}$$

$$\Leftrightarrow 2n+1 = An(n^2+2n+1) + B(n^2+2n+1) + C(n^2+n) + Dn^2$$
$$\Leftrightarrow 2n+1 = A(n^3+2n^2+n) + B(n^2+2n+1) + C(n^2+n) + Dn^2$$

$$\begin{cases} A = 0 \\ \cancel{2A}+B+\cancel{C}+D=0 \\ \cancel{A}+2B+C = 2 \\ B = 1 \end{cases} \qquad \begin{cases} A = 0 \\ D = -1 \\ C = 0 \\ B = 1 \end{cases}$$

$$= \sum_{n=1}^{+\infty} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) =$$

$$S_N = \sum_{n=1}^{N} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$S = \lim_{N\to+\infty} S_N$$

$$S_n = \frac{1}{1} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{9}} + \dots + \cancel{\frac{1}{n^2}}$$
$$- \cancel{\frac{1}{4}} - \cancel{\frac{1}{9}} - \dots - \cancel{\frac{1}{n^2}} - \frac{1}{(n+1)^2}$$
$$= 1 - \frac{1}{(n+1)^2}$$

$$\lim_{n\to+\infty} (S_n) = 1$$

$\boxed{2}$

$$\sum_{n=1}^{+\infty}\left[\left(\frac{1}{2}\right)^{2n-1}+b_n\right] = \sum_{n=1}^{+\infty}\left[\left(\frac{1}{2}\right)^{2n-1}\right] + \sum_{n=1}^{+\infty} b_n$$

$$\sum_{n=1}^{+\infty}\left(\frac{1}{4}\right)^n \times 2 = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$$\sum_{n=1}^{+\infty} b_n = \lim_{k\to+\infty} S_k = \lim_{k\to+\infty} \sqrt[k]{\frac{e}{k^k}} = \lim_{k\to+\infty}\frac{e^{1/k}}{k} = \frac{1}{+\infty} = 0$$

Logo, a soma da série dada é $\frac{2}{3}+0=\frac{2}{3}$

$\boxed{3}$

$$\sum_{n=1}^{+\infty}\left[3a_n+\frac{2}{3^n}\right] = 3\sum_{n=1}^{+\infty} a_n + \sum_{n=1}^{+\infty}\frac{2}{3^n}$$

Pelo enunciado: $\sum_{n=1}^{+\infty} a_n = S$

$$\sum_{n=1}^{+\infty}\frac{2}{3^n} = \frac{\frac{2}{3}}{\frac{3}{3}-\frac{1}{3}} = \frac{\cancel{\frac{2}{3}}}{\cancel{\frac{2}{3}}} = 1$$

$$r = \frac{\frac{\cancel{2}}{3^{n+1}}}{\frac{\cancel{2}}{3^n}} = \frac{1}{3}$$

Logo, a soma da série dada é $3S+1$

$\boxed{4}$

$$\sum_{n=1}^{+\infty} u_n = \sum_{n=1}^{3} 1+2(n-1) + \sum_{n=4}^{+\infty}\left(\frac{2}{3}\right)^n$$

$$\sum_{n=1}^{3} = 1+2(1-1)+1+2(2-1)+1+2(3-1)$$
$$= 1+0+1+2+1+4$$
$$= 9$$

$$\sum_{n=4}^{+\infty}\left(\frac{2}{3}\right)^n = \frac{\frac{2^4}{3^4}}{\frac{3}{3}-\frac{2}{3}} = \frac{\frac{2^4}{3^4}}{\frac{1}{3}} = \frac{16}{27}$$

Logo, a série dada é $9+\frac{16}{27} = \frac{243+16}{27} = \frac{259}{27}$

$\boxed{5}$

a)

$$\sum_{n=1}^{+\infty} \frac{5^n}{(a+1)^n} = \sum_{n=1}^{+\infty} \left(\frac{5}{a+1}\right)^n$$

A série é convergente se $\frac{5}{a+1} \in]-1,1[$

$$\frac{5}{a+1} > -1 \ \wedge \ \frac{5}{a+1} < 1 \ , \ a \in \mathbb{R} \setminus \{-1\}$$

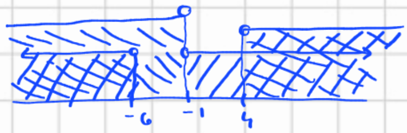
$$\Leftrightarrow \frac{5}{a+1} + \frac{a+1}{a+1} > 0 \ \wedge \ \frac{5}{a+1} - \frac{a+1}{a+1} < 0$$

$$\Leftrightarrow \frac{a+6}{a+1} > 0 \ \wedge \ \frac{4-a}{a+1} < 0 \ , \ a \in \mathbb{R} \setminus \{-1\}$$

	$-\infty$	-6		-1	$+\infty$
$a+6$	$-$	0	$+$	S.S.	$+$
$a+1$	$-$	$-$	$-$	S.S.	$+$
$\frac{a+6}{a+1}$	$+$	0	$-$	S.S.	$+$

	$-\infty$	-1		4	$+\infty$
$4-a$	$+$	S.S.	$+$	0	$-$
$a+1$	$-$	S.S.	$+$	$+$	$+$
$\frac{4-a}{a+1}$	$-$	S.S.	$+$	0	$-$

$$\Leftrightarrow a \in \left[\left(]-\infty,-6[\ \cup \ ]-1,+\infty[\right) \cap \left(]-\infty,-1[\ \cup \ ]4,+\infty[\right)\right]$$



$$\Leftrightarrow a \in [-\infty,-6[\ \cup \ ]4,+\infty[$$

b)

$a = 5$

$$\sum_{n=1}^{+\infty} \left(\frac{5}{6}\right)^n = \frac{\frac{5}{6}}{1-\frac{5}{6}} = \frac{\frac{5}{\cancel{6}}}{\frac{1}{\cancel{6}}} = 5$$

$\boxed{7}$

a)

i) Falso, prova-se que se $\lim_{n\to+\infty} a_n \neq 0$ então ela é divergente

ii) Verdadeiro,

iii) Verdadeiro,

b)

i) Falso, a soma da série pode ser qualquer valor real

ii) Falso,

iii) Verdadeiro,

a)

$$\sum_{n=1}^{+\infty} \operatorname{sen}\left(\frac{n^2\pi}{2}\right) \quad , \; a_n = \operatorname{sen}\left(\frac{n^2\pi}{2}\right)$$

Como $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \operatorname{sen}\left(n^2\frac{\pi}{2}\right) = \underbrace{\operatorname{sen}(+\infty)}_{\in[-1,1]} \neq 0$, logo a série diverge

b)

$$\sum_{n=1}^{+\infty} \frac{1}{\ln n}$$

$$L = \lim_{n\to+\infty} \frac{\frac{1}{\ln n}}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{n}{\ln n} \underset{R.C.}{\overset{\left(\frac{+\infty}{+\infty}\right)}{=}} \lim_{n\to+\infty} \frac{1}{\frac{1}{n}} = +\infty$$

Pelo critério do limite, se $L = +\infty$ e $\sum_{n=1}^{+\infty} \frac{1}{n}$ divergir, então a série dada diverge

Como $L = +\infty$ e $\sum_{n=1}^{+\infty} \frac{1}{n}$ diverge (Dirichlet $\alpha = 1 \leq 1$), logo a dada diverge

c)

$$\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \frac{\ln x}{x} \underset{R.C.}{\overset{\left(\frac{+\infty}{+\infty}\right)}{=}} \lim_{n\to+\infty} \frac{\frac{1}{x}}{1} = 0 \checkmark$$

$$\sum_{n=1}^{+\infty} \frac{\ln n}{n}$$

$$L = \lim_{n\to+\infty} \frac{\frac{\ln n}{\cancel{n}}}{\frac{1}{\cancel{n}}} = +\infty$$

$L = +\infty$ e $\sum_{n=1}^{+\infty} \frac{1}{n}$ diverge (Dirichlet $\alpha = 1 \leq 1$) então a série dada diverge

d)

$$\sum_{n=1}^{+\infty} \frac{n+1}{\sqrt{2n^5+n^3}}$$

$$L = \lim_{n\to+\infty} \frac{\frac{n+1}{\sqrt{2n^5+n^3}}}{\frac{n}{\sqrt{n^5}}} = \lim_{n\to+\infty} \frac{n^{5/2}(n+1)}{n\sqrt{2n^5+n^3}} = \lim_{n\to+\infty} \frac{(n^5)^{1/2}\left(1+1/n\right)}{(2n^5+n^3)^{1/2}}$$

$$= \lim_{n\to+\infty}\left[\left(1+\frac{1}{n}\right)\times\left(\frac{n^5}{2n^5+n^3}\right)^{1/2}\right] = \lim_{n\to+\infty}\left[\left(1+\frac{1}{n}\right)\times\left(\frac{1}{2+1/n^2}\right)^{1/2}\right]$$

$$= \left(1+\overset{0}{\frac{1}{+\infty}}\right)\times\left(\frac{1}{2+\overset{0}{1/+\infty}}\right)^{1/2} = \left(\frac{1}{2}\right)^{1/2} = \frac{\sqrt{2}}{2} \in \mathbb{R}^+$$, logo $\sum_{n=1}^{+\infty} \frac{n}{\sqrt{n^5}}$ tem a mesma natureza da série dada

$\sum_{n=1}^{+\infty} \frac{n}{\sqrt{n^5}} = \sum_{n=1}^{+\infty} \frac{1}{n^{3/2}}$ converge (Dirichlet $\alpha = 3/2 > 1$) logo a dada converge e)

$$\sum_{n=1}^{+\infty} \frac{n^{1/3}}{(n+1)\, n^{1/2}}$$

$$L = \lim_{n\to+\infty} \frac{\frac{\cancel{n^{1/3}}}{(n+1)\, n^{1/2}}}{\frac{\cancel{n^{1/3}}}{n^{3/2}}} = \lim_{n\to+\infty} \frac{n^{3/2}}{n^{3/2}+n^{1/2}} = \lim_{n\to+\infty} \frac{1}{1+1/n} = \frac{1}{1+0} = 1 \in \mathbb{R}^+,$$ logo tem a mesma nat.

$$\sum_{n=1}^{+\infty} \frac{n^{1/3}}{n^{3/2}} = \sum_{n=1}^{+\infty} \frac{1}{n^{7/6}},$$ converge (Dirichlet $\alpha = 7/6 > 1$) logo a dada conv.

f)

$$\sum_{n=1}^{+\infty} \frac{1}{[(-1)^n+5]^n},$$ série de termos não nulos.

$$\begin{cases} \sum_{n=1}^{+\infty} \left(\frac{1}{6}\right)^n, & \text{se } n \text{ par} \\ \sum_{n=1}^{+\infty} \left(\frac{1}{4}\right)^n, & \text{se } n \text{ impar} \end{cases}$$

Ambos convergem logo a dada converge

g)

$$\sum_{n=1}^{+\infty} \left(\frac{2n+1}{3n+1}\right)^{\frac{n}{2}}$$

$$L = \lim_{n\to+\infty} \sqrt[n]{\left|\left(\frac{2n+1}{3n+1}\right)^{\frac{n}{2}}\right|} = \lim_{n\to+\infty} \left(\left(\frac{2n+1}{3n+1}\right)^{\frac{\cancel{x}}{2}}\right)^{\frac{1}{\cancel{x}}} = \lim_{n\to+\infty} \left(\frac{\cancel{x}(2+1/n)}{\cancel{x}(3+1/n)}\right)^{1/2} = \sqrt{\frac{2+0}{3+0}} = \sqrt{\frac{2}{3}}$$

$L = \sqrt{\frac{2}{3}} \in [0,1[$, logo pelo Critério de Cauchy a série dada converge absolutamente

h)

$$\sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2}, \quad a_n = \left(\frac{n}{n+1}\right)^{n^2} \qquad -2$$

$$\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \left(\frac{1}{1+1/n}\right)^{n^2} \overset{(1^{\infty})}{=} \lim_{n\to+\infty} e^{\ln\left(\frac{1}{1+1/n}\right)^{n^2}}$$

$$= \lim_{n\to+\infty} e^{\frac{\ln\left(\frac{1}{1+1/n}\right)}{\frac{1}{n^2}}} \overset{\left(\frac{0}{0}\right)}{\underset{R.C.}{=}} \lim_{n\to+\infty} e^{\frac{\frac{1}{(n+1)^2}}{\frac{-2}{n^3}}} = \lim_{n\to+\infty} e^{-\frac{1}{2}\times \frac{n^3}{n^2+2n+1}} = e^{-1/2\times+\infty} = e^{-\infty}$$

$$= 0$$

$$L = \lim_{n\to+\infty} \sqrt[n]{\left|\left(\frac{n}{n+1}\right)^{n^2}\right|} = \lim_{n\to+\infty} \left(\left(\frac{n}{n+1}\right)^{n^2}\right)^{1/x} = \boxed{\lim_{n\to+\infty} \left(\frac{1}{1+1/n}\right)^n}$$

$$\left[\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n\right]^{-1} = e^{-1} = \frac{1}{e}$$

ou

$$= \lim_{n\to+\infty} e^{\ln\left(\frac{1}{1+1/n}\right)^n} = e^{\lim_{n\to+\infty}\left(\frac{\ln\left(\frac{1}{1+1/n}\right)}{1/n}\right)}$$

$$A = \lim_{n\to+\infty} \left(\frac{\ln\left(\frac{1}{1+1/n}\right)}{1/n} \right) \underset{R.C.}{\overset{\left(\frac{0}{0}\right)}{=}} \lim_{n\to+\infty} \frac{\frac{1}{(n+1)^2}}{\frac{-1}{n^2}} = -\lim_{n\to+\infty} \frac{n^2}{n^2+2n+1} = -\lim_{n\to+\infty} \frac{1}{1+2/n+1/n^2}$$

$$= -1$$

Logo, $\lim_{n\to+\infty} e^A = \lim_{n\to+\infty} e^{-1} = \frac{1}{e} \in [0,1[$ logo a série dada é absolutamente convergente

i)

$$\sum_{n=1}^{+\infty} \left(\frac{2n}{n+1}\right)^{n^3}$$

$$L = \lim_{n\to+\infty} \sqrt[n]{\left|\left(\frac{2n}{n+1}\right)^{n^3}\right|} = \lim_{n\to+\infty} \left(\frac{2n}{n+1}\right)^{n^2} = \lim_{n\to+\infty} \left(2 - \frac{\cancel{2}^{\,0}}{\cancel{n+1}}\right)^{n^2}$$

$$= 2^{+\infty} = +\infty$$

$$\begin{array}{r|l} 2n & n+1 \\ \hline -2n-2 & 2 \\ -2 & \end{array}$$

Logo, como $L = +\infty$, pelo Critério de Cauchy a série é divergente

j)

$$\sum_{n=1}^{+\infty} \frac{b^n}{n} \quad , \quad b \in]0,1[$$

$$L = \lim_{n\to+\infty} \sqrt[n]{\left|\left(\frac{b^n}{n}\right)\right|} = \lim_{n\to+\infty} \frac{(b^{\cancel{n}})^{1/\cancel{n}}}{\sqrt[n]{n}} = \lim_{n\to+\infty} \frac{b}{\sqrt[n]{n}}$$

$$\lim_{n\to+\infty} n^{1/n} = \lim_{n\to+\infty} e^{\ln(n)^{1/n}} = \lim_{n\to+\infty} e^{\frac{\ln(n)}{n}} = \lim_{n\to+\infty} e^A$$

$$A = \lim_{n\to+\infty} \frac{\ln n}{n} \underset{R.C.}{\overset{\left(\frac{+\infty}{+\infty}\right)}{=}} \lim_{n\to+\infty} \frac{\frac{1}{x}}{1} = 0$$

$$= \lim_{n\to+\infty} e^A = e^0 = 1$$

Logo, $L = \frac{b}{1} = b \in]0,1[$, logo pelo Critério de Cauchy converge. absolutamente

k) $\displaystyle\sum_{n=1}^{+\infty} \frac{n!}{d^n}$, série de termos não nulos

$$L = \lim_{n \to +\infty} \left| \frac{\frac{(n+1)!}{d^{\cancel{n}+1}}}{\frac{n!}{\cancel{d^n}}} \right| = \lim_{n\to+\infty} \left| \frac{n+1}{d} \right| = \frac{+\infty}{d} , \text{ como } d \in \mathbb{R}^+$$
$$= +\infty$$

Como $L = +\infty$, pelo Critério D'Alembert a série dada diverge.

l) $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{\ln n}{n}\right)^n$

$$L = \lim_{n\to+\infty} \sqrt[\cancel{n}]{\left|\left(\frac{\ln n}{n}\right)^{\cancel{n}}\right|} = \lim_{n\to+\infty} \frac{\ln n}{n} \overset{R.C.}{=} \lim_{n\to+\infty} \frac{1}{x} = 0 \in [0,1[$$

logo pelo Critério Cauchy converge absolutamente

m) $\displaystyle\sum_{n=1}^{+\infty} \frac{\pi^n n!}{n^n}$

$$L = \lim_{n\to+\infty} \left| \frac{\frac{\pi^{\cancel{n}+1}(n+1)!}{(n+1)^{n+1}}}{\frac{\cancel{\pi^n}\, n!}{n^n}} \right| = \lim_{n\to+\infty} \left| \frac{\pi\, \cancel{(n+1)} \times n^n}{(n+1)^n \times \cancel{(n+1)}} \right| = \pi \lim_{n\to+\infty} \left| \left(\frac{n}{n+1}\right)^n \right|$$

$$= \pi \lim_{n\to+\infty} \left| \left(\frac{1}{1+1/n}\right)^n \right| = \pi \times \lim_{n\to+\infty} \left| \left[\left(1 + 1/n\right)^n\right]^{-1} \right| = \pi \times \frac{1}{e} = \frac{\pi}{e} > 1$$

logo pelo Critério D'Alembert diverge!

n) $\displaystyle\sum_{n=1}^{+\infty} \frac{(n!)^2}{(2n)!}$

$$L = \lim_{n\to+\infty} \left| \frac{\frac{((n+1)!)^2}{(2(n+1))!}}{\frac{(n!)^2}{(2n)!}} \right| = \lim_{n\to+\infty} \left| \left(\frac{(n+1)!}{n!}\right)^2 \times \frac{(2n)!}{(2n+2)!} \right|$$

$$= \lim_{n\to+\infty} \left| \frac{(n+1)^2 \times \cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}} \right| = \lim_{n\to+\infty} \left| \frac{n^2+2n+1}{4n^2+6n+2} \right|$$

$$= \lim_{n\to+\infty} \left| \frac{1 + 2/n + 1/n^2}{4 + 6/n + 2/n^2} \right| = \frac{1}{4} \in [0,1[$$

, logo pelo Critério D'Alembert a série converge absolutamente o)

$$\sum_{n=1}^{+\infty}\left(\frac{1}{8^n}+\frac{1}{n(n+1)}\right)$$

A série dada converge se ambas as séries seguintes convergirem:

$$\sum_{n=1}^{+\infty}\left(\frac{1}{8}\right)^n \quad e \quad \sum_{n=1}^{+\infty}\frac{1}{n(n+1)}$$

Série geométrica de $r=\frac{1}{8}\in]-1,1[$ logo converge

$$L=\lim_{n\to+\infty}\frac{\frac{1}{n^2+n}}{\frac{1}{n^2}}=\lim_{n\to+\infty}\frac{1}{1+1/n}=1\in\mathbb{R}^+$$

Logo, a série tem a mesma natureza de $\sum_{n=1}^{+\infty}\frac{1}{n^2}$ que converge (Dirichlet $\alpha=2>1$)

- Logo, como ambas converge a dada converge.

p)

$$\sum_{n=1}^{+\infty}\frac{sen\left(\frac{\pi}{50}\right)}{2^n}$$

Estudamos a série dos módulos : $\sum_{n=1}^{+\infty}\left|\frac{sen\left(\frac{\pi}{50}\right)}{2^n}\right|=\sum_{n=1}^{+\infty}\frac{|sen(\frac{\pi}{50})|}{2^n}$

$$0\leq\frac{|sen\left(\frac{\pi}{50}\right)|}{2^n}\leq\frac{1}{2^n}=\left(\frac{1}{2}\right)^n$$

E como $\sum_{n=1}^{+\infty}\left(\frac{1}{2}\right)^n$ converge (razão $=\frac{1}{2}\in]-1,1[$), logo a dada converge absolutamente

q)

$$\sum_{n=1}^{+\infty}\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}$$

$$\cancel{2(n+2)}(n+1)\over(n+2)^{\cancel{2}}}\left(\frac{n+1}{n+2}\right)^n$$

$$L=\lim_{n\to+\infty}\left|\frac{\frac{2^{n+2}(n+2)!}{(n+2)^{n+2}}}{\frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}}\right|=\lim_{n\to+\infty}\left|\frac{\cancel{2^n}\times2^{\cancel{2}}(n+2)\cancel{(n+1)!}(n+1)(n+1)^n}{\cancel{2^n}\times\cancel{2}\,\cancel{(n+1)!}(n+2)^2(n+2)^n}\right|$$

$$=\lim_{n\to+\infty}\left|\frac{2(n^2+3n+2)}{n^2+4n+4}\left(1-\frac{1}{n+2}\right)^n\right|=2\times\lim_{n\to+\infty}\left|\left(1-\frac{1}{n+2}\right)^n\right|$$

$$\begin{array}{rr|l} n+1 & & n+2 \\ -n-2 & & 1 \\ -1 & & \end{array}$$

$$\lim_{n\to+\infty}\left(1-\frac{1}{n+2}\right)^n=\lim_{n\to+\infty}e^{\ln\left(1-\frac{1}{n+2}\right)^n}=\lim_{n\to+\infty}e^{x\ln\left(1-\frac{1}{n+2}\right)}$$

$$=e^{\lim_{n\to+\infty}\left[\frac{\ln\left(1-\frac{1}{n+2}\right)}{\frac{1}{x}}\right]}\overset{R.C.}{=}e^{-1}=\frac{1}{e}$$

$$=\frac{2}{e}\in[0,1[$$ logo pelo critério D'Alembert é abs. convergente

$$\left[\ln\left(1-\frac{1}{n+2}\right)\right]' = \left[\ln\left(\frac{n+1}{n+2}\right)\right]' = \frac{n+2}{n+1} \times \frac{(n+2)-(n-1)}{(n+2)^2} = \frac{1}{(n+1)(n+2)}$$

$$\left(\frac{1}{n}\right)' = -\frac{1}{n^2}$$

$$(*) \quad \frac{\frac{1}{n^2+3n+2}}{-\frac{1}{n^2}} = -\frac{1}{1+3/n+2/n^2}$$

r)

$$\sum_{n=1}^{+\infty} \frac{1}{n!+n-1}$$, série de termos não nulos

$$L = \lim_{n\to+\infty} \left| \frac{\frac{1}{(n+1)!+n+\cancel{1}-\cancel{1}}}{\frac{1}{n!+n-1}} \right| = \lim_{n\to+\infty} \left| \frac{n!+n-1}{(n+1)!+n} \right|$$

$$= \lim_{n\to+\infty} \left| \cancel{\frac{1}{2n+1}}^{\,0} + \frac{n}{(n+1)(n)(n-1)!+n} - \cancel{\frac{1}{(n+1)!+n}}^{\,0} \right|$$

$$= \lim_{n\to+\infty} \left| \cancel{\frac{\cancel{n}}{\cancel{n}((n+1)(n-1)!+1)}}^{\,0} \right| = 0 \in [0,1[$$ logo pelo critério D'Alembert a série dada converge absolutamente

s)

$$\sum_{n=1}^{+\infty} \frac{n^n+1}{2^n-1}$$

$$L = \lim_{n\to+\infty} \frac{\frac{n^n+1}{2^n-1}}{\frac{n^n}{2^n}} = \lim_{n\to+\infty} \frac{\cancel{2^n}\,\cancel{n^n}\,(1+1/n^n)}{\cancel{n^n}\,\cancel{2^n}\,(1-1/2^n)} = \frac{1}{1} = 1 \in \mathbb{R}^+,$$ logo

$$\sum_{n=1}^{+\infty} \frac{n^n}{2^n} = \sum_{n=1}^{+\infty} \left(\frac{n}{2}\right)^n$$ tem a mesma natureza da serie dada

$$L = \lim_{n\to+\infty} \sqrt[\cancel{n}]{\left|\left(\frac{n}{2}\right)^{\cancel{n}}\right|} = +\infty,$$ logo é divergente. Assim a dada é divergente também!

t)

$$\sum_{n=1}^{+\infty} \frac{(n+1)^n \cos(n\alpha)}{n^{2n}}, \quad \alpha \in \mathbb{R}$$

$$= \sum_{n=1}^{+\infty} \left(\frac{n+1}{n^2}\right)^n \cos(n\alpha)$$

Estudando a série dos módulos:

$$\sum_{n=1}^{+\infty} \left(\frac{n+1}{n^2}\right)^n |\cos(n\alpha)|$$

$$0 \leq \left(\frac{n+1}{n^2}\right)^n \underbrace{|\cos(n\alpha)|}_{\in[0,1]} \leq \left(\frac{n+1}{n^2}\right)^n$$

Pelo critério da comparação:

- Se $\sum_{n=1}^{+\infty} \left(\frac{n+1}{n^2}\right)^n$ convergir então a série dada converge absolutamente

$$L = \lim_{n \to +\infty} \sqrt[\cancel{n}]{\left|\left(\frac{n+1}{n^2}\right)\right|^{\cancel{n}}} = \lim_{n \to +\infty} \frac{1 + 1/n}{n} = \frac{1}{+\infty} = 0 \in [0,1[\text{ , logo}$$

pelo Critério de Cauchy a série converge absolutamente logo a série dada converge absolutamente

u)

$$\sum_{n=1}^{+\infty} \frac{-\operatorname{arctg} n}{n^2+1} = -\underbrace{\sum_{n=1}^{+\infty} \frac{\operatorname{arctg} n}{n^2+1}}$$

$$L = \lim_{n \to +\infty} \frac{\frac{\operatorname{arctg} x}{\cancel{n^2+1}}}{\frac{1}{\cancel{n^2+1}}} = \operatorname{arctg}(+\infty) = \frac{\pi}{2} \in \mathbb{R}^+ \text{ logo tem a mesma}$$

natureza de $\sum_{n=1}^{+\infty} \frac{1}{n^2+1}$

$$0 \leq \frac{1}{n^2+1} \leq \frac{1}{n^2} \text{ , e } \sum_{n=1}^{+\infty} \frac{1}{n^2} \text{ converge (Dirichlet } \alpha = 2 > 1)$$

Logo $\sum_{n=1}^{+\infty} \frac{1}{n^2+1}$ converge $\Rightarrow \sum_{n=1}^{+\infty} \frac{\operatorname{arctg} n}{n^2+1}$ converge $\Rightarrow \sum_{n=1}^{+\infty} \frac{-\operatorname{arctg} n}{n^2+1}$ converge

v)

$$\sum_{n=0}^{+\infty} \frac{n! + 3n}{((n+1)!)^2}$$

$$L = \lim_{n \to +\infty} \frac{\frac{n! + 3n}{\cancel{((n+1)!)^2}}}{\frac{n!}{\cancel{((n+1)!)^2}}} = \lim_{n \to +\infty} \frac{\cancel{n!}(1 + 3/(n-1)!)}{\cancel{n!}} = 1 + \frac{3}{(+\infty)!} = 1 + 0 = 1 \in \mathbb{R}^+$$

Como $L = 1 \in \mathbb{R}^+$ pelo critério do limite $\sum_{n=1}^{+\infty} \frac{n!}{((n+1)!)^2}$ tem a mesma natureza da série dada

$(n!)^2 = n^2 \times (n-1)^2 \times (n-2)^2 \ldots$

$$L = \lim_{n \to +\infty} \left| \frac{\frac{(n+1)!}{((n+2)!)^2}}{\frac{n!}{((n+1)!)^2}} \right| = \lim_{n \to +\infty} \left| \frac{(n+1)\cancel{((n+1)!)^2}}{(n+2)^2\cancel{((n+1)!)^2}} \right| = \lim_{n \to +\infty} \frac{n+1}{n^2+4n+4}$$

$$= \lim_{n \to +\infty} \frac{1 + 1/n}{n + 4 + 4/n} = \frac{1+0}{(+\infty)+4+0} = \frac{1}{+\infty} = 0 \in [0,1[\text{ , logo pelo Critério}$$

D'Alembert a série $\sum_{n=1}^{+\infty} \frac{n!}{((n+1)!)^2}$ converge absolutamente

• Assim, como tem a mesma natureza da série dada, a série dada converge.

9

• Séries de termos não negativos

$$\sum_{n=1}^{+\infty} (a_n + b_n) = \sum_{n=1}^{+\infty} \overset{a_n \geq 0}{a_n} + \sum_{n=1}^{+\infty} \overset{b_n \geq 0}{b_n}$$, em caso de convergência da série dada

Como $\lim\limits_{n \to +\infty} \sqrt[n]{\underset{b_n \geq 0}{|b_n|}} = \lim\limits_{n \to +\infty} \sqrt[n]{b_n} = \frac{1}{3} \in [0,1[$ pelo Critério de Cauchy a série $\sum_{n=1}^{+\infty} b_n$ converge absolutamente

$$\sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} \left(b_n + \frac{1}{3}\right)$$

$\lim\limits_{n \to +\infty} a_n = \underbrace{\lim\limits_{n \to +\infty} b_n}_{=0,\text{ pois } \sum_{n=1}^{+\infty} b_n \text{ é convergente}} + \lim\limits_{n \to +\infty} \frac{1}{3} = \frac{1}{3} \neq 0$, logo pelo CNC a série $\sum_{n=1}^{+\infty} a_n$ é divergente

• Logo a série dada é divergente.

$\boxed{10}$

a) $\displaystyle\sum_{n=1}^{+\infty} (-1)^{n-1} \frac{1}{2n-1}$

Estudando a série dos módulos:

$$\sum_{n=1}^{+\infty} \frac{1}{2n-1}$$

$$L = \lim_{n\to+\infty} \frac{\frac{1}{2n-1}}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{\cancel{n}}{\cancel{n}(2-1/n)} = \frac{1}{2} \in \mathbb{R}^+$$

Logo pelo critério do limite a série $\sum_{n=1}^{+\infty} \frac{1}{n}$ que diverge (Dirichlet $\alpha = 1 \leq 1$) tem a mesma natureza da série dos módulos, logo a série dada não pode ser absolutamente convergente

Série alternada, onde $a_n = \frac{1}{2n-1} > 0$

Como :

- $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \frac{1}{2n-1} = 0$
- a_n é monótona decrescente:
  $n \in \mathbb{N} \wedge n \in [1, +\infty[\,, n > 0$

$$\frac{1}{(2n+1)-1} < \frac{1}{2n-1} \Leftrightarrow \frac{1}{2n} < \frac{1}{2n-1} \overset{n>0}{\Leftrightarrow} \underbrace{2n > 2n-1}_{\text{Verdadeiro}}$$

Então:

Pelo critério de Leibniz a série dada é convergente.

Assim, a série é simplesmente convergente

b) $\displaystyle\sum_{n=2}^{+\infty} (-1)^{n} \frac{1}{\ln(n)}$

Estudando a série dos módulos:

$$\sum_{n=2}^{+\infty} \frac{1}{\ln(n)}$$

$$L = \lim_{n\to+\infty} \frac{\frac{1}{\ln(n)}}{\frac{1}{n}} = \lim_{n\to+\infty} \frac{n}{\ln n} \overset{R.C.}{=} \lim_{n\to+\infty} n = +\infty$$

Logo, como $\sum_{n=2}^{+\infty} \frac{1}{n}$ diverge (Dirichlet $\alpha = 1 \leq 1$) pelo critério do limite ($L = +\infty$) a série dos módulos diverge

Série alternada, onde $a_n = \frac{1}{\ln n} > 0$, $\forall n \in \mathbb{N} \wedge \underbrace{n \in [2, +\infty[}_{I}$

Como:

- $\lim_{n \to +\infty} a_n = \frac{1}{+\infty} = 0$

- a_n é monotona decrescente:

$$a_{n+1} < a_n \Leftrightarrow \frac{1}{\ln(n+1)} < \frac{1}{\ln(n)} \overset{\ln(n)>0,\ \forall n \in I}{\Leftrightarrow} \underbrace{\ln(n) < \ln(n+1)}_{\text{verdadeiro},\ \forall n \in I}$$

Então, pelo critério de Leibniz a série dada é convergente

- Como a série dos módulos diverge $\Rightarrow$ a série dada é simplesmente convergente

c)

$$\sum_{n=1}^{+\infty} (-1)^n \times \frac{1}{2^{n-1}} = \sum_{n=1}^{+\infty} \underbrace{\left(-\frac{1}{2}\right)}^n \times 2$$

$r = -\frac{1}{2} \in]-1, 1[$, logo a série dada é convergente

$$L = \lim_{n \to +\infty} \sqrt[n]{\left|\left(-\frac{1}{2}\right)^n \times 2\right|} = \lim_{n \to +\infty} \frac{1}{2} \times 2^{\frac{1}{n}} = \frac{1}{2} \times 2^0 = \frac{1}{2} \in [0, 1[$$

$L \in [0,1[\Rightarrow$ Logo pelo Critério de Cauchy a série dada converge absolutamente.

d)

$$\sum_{n=1}^{+\infty} (-1)^n \times \frac{1}{e^n + 1}$$

Estudando a série dos módulos:

$$\sum_{n=1}^{+\infty} \frac{1}{e^n + 1}$$

$$0 \leq \frac{1}{e^n + 1} \leq \frac{1}{e^n} = \left(\frac{1}{e}\right)^n$$

Como $\sum_{n=1}^{+\infty} \left(\frac{1}{e}\right)^n$ converge (razão $\in]-1, 1[$) então a série dos módulos converge.

- A série dada é absolutamente convergente

e) $\sum_{n=1}^{+\infty} (-1)^n \left(\frac{n}{n+1}\right)^2$, série alternada, onde $a_n = \left(\frac{n}{n+1}\right)^2 > 0$

$$\lim_{n\to+\infty} \left[(-1)^n \left(\frac{n}{n+1}\right)^2\right] = \lim_{n\to+\infty} \left[(-1)^n \times \frac{1}{1+2/n+1/n^2}\right]$$

$$= \lim_{n\to+\infty} \underbrace{(-1)^n}_{-1 \text{ ou } 1} \times 1$$

-1 ou 1, logo e $\lim_{n\to+\infty} a_n \neq 0$, logo é DIV

f) $\sum_{n=1}^{+\infty} (-1)^n \left(\frac{n}{n+1}\right)^{n^2}$

Estudando a série dos módulos:

$$\sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2}$$

$$L = \lim_{n\to+\infty} \sqrt[n]{\left|\left(\frac{n}{n+1}\right)^{n^2}\right|} = \lim_{n\to+\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to+\infty} e^{\frac{\ln\left(\frac{1}{1+1/n}\right)}{\frac{1}{n}}} \quad \left(\frac{0}{0}\right)$$

Deveria ter criado uma função f, extensão, para derivar...

$$\overset{R.C.}{=} \lim_{n\to+\infty} e^{\frac{\frac{1}{n^2+n}}{\frac{-1}{n^2}}} = \lim_{n\to+\infty} e^{-\frac{1}{1+1/n}} = e^{-1} = \frac{1}{e} \in [0,1[$$

, logo pelo Critério de Cauchy converge absolutamente

$\Downarrow$

a série dada converge absolutamente

$$\left[\ln\left(\frac{n}{n+1}\right)\right]' = \frac{\cancel{n}+1-\cancel{n}}{(n+1)^{\cancel{2}}} \times \frac{\cancel{(n+1)}}{n} = \frac{1}{n^2+n}$$

g)

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{(n+1)(n+2)} = \sum_{n=1}^{+\infty} (-1)^n \frac{1}{(n+1)(n+2)} = \frac{1}{n^2+3n+2}$$

Estudamos a série dos módulos:

$$\sum_{n=1}^{+\infty} \frac{1}{n^2+3n+2}$$

$$L = \lim_{n\to+\infty} \frac{\frac{1}{n^2+3n+2}}{\frac{1}{n^2}} = \lim_{n\to+\infty} \frac{1}{1+3/n+2/n^2} = 1 \in \mathbb{R}^+$$

Logo pelo critério do limite, como $L \in \mathbb{R}^+$ e $\sum_{n=1}^{+\infty} \frac{1}{n^2}$ converge (Dirichlet $\alpha = 2 > 1$) então a série dada converge absolutamente

h) $\sum_{n=1}^{+\infty} (-1)^{n+1} \ln\left(1+\frac{1}{n}\right)$

Estudando a série dos módulos:

$$\sum_{n=1}^{+\infty} \ln\left(1+\frac{1}{n}\right)$$

Seja $f: [1,+\infty[\longrightarrow \mathbb{R}$

$x \longmapsto f(x) = \ln\left(1+\frac{1}{x}\right)$, onde $f'(x) = \frac{-\frac{1}{x^2}}{1+\frac{1}{x}} < 0, \forall x \in D_f$ (f é estritamente decrescente)

Nota: $f(n) = \ln\left(1+\frac{1}{n}\right) = a_n, \forall n \in \mathbb{N} \land n \in [1,+\infty[$

$$L = \lim_{x\to+\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \overset{\left(\frac{0}{0}\right)}{\underset{R.C.}{=}} \lim_{n\to+\infty} \frac{\frac{\cancel{-}\frac{1}{\cancel{x^2}}}{1+\frac{1}{x}}}{\cancel{-}\frac{1}{\cancel{x^2}}} = \lim_{n\to+\infty} \frac{1}{1+\frac{1}{x}} = 1 \in \mathbb{R}^+$$

Logo como $\sum_{n=1}^{+\infty} \frac{1}{x}$ diverge (Dirichlet $\alpha = 1 \leq 1$) então a série dos módulos diverge

série alternada, onde $a_n = \ln\left(1+\frac{1}{n}\right) > 0$, $n \in \mathbb{N} \land n \in [1,+\infty[$

<u>Como</u>:

- $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \ln\left(1+\frac{1}{n}\right) = \ln(1) = 0$ ✓
- a_n é monotona decrescente:

  Já provamos que f (extensão de a_n), tal que $f(n) = a_n, \forall n \in \mathbb{N} \land n \in [1,+\infty[$ e estritamente decrescente, logo a_n é monotona decrescente ✓

<u>Então</u>, pelo critério de Leibniz a série dada é convergente

- Logo é simplesmente convergente

$\boxed{11}$

a) $\sum_{n=2}^{+\infty} (-1)^n \times \frac{1}{n \ln(n)}$

Estudando a série dos módulos:

$$\sum_{n=2}^{+\infty} \frac{1}{n\ln(n)}, \quad a_n = \frac{1}{n\ln(n)}$$

Seja $f: [2,+\infty[\longrightarrow \mathbb{R}$

$x \longmapsto f(x) = x\ln(n)$

Nota: $f(n) = a_n$, $n = 2, 3, 4, \ldots$

Como:

- $f(x) > 0 \Leftrightarrow \underbrace{x \ln(x) > 0}_{\text{Verdadeiro}}, \quad x \in [2, +\infty[$ ✓

- f é decrescente:

$$f'(x) = \left[\frac{\frac{1}{x}}{\ln(x)}\right]' = \frac{-\frac{1}{x^2} \times \ln(x) - \frac{1}{x} \times \frac{1}{x}}{\ln^2(x)} = \frac{-\frac{1}{x^2}\left(\ln(x) + 1\right)}{\ln^2(x)}$$

$f'(x) < 0, \forall x \in [2, +\infty[$, logo f é decrescente ✓

Então, pelo critério do Integral a série dos módulos tem a mesma natureza do integral $\int_2^{+\infty} \frac{1}{x \ln(x)}$.

$$= \lim_{t \to +\infty} \int_2^t \frac{\frac{1}{x}}{\ln(x)} \, dx = \lim_{t \to +\infty} \left[\ln(\ln(x))\right]_2^t = \ln(\ln(+\infty)) - \ln(\ln(2))$$

$$= +\infty - \underbrace{\ln(\ln(2))}_{\in \mathbb{R}}$$

$$= +\infty$$

Logo, como o integral diverge a série dos módulos diverge.

Voltando à série dada:

série alternada, onde $a_n = \frac{1}{n \ln(n)} > 0$, $n = 2, 3, 4, \ldots$

Como:

- $\lim_{n \to +\infty} a_n = \lim_{n \to +\infty} \frac{\frac{1}{n}}{\ln(n)} = \frac{0}{+\infty} = 0$

- a_n é monotona decrescente:

$$a_{n+1} < a_n \Leftrightarrow \frac{1}{(n+1)\ln(n+1)} < \frac{1}{n \ln(n)} \overset{n\ln(n) > 0,\, n = 2,3,4,\ldots}{\Leftrightarrow} \underbrace{(n+1)\ln(n+1) > n \ln(n)}_{\text{Verdadeiro}}$$ ✓

Então, pelo Critério de Leibniz a série é convergente.
Como a série dos módulos é DIV então a série dada é simplesmente convergente.

b)

Faziamos por comparação da série dos módulos: $0 \leq \underbrace{\frac{1}{\ln(n^n)}}_{\text{Diverge}} \leq \frac{1}{\ln(n!)}$ Diverge $\Rightarrow$ Diverge
E o critério de Leibniz

c)

Comparavamos com $\frac{1}{n}$: $0 \leq \underbrace{\frac{1}{n}}_{\text{DIV}} \leq \frac{(\ln n)^{-\alpha}}{n}$ $-\alpha > 0$ DIV $\rightarrow$ DIV

12

$$\sum_{n=1}^{8} a_n = 15$$

$$a_n = \left(\frac{3}{2}\right)^n, \quad n \geq 9$$

$$b_n > a_n, \quad n > 20$$

$\sum_{n=1}^{+\infty} a_n$ tem a mesma natureza de $\sum_{n=9}^{+\infty} \left(\frac{3}{2}\right)^n$ que diverge $(r = \frac{3}{2} \notin]-1,1[)$

$\sum_{n=1}^{+\infty} b_n$ tem a mesma natureza de $\sum_{n=21}^{+\infty} b_n$

Como $a_n > 0$, $n \geq 21$ e $0 \leq a_n \leq b_n$, $n \geq 21$

então pelo critério da comparação se $\sum_{n=21}^{+\infty} a_n$ divergir $\Rightarrow \sum_{n=21}^{+\infty} b_n$ diverge

$\Rightarrow \sum_{n=1}^{+\infty} b_n$ diverge

Como $\sum_{n=21}^{+\infty} a_n$ tem a mesma natureza de $\sum_{n=1}^{+\infty} a_n$ que diverge,

logo $\sum_{n=1}^{+\infty} b_n$ diverge

- Ambos divergem!

14

a)

(i) $\sum_{n=1}^{+\infty} (-1)^n \times \frac{n!}{n^n}$ $\boxed{n^n > n!}$

Estudando a série dos módulos:

$$\sum_{n=1}^{+\infty} \frac{n!}{n^n}$$

$$L = \lim_{n\to+\infty} \left| \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} \right| = \lim_{n\to+\infty} \frac{(n+1) \times n^n}{(n+1)^{n+1}} = \lim_{n\to+\infty} \frac{\cancel{(n+1)}}{\cancel{(n+1)}} \times \left(\frac{n}{n+1}\right)^n = \frac{1}{e} \in [0,1[$$

logo a série dada converge absolutamente

(ii) $$\sum_{n=1}^{+\infty} \frac{\sqrt[3]{n}}{n^2+1} = \sum_{n=1}^{+\infty} \frac{n^{1/3}}{n^2+1}$$

A série dada é igual à série dos módulos (*)

$$L = \lim_{n\to+\infty} \frac{\frac{n^{1/3}}{n^2+1}}{\frac{n^{1/3}}{n^2}} = \lim_{n\to+\infty} \frac{\cancel{n^2}\, \cancel{n^{1/3}} \times 1}{\cancel{n^{1/3}}\, \cancel{n^2}(1+1/n^2)} = \frac{1}{1+0} = 1 \in \mathbb{R}^+$$

Logo $\sum_{n=1}^{+\infty} \frac{n^{1/3}}{n^2} = \sum_{n=1}^{+\infty} \frac{1}{n^{5/3}}$ que converge (Dirichlet $\alpha = 5/3 > 1$) tem a mesma natureza da série ⊛ dos módulos

• Logo ambos são abs. convergentes

b) Como ambos os séries são convergentes os limites do termo geral dos series (pela condição necessária de convergência) são iguais a zero.

c)

$$\sum_{n=1}^{+\infty}\left[(-1)^n \frac{n!}{n^n} + b_n\right] = \sum_{n=1}^{+\infty} \frac{\sqrt[3]{n}}{n^2+1}$$

$$\Leftrightarrow \underbrace{\sum_{n=1}^{+\infty} (-1)^n \frac{n!}{n^n}}_{= a \in \mathbb{R}, \text{ pois é convergente}} + \sum_{n=1}^{+\infty} b_n = \underbrace{\sum_{n=1}^{+\infty} \frac{\sqrt[3]{n}}{n^2+1}}_{= c \in \mathbb{R}, \text{ pois é convergente}}$$

$$\Leftrightarrow a + \sum_{n=1}^{+\infty} b_n = c$$

$$\Leftrightarrow \underbrace{a - c}_{\text{subtração em } \mathbb{R}} = \sum_{n=1}^{+\infty} b_n$$

$\in \mathbb{R}$, logo a soma da série existe e é finita $\Longrightarrow \sum_{n=1}^{+\infty} b_n$ é convergente