Funções Inversas
(Slides Ana B.)

Slide #2 + #04

Determine o domínio e o contradomínio da função (f definida analiticamente por) $f(x) = 3 - \sqrt{x+1}$.

$\sqrt{(\cdot)}$ tem significado sse o radicando $(\cdot)$ for não negativo.

$x+1 \geqslant 0 \Leftrightarrow x \geqslant -1$

$D_f = [-1, +\infty[$ ✓

Para valores $x \in D_f$ sabemos que $\sqrt{x+1}$ toma qualquer valor de $\mathbb{R}_0^+$

$\sqrt{x+1} \geqslant 0$

$-\sqrt{x+1} \leqslant 0$

$3 - \sqrt{x+1} \leqslant 3$

$CD_f =]-\infty, 3]$ ✓

$\boxed{\text{Slide \#04}}$ 1.9

Caracterize a inversa das funções

$f(x) = 3x - \pi$ e $g(x) = \sqrt{x-1}$.

$\mathcal{D}_f = \mathbb{R}$, $CD_f = \mathbb{R}$

f é uma função afim estritamente crescente

f é injectiva (f diferenciável

f é invertível e $f'(x) > 0$, $\forall x \in \mathcal{D}_f$)

$y = 3x - \pi \Leftrightarrow 3x = y + \pi \Leftrightarrow x = \frac{y+\pi}{3}$

f^{-1} será:

$$f^{-1}: \underset{CD_f}{\mathbb{R}} \longrightarrow \mathbb{R}$$
$$x \longmapsto \frac{x+\pi}{3}$$

(Nota: É uso generalizado usar "x" como variável independente)

Então, $f^{-1}(x) = \frac{x+3}{\pi}$, sendo $\mathcal{D}_{f^{-1}} = CD_f = \mathbb{R}$

$\mathcal{D}g = [1, +\infty[$, $CD_g = \mathbb{R}_0^+$

g é estrit. crescente em $\mathcal{D}_f$

Para $x \in \mathcal{D}_f$:

(g diferenciável, $g'(x) = \frac{1}{2}(x-1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x-1}}$

$g'(x) > 0, \forall x \in \mathbb{R}^+$)

$y = \sqrt{x-1} \Leftrightarrow y^2 = x - 1 \Leftrightarrow x = y^2 + 1$

$$g^{-1}: \mathbb{R}_0^+ \longrightarrow [1, +\infty[\subset \mathbb{R}$$
$$x \longmapsto x^2 + 1$$

sendo $\mathcal{D}g^{-1} = [1, +\infty[$

$\boxed{\text{Slide \#08}}$ (1.17)

indicar: expressão, domínio, contradomínio

Caracterize a inversa das funções:

(a) $f(x) = e^{1-2x}$

$D_f = \mathbb{R}$

$u = 1-2x$ toma qq valor real quando x percorre $D_f = \mathbb{R}$

Assim

$f(x) = e^{u} = e^{1-2x}$ toma todos os valores de $\mathbb{R}^+$

$CD_f = \mathbb{R}^+$

f é diferenciável e $f'(x) = -2e^{1-2x}$

$f'(x) < 0, \ \forall x \in D_f$

f é estrit. decrescente em D_f

f é injetiva em D_f

f é invertível em D_f

$y = e^{1-2x} \Leftrightarrow \ln y = \ln(e^{1-2x})$

$\ln y = 1-2x \Leftrightarrow 2x = 1 - \ln y$

$$x = \frac{1-\ln y}{2}$$

$$f^{-1}: \mathbb{R}^+ \longrightarrow \mathbb{R},$$
$$x \longmapsto y = \frac{1-\ln x}{2}$$

sendo $CD_{f^{-1}} = \mathbb{R}$.

$$b)\quad f(x) = \frac{5\ln(x-3)-1}{4}$$

$x-3>0 \Leftrightarrow x>3$

$D_f =]3,+\infty[$

f é diferenciável

$$f'(x) = \left(\frac{5}{4}\ln(x-3) - \frac{1}{4}\right)'$$

$$f'(x) = \frac{5}{4}\,\frac{1}{x-3} > 0, \ \forall x \in D_f$$

$u = x-3$, percorre todos os valores de D_f =

$\ln u$ percorre todos os valores de $\mathbb{R}$

$f(x) = \frac{5}{4}\ln(x-3) - \frac{1}{4}$ percorre tb todos os valores de

$CD_f = \mathbb{R}$

$$y = \frac{5}{4}\ln(x-3) - \frac{1}{4}$$

$$\frac{5}{4}\ln(x-3) = y + \frac{1}{4}$$

$$\ln(x-3) = \frac{4}{5}y + \frac{1}{5}$$

$$e^{\ln(x-3)} = e^{\frac{4}{5}y + \frac{1}{5}}$$

$$x-3 = e^{\frac{4}{5}y} \cdot \sqrt[5]{e}$$

$$x = \sqrt[5]{e}\, e^{\frac{4}{5}y} + 3$$

$$f^{-1}: \underset{CD_f}{\mathbb{R}} \longrightarrow \mathbb{R}$$

$$x \longmapsto y = \sqrt[5]{e}\, e^{\frac{4y}{5}} + 3$$

sendo $CD_{f^{-1}} = D_f =]3, +\infty[$

c) $f(x) = \log_3(2-x)$

$2 - x > 0 \Leftrightarrow x < 2$

$D_f =]-\infty, 2[$, $CD_f = \mathbb{R}$

$y = \log_3(2-x) \Leftrightarrow 3^y = 3^{\log_3(2-x)}$

$2 - x = 3^y \Leftrightarrow x = 2 - 3^y$

$$f^{-1}: \underset{CD_f}{\mathbb{R}} \longrightarrow \mathbb{R}$$

$$x \longmapsto y = 2 - 3^x$$

sendo $CD_{f^{-1}} =]-\infty, 2[$

d) $f(x) = \dfrac{e^x}{e^x + 1}$ $\quad e^x > 0, \forall x \in \mathbb{R}$, $\quad e^x + 1 > 1, \forall x \in \mathbb{R}$

$f(x) = \dfrac{1}{1 + e^{-x}}$

$D_f = \mathbb{R}$

$CD_f =]0, 1[$,

$1 + e^{-x} > 1$

$\dfrac{1}{1+e^{-x}} < 1$

$\dfrac{1}{1+e^{-x}} > 0$

para $x \in D_f$:

$$y = \frac{1}{1+e^{-x}} \Leftrightarrow 1+e^{-x} = \frac{1}{y}$$

$$e^{-x} = \frac{1}{y} - 1 = \frac{1-y}{y}$$

$$\ln\left(e^{-x}\right) = \ln\left(\frac{1-y}{y}\right)$$

$$-x = \ln\left(\frac{1-y}{y}\right)$$

$$x = -\ln\left(\frac{1-y}{y}\right)$$

$$x = \ln\left(\left(\frac{1-y}{y}\right)^{-1}\right)$$

$$x = \ln \frac{y}{1-y}$$

$$f^{-1}:]0,1[\longrightarrow \mathbb{R}$$

$$x \longmapsto y = \ln \frac{x}{1-x}$$

sendo $CD_{f^{-1}} = D_f = \mathbb{R}$

Slide#11

Caracterize inversas — restrição principal da função seno

Domínio $= [-\pi/2, \pi/2]$

a) $f(x) = \frac{1}{2} \sin\left(x + \frac{\pi}{2}\right)$

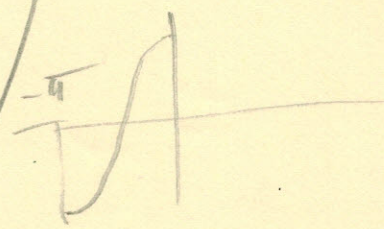
$$-\frac{\pi}{2} \leq x + \frac{\pi}{2} \leq \frac{\pi}{2}$$

$$-\pi \leq x \leq 0 \quad , \quad D_f = [-\pi, 0]$$

$$-1 \leq \sin\left(x+\frac{\pi}{2}\right) \leq 1$$

$$-\frac{1}{2} \leq \frac{1}{2} \sin\left(x+\frac{\pi}{2}\right) \leq \frac{1}{2}$$

$$CD_f = \left[-\frac{1}{2}, \frac{1}{2}\right]$$



$$f'(x) = \frac{1}{2} \cos\left(x + \frac{\pi}{2}\right)$$

$f'(x) > 0, \forall x \in]-\pi, 0[$

f estrit. crescente em $[-\pi, 0]$

f injetiva em $[-\pi, 0]$

f invertível

$$y = \frac{1}{2} \sin\left(x + \frac{\pi}{2}\right)$$

$$2y = \sin\left(x + \frac{\pi}{2}\right) \Leftrightarrow x + \frac{\pi}{2} = \arcsin(2y)$$

$$x = \arcsin(2y) - \frac{\pi}{2}$$

$$f^{-1} : \left[-\frac{1}{2}, \frac{1}{2}\right] \longrightarrow \mathbb{R}$$

$$x \longmapsto y = \arcsin(2x) - \frac{\pi}{2}$$

com $CD_{f^{-1}} = D_f = [-\pi, 0]$

b) inversa de

é um seno

$$f(x) = \frac{\pi}{2} - \frac{2 \arcsin(1-x)}{3}$$

$$-1 \leq 1-x \leq 1$$

$$-2 \leq -x \leq 0$$

$$2 \geqslant x \geqslant 0 \quad , \quad x \in [0,2] = D_f$$

$$-\frac{\pi}{2} \leq \arcsin(1-x) \leq \frac{\pi}{2}$$

$$-\frac{\pi}{3} \leq \frac{2}{3} \arcsin(1-x) \leq \frac{\pi}{3}$$

$$\frac{\pi}{3} \geqslant -\frac{2}{3} \arcsin(1-x) \geqslant -\frac{\pi}{3}$$

$$\frac{\pi}{2} + \frac{\pi}{3} \geqslant \frac{\pi}{2} - \frac{2}{3} \arcsin(1-x) \geqslant \frac{\pi}{2} - \frac{\pi}{3}$$

$$\frac{5\pi}{6} \geqslant f(x) \geqslant \frac{\pi}{6} \quad , \quad CD_f = \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$$

$$y = \frac{\pi}{2} - \frac{2}{3} \arcsin(1-x)$$

$$-\frac{2}{3} \arcsin(1-x) = y - \frac{\pi}{2}$$

$$\arcsin(1-x) = -\frac{3}{2}\left(y - \frac{\pi}{2}\right)$$

$$\arcsin(1-x) = -\frac{3y}{2} + \frac{3\pi}{4}$$

arco

$$\sin\left(-\frac{3y}{2}+\frac{3\pi}{4}\right) = 1-x$$

$$x = 1-\sin\left(-\frac{3y}{2}+\frac{3\pi}{4}\right)$$

$$f^{-1}: \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] \longrightarrow \mathbb{R}$$

$$x \longmapsto y = 1-\sin\left(\frac{3\pi}{4}-\frac{3x}{2}\right)$$

com $CD_{f^{-1}} = D_f = [0,2]$

c) $f(x) = 2\arcsin(\sqrt{x}) - \pi$

seno

para $\sqrt{x}$ ser um seno

$\boxed{-1 \leq \sqrt{x} \leq 1} \wedge x \geq 0$ (para a $\sqrt{x}$ estar definida)

$$x \in [0,1] = D_f$$

$$\arcsin(0) \leq \arcsin(\sqrt{x}) \leq \arcsin(1)$$

$$0 \leq \arcsin(\sqrt{x}) \leq \frac{\pi}{2}$$

$$0 \leq 2\arcsin(\sqrt{x}) \leq \pi$$

$$-\pi \leq 2\arcsin(\sqrt{x}) - \pi \leq 0$$

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$f(x) \in [-\pi, 0]$, $CD_f = [-\pi, 0]$

$$y = 2\arcsin(\sqrt{x}) - \pi$$

$$2\arcsin(\sqrt{x}) = y + \pi$$

$$\arcsin(\sqrt{x}) = \frac{y+\pi}{2}$$

$$\sqrt{x} = \sin\left(\frac{y+\pi}{2}\right)$$

$$x = \sin^2\left(\frac{y+\pi}{2}\right)$$

notar que $x \in [0,1]$ $x > 0$

$$f^{-1}: [-\pi, 0] \longrightarrow \mathbb{R}$$

$$x \longmapsto y = \sin^2\left(\frac{x+\pi}{2}\right)$$

com $CD_{f^{-1}} = D_f = [0, 1]$.

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$\boxed{\text{Slide \#14}}$

Caracterize a inversa, de

a) $f(x) = \dfrac{1}{2+\cos x}$, $2+\cos x \neq 0$

restrição principal da função coseno

$D_f = [0, \pi]$

$-1 \leq \cos x \leq 1$

$1 \leq 2+\cos x \leq 3$

$\frac{1}{3} \leq \dfrac{1}{2+\cos x} \leq 1$

$f(x) \in \left[\frac{1}{3}, 1\right] = CD_f$

$y = \dfrac{1}{2+\cos x} \Leftrightarrow 2+\cos x = \dfrac{1}{y}$

$\cos x = \dfrac{1}{y} - 2 \Leftrightarrow x = \arccos\left(\dfrac{1-2y}{y}\right)$

$$\begin{aligned} f^{-1}: \left[\tfrac{1}{3}, 1\right] &\longrightarrow \mathbb{R} \\ x &\longmapsto y = \arccos\left(\frac{1-2x}{x}\right) \end{aligned}$$

com $CD_{f^{-1}} = D_f = [0, \pi]$

b) $f(x) = 2\pi - \arccos\left(\frac{x}{2}\right)$

cosseno

$-1 \leq \frac{x}{2} \leq 1$

$-2 \leq x \leq 2$, $x \in [-2,2] = D_f$

$0 \leq \arccos\left(\frac{x}{2}\right) \leq \pi$

$0 \geq -\arccos\left(\frac{x}{2}\right) \geq -\pi$

$2\pi \geq 2\pi - \arccos\left(\frac{x}{2}\right) \geq \pi$

$CD_f = [\pi, 2\pi]$

$y = 2\pi - \arccos\left(\frac{x}{2}\right)$

$+\arccos\left(\frac{x}{2}\right) = 2\pi - y$

$\frac{x}{2} = \cos(2\pi - y)$

$x = 2\cos(2\pi - y) = 2\cos y$

$f^{-1}: [\pi, 2\pi] \longrightarrow \mathbb{R}$

$x \longrightarrow y = 2\cos x$

com $CD_{f^{-1}} = D_f = [-2,2]$

$\boxed{\text{Slide \#17}}$

Caracterize a inversa das funções definidas por:

(a) $f(x) = tg\left(\frac{\pi}{2-x}\right)^{z}$

$z \in \underbrace{\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[}_{\text{restrição principal da função } tg\, z}$

$-\frac{\pi}{2} < \frac{\pi}{2-x} < \frac{\pi}{2}$

Vamos determinar o domínio de f

$$D_f = \left\{ x \in \mathbb{R} : \left| \frac{\pi}{2-x} \right| < \frac{\pi}{2} \wedge x \neq 2 \right\}$$

• Para $x > 2$:

$$\begin{cases} \frac{\pi}{2-x} > -\frac{\pi}{2} \\ \frac{\pi}{2-x} < 0 \end{cases} \quad \begin{cases} \frac{1}{2-x} > -\frac{1}{2} \\ \end{cases} \quad \begin{cases} 2 < -(2-x) \\ \end{cases}$$

$$\begin{cases} x > 4 \\ x > 2 \end{cases}$$, vindo neste caso, $x \in]4, +\infty[$

• Para $x < 2$:

$$\begin{cases} \frac{\pi}{2-x} < \frac{\pi}{2} \\ \frac{\pi}{2-x} > 0 \end{cases} \quad \begin{cases} 2-x > 2 \\ x < 2 \end{cases} \quad \begin{cases} x < 0 \\ x < 2 \end{cases}$$

vindo, neste caso, $x \in]-\infty, 0[$

$$D_f =]-\infty, 0[\cup]4, +\infty[$$

Quando x percorre D_f

$\frac{\pi}{2-x}$ percorre ~~$\mathbb{R}\setminus\{0\}$~~ $\left]-\frac{\pi}{2},0\right[\cup\left]0,\frac{\pi}{2}\right[$, logo f

percorre $\mathbb{R}\setminus\{0\}$

$$CD_f = \mathbb{R}\setminus\{0\}$$

Expressão analítica de f^{-1}:

$$y = \operatorname{tg}\left(\frac{\pi}{2-x}\right) \Leftrightarrow \frac{\pi}{2-x} = \operatorname{arctg} y$$

$$2-x = \frac{\pi}{\operatorname{arctg} y}$$

$$x = 2 - \frac{\pi}{\operatorname{arctg} y}$$

Defino agora f^{-1}:

$$f^{-1}: \mathbb{R}\setminus\{0\} \longrightarrow \mathbb{R}$$

$$x \longmapsto y = f^{-1}(x) = 2 - \frac{\pi}{\operatorname{arctg} x}$$

com $CD_{f^{-1}} = D_f =]-\infty,0[\,\cup\,]4,+\infty[$

b) $f(x) = \frac{\pi}{2} - \operatorname{arctg}(1-x)$

$1 - x \in \mathbb{R} \Leftrightarrow x \in \mathbb{R}$ $\quad z \in \mathbb{R} = D_{tg\,z}$

$D_f = \mathbb{R}$

Quando x percorre $\mathbb{R}$

$$-\frac{\pi}{2} < \operatorname{arctg}(1-x) < \frac{\pi}{2}$$

$$\frac{\pi}{2} > -\operatorname{arctg}(1-x) > -\frac{\pi}{2}$$

$$\pi > \frac{\pi}{2} - \operatorname{arctg}(1-x) > 0$$

$$f(x) \in [0, \pi] = CD_f$$

Expressão de f^{-1}

$$y = \frac{\pi}{2} - \operatorname{arctg}(1-x)$$

$$\operatorname{arctg}(1-x) = \frac{\pi}{2} - y$$

$$1 - x = tg\left(\frac{\pi}{2} - y\right)$$

$$-x = tg\left(\frac{\pi}{2} - y\right) - 1$$

$$x = 1 - tg\left(\frac{\pi}{2} - y\right)$$

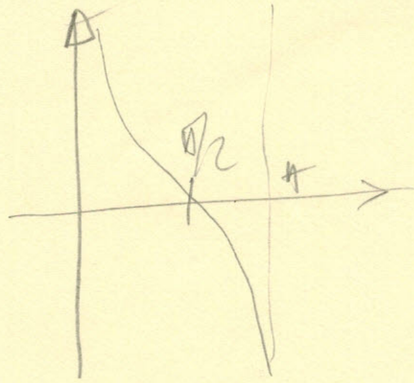
$$f^{-1} :]0, \pi[\longrightarrow \mathbb{R}$$

$$x \longmapsto y = 1 - tg\left(\frac{\pi}{2} - x\right)$$

Slide #20

caracterize a inversa das funções definidas por:

a) $f(x) = 2\,\text{cotg}\left(\frac{x}{3}\right)$



$\frac{x}{3} \in \left]0, \pi\right[$ ← função def na restrição principal

$x \in \left]0, 3\pi\right[= D_f$

Quando x percorre D_f

$\text{cotg}\left(\frac{x}{3}\right)$ percorre $\mathbb{R}$

$CD_f = \mathbb{R}$

$y = 2\,\text{cotg}\left(\frac{x}{3}\right)$

$\text{cotg}\left(\frac{x}{3}\right) = \frac{y}{2}$

$\frac{x}{3} = \text{arccotg}\,\frac{y}{2}$

$x = 3\,\text{arccotg}\,\frac{y}{2}$

$$f^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto y = f^{-1}(x) = 3\,\text{arccotg}\,\frac{x}{2}$$

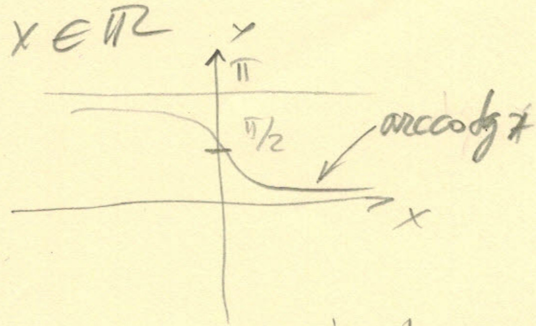
com $CD_{f^{-1}} = D_f = \left]0, 3\pi\right[$

(b) $f(x) = \pi + \text{arcotg}\left(\frac{x-1}{2}\right)$

$\frac{x-1}{2} \in \mathbb{R}$

$x-1 \in \mathbb{R}$

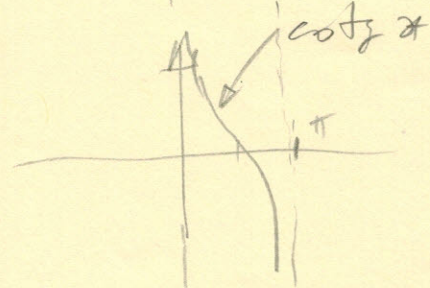
$x \in \mathbb{R}$



$D_f = \mathbb{R}$

Quando $\frac{x-1}{2}$ percorre $\mathbb{R}$

$\text{arcotg}\left(\frac{x-1}{2}\right)$ percorre $]0, \pi[$



$f(x)$ percorre $]\pi, 2\pi[$

$CD_f =]\pi, 2\pi[$

$y = \pi + \text{arcotg}\left(\frac{x-1}{2}\right)$

$\text{arcotg}\left(\frac{x-1}{2}\right) = y - \pi$

$\frac{x-1}{2} = \text{cotg}(y-\pi)$

$x - 1 = 2\,\text{cotg}(y-\pi)$

$x = 1 + 2\,\text{cotg}(y-\pi)$

$f^{-1}:]\pi, 2\pi[\longrightarrow \mathbb{R}$

$x \longmapsto y = f^{-1}(x) = 1 + 2\,\text{cotg}(x-$

com $CD_{f^{-1}} = D_f = \mathbb{R}$.

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Slide #27

Propriedades das inversas de funções trigonométricas:

$\boxed{1}$ $\sin(\arccos(x)) = \sqrt{1-x^2} = \cos(\arcsin(x))$

$y \in [0,\pi]$

x é um coseno
$|x| \leq 1$

Sabendo que

$$\cos(\underbrace{\arccos(x)}_{y}) = x \qquad \Big| \; f^{-1}(f(x)) = x$$

$$\sin(\underbrace{\arccos(x)}_{y}) = \sqrt{1-x^2}$$

$$\left[\cos^2 y + \sin^2 y = 1 \Leftrightarrow \sin y = +\sqrt{1-x^2}\right]$$

$y \in [0,\pi] \Rightarrow \sin y \geq 0$

Da mesma forma,

$$\cos(\arcsin(x)) = \sqrt{1-x^2}$$

$z \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

x é um seno
$|x| \leq 1$

Sabendo que

$$\sin(\underbrace{\arcsin(x)}_{z}) = x \qquad \Big| \; f^{-1}(f(x)) = x$$

$$\cos(\arcsin x) = \sqrt{1-x^2}$$

$z \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos z \geq 0$

$$\left[\sin^2 z + \cos^2 z = 1 \Leftrightarrow \cos z = \sqrt{1-x^2}\right.$$

$\boxed{2}$ $\operatorname{arccotg}(x) = \operatorname{arctg}\left(\frac{1}{x}\right)$ $, x \neq 0$

$x \in \mathbb{R} \setminus \{0\}$

$$\operatorname{tg}\left(\underbrace{\operatorname{arctg}\left(\frac{1}{x}\right)}_{y}\right) = \frac{1}{x}$$

$$\frac{\sin y}{\cos y} = \frac{1}{x}$$

$$\frac{\cos y}{\sin y} = x = \operatorname{cotg}(y)$$

$$y = \operatorname{arccotg}(x) = \operatorname{arctg}\left(\frac{1}{x}\right)$$

$\boxed{3}$ $\operatorname{arcsec}(x) = \arccos\left(\frac{1}{x}\right)$ $, x \neq 0$

$\frac{1}{x} \in [-1, 0[\cup]0, 1[$

$$\cos\left(\underbrace{\arccos\left(\frac{1}{x}\right)}_{y}\right) = \frac{1}{x} \qquad \left[\, f^{-1}(f(x)) = x\right.$$

$$\cos y = \frac{1}{x} \Leftrightarrow \frac{1}{\cos y} = x$$

$$\sec y = x$$

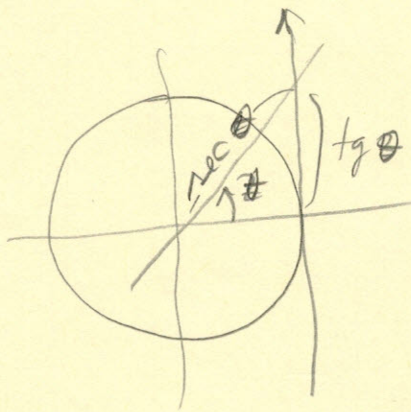
$$y = \operatorname{arcsec} x = \arccos\left(\frac{1}{x}\right)$$

$\boxed{4}$ $\text{arccosec}(x) = \text{arcsin}\left(\frac{1}{x}\right) \quad , x \neq 0$

exercício.

$\boxed{5}$ $\sec(\text{arctg}(x)) = \sqrt{1+x^2}$

$y = \sec(\text{arctg}(x))$



$\text{tg}\,(\underbrace{\text{arctg}(x)}_{y}) = x$

$1 + \text{tg}^2\theta = \sec^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\underbrace{1 + \text{tg}^2(\overset{y}{\text{arctg}\, x})}_{x^2} = \sec^2(\overset{y}{\text{arctg}\, x})$

$\sec(\text{arctg}\, x) = \sqrt{1+x^2}$

$x \in \mathbb{R} \Rightarrow x^2 \in \mathbb{R}^+$

$\boxed{\text{Slide } \#28} + \boxed{\#29}$

Considere as f.r.v.r. f e g definidas por $f(x) = x^3$ e $g(x) = \sin x$.

$D_f = \mathbb{R}$ $\quad D_f = \mathbb{R}$

Determine usando a regra da cadeia as derivadas seguintes:

$\boxed{1}$ $(g \circ f)'(x)$, $x \in \mathbb{R}$

$$\varphi(x) = g(f(x))$$

$$\varphi(x) = \sin x^3$$

$$\varphi'(x) = 3x^2 \cos x^3$$

$\boxed{2}$ $(f \circ g)'(x)$, $x \in \mathbb{R}$

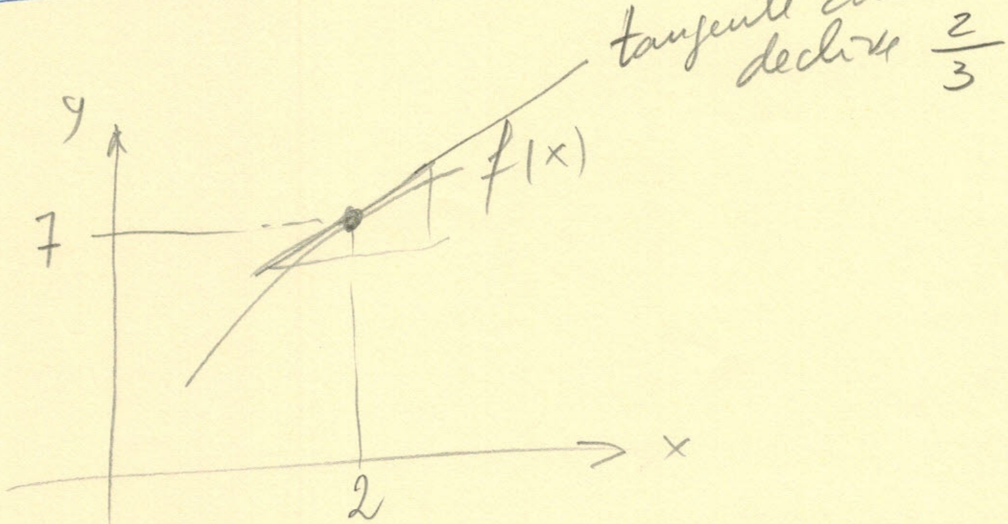
$$\phi(x) = f(g(x))$$

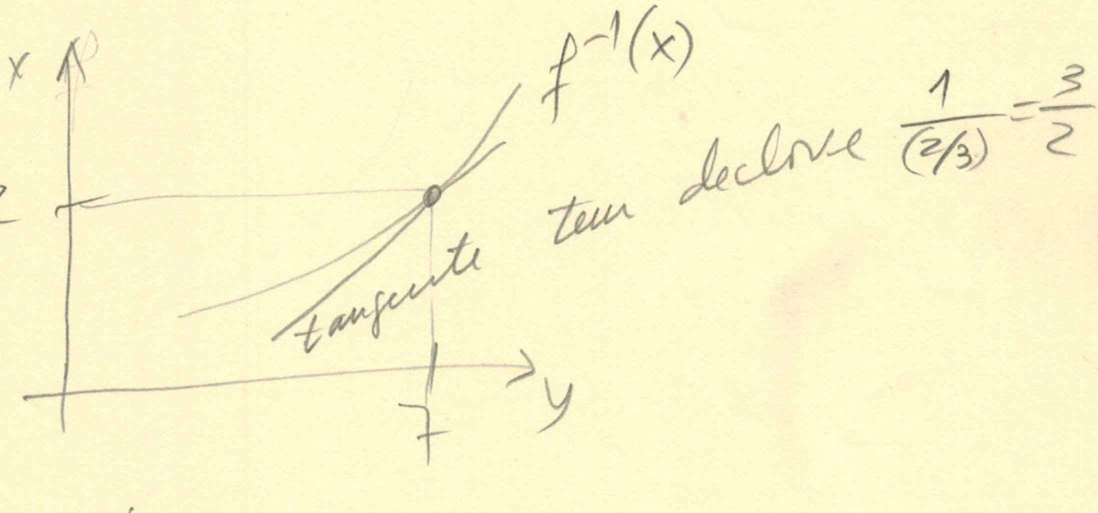
$$\phi(x) = \sin^3 x$$

$$\phi'(x) = 3 \sin^2 x \cos x$$

Slide #29

1 Sendo $f:[1,4]\to\mathbb{R}$ contínua e estritamente crescente tal que $f(2)=7$ e $f'(2)=\frac{2}{3}$, calcule, caso exista, $(f^{-1})'(7)$.





$$(f^{-1})'(7)=\frac{3}{2}$$

$\boxed{2}$ Sabendo que $f(x) = 4x^3 + x + 2$ é invertível, calcule $(f^{-1})'(2)$.

$$4x^3 + x + 2 = 2$$

$f'(x) = 12x^2 + 1 > 0 \quad \forall x \in \mathbb{R}$

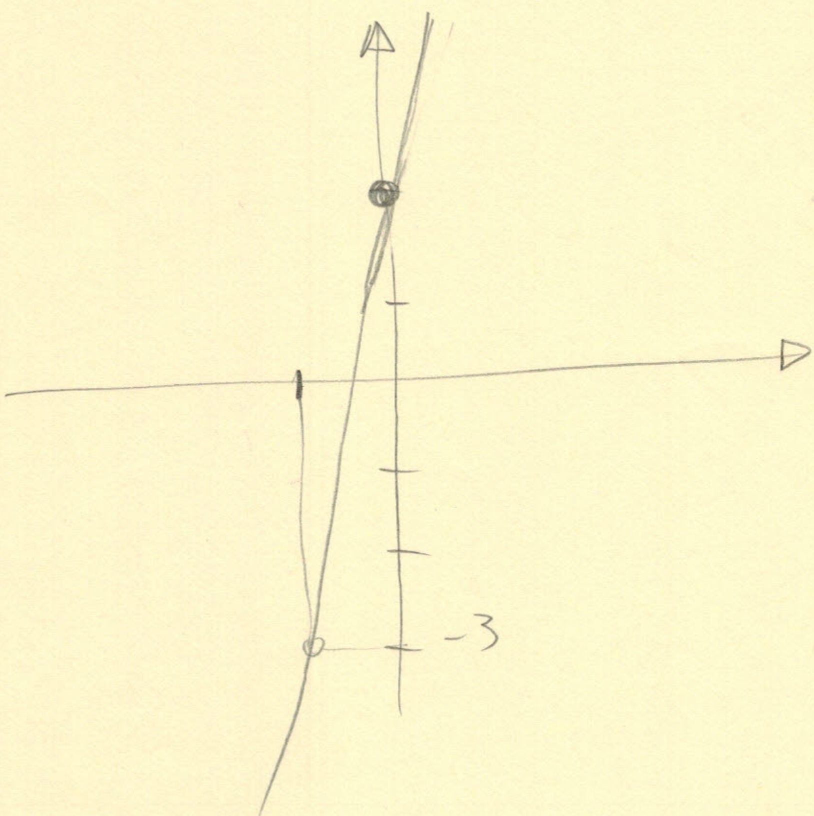
$$4x^3 + x = 0$$

$$x(4x^2 + 1) = 0$$

$$x = 0$$

$$f'(x) = 12x^2 + 1 \Rightarrow f'(0) = 1$$

$$(f^{-1})'(2) = \frac{1}{f^{-1}(0)} = 1$$



$\boxed{3}$ Seja $f(x)=x^3$. Determine a derivada de f^{-1} utilizando o teorema da função inversa

$f(x)=x^3$, contínua, dif.

$f'(x)=3x^2>0, \ \forall x \in \mathbb{R}\setminus\{0\}$

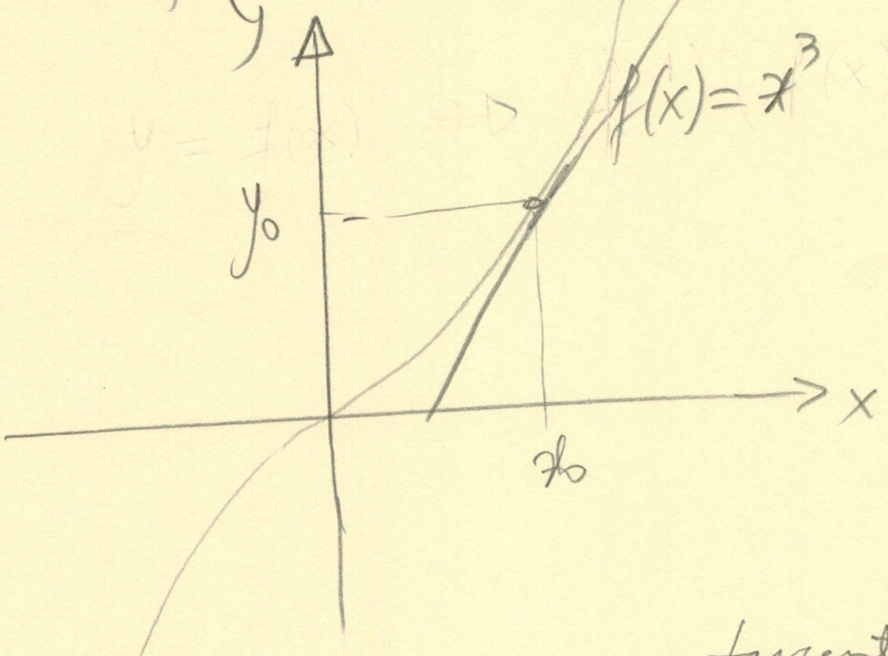
f é estritamente crescente em $\mathbb{R}$

f é injetiva em $\mathbb{R}$

f é invertível

tangente com declive

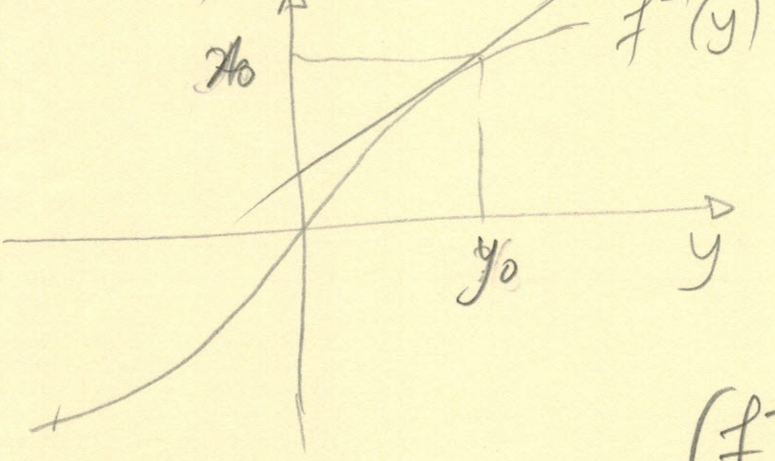
$3x_0^2 = f'(x_0)$



inversa

tangente com declive

$\frac{1}{3x_0^2} = (f^{-1})'(y_0)$



mas como

$y_0 = x_0^3$

$x_0 = \sqrt[3]{y_0}$

$$(f^{-1})'(y_0) = \frac{1}{3\sqrt[3]{y_0^2}}$$

Confirmo definindo a inversa

$$y = x^3 \Leftrightarrow x = \sqrt[3]{y} = y^{1/3}$$

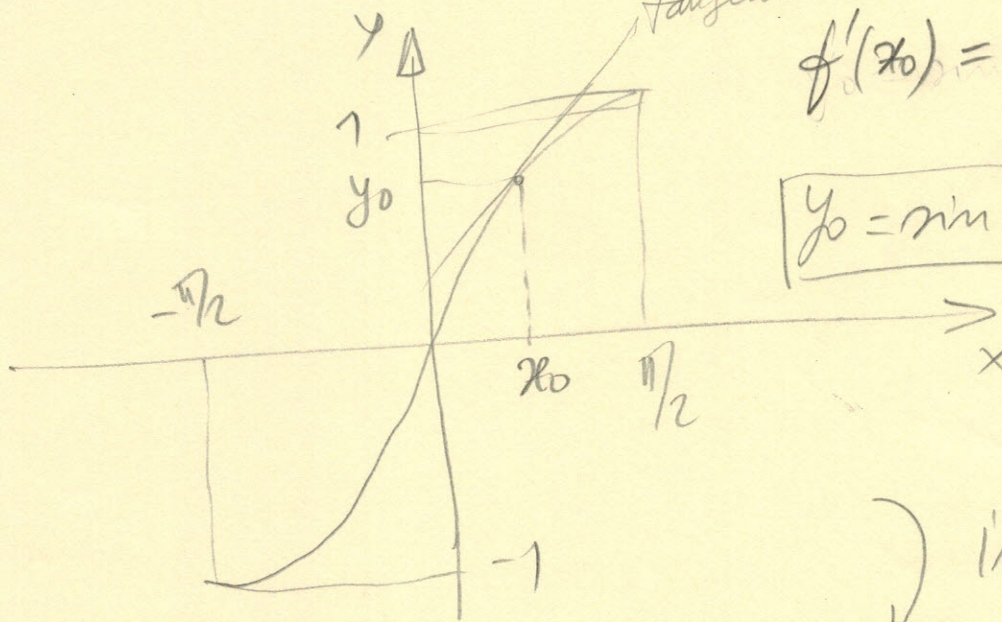
$$f^{-1}(y) = y^{1/3}$$

$$(f^{-1})'(y) = \frac{1}{3} y^{-2/3} = \frac{1}{3\sqrt[3]{y^2}}$$ ✓

$\boxed{\text{Slide \#30}}$

Utilizando o T. da derivada da função inversa mostre que

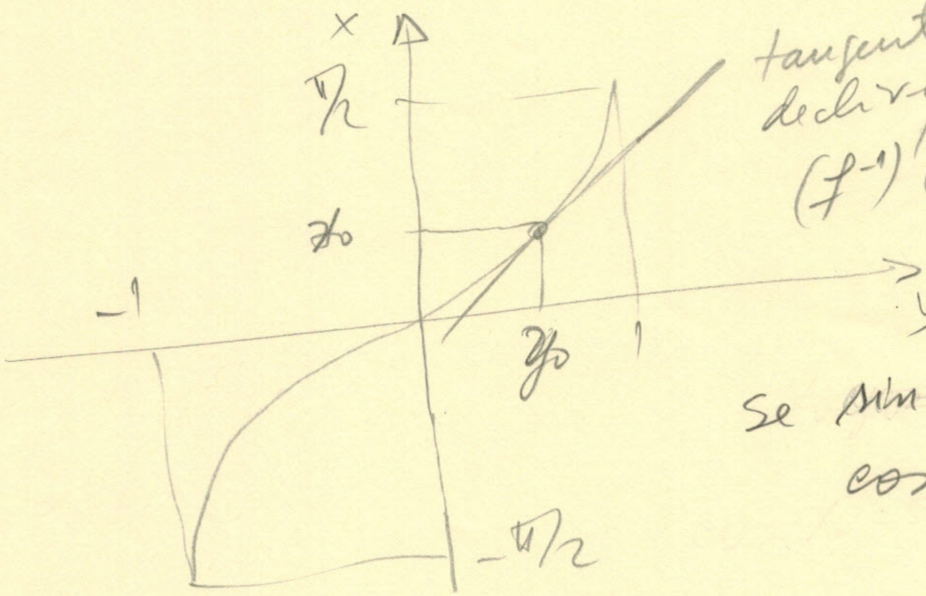
$\boxed{1}$ $$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \ \forall x \in]-1,1[$$



tangente tem declive $f'(x_0) = \cos x_0$

$\boxed{y_0 = \sin x_0}$

invento

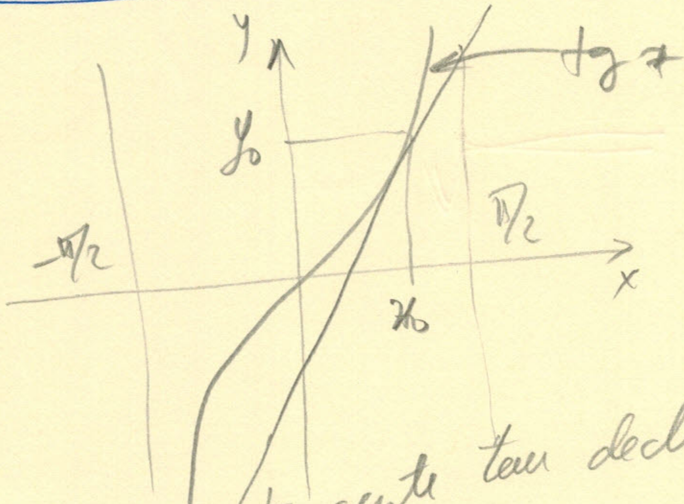


tangente tem declive $(f^{-1})'(y_0) = \frac{1}{\cos x_0}$

Se $\sin x_0 = y_0$
$\cos x_0 = \sqrt{1-y_0^2}$

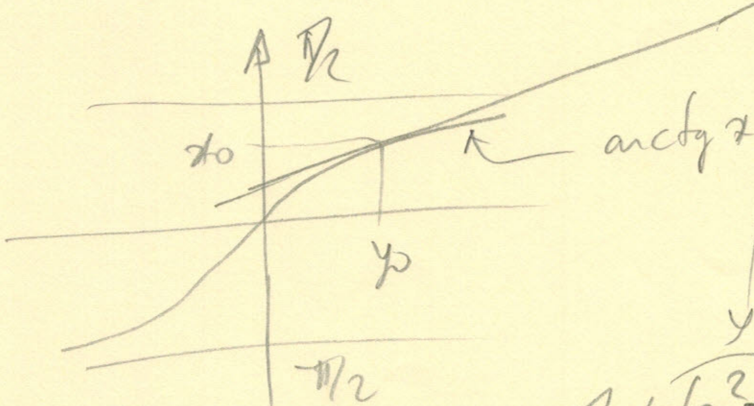
$$(f^{-1})'(y_0) = \frac{1}{\sqrt{1-y_0^2}}, \ \forall x \in]-1,1[$$

$$\boxed{3} \quad (\mathrm{arctg}\, x)' = \frac{1}{1+x^2}, \quad x \in \mathbb{R}$$



$y_0 = \mathrm{tg}\, x_0$

tangente tem declive $f'(x_0) = \frac{1}{\cos^2 x_0}$



tangente tem declive

$(f^{-1})'(y_0) = \cos^2 x_0$

$1 + \mathrm{tg}^2 x = \sec^2 x$

$$y_0 = \mathrm{tg}\, x_0 \Rightarrow \boxed{1 + y_0^2 = \frac{1}{\cos^2 x_0}}$$

$$\cos^2 x_0 = \frac{1}{1+y_0^2}$$

$$(f^{-1})'(y_0) = \frac{1}{1+y_0^2}$$

Slide #32

$\boxed{1}$ Seja $f(x) = \ln(\arcsin x)$, $x \in]0,1[$
Calcule $(f^{-1})'$ utilizando o teorema da função inversa.

$y_0 = \ln(\arcsin x_0) = f(x_0)$

$f'(x_0) = \frac{1}{\arcsin x_0} \frac{1}{\sqrt{1-x_0^2}}$

$(f^{-1})'(y_0) = \sqrt{1-x_0^2} \arcsin x_0$

$y_0 = \ln(\arcsin x_0)$

$e^{y_0} = \arcsin x_0 \Leftrightarrow x_0 = \sin(e^{y_0})$

$(f^{-1})'(y_0) = \sqrt{1-\sin^2(e^{y_0})} \arcsin(\sin e^{y_0})$

$(f^{-1})'(y_0) = \cos e^{y_0} . e^{y_0}$

Defino a inversa

$y = \ln(\arcsin x)$

$e^y = (\arcsin x)$

$\sin e^y = x$

$x = f^{-1}(y) = \sin e^y$

$(f^{-1})'(y) = e^y \cos e^y$

$\boxed{2}$ Calcule a derivada das seguintes funções

a) $f(x) = (1+x^2) \operatorname{arctg} x$

$f'(x) = (1+x^2)' \operatorname{arctg} x + (1+x^2) \frac{1}{1+x^2}$

$f'(x) = 2x \operatorname{arctg} x + 1$

b) $f(x) = \arcsin\left(\frac{1}{x^2}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{1}{x^2}\right)^2}}\left(\frac{1}{x^2}\right)'$$

$$f'(x) = \frac{1}{\sqrt{1-\frac{1}{x^4}}}\left(-\frac{2x}{x^4}\right)$$

$$f'(x) = \frac{1}{\sqrt{\frac{x^4-1}{x^4}}}\left(-\frac{2}{x^3}\right)$$

$$f'(x) = \frac{1}{\frac{1}{x^2}\sqrt{x^4-1}}\left(-\frac{2}{x^3}\right)$$

$$f'(x) = -\frac{x^2}{\sqrt{x^4-1}}\,\frac{2}{x^3}$$

$$f'(x) = -\frac{2}{x\sqrt{x^4-1}} \checkmark$$

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c) $f(x) = \operatorname{arccotg}\left(\sin(4x^3)\right)$

$$f'(x) = -\frac{\left[\sin(4x^3)\right]'}{1+\sin^2(4x^3)}$$

$$f'(x) = -\frac{12x^2\cos(4x^3)}{1+\sin^2(4x^3)}$$

(d) $f(x) = \sqrt[3]{\arccos x}$

$$f'(x) = \frac{1}{3}\left(\arccos x\right)^{-\frac{2}{3}}\left(\arccos x\right)'$$

$$f'(x) = \frac{1}{3\sqrt[3]{\arccos^2 x}}\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$f'(x) = -\frac{1}{3\sqrt{1-x^2}\,\sqrt[3]{\arccos^2 x}}$$

$\boxed{3}$ Considere a função

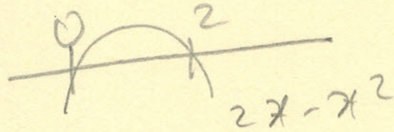
$$f(x) = \arcsin(1-x) + \sqrt{2x - x^2}$$

seno $\quad\geqslant 0$

a) D_f

$$\begin{cases} |1-x| \leq 1 \\ 2x - x^2 \geqslant 0 \end{cases} \qquad \begin{cases} 1-x \leq 1 \\ 1-x \geqslant -1 \\ 2x - x^2 \geqslant 0 \end{cases} \qquad \begin{matrix} x \geqslant 0 \\ x \leq 2 \\ x \in [0,2] \end{matrix}$$

$$2x - x^2 = 0 \Leftrightarrow x(2-x) = 0 \Leftrightarrow x = 0 \vee x = 2$$



$$D_f = [0,2]$$

b) Mostre que $f'(x) = -\dfrac{x}{\sqrt{2x - x^2}}$

$$f'(x) = \frac{-1}{\sqrt{1-(1-x)^2}} + \left(\frac{1}{2}\right)(2x - x^2)^{-\frac{1}{2}}(2-2x)$$

$$f'(x) = \frac{-1}{\sqrt{1-(x-1)^2}} + \frac{2-2x}{2\sqrt{2x - x^2}}$$

$$f'(x) = \frac{-1}{\sqrt{1-(x^2-2x+1)}} + \frac{\cancel{2}(1-x)}{\cancel{2}\sqrt{2x-x^2}}$$

$$f'(x) = \frac{-1}{\sqrt{2x-x^2}} + \frac{1-x}{\sqrt{2x-x^2}}$$

$$f'(x) = \frac{-x}{\sqrt{2x-x^2}}$$

//