$\boxed{1}$

a) $$\int (3x^2+5x+7)\,dx = \int 3x^2\,dx + \int 5x\,dx + \int 7\,dx$$

$$= \cancel{3}\,\frac{x^3}{\cancel{3}} + 5\times\frac{x^2}{2} + 7x + c$$

$$= x^3 + \frac{5}{2}x^2 + 7x + c,\ c\in\mathbb{R}$$ em intervalos

b) $$\int \sqrt[3]{x}\,dx = \int (x)^{1/3}\,dx = \frac{x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}} = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3}{4}\times\sqrt[3]{x^4}$$

c) $$\int (x^3+1)^2\,dx = \int x^6\,dx + \int 2x^3\,dx + \int 1\,dx$$

$$= \frac{x^7}{7} + 2\times\frac{x^4}{4} + x + c$$

$$= \frac{x^7}{7} + \frac{x^4}{2} + x + c,\ c\in\mathbb{R}$$ em intervalos

d) $$\int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\times\operatorname{arctg} x\,dx$$

$$= \int (\operatorname{arctg} x)'\times\operatorname{arctg} x\,dx$$

$$= \frac{\operatorname{arctg}^2 x}{2} + c,\ c\in\mathbb{R}$$ em intervalos

e) $$\int \frac{3x^2}{1+x^3} = \int \frac{(1+x^3)'}{1+x^3}\,dx = \ln|1+x^3| + c,\ c\in\mathbb{R}$$ em intervalos

f) $$\int \frac{1}{x^7}\,dx = \int x^{-7}\,dx = \frac{x^{-6}}{-6} + c,\ c\in\mathbb{R}$$ em intervalos

g) $$\int \frac{x+1}{2+4x^2}\,dx = \int \frac{x+1}{2\times(1+2x^2)}\,dx = \frac{1}{2}\int\frac{x+1}{1+2x^2}\,dx$$

$$= \frac{1}{2}\times\int\frac{4\times\frac{1}{4}\times(x+1)}{1+2x^2}\,dx = \frac{1}{2}\times\frac{1}{4}\times\int\frac{4x+4}{1+2x^2}\,dx$$

$$= \frac{1}{8}\times\int\frac{(1+2x^2)'}{1+2x^2}\,dx + \frac{1}{8}\times\int\frac{4}{1+2x^2}\,dx$$

$$= \frac{1}{8} \times \ln\left(1+2x^2\right) + C + \frac{1}{2} \times \frac{1}{\sqrt{2}} \int \frac{\sqrt{2}}{1+\left(\sqrt{2}\,x\right)^2}\, dx$$

$$= \frac{1}{8} \times \ln\left(1+2x^2\right) + \frac{\sqrt{2}}{4} \operatorname{arctg}\left(\sqrt{2}\,x\right) + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

h) $$\int 4x^3 \cos(x^4)\, dx = \operatorname{sen}(x^4) + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

$\operatorname{sen}(\operatorname{arcsen} x) = x$

i) $$\int \frac{x}{\sqrt{1-x^2}}\, dx = \int \frac{1}{\sqrt{1-x^2}} \times x\, dx = \int (\operatorname{arcsen} x)' \times x\, dx$$

$$= \int (\operatorname{arcsen} x)' \times \operatorname{sen}(\operatorname{arcsen} x)\, dx$$

Como: $\cos y = \pm\sqrt{1-\operatorname{sen}^2 y}$

$\cos y \geq 0 \;\; \forall y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \sqrt{1-\operatorname{sen}^2 y}$

$$= -\cos\big(\underbrace{\overbrace{\operatorname{arcsen} x}^{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}}_{=y}\big) + C$$

$\boxed{\text{Seja } y = \operatorname{arcsen} x}$

$$= -\cos y + C$$

$$= -\sqrt{1-\operatorname{sen}^2 y} + C$$

$\boxed{\text{Substituindo...}}$

$$= -\sqrt{1-\left[\operatorname{sen}(\operatorname{arcsen} x)\right]^2} + C$$

$$= -\sqrt{1-x^2} + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

j) $$\int \operatorname{sen} x \cos^5 x\, dx = -\int (\cos x)' \times \cos^5 x\, dx$$

$$= -\frac{\cos^6 x}{6} + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

k) $$\int \operatorname{tg} x\, dx = \int \frac{\operatorname{sen} x}{\cos x}\, dx = -\int \frac{(\cos x)'}{\cos x}\, dx = -\ln|\cos x| + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

l) $$\int \frac{\ln(x)}{x}\, dx = \int \frac{1}{x} \times \ln(x)\, dx = \int \left[\ln(x)\right]' \times \ln(x)\, dx$$

$$= \frac{\left[\ln(x)\right]^2}{2} + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

m) $$\int e^{\operatorname{tg} x} \sec^2 x\, dx = \int \left[\operatorname{tg}(x)\right]' \times e^{\operatorname{tg} x}\, dx = e^{\operatorname{tg} x} + C, \quad C \in \mathbb{R} \text{ em intervalos}$$

n) $\displaystyle\int x\,7^{x^2}\,dx = \frac{1}{2}\times\int 2x\times 7^{x^2}\,dx = \frac{1}{2}\int (x^2)'\times 7^{x^2}\,dx = \frac{1}{2}\times\left[\frac{7^{x^2}}{\ln(7)}+c\right]$

$\displaystyle = \frac{7^{x^2}}{\ln(49)} + c \;,\; c\in\mathbb{R}$ em intervalos

o) $\displaystyle\int \operatorname{sen}(\sqrt{2}\,x)\,dx = \frac{1}{\sqrt{2}}\times\int(\sqrt{2}\,x)'\operatorname{sen}(\sqrt{2}\,x)\,dx = -\frac{\sqrt{2}}{2}\times\cos(\sqrt{2}\,x)+c \;,\; c\in\mathbb{R}$ em intervalos

p) $\displaystyle\int \frac{x^2+1}{x}\,dx = \int\frac{x^2}{x}\,dx + \int\frac{1}{x}\,dx = \int x\,dx + \ln|x| + c$

$\displaystyle = \frac{x^2}{2} + \ln|x| + c \;,\; c\in\mathbb{R}$ em intervalos

q) $\displaystyle\int \frac{x}{(7+5x^2)^{\frac{3}{2}}}\,dx = \frac{1}{10}\int(5x^2)'\times(7+5x^2)^{-\frac{3}{2}}\,dx = \frac{1}{10}\times\frac{(7+5x^2)^{-\frac{1}{2}}}{-\frac{1}{2}}+c$

$\displaystyle = -\frac{1}{5\times\sqrt{7+5x^2}} + c \;,\; c\in\mathbb{R}$ em intervalos

r) $\displaystyle\int\frac{x^3}{1+x^8}\,dx = \frac{1}{4}\times\int\frac{(x^4)'}{1+(x^4)^2}\,dx = \frac{1}{4}\times\operatorname{arctg}(x^4)+c \;,\; c\in\mathbb{R}$ em intervalos

s) $\displaystyle\int\frac{5x^2}{\sqrt{1-x^6}}\,dx = \frac{5}{3}\int\frac{(x^3)'}{\sqrt{1-[x^3]^2}}\,dx = \frac{5}{3}\operatorname{arcsen}(x^3)+c \;,\; c\in\mathbb{R}$ em intervalos

t) $\displaystyle\int\frac{1}{x^2+7}\,dx = \int\frac{1}{7\left(\frac{x^2}{7}+1\right)}\,dx = \frac{1}{7}\int\frac{1}{\left(\frac{x}{\sqrt{7}}\right)^2+1}\,dx$

$\displaystyle = \frac{1}{\sqrt{7}}\times\int\frac{\left(\frac{x}{\sqrt{7}}\right)'}{1+\left(\frac{x}{\sqrt{7}}\right)^2}\,dx = \frac{\sqrt{7}}{7}\times\operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right)+c \;,\; c\in\mathbb{R}$ em intervalos

$\boxed{2}$

$F(-1) = 1$

$\displaystyle F(x) = \int\frac{2}{x}+\frac{3}{x^2}\,dx = 2\times\int\frac{1}{\underset{<0}{x}}\,dx + 3\times\int\frac{1}{x^2}\,dx$

$\displaystyle = 2\times\ln|x| + 3\times\frac{x^{-1}}{-1}+c$

$\displaystyle = 2\times\ln|x| - \frac{3}{x}+c \;,\; c\in\mathbb{R}$

Como $F(-1) = 1$:

$$2 \times \ln|-1| + 3 + C = 1$$

$$\Leftrightarrow 2\ln(1) + 2 = -C$$

$$\Leftrightarrow 2 \times 0 + 2 = -C$$

$$\Leftrightarrow C = -2$$

Logo, $\boxed{F(x) = 2 \times \ln|x| - \frac{3}{x} - 2}$

3

$$\int f(x)\,dx = \operatorname{sen} x - x\cos x - \frac{1}{2}x^2 + C, \; C \in \mathbb{R}$$

$$f\left(\frac{\pi}{4}\right) = ?$$

$$f(x) = \left(\operatorname{sen} x - x \cos x - \frac{1}{2}x^2 + \cancel{C}\right)'$$

$$= (\operatorname{sen} x)' - (x\cos x)' - \left(\frac{x^2}{2}\right)'$$

$$= \cancel{\cos x} - \left(\cancel{\cos} x + x(-\operatorname{sen} x)\right) - \frac{\cancel{2}x \times \cancel{2} - 0}{\cancel{4}}$$

$$= x \operatorname{sen} x - x$$

$$= x(\operatorname{sen} x - 1)$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \times \left(\operatorname{sen}\frac{\pi}{4} - 1\right)$$

$$= \frac{\pi}{4} \times \left(\frac{\sqrt{2}}{2} - \frac{2}{2}\right)$$

$$= \frac{\pi}{8} \times \left(\sqrt{2} - 2\right)$$

4

$$\int \frac{1}{x^2} + 1\,dx = \int x^{-2}\,dx + x + C = \frac{x^{-1}}{-1} + x + C$$

$x \in \mathbb{R}^+$

$$= -\frac{1}{x} + x + C$$

$$= -\frac{1}{x} + \frac{x^2}{x} + C$$

$$= \frac{x^2 - 1}{x} + C, \; C \in \mathbb{R} \text{ em intervalos}$$

$$g(x) = \frac{x^2 - 1}{x} + c \,, \quad c \in \mathbb{R} \text{ em intervalos}$$

$$g(2) = 0$$

$$\Leftrightarrow \frac{4-1}{2} + c = 0$$

$$\Leftrightarrow c = -\frac{3}{2}$$

$$g(x) = \frac{x^2 - 1}{x} - \frac{3}{2}$$

$\boxed{5}$

$$g(x) = ?$$

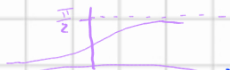
$$g(x) = \int \frac{1}{(1 + \operatorname{arctg}^2 x)(1 + x^2)} \, dx \qquad \boxed{\lim_{x \to +\infty} g(x) = 0}$$

$$(\operatorname{arctg} x)' = \frac{1}{1 + x^2}$$

$$= \int \frac{1}{(1 + \operatorname{arctg}^2 x)} \times (\operatorname{arctg} x)' \, dx$$

$$= \int \frac{(\operatorname{arctg} x)'}{1 + (\operatorname{arctg} x)^2} \, dx = \operatorname{arctg}(\operatorname{arctg} x) + c \,, \quad c \in \mathbb{R} \text{ em intervalos}$$

$$\lim_{x \to +\infty} g(x) = 0 \Leftrightarrow \lim_{x \to +\infty} \left[\operatorname{arctg}(\operatorname{arctg} x) + c \right] = 0$$



$$\Leftrightarrow \operatorname{arctg}\left(\lim_{x \to +\infty} \operatorname{arctg} x \right) + c = 0$$

$$\Leftrightarrow \operatorname{arctg}\left(\frac{\pi}{2} \right) + c = 0$$

$$\Leftrightarrow c = -\operatorname{arctg}\left(\frac{\pi}{2} \right)$$

$$g(x) = \operatorname{arctg}(\operatorname{arctg} x) - \operatorname{arctg}\left(\frac{\pi}{2} \right)$$

$\boxed{6}$

a) $$\int \underbrace{x}_{u} \underbrace{\cos x}_{v'} \, dx = x \times \operatorname{sen} x - \int (1) \times \operatorname{sen} x \, dx$$

$$= x \operatorname{sen} x + \int (-\operatorname{sen} x) \, dx$$

$u = x \qquad v = \operatorname{sen} x$

$u' = 1 \qquad v' = \cos x$

$$= x \operatorname{sen} x + \cos x + c, \ c \in \mathbb{R} \text{ em intervalos}$$

b) $$\int \underbrace{x^2}_{u} \underbrace{\cos x}_{v'} \, dx = x^2 \times \operatorname{sen} x - \int 2x \operatorname{sen} x \, dx$$

$u = x^2 \qquad v = \operatorname{sen} x$

$u' = 2x \qquad v' = \cos x$

$$= x^2 \operatorname{sen} x - 2 \int \underbrace{x}_{u_0} \underbrace{\operatorname{sen} x}_{v_0'} \, dx$$

$u_0 = x \qquad v_0 = -\cos x$

$u_0' = 1 \qquad v_0' = \operatorname{sen} x$

$$= x^2 \operatorname{sen} x - 2 \times x \times (-\cos x) - (-2) \int (1) \times (-\cos x) \, dx$$

$$= x^2 \operatorname{sen} x + 2x \cos x - 2 \int \cos x \, dx$$

$$= x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + c, \ c \in \mathbb{R} \text{ em intervalos}$$

c) $$\int \underbrace{e^{-3x}}_{u'} \underbrace{(2x+3)}_{v} \, dx = -\frac{1}{3} e^{-3x} \times (2x+3) - \int -\frac{1}{3} e^{-3x} \times 2 \, dx$$

$$\boxed{\begin{array}{ll} u = -\frac{1}{3} e^{-3x} & v = 2x+3 \\ u' = e^{-3x} & v' = 2 \end{array}}$$

$$= -\frac{2}{3} x e^{-3x} - e^{-3x} + \frac{2}{3} \int e^{-3x} \, dx$$

$$= e^{-3x} \left(-\frac{2}{3} x - 1 \right) + \frac{2}{3} \times \left(-\frac{1}{3} \right) \int (-3) \times e^{-3x} \, dx$$

$$= e^{-3x} \left(-\frac{2}{3} x - 1 \right) - \frac{2}{9} e^{-3x} + c, \ c \in \mathbb{R} \text{ em intervalos}$$

d) $$\int \ln^2 x \, dx = \int \underbrace{(1)}_{u'} \times \underbrace{\ln^2 x}_{v} \, dx = x \times \ln^2 x - \int \cancel{x} \times \frac{2 \ln x}{\cancel{x}} \, dx$$

$$\boxed{\begin{array}{ll} u = x & v = \ln^2 x \\ u' = 1 & v' = \frac{2 \ln x}{x} \end{array}}$$

$$= x \times \ln^2 x - 2 \int \underbrace{(1)}_{u_0'} \times \underbrace{\ln x}_{v_0} \, dx$$

$u_0 = x \qquad v_0 = \ln x$

$u_0' = 1 \qquad v_0' = \frac{1}{x}$

$$= x \ln^2 x - 2x \ln x + 2 \int \cancel{x} \times \frac{1}{\cancel{x}} \, dx$$

$$= x \ln^2 x - 2x \ln x + 2x + c$$

$$= x \ln^2 x + 2x (x - \ln x) + c$$

$$= x\left(\ln^2(x) - 2\ln(x) + 2\right) + c,\ c \in \mathbb{R} \text{ em intervalos}$$

e)

$$\int \underbrace{e^{2x}}_{u} \underbrace{\text{sen}(x)}_{v'}\,dx = e^{2x} \times (-\cos x) - \int 2e^{2x} \times (-\cos x)\,dx$$

$$= -e^{2x}\cos x + 2\int \underbrace{e^{2x}}_{u_0} \underbrace{\cos x}_{v_0'}\,dx$$

$$= -e^{2x}\cos x + 2e^{2x}\,\text{sen}\,x - 2\int 2e^{2x}\,\text{sen}\,x\,dx$$

$$= e^{2x}(-\cos x + 2\,\text{sen}\,x) - 4\boxed{\int e^{2x}\,\text{sen}\,x\,dx}$$

É igual à inicial

$$\boxed{\begin{array}{ll} u = e^{2x} & v = -\cos x \\ u' = 2e^{2x} & v' = \text{sen}\,x \end{array}}$$

$$\boxed{\begin{array}{ll} u_0 = e^{2x} & v_0 = \text{sen}\,x \\ u_0' = 2e^{2x} & v_0' = \cos x \end{array}}$$

$$\int e^{2x}\,\text{sen}(x)\,dx = e^{2x}(-\cos x + 2\,\text{sen}\,x) - 4\int e^{2x}\,\text{sen}\,x\,dx$$

$$\Leftrightarrow \int e^{2x}\,\text{sen}\,x\,dx = \frac{e^{2x}(2\,\text{sen}\,x - \cos x)}{5} + c,\ c \in \mathbb{R} \text{ em intervalos}$$

f)

$$\int \text{sen}(\ln x)\,dx = \int \underbrace{1}_{u'} \times \underbrace{\text{sen}(\ln(x))}_{v}\,dx = x\,\text{sen}(\ln x) - \int \cancel{x} \times \frac{1}{\cancel{x}} \times \cos(\ln x)\,dx$$

$$= x\,\text{sen}(\ln x) - \int \underbrace{1}_{u_0'} \times \underbrace{\cos(\ln x)}_{v_0}\,dx$$

$$= x\,\text{sen}(\ln x) - x\cos(\ln x) + \int \cancel{x} \times \left(-\frac{1}{\cancel{x}}\,\text{sen}(\ln x)\right)dx$$

$$= x\left(\text{sen}(\ln x) - \cos(\ln x)\right) - \boxed{\int \text{sen}(\ln x)\,dx}$$

Igual à inicial!

$$\boxed{\begin{array}{ll} u = x & v = \text{sen}(\ln x) \\ u' = 1 & v' = \frac{1}{x} \times \cos(\ln x) \end{array}}$$

$(f \circ g)'(x) = g'(x) \times f'(g(x))$

$$\boxed{\begin{array}{ll} u_0 = x & v_0 = \cos(\ln x) \\ u_0' = 1 & v_0' = -\frac{1}{x} \times \text{sen}(\ln x) \end{array}}$$

$$\int \text{sen}(\ln x)\,dx = x\left(\text{sen}(\ln x) - \cos(\ln x)\right) - \int \text{sen}(\ln x)\,dx$$

$$\Leftrightarrow \int \text{sen}(\ln x)\,dx = \frac{x\left[\text{sen}(\ln x) - \cos(\ln x)\right]}{2} + c,\ c \in \mathbb{R} \text{ em intervalos}$$

g)

$$\int \arcsin x \, dx = \int \underbrace{1}_{u'} \times \underbrace{\text{arcsen}\, x}_{v} \, dx = x \,\text{arcsen}\, x - \int x \times \frac{1}{\sqrt{1-x^2}} \, dx$$

$$\boxed{\begin{array}{ll} u = x & v = \text{arcsen}\, x \\ u' = 1 & v' = \dfrac{1}{\sqrt{1-x^2}} \end{array}}$$

$$= x \,\text{arcsen}\, x + \int -\frac{x}{\sqrt{1-x^2}} \, dx$$

$$= x \,\text{arcsen}\, x + \int \left(\sqrt{1-x^2}\right)' dx$$

$$= x \,\text{arcsen}\, x + \sqrt{1-x^2} + C, \; C \in \mathbb{R} \text{ em intervalos}$$

$$\left(\sqrt{1-x^2}\right)' = \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{1-x^2}} \times (-\cancel{2}x)$$

$$= -\frac{x}{\sqrt{1-x^2}}$$

h)

$$\int \underbrace{x}_{u'} \underbrace{\text{arcsen}(x^2)}_{v} \, dx = \frac{x^2}{2} \times \text{arcsen}(x^2) - \int \frac{x^2}{\cancel{2}} \times \frac{\cancel{2}x}{\sqrt{1-(x^2)^2}} \, dx$$

$$\boxed{\begin{array}{ll} u = \dfrac{x^2}{2} & v = \text{arcsen}(x^2) \\ u' = x & v' = \dfrac{2x}{\sqrt{1-(x^2)^2}} \end{array}}$$

$$= \frac{x^2}{2} \,\text{arcsen}(x^2) - \int \frac{x^3}{\sqrt{1-x^4}} \, dx$$

$$= \frac{x^2}{2} \,\text{arcsen}(x^2) + \frac{1}{2} \int \frac{-2x^3}{\sqrt{1-x^4}} \, dx$$

$$= \frac{x^2}{2} \,\text{arcsen}(x^2) + \frac{1}{2} \int \left(\sqrt{1-x^4}\right)' dx$$

$$= \frac{x^2}{2} \,\text{arcsen}(x^2) + \frac{1}{2} \sqrt{1-x^4} + C, \; C \in \mathbb{R} \text{ em intervalos}$$

$$\left(\sqrt{1-x^4}\right)' = \frac{1}{2} \times \frac{1}{\sqrt{1-x^4}} \times -4x^3$$

$$= -\frac{2x^3}{\sqrt{1-x^4}}$$

i)

$$\int \text{arctg}\, x \, dx = \int \underbrace{1}_{u'} \times \underbrace{\text{arctg}\, x}_{v} \, dx = x \times \text{arctg}\, x - \int x \times \frac{1}{1+x^2} \, dx$$

$$\boxed{\begin{array}{ll} u = x & v = \text{arctg}\, x \\ u' = 1 & v' = \dfrac{1}{1+x^2} \end{array}}$$

$$= x \times \text{arctg} - \frac{1}{2} \int \frac{(1+x^2)'}{1+x^2} \, dx$$

$$= x \,\text{arctg} - \frac{1}{2} \times \ln(1+x^2) + C, \; C \in \mathbb{R} \text{ em intervalos}$$

j)

$$\int \operatorname{arctg}\frac{1}{x}\,dx = \int \underbrace{1}_{u'} \times \underbrace{\operatorname{arctg}\left(\frac{1}{x}\right)}_{v} dx = x\operatorname{arctg}\left(\frac{1}{x}\right) - \int x \times \frac{-1}{x^2+1}\,dx$$

$$\boxed{\begin{array}{ll} u = x & v = \operatorname{arctg}\left(\frac{1}{x}\right) \\ u' = 1 & v' = -\frac{1}{x^2+1} \end{array}}$$

$$= x\operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$$

$$= x\operatorname{arctg}\left(\frac{1}{x}\right) + \frac{1}{2} \times \ln\left(1+x^2\right) + c, \; c \in \mathbb{R} \text{ em intervalos}$$

k)

$$\int \underbrace{\sqrt{x}}_{u'}\,\underbrace{\ln x}_{v}\,dx = \frac{2}{3} \times \sqrt{x^3} \times \ln x - \int \frac{2\sqrt{x^3}}{3} \times \frac{1}{x}\,dx$$

$$\int x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \times \sqrt{x^3}$$

$$\boxed{\begin{array}{ll} u = \frac{2x\sqrt{x}}{3} & v = \ln x \\ u' = \sqrt{x} & v' = \frac{1}{x} \end{array}}$$

$$= \frac{2}{3}\sqrt{x^3}\ln x - \frac{2}{3}\int \frac{x^{\frac{3}{2}}}{x}\,dx$$

$$= \frac{2}{3}\sqrt{x^3}\ln x - \frac{2}{3}\int \sqrt{x}\,dx$$

$$= \frac{2}{3}\sqrt{x^3}\ln x - \frac{2}{3} \times \frac{2}{3} \times \sqrt{x^3} + c$$

$$= \frac{2}{3}\sqrt{x^3}\left(\ln x - \frac{2}{3}\right) + c, \; c \in \mathbb{R} \text{ em intervalos}$$

l)

$$\int \underbrace{\sin x}_{u}\,\underbrace{\cos x}_{v'}\,dx = \sin^2 x - \int \cos x \times \sin x\,dx$$

$$\boxed{\begin{array}{ll} u = \sin x & v = \sin x \\ u' = \cos x & v' = \cos x \end{array}}$$

$$= \sin^2 x - \int (\sin x)' \times \sin x\,dx$$

$$= \frac{2\sin^2 x}{2} - \frac{\sin^2 x}{2} + c$$

$$= \frac{\sin^2 x}{2} + c, \; c \in \mathbb{R} \text{ em intervalos}$$

$\boxed{7}$

a)

$$\int \operatorname{cosec} x \, dx = \int \frac{\operatorname{cosec} x \left(\operatorname{cosec} x + \operatorname{cotan} x \right)}{\operatorname{cosec} x + \operatorname{cotan} x} = \int \frac{\operatorname{cosec}^2 x + \operatorname{cosec} x \operatorname{cotan} x \, dx}{\operatorname{cosec} x + \operatorname{cotan}}$$

$$= - \ln \left| \operatorname{cosec} x + \operatorname{cotan} \right| + C, \ C \in \mathbb{R} \text{ em int.}$$

b) $$\int \frac{tg^3 x}{\cos^2 x} \, dx = \int tg^3 x \sec^2 x \, dx = \frac{tg^4 x}{4} + C, \ C \in \mathbb{R} \text{ em int}$$

c)

$$\int \operatorname{cotg}^2 x \, dx = \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1 - \operatorname{sen}^2 x}{\sin^2 x} \, dx = \int \frac{1}{\sin^2 x} \, dx - \int 1 \, dx$$

$$= - \int - \operatorname{cosec}^2 x \, dx - x + C = - \operatorname{cotan} x - x + C, \ C \in \mathbb{R} \text{ em int.}$$

d)

$$\int \cos^2 \theta \, d\theta = \int \frac{1 + \cos(2\theta)}{2} \, d\theta = \frac{1}{2} \theta + \frac{1}{2} \times \int \cos(2\theta) \, d\theta$$

$$= \frac{1}{2} \theta + \frac{1}{4} \times \operatorname{sen}(2\theta) + C, \ C \in \mathbb{R} \text{ em int.}$$

e)

$$\int \operatorname{sen}^2 x \, dx = \int \frac{1 - \cos(2x)}{2} \, dx = \frac{1}{2} x - \frac{1}{4} \operatorname{sen}(2x) + C, \ C \in \mathbb{R}$$

f)

$$\int \operatorname{sen}^3 t \, dt = \int (1 - \cos^2 t) \operatorname{sen} t \, dt = \int \operatorname{sen} t \, dt + \int (-\operatorname{sen} t) \cos^2 t \, dt$$

$$= - \cos t + \frac{\cos^3 t}{3} + C, \ C \in \mathbb{R} \text{ em int.}$$

g)

$$\int tg^4 x \, dx = \int (\sec^2 x - 1) \, tg^2 x \, dx = \int tg^2 x \sec^2 x \, dx - \int tg^2 x \, dx$$

$$= \frac{tg^3}{3} - \int \sec^2 x - 1 \, dx + C = \frac{tg^3}{3} - tg \, x + x + C, \ C \in \mathbb{R} \text{ em int}$$

h)

$$\int \operatorname{sen}(3x) + \cos(5x) \, dx = \frac{1}{3} \int 3 \operatorname{sen}(3x) \, dx + \frac{1}{5} \int 5 \cos(5x) \, dx$$

$$= - \frac{\cos(3x)}{3} + \frac{\operatorname{sen}(5x)}{5} + C, \ C \in \mathbb{R} \text{ em int.}$$

i) $\int tg\,x \sec^2 x \, dx = \int (tg\,x)' \, tg(x) \, dx = \frac{tg^2 x}{2} + c, c \in \mathbb{R}$ em intervalos

j) $\int sen^5 x \cos^2 x \, dx = \int sen^5 x \,(1 - sen^2 x)\, dx = \int sen^5 x - sen^7 x \, dx$

$= \int sen^5 x \, dx - \int sen^7 x \, dx$

$\int sen^5 x \, dx = \int (1 - \cos^2 x)\, sen^3 x \, dx = \int sen^3 x \, dx - \int \cos^2 x \, sen^3 x \, dx$

$= \int (1 - \cos^2 x)\, sen\, x \, dx - \int \cos^2 x \,(1 - \cos^2 x)\, sen\, x \, dx$

$= \int sen\, x \, dx + \int (-sen\, x) \cos^2 x \, dx - \int \cos^2 x \, sen\, x \, dx + \int \cos^4 x \, sen\, x \, dx$

$= -\cos x + \frac{\cos^3 x}{3} + \underbrace{\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5}}_{-\int \cos^2 x \, sen^3 x \, dx} + c$

$= \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} - \cos x + c$

$\int sen^7 x \, dx = \int (1 - \cos^2 x)\, sen^5 x \, dx = \boxed{\int sen^5 x \, dx} - \int \cos^2 x \, sen^5 x \, dx$

A

$= A - \int \cos^2 x \,(1 - \cos^2 x)\, sen^3 x \, dx$

$\cos^2 x - \cos^4 x$

$= A - \int \cos^2 x \, sen^3 x \, dx + \int \cos^4 x \, sen^3 x \, dx$

$= A + \frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + \int \cos^4 x \,(1 - \cos^2 x)\, sen\, x$

$= A + \frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + \int \cos^4 x \, sen\, x \, dx - \int \cos^6 x \, sen\, x \, dx$

$= A + \frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} - \frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$

$$\int sen^5 x \cos^2 x \, dx = \cancel{A} - \left(\cancel{A} + \frac{\cos^3 x}{3} - \frac{2\cos^5 x}{5} + \frac{\cos^7 x}{7} + c \right)$$

$$= -\frac{\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + c, \; c \in \mathbb{R} \text{ em intervalos}$$

k) Já chega! Muitos cálculos

l)

$\cos(3x+2x)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\int \cos x \cos(5x) \, dx =$$

$$= \int \frac{1}{2}\left(\cos 6x + \cos(4x)\right) dx$$

$$\boxed{\cos A \cos B = \frac{1}{2}\left(\cos(B+A) + \cos(B-A)\right)}$$

$$= \frac{1}{2} \times \frac{1}{6} \times \int 6 \cos 6x \, dx + \frac{1}{2} \times \frac{1}{4} \times \int 4 \cos 4x \, dx$$

$$= \frac{\sin 6x}{12} + \frac{\sin 4x}{8} + c, \; c \in \mathbb{R} \text{ em intervalos}$$

m)

$$\int \frac{1}{x} \cos(\ln x) \, dx = \int (\ln x)' \cos(\ln x) \, dx$$

$$= \sin(\ln x) + c, \; c \in \mathbb{R} \text{ em int.}$$

n)

$$\int x^5 sen(x^6) \, dx = -\frac{1}{6} \cos(x^6) + c, \; c \in \mathbb{R} \text{ em int.}$$

$(x^6)' = 6x^5$

o)

$$\int \frac{\arccos x - x}{\sqrt{1-x^2}} \, dx = \int \frac{\arccos x}{\sqrt{1-x^2}} \, dx + \int \frac{-x}{\sqrt{1-x^2}} \, dx$$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$$= -\frac{\arccos^2 x}{2} + c + \int \left(\sqrt{1-x^2}\right)' dx$$

$$= -\frac{\arccos^2 x}{2} + \sqrt{1-x^2} + c, \; c \in \mathbb{R} \text{ em int.}$$

$\left(\sqrt{1-x^2}\right)' = \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{1-x^2}} \times (-\cancel{2}x)$

$= -\frac{x}{\sqrt{1-x^2}}$

p)

$$\int \frac{\cos(\ln(x^2))}{x} \, dx = \int \frac{1}{x} \times \cos(\ln(x^2)) \, dx = \frac{1}{2} \times \sin(\ln(x^2)) + c, \; c \in \mathbb{R} \text{ em int.}$$

$[\ln(x^2)]' = \frac{2\cancel{x}}{x^{\cancel{2}}} = \frac{2}{x}$

$\boxed{8}$

a) $$\int \frac{x+2}{x^2+5x-6}\,dx = \int \frac{x+2}{(x-1)(x+6)}\,dx = \int \frac{A}{(x-1)}\,dx + \int \frac{B}{(x+6)}\,dx$$

$x^2+5x-6=0$

$\Leftrightarrow x = \frac{-5 \pm \sqrt{25-4\times1\times(-6)}}{2}$

$\Leftrightarrow x = \frac{-5\pm 7}{2}$

$\Leftrightarrow x = 1 \vee x = -6$

$$\frac{x+2}{(x-1)(x+6)} = \frac{A}{(x-1)} + \frac{B}{(x+6)}$$

$$\Leftrightarrow \frac{x+2}{\cancel{(x-1)(x+6)}} = \frac{A(x+6)+B(x-1)}{\cancel{(x-1)(x+6)}}$$

$$\Leftrightarrow x+2 = Ax+6A+Bx-B$$

$$\Leftrightarrow x+2 = (A+B)x + 6A-B$$

$$\begin{cases} A+B=1 \\ 6A-B=2 \end{cases} \quad \begin{cases} A=1-B \\ 6-6B-B=2 \end{cases} \quad \begin{cases} A=\frac{3}{7} \\ B=\frac{4}{7} \end{cases}$$

Voltando ao integral:

$$\int \frac{x+2}{(x-1)(x+6)}\,dx = \int \frac{\frac{3}{7}}{x-1}\,dx + \int \frac{\frac{4}{7}}{x+6}\,dx$$

$$= \frac{3}{7}\ln(x-1) + \frac{4}{7}\ln(x+6) + C \text{ , } C \in \mathbb{R} \text{ em intervalos}$$

b) $$\int \frac{1}{(x-1)(x+1)^3}\,dx = \int \frac{A}{x-1}\,dx + \int \frac{B}{x+1}\,dx + \int \frac{C}{(x+1)^2}\,dx + \int \frac{D}{(x+1)^3}\,dx$$

$$\frac{1}{(x-1)(x+1)^3} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

$$\Leftrightarrow \frac{1}{\cancel{(x-1)(x+1)^3}} = \frac{A(x+1)^3 + B(x+1)^2(x-1) + C(x+1)(x-1) + D(x-1)}{\cancel{(x-1)(x+1)^3}}$$

$$\Leftrightarrow 1 = A(x^3+3x^2+3x+1) + B(x^2+2x+1)(x-1) + C(x^2-1) + D(x-1)$$

$$\Leftrightarrow 1 = A(x^3+3x^2+3x+1) + B(x^3+x^2-x-1) + C(x^2-1) + D(x-1)$$

$$\Leftrightarrow 1 = (A+B)x^3 + (3A+B+C)x^2 + (3A-B+D)x + A-B-C-D$$

$$\begin{cases} A+B=0 \\ 3A+B+C=0 \\ 3A-B+D=0 \\ A-B-C-D=1 \end{cases} \quad \begin{cases} A=-B \\ -3B+B+C=0 \\ -3B-B+D=0 \\ -B-B-C-D=1 \end{cases} \quad \begin{cases} A=-B \\ 2B=C \\ 4B=D \\ -2B-C-D=1 \end{cases}$$

$$\begin{cases} A=-B \\ 2B=C \\ 4B=D \\ -2B-2B-4B=1 \end{cases} \quad \begin{cases} A=1/8 \\ 2\times(-1/8)= \\ 4\times(-1/8)=D \\ B=-1/8 \end{cases} \quad \begin{cases} A=1/8 \\ B=-1/8 \\ C=-1/4 \\ D=-1/2 \end{cases}$$

__Voltando ao integral:__

$$\int \frac{1}{(x-1)(x+1)^3}\,dx = \int \frac{\frac{1}{8}}{x-1}\,dx + \int \frac{-\frac{1}{8}}{x+1}\,dx + \int \frac{-1/4}{(x+1)^2}\,dx + \int \frac{-\frac{1}{2}}{(x+1)^3}\,dx$$

$$\int (x+1)^{-2}\,dx = -\frac{1}{x+1}$$

$$\int (x+1)^{-3}\,dx = -\frac{1}{2(x+1)^2}$$

$$= \frac{1}{8}\ln|x-1| - \frac{1}{8}\ln|x+1| - \frac{1}{4}\int \frac{1}{(x+1)^2}\,dx - \frac{1}{2}\int \frac{1}{(x+1)^3}\,dx$$

$$= \frac{1}{8}\ln|x-1| - \frac{1}{8}\ln|x+1| + \frac{1}{4}\times\frac{1}{x+1} + \frac{1}{4}\times\frac{1}{(x+1)^2} + C,\ C\in\mathbb{R}$$ em intervalo

c)

$$\int \frac{1}{x^3+8}\,dx = \int \frac{1}{(x+2)(x^2-2x+4)}\,dx = \int \frac{A}{x+2}\,dx + \int \frac{Bx+C}{x^2-2x+4}\,dx$$

$x^3+8=0$
$\Leftrightarrow x^3=-8$
$\Leftrightarrow x=-2$ é 1 das raízes

$$\frac{1}{\cancel{(x+2)(x^2-2x+4)}} = \frac{A(x^2-2x+4)+B(x^2+2x)+C(x+2)}{\cancel{(x+2)(x^2-2x+4)}}$$

	1	0	0	8
-2		-2	4	-8
	1	-2	4	0

$$\Leftrightarrow 1 = (A+B)x^2 + (-2A+2B+C)x + 4A+2C$$

$x^3+8=(x+2)(x^2-2x+4)$

$$\begin{cases} A+B=0 \\ -2A+2B+C=0 \\ 4A+2C=1 \end{cases} \quad \begin{cases} A=-B \\ 2B+2B=-C \\ -4B+2C=1 \end{cases} \quad \begin{cases} A=-B \\ -4B=C \\ -4B-8B=1 \end{cases}$$

$x^2-2x+4=0$

$\Leftrightarrow x = \frac{2\pm\sqrt{4-4\times1\times4}}{2}$ não tem raízes reais!

$$\begin{cases} A=1/12 \\ C=1/3 \\ B=-\frac{1}{12} \end{cases}$$

$$\int \frac{1}{(x+2)(x^2-2x+4)}\,dx = \int \frac{\frac{1}{12}}{x+2}\,dx + \int \frac{-\frac{1}{12}x+\frac{1}{3}}{x^2-2x+4}\,dx$$

$$= \frac{1}{12} \times \ln|x+2| - \frac{1}{24}\int \frac{2x-8}{x^2-2x+4}\,dx$$

$$= \frac{1}{12} \times \ln|x+2| - \frac{1}{24}\int \frac{2x-2}{x^2-2x+4}\,dx + \frac{1}{24}\boxed{\int \frac{-6}{(x-1)^2+3}\,dx}$$

$$\int \frac{-2}{\frac{(x-1)^2}{3}+1}\,dx = \int \frac{-2}{1+\left(\frac{x-1}{\sqrt{3}}\right)^2}\,dx = -2 \times \frac{3}{\sqrt{3}} \int \frac{\frac{\sqrt{3}}{3}}{1+\left(\frac{x-1}{\sqrt{3}}\right)^2}\,dx$$

$$\left(\frac{x-1}{\sqrt{3}}\right)' = \frac{\sqrt{3}}{3}$$

$$= \frac{-2\times\cancel{3}\sqrt{3}}{\cancel{3}} \times \operatorname{atg}\left(\frac{x-1}{\sqrt{3}}\right) + c,\ c \in \mathbb{R}$$

$$= \frac{1}{12} \times \ln|x+2| - \frac{1}{24} \times \ln(x^2-2x+4) - \frac{1}{24} \times 2\sqrt{3}\,\operatorname{arctg}\left(\frac{x-1}{\sqrt{3}}\right)$$

$$= \frac{2\ln|x+2| - \ln(x^2-2x+4) - 2\sqrt{3}\,\operatorname{arctg}\left(\frac{x-1}{\sqrt{3}}\right)}{24} + c,\ c \in \mathbb{R} \text{ em intervalos}$$

d)

$$\int \frac{x^4-4x^2+3}{x^2-9}\,dx$$

$$\begin{array}{r|l} x^4-4x^2+3 & x^2-9 \\ \cline{2-2} \underline{x^4+9x^2} & x^2+5 \\ 5x^2+3 & \\ \underline{5x^2+45} & \\ 48 & \end{array}$$

$$\frac{x^4-4x^2+3}{x^2-9} = x^2+5 + \frac{48}{x^2-9}$$

$$\int \frac{x^4-4x^2+3}{x^2-9}\,dx = \int x^2\,dx + \int 5\,dx + \int \frac{48}{x^2-9}\,dx$$

$x^2-9=0$

$\Leftrightarrow x=-3 \vee x=3$

$$= \frac{x^3}{3} + 5x + C + 48\int \frac{1}{(x+3)(x-3)}\,dx$$

$$\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$$

$$\Leftrightarrow 1 = A(x-3) + B(x+3)$$

$$\Leftrightarrow 1 = (A+B)x - 3A + 3B$$

$$\begin{cases} A+B=0 \\ -3A+3B=1 \end{cases} \quad \begin{cases} A=-B \\ 3B+3B=1 \end{cases} \quad \begin{cases} A=-1/6 \\ B=1/6 \end{cases}$$

Voltando ao integral:

$$\frac{x^3}{3} + 5x + C + 48\int \frac{1}{(x+3)(x-3)}\,dx$$

$$= \frac{x^3}{3} + 5x + C + \int \frac{48\times\left(-\frac{1}{6}\right)}{x+3}\,dx + \int \frac{48\times\frac{1}{6}}{x-3}\,dx$$

$$= \frac{x^3}{3} + 5x + C - 8\int \frac{1}{x+3}\,dx + 8\int \frac{1}{x-3}\,dx$$

$$= \frac{x^3}{3} + 5x + C - 8\ln|x+3| + 8\ln|x-3|, \; C \in \mathbb{R} \text{ em intervalos}$$

$$= \frac{x^3}{3} + 5x + 8\ln\left|\frac{x-3}{x+3}\right| + C, \; C \in \mathbb{R} \text{ em intervalos}$$

e)

$$\int \frac{x^3+3x-1}{x^4-4x^2}\,dx = \int \frac{x^3+3x-1}{x^2(x-2)(x+2)}\,dx$$

$$\frac{x^3+3x-1}{x^2(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2}$$

$$\Leftrightarrow x^3+3x-1 = Ax(x-2)(x+2) + B(x-2)(x+2) + C(x+2)x^2 + D(x-2)x^2$$

$$\Leftrightarrow x^3+3x-1 = A(x^3-4x) + B(x^2-4) + C(x^3+2x^2) + D(x^3-2x^2)$$

$$\Leftrightarrow x^3+3x-1 = (A+C+D)x^3 + (B+2C-2D)x^2 - 4Ax - 4B$$

$$\begin{cases} A+C+D=1 \\ B+2C-2D=0 \\ -4A=3 \\ -4B=-1 \end{cases} \quad \begin{cases} C = 1+3/4-\frac{1}{8}-C \\ 1/8 + C = D \\ A=-3/4 \\ B=1/4 \end{cases} \quad \begin{cases} C = 13/16 \\ D = 15/16 \\ A=-3/4 \\ B=1/4 \end{cases}$$

$$\int \frac{x^3+3x-1}{x^2(x-2)(x+2)}\,dx = -\frac{3}{4}\int \frac{1}{x}\,dx + \frac{1}{4}\int \frac{1}{x^2}\,dx + \frac{13}{16}\int \frac{1}{x-2}\,dx + \frac{15}{16}\int \frac{1}{x+2}\,dx$$

$$= -\frac{3}{4}\ln|x| - \frac{1}{4x} + \frac{13}{16}\times \ln|x-2| + \frac{15}{16}\times\ln|x+2| + c,\ c\in\mathbb{R} \text{ em intervalos}$$

f)

$$\int \frac{x^4}{x^4-1}\,dx = \int 1 + \frac{1}{x^4-1}\,dx$$

$$= x + c + \int \frac{1}{x^4-1}\,dx$$

$$= x + c + \int \frac{1}{(x+1)(x-1)(x^2+1)}\,dx \text{ , } c\in\mathbb{R} \text{ em intervalos}$$

$$\begin{array}{r|l} x^4+0 & x^4-1 \\ \hline -x^4+1 & 1 \\ 1 & \end{array}$$

$x^4-1=0$

$\Leftrightarrow x=-1 \vee x=1$

$$\begin{array}{r|rrrrr} & 1 & 0 & 0 & 0 & -1 \\ -1 & & -1 & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -1 & \boxed{0} \\ 1 & & 1 & 0 & 1 & \\ \hline & 1 & 0 & 1 & \boxed{0} & \end{array}$$

$x^4-1=(x+1)(x-1)(x^2+1)$

$x^4-1=(x+1)(x-1)(x^2+1)$

$$\frac{1}{(x+1)(x-1)(x^2+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x^2+1}$$

$$\Leftrightarrow 1 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + C(x+1)(x-1)$$

$$\Leftrightarrow 1 = A(x^3-x^2+x-1) + B(x^3+x^2+x+1) + C(x^2-1)$$

$$\Leftrightarrow 1 = (A+B)x^3 + (-A+B+C)x^2 + (A+B)x - A+B-C$$

$$\begin{cases} A+B=0 \\ -A+B+C=0 \\ \cancel{A+B=0} \\ -A+B-C=1 \end{cases} \quad \begin{cases} A=-B \\ C=-2B \\ B+B+2B=1 \end{cases} \quad \begin{cases} A=-1/4 \\ C=-1/2 \\ B=1/4 \end{cases}$$

Voltando ao integral:

$$x + c + \int \frac{1}{(x+1)(x-1)(x^2+1)}\, dx = x + c - \frac{1}{4}\int \frac{1}{x-1}\, dx + \frac{1}{4}\int \frac{1}{x+1}\, dx - \frac{1}{2}\int \frac{1}{x^2+1}$$

$$= x - \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| - \frac{1}{2}\operatorname{arctg} x + c,\ c \in \mathbb{R} \text{ em intervalos}$$

g)

$$\frac{1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$\Leftrightarrow 1 = A(x^2+1)^2 + Bxx(x^2+1) + Cx(x^2+1) + Dxx + Ex$$

$$\Leftrightarrow 1 = A(x^4+2x^2+1)\ B(x^4+x^2) + C(x^3+x) + Dx^2 + Ex$$

$$\Leftrightarrow 1 = (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A$$

$$\begin{cases} A+B=0 \\ C=0 \\ 2A+B+D=0 \\ C+E=0 \\ A=1 \end{cases} \qquad \begin{cases} B=-1 \\ C=0 \\ D=-1 \\ E=0 \\ A=1 \end{cases}$$

$$\int \frac{1}{x(x^2+1)^2}\, dx = \int \frac{1}{x}\, dx + \int \frac{-x}{x^2+1}\, dx + \int \frac{-x}{(x^2+1)^2}\, dx$$

$$= \ln|x| - \frac{1}{2} \times \ln(x^2+1) - \frac{1}{2} \times \int \frac{(x^2+1)'}{(x^2+1)^2}\, dx$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + \frac{1}{2} \times \frac{1}{x^2+1} + c$$

h)

$$\int \frac{x+1}{x^2+4x+5}\,dx = \frac{1}{2}\int \frac{2x+2+2-2}{x^2+4x+5}\,dx$$

$x^2+4x+5=0$

$\Leftrightarrow x = \frac{-4 \pm \sqrt{16-4\times1\times5}}{2}$

Não tem raízes reais!

$(x^2+4x+5)' = 2x+4$

$$= \frac{1}{2} \times \int \frac{2x+4}{x^2+4x+5}\,dx - \int \frac{1}{x^2+4x+5}\,dx$$

$$= \frac{1}{2} \times \ln(x^2+4x+5) - \int \frac{1}{(x+2)^2+1}\,dx$$

$$= \frac{1}{2}\ln(x^2+4x+5) - \operatorname{arctg}(x+2) + c, \; c \in \mathbb{R}$$

$x^2+4x+5 = (x+p)^2+q$

$\Leftrightarrow x^2+4x+5 = x^2+2px+(p^2+q)$

$\begin{cases} 1=1 \\ 4=2p \\ 5=p^2+q \end{cases} \quad \begin{cases} p=2 \\ q=1 \end{cases} \Rightarrow x^2+4x+5 = (x+2)^2+1$

$\boxed{9}$

a) $\int x^2\sqrt{1-x}\,dx = \int (1-t^2)^2 \times t \times (-2t)\,dt = \int (1-2t^2+t^4)\times(-2t^2)\,dt$

$x \in]-\infty, 1]$

$t = \sqrt{1-x}$

$\Leftrightarrow x = 1-t^2$

$$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = 1-t^2 \\ g'(t) = -2t < 0,\; t \in [1, +\infty[\\ g \text{ é inv. e dif. em} \\ dx = -2t\,dt \end{array}}$$

$$= -2\int t^2 - 2t^4 + t^6\,dt$$

$$= \frac{-2}{3}t^2 \; \frac{-2\times-2}{5}t^5 + \frac{-2}{7}t^7 + c$$

$$= -\frac{2}{3}t^2 - \frac{4}{5}t^5 - \frac{2}{7}t^7 + c,\; c \in \mathbb{R} \text{ em intervalos}$$

$$\boxed{\begin{array}{l} t \longrightarrow x \\ t = \sqrt{1-x} \end{array}}$$

$$\int x^2\sqrt{1-x}\,dx = -\frac{2}{3}\times(1-x) - \frac{4}{5}\times(1-x)^{5/2} - \frac{2}{7}(1-x)^{7/2} + c,$$

$c \in \mathbb{R}$ em intervalos

b)

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \int \frac{t^3}{1+t^2} \times 6t^5\,dt = 6\int \frac{t^8}{1+t^2}\,dt$$

$$= 6\int t^6 dt - 6\int t^4 dt + 6\int t^2 dt - 6\int 1 dt + 6\int \frac{1}{1+t^2} dt$$

$$= 6\frac{t^7}{7} - \frac{6t^5}{5} + \frac{6t^3}{3} - 6t + 6\operatorname{arctg}(t) + c, c \in \mathbb{R} \text{ em intervalos}$$

m.d.c.$(3,2) = 6$

$t = x^{1/6}$

$\Leftrightarrow t^6 = x$

$t^2 = x^{1/3} = \sqrt[3]{x}$

$t^3 = x^{1/2} = \sqrt{x}$

$x \longrightarrow t$

$x = g(t) = t^6$

$g'(t) = 6t^5 > 0,\ t \in \mathbb{R}^+$

g é invertível e diferenciável em $\mathbb{R}^+$

$dx = 6t^5 dt$

— // —

$$\begin{array}{r|l} t^8 & 1+t^2 \\ \hline -t^8 - t^6 & t^6 - t^4 + t^2 - 1 \\ -t^6 & \\ +t^6 + t^4 & \\ t^4 & \\ -t^4 - t^2 & \\ -t^2 & \\ +t^2 + 1 & \\ 1 & \end{array}$$

$$\frac{t^8}{1+t^2} = t^6 - t^4 + t^2 - 1 + \frac{1}{1+t^2}$$

$t \longrightarrow x$

$t = x^{1/6}$

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = 6\frac{x^{7/6}}{7} - \frac{6x^{5/6}}{5} + \frac{6x^{1/2}}{3} - 6x^{1/6} + 6\operatorname{arctg}(x^{1/6}) + c, c \in \mathbb{R} \text{ em intervalos}$$

c)

$$\int x(2x+5)^{10}\,dx = \int \frac{t-5}{2} \times t^{10} \times \frac{1}{2}\,dt = \frac{1}{4} \times \int t^{11} dt - \frac{1}{4}\times 5\int t^{10} dt$$

$$= \frac{1}{4} \times \frac{t^{12}}{12} - \frac{5}{4} \times \frac{t^{11}}{11} + c, c \in \mathbb{R}$$

$t = 2x+5$

$\Leftrightarrow x = \frac{t-5}{2}$

$\left(\frac{t-5}{2}\right)' = \frac{2}{4} = \frac{1}{2}$

$x \longrightarrow t$

$x = g(t) = \frac{t-5}{2}$

$g'(t) = \frac{1}{2} > 0, \forall t \in \mathbb{R}$

g é inv. e difer. em

$dx = \frac{1}{2} dt$

$t \longrightarrow x$

$t = 2x+5$

$$\int x(2x+5)^{10}\,dx = \frac{1}{48} \times (2x+5)^{12} - \frac{5}{44}(2x+5)^{11} + c,\ c \in \mathbb{R} \text{ em intervalos}$$

d)

$$\int \frac{1}{x^2\sqrt{9-x^2}}\,dx = \int \frac{3\cos t}{9\sin^2 t\sqrt{9-9\sin^2 t}}\,dt = \frac{1}{9}\int \frac{\cancel{\cos t}}{\sin^2 t\ \cancel{\cos t}}\,dt$$

$$= \frac{1}{9}\int (\operatorname{cosec} t)^2\,dt = -\frac{1}{9}\times \operatorname{cotg} t + c,\ c\in\mathbb{R} \text{ em intervalos}$$

$x \longrightarrow t$
$x = g(t) = 3\sin t$
$g'(t) = 3\cos t > 0,\ t\in\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$
g é inv. e dif. em
$dx = 3\cos t\,dt$

$t \longrightarrow x$
$t = \arcsin\left(\frac{x}{3}\right)$

$$\int \frac{1}{x^2\sqrt{9-x^2}}\,dx = -\frac{1}{9}\times \operatorname{cotg}\left(\arcsin\left(\frac{x}{3}\right)\right) + c$$

$$= -\frac{1}{9}\times\frac{\cos\left(\arcsin\left(\frac{x}{3}\right)\right)}{\sin\left(\arcsin\left(\frac{x}{3}\right)\right)} + c$$

$$= -\frac{3}{9}\times\frac{\cos\left(\arcsin\left(\frac{x}{3}\right)\right)}{x} + c$$

$$= -\frac{\sqrt{9-x^2}}{9x} + c,\ C\in\mathbb{R} \text{ em intervalos}$$

$\cos\alpha = (+)\sqrt{1+\sin^2\alpha}$

$\cos\alpha > 0, \forall\alpha\in\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$

$$\cos\left(\arcsin\left(\frac{x}{3}\right)\right) = \sqrt{1-\left(\frac{x}{3}\right)^2} = \frac{\sqrt{9-x^2}}{3}$$

e)

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \int \frac{\cancel{\sec t}\ \cancel{\operatorname{tg} t}}{\cancel{\sec t}\ \cancel{\sqrt{\sec^2 t-1}}}\,dt = t + c,\ c\in\mathbb{R}$$

$x \longrightarrow t$
$x = g(t) = \sec t$
$g'(t) = \sec t\ \operatorname{tg} t,\ t\in\left]0,\frac{\pi}{2}\right[$
g é inv. def. em
$dx = \sec t\ \operatorname{tg} t$

$t \longrightarrow x$
$t = \operatorname{arcsec}(x)$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec}(x) + c,\ c\in\mathbb{R} \text{ em intervalos}$$

$$= \arccos\left(\frac{1}{x}\right) + c,\ c\in\mathbb{R} \text{ em intervalos}$$

f)

$$\int \frac{1}{x\sqrt{x^2+4}}\,dx = \int \frac{\frac{\cancel{t}}{\sqrt{t^2-4}}}{\sqrt{t^2-4}\times \cancel{t}}\,dt = \int \frac{1}{t^2-4}\,dt$$

$$= \int \frac{1}{(t-2)(t+2)}\,dt = \int \frac{A}{t-2}\,dt + \int \frac{B}{t+2}\,dt$$

$t = \sqrt{x^2+4}$

$x = \sqrt{t^2-4}$

$g'(t) = \frac{1}{2}\times \frac{1}{\sqrt{t^2-4}}\times 2t$

$= \frac{t}{\sqrt{t^2-4}}$

$$\boxed{\begin{array}{l} x \to t \\ x = g(t) = \sqrt{t^2-4} \\ g'(t) = \frac{t}{\sqrt{t^2+4}} > 0,\ t \in \mathbb{R}^+ \\ g \text{ é inv. e def. em} \\ dx = \frac{t}{\sqrt{t^2-4}}\,dt \end{array}}$$

$t^2 - 4 = 0$

$\Leftrightarrow t = -2 \lor t = 2$

$$\frac{1}{t^2-4} = \frac{A}{t-2} + \frac{B}{t+2} \Leftrightarrow \frac{1}{t^2-4} = A(t+2) + B(t-2)$$

$$\Leftrightarrow 1 = (A+B)t + 2A - 2B$$

$$\begin{cases} A+B=0 \\ 2A-2B=1 \end{cases} \quad \begin{cases} A=-B \\ -2B-2B=1 \end{cases} \quad \begin{cases} A=1/4 \\ B=-1/4 \end{cases}$$

<u>Logo</u>:

$$\int \frac{1}{(t-2)(t+2)}\,dt = \frac{1}{4}\int \frac{1}{t-2}\,dt - \frac{1}{4}\int \frac{1}{t+2}\,dt$$

$$= \frac{1}{4}\times \ln|t-2| - \frac{1}{4}\times \ln|t+2| + c,\ c \in \mathbb{R} \text{ em intervalos}$$

Voltando à variável x:

$$\boxed{\begin{array}{l} t \to x \\ t = \sqrt{x^2+4} \end{array}}$$

$$\int \frac{1}{x\sqrt{x^2+4}}\,dx =$$

$$= \frac{1}{4}\times \ln\left|\sqrt{x^2+4}-2\right| - \frac{1}{4}\times \ln\left|\sqrt{x^2+4}+2\right| + c,\ c \in \mathbb{R}$$

g)

$$\int \sqrt{3-2x^2}\,dx = \int \sqrt{3 - \cancel{2} \times \frac{3}{\cancel{2}} \sin^2 t}\ \frac{\sqrt{3}}{\sqrt{2}} \cos t\ dt =$$

$2 \times a^2 = 3$

$\Leftrightarrow a^2 = \frac{3}{2}$

$\Leftrightarrow a = \sqrt{\frac{3}{2}},\ a>0$

$$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = \sqrt{\frac{3}{2}} \sin t \\ g'(t) = \sqrt{\frac{3}{2}} \cos t,\ t \in]0, \frac{\pi}{2}[\\ g \text{ é inv. e dif. em } \nearrow \\ dx = \sqrt{\frac{3}{2}} \cos t\, dt \end{array}}$$

$$= \sqrt{3} \times \frac{\sqrt{3}}{\sqrt{2}} \int \sqrt{1-\sin^2 t}\ \cos t\ dt$$

$$= \frac{3}{\sqrt{2}} \times \int \cos^2 t\ dt$$

$$= \frac{3}{\sqrt{2}} \times \int \frac{1}{2} + \frac{1}{2}\cos(2t)\,dt$$

$$= \frac{3}{\sqrt{2}} \times \frac{1}{2} t + \frac{3}{\sqrt{2}} \times \frac{1}{2} \int \cos(2t)\,dt$$

$$= \frac{3}{2\sqrt{2}} t + \frac{3}{4\sqrt{2}} \int (2t)' \cos(2t)\,dt$$

$$= \frac{3}{2\sqrt{2}} t + \frac{3}{4\sqrt{2}} \times \sin(2t) + c,\ c \in \mathbb{R} \text{ em int.}$$

$x = \sqrt{\frac{3}{2}} \sin t$

$\Leftrightarrow \sin t = \frac{\sqrt{2}}{\sqrt{3}} x$

$\Leftrightarrow t = \arcsin\left(\frac{\sqrt{2}}{\sqrt{3}} x\right)$

$\Leftrightarrow \cos t = \cos\left(\arcsin\left(\frac{\sqrt{2}}{\sqrt{3}} x\right)\right)$

$\Leftrightarrow \cos t = \sqrt{1 - \left[\cancel{\sin}\left(\cancel{\arcsin}\left(\frac{\sqrt{2}}{\sqrt{3}} x\right)\right)\right]^2}$

$\Leftrightarrow \cos t = \sqrt{1 - \frac{2}{3} x^2}$

$\boxed{\sin 2t = 2 \cos t \sin t}$

$\Leftrightarrow \sin 2t = 2\sqrt{1 - \frac{2}{3}x^2} \times \frac{\sqrt{2}}{\sqrt{3}} x$

$$\boxed{\begin{array}{l} t \longrightarrow x \\ t = \arcsin\left(\frac{2}{\sqrt{3}} x\right) \\ \sin 2t = 2\sqrt{1-\frac{4}{3}x^2} \times \frac{2}{\sqrt{3}} x \end{array}}$$

$$\int \sqrt{3-2x^2}\,dx = \frac{3 \arcsin\left(\frac{\sqrt{2}}{\sqrt{3}} x\right)}{2\sqrt{2}} + \frac{3}{4\cancel{\sqrt{2}}} \times 2\sqrt{1-\frac{2}{3}x^2} \times \frac{\cancel{\sqrt{2}}}{\sqrt{3}} x$$

$$= \frac{3 \arcsin\left(\frac{\sqrt{6}}{3} x\right)}{2\sqrt{2}} + \frac{3x\sqrt{1-\frac{2}{3}x^2}}{2\sqrt{3}}$$

h)

$$\int \frac{x^2}{\sqrt{1-2x-x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x+1)^2+2}}\,dx = \int \frac{x^2}{\sqrt{(\sqrt{2})^2 - (x+1)^2}}\,dx$$

$\sqrt{a^2 - x^2}$

$x + 1 = \sqrt{2} \sin t$

$\Leftrightarrow x = \sqrt{2} \sin t - 1$

$$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = \sqrt{2}\sin t - 1,\ t \in]0, \frac{\pi}{2}[\\ g'(t) = \sqrt{2} \cos t,\ t \in]0, \frac{\pi}{2}[\\ g \text{ é dif e inv. em } \nearrow,\ dx = \sqrt{2} \cos t\, dt \end{array}}$$

$$\int \frac{(\sqrt{2}\sin t - 1)^2 \times \sqrt{2}\cos t}{\sqrt{(\sqrt{2})^2 - (\sqrt{2}\sin t)^2}}\,dt$$

$$= \int \frac{(2\sin^2 t - 2\sqrt{2}\sin t + 1) \times \cancel{\sqrt{2}}\ \cancel{\cos t}}{\cancel{\sqrt{2}} \times \cancel{\sqrt{1-\sin^2 t}}}\,dt$$

$$= \int 2\sin^2 t - 2\sqrt{2}\sin t + 1 \, dt = \frac{\cancel{2}}{\cancel{2}}\int 1-\cos(2t)\,dt - 2\sqrt{2}\int \sin t \, dt + t + c$$

$$= t - \frac{1}{2}\sin(2t) + 2\sqrt{2}\cos t + t + c, \; c \in \mathbb{R}$$

$$= 2t - \frac{\sin(2t)}{2} + 2\sqrt{2}\cos t + c, \; c \in \mathbb{R} \text{ em intervalos}$$

$x = \sqrt{2}\sin t - 1$

$\Leftrightarrow t = \arcsin\left(\frac{x+1}{\sqrt{2}}\right)$

$\boxed{\begin{array}{l}\sin(2t) = 2\sin\cos \\ \cos(2t) = \cos^2 x - \text{sen}^2 x\end{array}}$

$\Leftrightarrow \sin t = \frac{x+1}{\sqrt{2}}$

$\Leftrightarrow \cos t = \sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}$

$\Leftrightarrow \sin(2t) = 2 \times \frac{x+1}{\sqrt{2}} \times \sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}$

$= \frac{\cancel{2} \times (x+1) \times \sqrt{2}\sqrt{1-\frac{(x+1)^2}{2}}}{\cancel{2}}$

$= (x+1) \times \sqrt{2-(x+1)^2}$

$\boxed{\begin{array}{c} t \longrightarrow x \\ t = \arcsin\left(\frac{x+1}{\sqrt{2}}\right)\end{array}}$

$$\int \frac{x^2}{\sqrt{1-2x-x^2}}\,dx =$$

$$= 2\arcsin\left(\frac{x+1}{\sqrt{2}}\right) - \frac{(x+1)\sqrt{2-(x+1)^2}}{2} + 2\sqrt{2-(x+1)^2} + c, \; c \in \mathbb{R} \text{ em intervalos}$$

(i)

$$\int \frac{1}{x^2\sqrt{x^2-7}}\,dx = \int \frac{\sqrt{7}\tan t \sec t}{7\sec^2 t\sqrt{7\sec^2 t - 7}}\,dt = \frac{1}{7}\int \frac{\tan t}{\sec t\sqrt{\cancel{\sec^2 t - 1}}}\,dt$$

$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = \sqrt{7}\sec t, \; t \in \left]0, \frac{\pi}{2}\right[\\ g'(t) = \sqrt{7}\tan t \sec t, \; t \in \left]0, \frac{\pi}{2}\right[\\ g \text{ é dif e inv em} \\ dx = \sqrt{7}\tan t \sec t \, dt \end{array}}$

$$= \frac{1}{7} \times \int \frac{1}{\sec t}\,dt$$

$$= \frac{1}{7} \times \int \cos t \, dt$$

$$= \frac{1}{7}\text{sen}\, t + c, \; c \in \mathbb{R} \text{ em intervalos}$$

$x = \sqrt{7}\sec t \Leftrightarrow \sec t = \frac{x}{\sqrt{7}}$

$\Leftrightarrow \frac{1}{\cos t} = \frac{x}{\sqrt{7}}$

$\Leftrightarrow \cos t = \frac{\sqrt{7}}{x}$

$\Leftrightarrow \sin t = \sqrt{1-\left(\frac{\sqrt{7}}{x}\right)^2}$

$\Leftrightarrow \sin t = \sqrt{1 - \frac{7}{x^2}}$

$\Leftrightarrow \sin t = \sqrt{\frac{x^2-7}{x^2}}$

$\sin t > 0, \forall t \in]0, \frac{\pi}{2}[$

$\Leftrightarrow \sin t = \frac{\sqrt{x^2-7}}{x}$

$$\boxed{\begin{array}{l} t \longrightarrow x \\ \sin t = \frac{\sqrt{x^2-7}}{x} \end{array}}$$

$$\int \frac{1}{x^2\sqrt{x^2-7}}\,dt = \frac{\sqrt{x^2-7}}{7x} + c, \; c \in \mathbb{R} \text{ em intervalos}$$

$\boxed{j}$

$$\int \frac{1}{\sqrt{2x+3} + \sqrt[3]{(2x+3)^2}}\,dx = \int \frac{3t^5}{t^3+t^4}\,dt = 3\int \frac{t^2}{1+t}\,dt$$

$(2x+3)^{1/2} + \left[(2x+3)^2\right]^{1/3}$

$m.m.c(2,3) = 6$

$(t^6)^{1/2} + ((t^6)^2)^{1/3}$

$= t^3 + t^4$

$$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = \frac{t^6-3}{2} \\ dx = 3t^5 \end{array}}$$

$$= 3\int t\,dt - 3\int 1\,dt + 3\int \frac{1}{t+1}\,dt$$

$$= 3\frac{t^2}{2} - 3t + 3\ln|t+1| + c, \; c \in \mathbb{R} \text{ em intervalos}$$

$2x+3 = t^6 \Leftrightarrow$

$\Leftrightarrow x = \frac{t^6-3}{2}$

$\Leftrightarrow t = \sqrt[6]{2x+3}$

$$\begin{array}{r|l} t^2 + 0 & t+1 \\ \hline -t^2 - t & t-1 \\ -t & \\ t+1 & \\ \hline 1 & \end{array}$$

$\frac{t^2}{t+1} = t - 1 + \frac{1}{t+1}$

$$\boxed{\begin{array}{l} t \longrightarrow x \\ t = \sqrt[6]{2x+3} \end{array}}$$

$$\int (\ldots)\,dx = \frac{3}{2}\sqrt[3]{2x+3} - 3\sqrt[6]{2x+3} + 3\ln\left(\sqrt[6]{2x+3}+1\right) + c, \; c \in \mathbb{R} \text{ em intervalos}$$

$\boxed{K}$

$$\int e^{\sqrt{x}}\,dx = \int \underbrace{e^t}_{u'} \times \underbrace{2t}_{v}\,dx = e^t \times 2t - \int e^t \times 2\,dt$$

$$= 2t\,e^t - 2e^t + c$$

$$= 2e^t(t-1) + c, \; c \in \mathbb{R} \text{ em int.}$$

$\sqrt{x} = t$

$\Leftrightarrow x = t^2$

$$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = t^2 \\ dx = 2t\,dt \end{array}}$$

$$\boxed{\begin{array}{ll} u = e^t & v = 2t \\ u' = e^t & v' = 2 \end{array}}$$

$$\boxed{\begin{array}{l} t \longrightarrow x \\ t = \sqrt{x} \end{array}}$$

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}(\sqrt{x}-1) + c, \; c \in \mathbb{R} \text{ em int.}$$

$\boxed{l}$

$$\int \frac{\ln(x)}{x\sqrt{1+\ln(x)}}\,dx = \int \frac{t \times \cancel{e^t}}{\cancel{e^t}\sqrt{1+t}}\,dt =$$

$t = \ln(x)$
$\Leftrightarrow x = e^t$

$$\boxed{\begin{array}{c} x \longrightarrow t \\ x = g(t) = e^t \\ dx = e^t\,dt \end{array}}$$

$u = 1+t$
$\Leftrightarrow t = u-1$

$$\boxed{\begin{array}{l} t \longrightarrow u \\ t = h(u) = u-1 \\ dt = du \end{array}}$$

$\begin{cases} u = 1+t \\ t = \ln(x) \end{cases}$

$\Rightarrow u = 1+\ln(x)$

$$\boxed{\begin{array}{l} u \longrightarrow x \\ u = 1+\ln(x) \end{array}}$$

$$= \int \frac{u-1}{\sqrt{u}}\,du$$

$$= \int \frac{u}{u^{1/2}}\,du - \int u^{-1/2}\,du$$

$$= \int u^{1/2}\,du - \frac{u^{1/2}}{\frac{1}{2}} + c$$

$$= \frac{u^{3/2}}{3/2} - 2u^{1/2} + c$$

$$= \frac{2}{3}u^{\frac{3}{2}} - 2u^{1/2} + c,\ c \in \mathbb{R} \text{ em int}$$

$$\int \frac{\ln(x)}{x\sqrt{1+\ln(x)}}\,dx = \frac{2}{3}\left(\sqrt{1+\ln(x)}\right)^3 - 2\sqrt{1+\ln(x)} + c,\ c \in \mathbb{R} \text{ em int}$$

$\boxed{10}$

a)

$$\int \frac{x+1}{\sqrt{3-x^2}}\,dx = \int \frac{\sqrt{3}\sin t + 1}{\cancel{\sqrt{3}}\sqrt{\cancel{1-\sin^2 x}}} \times \cancel{\sqrt{3}}\,\cancel{\cos t}\,dt = -\sqrt{3}\cos t + t + c,\ c \in \mathbb{R} \text{ em int.}$$

$$\boxed{\begin{array}{l} \quad x \longrightarrow t \\ x = g(t) = \sqrt{3}\sin t,\ t \in]0, \tfrac{\pi}{2}[\\ g'(t) = \sqrt{3}\cos t,\ t \in]0, \tfrac{\pi}{2}[\\ g \text{ é invertível e diferenciável em } t \in]0, \tfrac{\pi}{2}[\\ dx = \sqrt{3}\cos t\,dt \end{array}}$$

$$= -\cancel{\sqrt{3}}\frac{\sqrt{3-x^2}}{\cancel{\sqrt{3}}} + \arcsin\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= -\sqrt{3-x^2} + \arcsin\left(\frac{x}{\sqrt{3}}\right) + c,\ c \in \mathbb{R} \text{ em int.}$$

$x = \sqrt{3}\sin t$
$\Leftrightarrow t = \arcsin\left(\frac{x}{\sqrt{3}}\right)$

$$\boxed{\begin{array}{l} t \longrightarrow x \\ t = \arcsin\left(\frac{x}{\sqrt{3}}\right) \end{array}}$$

$\Leftrightarrow \sin t = \frac{x}{\sqrt{3}}$

$\Leftrightarrow \cos t = \sqrt{1-\left(\frac{x}{\sqrt{3}}\right)^2}$

$\Leftrightarrow \cos t = \frac{\sqrt{3-x^2}}{\sqrt{3}}$

b)

$$\int \operatorname{sen}^4 x \, dx = \frac{1}{4}\int \left[1-\cos(2x)\right]^2 dx = \frac{1}{4}\int 1 - 2\cos(2x) + \cos^2(2x)\, dx$$

$$= \frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{8}\int 1 + \cos(4x)\, dx + c$$

$$= \frac{x - \sin(2x)}{4} + \frac{1}{8}t + \frac{1}{32}\sin(4x)\,dx + c,\ c \in \mathbb{R} \text{ em int}$$

c)

$$\int \frac{1}{x^2+2x+5}\,dx = \int \frac{1}{(x+1)^2+4}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x+1}{2}\right)^2+1}\,dx$$

$$x^2+2x+5 = a(x+p)^2+q$$
$$= ax^2+2apx+(ap^2+q)$$

$$\begin{cases} a=1 \\ 2=2ap \\ 5=ap^2+q \end{cases} \quad \begin{cases} a=1 \\ p=1 \\ q=4 \end{cases}$$

$$x^2+2x+5 = (x+1)^2+4$$

$$= \frac{1}{2}\int \frac{\frac{1}{2}}{\left(\frac{x+1}{2}\right)^2+1}\,dx$$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + c,\ c \in \mathbb{R} \text{ em int.}$$

d)

$$\int \frac{1}{\sqrt{2+x^2}}\,dx = \int \frac{\cancel{\sqrt{2}}\sec t}{\cancel{\sqrt{2}}\sqrt{\cancel{1+\operatorname{tg}^2 x}}}\,dt = \int \sec t\,dt$$

$$\boxed{\begin{array}{c} x \longrightarrow t \\ x = g(t) = \sqrt{2}\tan t,\ t \in \left]0, \frac{\pi}{2}\right[\\ dx = \sqrt{2}\sec^2 t \end{array}}$$

$$\boxed{\begin{array}{c} t \longrightarrow x \\ t = \operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) \end{array}}$$

$$= \int \frac{\sec t(\tan t + \sec t)}{\tan t + \sec t}\,dt$$

$$= \int \frac{\sec t \tan t + \sec^2 t}{\tan t + \sec t}\,dt$$

$$= \int \frac{(\tan t + \sec t)'}{\tan t + \sec t}\,dt$$

$$= \ln|\tan t + \sec t| + c,\ c \in \mathbb{R} \text{ em int}$$

$$\tan t = \frac{x}{\sqrt{2}}$$

$$\sec t = \sqrt{1+\left(\frac{x}{\sqrt{2}}\right)^2}$$

$$= \frac{\sqrt{2+x^2}}{\sqrt{2}}$$

$$\int \frac{1}{\sqrt{2+x^2}}\,dx = \ln\left|\frac{x}{\sqrt{2}} + \frac{\sqrt{2+x^2}}{\sqrt{2}}\right| + c$$

$$= \ln\left|\frac{x+\sqrt{2+x^2}}{\sqrt{2}}\right| + c,\ c \in \mathbb{R} \text{ em int.}$$

e)

$$\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\operatorname{sen} t}{\cancel{t}} \times 2\cancel{t}\,dt = -2\cos t + c$$

$$\boxed{\begin{array}{c} x \longrightarrow t \\ x = g(t) = t^2,\ t \in \mathbb{R}^+ \\ dx = 2t\,dt \end{array}}$$

$$\boxed{\begin{array}{c} t \longrightarrow x \\ t = \sqrt{x} \end{array}}$$

$$\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx = -2\cos\sqrt{x} + c,\ c \in \mathbb{R} \text{ em int}$$

f)

$$\int \frac{x}{x^2-5x+6}\,dx = \int \frac{x}{(x-2)(x+3)}\,dx$$

$x^2-5x+6=0$

$\Leftrightarrow x = \frac{5 \pm \sqrt{25-4\times 6}}{2}$

$\Leftrightarrow x = \frac{5-1}{2} \vee x = \frac{5+1}{2}$

$\Leftrightarrow x = 2 \vee x = 3$

$$\frac{x}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)}$$

$$\Leftrightarrow x = A(x+3) + B(x-2)$$

$$\Leftrightarrow x = x(A+B) + 3A - 2B$$

$$\begin{cases} A+B=1 \\ 3A-2B=0 \end{cases} \quad \begin{cases} B = \frac{3}{5} \\ A = \frac{2}{3}B \end{cases} \quad \begin{cases} B = \frac{3}{5} \\ A = \frac{2}{5} \end{cases}$$

$$\int \frac{x}{x^2-5x+6}\,dx = \frac{2}{5}\int \frac{1}{x-2}\,dx + \frac{3}{5}\int \frac{1}{x+3}\,dx$$

$$= \frac{2}{5}\ln|x-2| + \frac{3}{5}\ln|x+3| + c,\ c \in \mathbb{R} \text{ em int.}$$

g)

$$\int \frac{1}{\sqrt{2x-x^2}}\,dx = \int \frac{1}{\sqrt{1-(x-1)^2}}\,dx = \arcsin(x-1) + c,\ c \in \mathbb{R} \text{ em int.}$$

$-x^2+2x = -(x+p)^2+q$

$\quad = -x^2-2px-p^2+q$

$$\begin{cases} p = -1 \\ q = 1 \end{cases}$$

h)

$$\int x\sqrt{(1+x^2)^3}\,dx = \frac{1}{2}\times\int (2x)\times(1+x^2)^{\frac{3}{2}}\,dx = \frac{1}{\cancel{2}}\times\frac{(1+x^2)^{\frac{5}{2}}}{\frac{5}{\cancel{2}}}+c$$

$$= \frac{(1+x^2)^2\sqrt{1+x^2}}{5}+c,\ c\in\mathbb{R} \text{ em int.}$$

i)

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = \int \frac{t\times 2t}{1+t}\,dt = 2\int\frac{t^2}{1+t}\,dt = 2\int t-1+\frac{1}{t+1}\,dt$$

$\sqrt{x}=t$

$\Leftrightarrow x = t^2$

$$\boxed{\begin{array}{l} x\longrightarrow t \\ x=g(t)=t^2 \\ g'(t)=2t>0,\ t\in\mathbb{R}^+ \\ g \text{ é dif e inv. em } \mathfrak{I} \\ dx = 2t\,dt \end{array}}$$

$$= \frac{\cancel{2}t^2}{\cancel{2}} - 2t + 2\ln|t+1| + c$$

$$\begin{array}{r|l} t^2 & t+1 \\ \hline -t^2-t & t-1 \\ \hline -t & \\ +t+1 & \\ \hline 1 & \end{array}$$

$$\boxed{\begin{array}{l} t\to x \\ t=\sqrt{x} \end{array}} \longrightarrow$$

$$= x - 2\sqrt{x} + 2\ln|\sqrt{x}+1| + c,\ c\in\mathbb{R} \text{ em int.}$$

j)

$$\int \underbrace{x}_{u'}\,\underbrace{\ln(x)}_{v}\,dx = \frac{x^2}{2}\ln(x) - \int\frac{x^{\cancel{2}}}{2}\times\frac{1}{\cancel{x}}\,dx$$

$$\boxed{\begin{array}{ll} u=\frac{x^2}{2} & v=\ln(x) \\ u'=x & v'=\frac{1}{x} \end{array}}$$

$$= \frac{x^2}{2}\ln(x) - \frac{1}{2}\frac{x^2}{2}+c$$

$$= \frac{x^2}{2}\left(\ln(x)-\frac{1}{2}\right)+c,\ c\in\mathbb{R} \text{ em int.}$$

k)

$$\int\frac{1+e^x}{e^{2x}+4}\,dx = \int\frac{1}{e^{2x}+4}\,dx + \frac{1}{2}\int\frac{\frac{1}{2}e^x}{1+\left(\frac{e^x}{2}\right)^2}\,dx$$

$$= \underbrace{\int\frac{1}{e^{2x}+4}\,dx}_{} + \frac{1}{2}\operatorname{arctg}\left(\frac{e^x}{2}\right)$$

$t=e^x$

$\Leftrightarrow x=\ln(t)$

$\Leftrightarrow 2x = 2\ln(t)$

$\Leftrightarrow 2x = \ln(t^2)$

$\Leftrightarrow t^2 = e^{2x}$

$$= \int\frac{\frac{1}{t}}{t^2+4}\,dt = \int\frac{1}{t^3+4t}\,dt = \int\frac{1}{t(t^2+4)}\,dt$$

$$\boxed{\begin{array}{l} x\to t \\ x=g(t)=\ln(t) \\ g'(t)=\frac{1}{t}>0,\ t\in\mathbb{R}^+ \\ g \text{ é inv. e dif em } \mathfrak{I} \\ dx=\frac{1}{t}\,dt \end{array}}$$

$$\frac{1}{t(t^2+4)} = \frac{A}{t}+\frac{Bt+C}{t^2+4} \Leftrightarrow 1 = A(t^2+4)+Bt^2+Ct$$

$$\begin{cases} A+B=0 \\ C=0 \\ 4A=1 \end{cases} \quad \begin{cases} B=-1/4 \\ C=0 \\ A=1/4 \end{cases}$$

Voltando ...

$$\int \frac{1}{t(t^2+4)}\,dt = \frac{1}{4}\int \frac{1}{t}\,dt - \frac{1}{4}\times\frac{1}{2}\int \frac{2t}{4+t^2}\,dt$$

$$= \frac{1}{4}\ln|t| - \frac{1}{8}\ln|t^2+4| + c$$

$$\boxed{\begin{matrix} t \to x \\ t = e^x \end{matrix}}$$

$$= \frac{1}{4}x - \frac{1}{8}\ln(e^{2x}+4) + c,\ c \in \mathbb{R} \text{ em int.}$$

Juntando tudo...

$$\int \frac{1+e^x}{e^{2x}+4}\,dx = \frac{1}{4}x - \frac{1}{8}\ln(e^{2x}+4) + \frac{1}{2}\operatorname{arctg}\left(\frac{e^x}{2}\right) + c,\ c\in\mathbb{R} \text{ em int.}$$

l)

$$\int \underbrace{x}_{u'}\,\underbrace{\operatorname{arctg} x}_{v}\,dx = \frac{x^2}{2}\times\operatorname{arctg} x - \int \frac{x^2}{2}\times\frac{1}{1+x^2}\,dx$$

$$\boxed{\begin{matrix} u = \dfrac{x^2}{2} & v = \operatorname{arctg} x \\ u' = x & v' = \dfrac{1}{1+x^2} \end{matrix}}$$

$$= \frac{x^2\operatorname{arctg} x}{2} - \frac{1}{2}\int 1 - \frac{1}{x^2+1}\,dx$$

$$= \frac{x^2\operatorname{arctg} x}{2} - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg}(x) + c,\ c\in\mathbb{R} \text{ em int.}$$

$$\begin{array}{r|l} x^2 & x^2+1 \\ \cline{2-2} -x^2-1 & 1 \\ \hline -1 & \end{array}$$

m)

$$\int \frac{\operatorname{sen} x}{(1-\cos x)^3}\,dx = \int (\operatorname{sen} x)(1-\cos x)^{-3}\,dx$$

∇ este é direto, mas é ganda "bait" ö

$$= \frac{(1-\cos x)^{-2}}{-2} + c = \frac{-1}{2(1+\cos x)^2} + c,\ c\in\mathbb{R} \text{ em int.}$$

n)

$$⊛ \int \underbrace{(2x^2+3)}_{u'}\,\underbrace{\operatorname{arctg} x}_{v}\,dx = \left(\frac{2x^3}{3}+3x\right)\operatorname{arctg} x - \int\left(\frac{2x^3}{3}+3x\right)\times\frac{1}{1+x^2}\,dx$$

$$\boxed{\begin{matrix} u = \dfrac{2x^3}{3}+3x & v = \operatorname{arctg} x \\ u' = 2x^2+3 & v' = \dfrac{1}{1+x^2} \end{matrix}}$$

$$\begin{array}{r|l} 2x^3+9x & x^2+1 \\ \cline{2-2} -2x^3-2x & 2x \\ \hline 7x & \end{array}$$

$$\frac{1}{3}\int \frac{2x^3+9x}{1+1x^2}\,dx = \frac{1}{3}\int 2x + \frac{7x}{x^2+1}\,dx$$

$$= \frac{\cancel{2}}{3}\frac{x^2}{\cancel{2}} + 7\times\frac{1}{3}\times\frac{1}{2}\times\ln(x^2+1) + c$$

$$= \frac{x^2}{3} + \frac{7}{6}\ln(1+x^2) + c,\ c\in\mathbb{R} \text{ em int}$$

Juntando ...

$$\circledast = \left(\frac{2}{3}x^3 + 3x\right)\operatorname{arctg} x - \frac{1}{3}x^2 - \frac{7}{6}\ln(1+x^2) + c,\ c \in \mathbb{R} \text{ em int.}$$

o)

$$\int \frac{1}{\sqrt{x^2+2x-3}}\,dx = \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx = \frac{1}{\cancel{2}}\int \frac{\cancel{2}\sec t \operatorname{tg} t}{\sqrt{\sec^2 t - 1}}\,dt$$

$$= \int \frac{\sec t\ \cancel{\operatorname{tg} t}}{\cancel{\operatorname{tg} t}}\,dt$$

Multiplicar e dividir por $(\sec t + \operatorname{tg} t)$ (...)

$$= \ln|\sec t + \tan t| + c$$

$x^2 + 2x - 3 = x^2 + 2px + p^2 + q$

$$\begin{cases} p = 1 \\ q = -4 \end{cases} \Rightarrow x^2 + 2x - 3 = (x+1)^2 - 4$$

$\frac{x+1}{2} = \sec t$

$\Leftrightarrow x = 2\sec t - \cancel{2}$

$x \to t$

$x = g(t) = 2\sec(t) - \cancel{2}$

$g'(t) = 2\sec(t)\operatorname{tg}(t) > 0,\ t \in]0, \frac{\pi}{2}[$

g é inv. e dif. em

$dx = 2\sec(t)\operatorname{tg}(t)\,dt$

$t \to x$

$t = \operatorname{arcsec}(x+1)$

$$= \ln\left|\frac{x+1 + \sqrt{(x+1)^2-4}}{2}\right| + c,\ c \in \mathbb{R} \text{ em int.}$$

$\frac{x+1}{2} = \sec(t)$

$\Leftrightarrow \frac{x+1}{2} = \frac{1}{\cos t}$

$\Leftrightarrow \cos(t) = \frac{2}{x+1}$

$t \in]0, \frac{\pi}{2}[$

$\Leftrightarrow \sin(t) = \sqrt{1 - \left(\frac{2}{x+1}\right)^2}$

$\Leftrightarrow \operatorname{tg}(t) = \frac{\sqrt{1-\left(\frac{2}{x+1}\right)^2}}{\frac{2}{x+1}}$

$\Leftrightarrow \operatorname{tg}(t) = \frac{\sqrt{(x+1)^2-4}}{2}$

p)

$$\int \sqrt{1+e^x}\,dx = \int t \times \frac{2t}{t^2-1}\,dt = 2\int 1 + \frac{1}{t^2-1}\,dt$$

$$= 2t + 2\int \frac{1}{(t-1)(t+1)}\,dt$$

$$= 2t + \cancel{2} \times \frac{1}{\cancel{2}} \int \frac{1}{(t-1)}\,dt + \cancel{2} \times \frac{1}{\cancel{2}} \int \frac{1}{(t+1)}\,dt$$

$$= 2t + \ln|t-1| + \ln|t+1|$$

$t = \sqrt{1+e^x}$

$\Leftrightarrow e^x = t^2 - 1$

$\Leftrightarrow x = \ln(t^2-1)$

$[\ln(t^2-1)]' = \frac{2t}{t^2-1}$

$x \to t$

$x = g(t) = \ln(t^2-1)$

$g'(t) = \frac{2t}{t^2-1} > 0,\ t \in]1, +\infty[$

g é dif. e inv. em

$dx = \frac{2t}{t^2-1}\,dt$

$t \to x$

$t = \sqrt{1+e^x}$

$$= 2\sqrt{1+e^x} + \ln\left|\sqrt{1+e^x} - 1\right| + \ln\left|\sqrt{1+e^x} - 1\right| + c,\ c \in \mathbb{R} \text{ em int.}$$

$1 = A(t+1) + B(t-1)$

$$\begin{cases} A + B = 0 \\ A - B = 1 \end{cases} \quad \begin{cases} A = -B \\ B = -\frac{1}{2} \end{cases} \quad \begin{cases} A = \frac{1}{2} \\ B = -\frac{1}{2} \end{cases}$$

q)

$$\int \frac{1}{\sqrt{e^x-1}}\,dx = \int \frac{\frac{2\cancel{t}}{t^2+1}}{\cancel{t}}\,dt = 2\int \frac{1}{1+t^2}\,dt = 2\,\text{arctg}(t)+c$$

$t = \sqrt{e^x-1}$

$\Leftrightarrow t^2 = e^x - 1$

$\Leftrightarrow x = \ln(t^2+1)$

$[\ln(t^2+1)]' = \frac{2t}{t^2+1}$

$$\boxed{\begin{array}{c} x \longrightarrow t \\ x = g(t) = \ln(t^2+1) \\ g'(t) = \frac{2t}{t^2+1} > 0,\ t \in \mathbb{R}^+ \\ g \text{ é dif. e inv em } \mathfrak{I} \\ dx = \frac{2t}{t^2+1}\,dt \end{array}}$$

$$\boxed{\begin{array}{c} t \to x \\ t = \sqrt{e^x-1} \end{array}}$$

$= 2\,\text{arctg}(\sqrt{e^x-1}) + c,\ c \in \mathbb{R}$ em int

r)

$$\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx = -\int \frac{[\sin(\arccos(t^2))]^3}{\cancel{t}} \times \frac{2\cancel{t}}{\sqrt{1-t^4}}\,dt = -2\int \frac{(1-t^4)^{3/2}}{(1-t^4)^{1/2}}\,dt$$

$$= -2\int 1 - t^4\,dt$$

$$= -2t + 2\frac{t^5}{5} + c,\ c \in \mathbb{R} \text{ em int}$$

$$= -2\sqrt{\cos x} + \frac{2(\sqrt{\cos x})^5}{5} + c,\ c \in \mathbb{R} \text{ em int}$$

$t = \sqrt{\cos x}$

$\Leftrightarrow t^2 = \cos x$

$\Leftrightarrow x = \arccos(t^2)$

$[\arccos(t^2)]' = -\frac{2t}{\sqrt{1-t^4}}$

$[\sin(\arccos(t^2))] = (1-t^4)^{3/2}$

$x \longrightarrow t$

$x = g(x) = \arccos(t^2)$

$g'(x) = -\frac{2t}{\sqrt{1-t^4}} < 0,\ t \in \mathbb{R}^+$

g é div e inv. em $\mathfrak{I}$

$dx = \frac{-2t}{\sqrt{1-t^4}}\,dt$

$$\boxed{\begin{array}{c} t \to x \\ t = \sqrt{\cos x} \end{array}}$$

s)

$$\int \frac{\ln(x)}{x(\ln^2 x+1)}\,dx = \int \frac{\frac{\ln(x)}{x}}{(\ln^2 x+1)}\,dx = \frac{1}{2}\int \frac{\frac{2\ln x}{x}}{(\ln^2 x+1)}\,dx$$

$(\ln^2 x+1)' = 2\ln x \times \frac{1}{x}$

$= \frac{1}{2}\ln(\ln^2 x+1) + c,\ c \in \mathbb{R}$ em int

t)

$$\int x^3 e^{x^2}\,dx = \int t\,\cancel{\sqrt{t}} \times e^t \times \frac{1}{2}\frac{1}{\cancel{\sqrt{t}}}\,dx = \frac{1}{2}\int t e^t\,dt$$

$$= \frac{1}{2} t e^t - \frac{1}{2}\int e^t\,dt$$

$$= \frac{1}{2} t e^t - \frac{1}{2} e^t + c$$

$t = x^2$

$\Leftrightarrow x = \sqrt{t}$, $x \in \mathbb{R}^+$

$$\boxed{\begin{array}{c} x \longrightarrow t \\ x = g(x) = \sqrt{t} \\ g'(t) = \frac{1}{2} \times \frac{1}{\sqrt{t}} > 0,\ t \in \mathbb{R}^+ \\ g \text{ é inv e dif em } \mathfrak{I} \end{array}}$$

$$\boxed{\begin{array}{c} t \to x \\ t = x^2 \end{array}}$$

$u = t$ $\quad u = e^t$

$u' = 1$ $\quad u' = e^t$

$$= \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + c$$

$$= \frac{1}{2}e^{x^2}(x^2-1) + c,\ c \in \mathbb{R} \text{ em int.}$$

u)

$$\int \frac{2x+1}{(x-2)(x-3)(x+1)}\,dx$$

$$\frac{2x-1}{(x-2)(x-3)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x-3)} + \frac{C}{(x+1)}$$

$$\Leftrightarrow 2x-1 = A(x^2+x-3x-3) + B(x^2+x-2x-2) + C(x^2-3x-2x+6)$$

$$\Leftrightarrow 2x-1 = x^2(A+B+C) + x(-2A-B-5C) - 3A-2B+6C$$

$$\begin{cases} A+B+C=0 \\ -2A-B-5C=2 \\ -3A-2B+6C=-1 \end{cases} \quad \begin{cases} A=-B-C \\ 2B+2C-B-5C=2 \\ 3B+3C-2B+6C=-1 \end{cases} \quad \begin{cases} A=-B-C \\ B=2+3C \\ 2+3C+9C=-1 \end{cases} \quad \begin{cases} A=-1 \\ B=5/4 \\ C=-1/4 \end{cases}$$

$$-\ln|x-2| + \frac{5}{4}\ln|x-3| - \frac{1}{4}\ln|x+1|$$

v)

$$\int \frac{1+tg^2x}{\sqrt{tgx-1}}\,dx = \int sec^2x \times (tgx-1)^{-1/2}\,dx = 2\sqrt{tgx-1} + c,\ c\in\mathbb{R} \text{ em int.}$$

w)

$$\int \frac{x^8}{1+x^2}\,dx = \int x^6 - x^4 + x^2 - 1 + \frac{1}{x^2+1}\,dx$$

$$\begin{array}{l|l} x^8 & x^2+1 \\ \hline -x^8-x^6 & x^6-x^4+x^2-1 \\ -x^6 & \\ +x^6+x^4 & \\ x^4 & \\ -x^4-x^2 & \\ -x^2 & \\ +x^2+1 & \\ 1 & \end{array}$$

$$= \frac{x^7}{7} - \frac{x^5}{5} + \frac{x^3}{3} - x + arctg(x) + c,\ c\in\mathbb{R} \text{ em int.}$$

x)

$$\int \frac{x+1}{x^3-1}\,dx = \int \frac{x+1}{(x-1)(x^2+x+1)}\,dx$$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

$$x^2+x+1 = (x+p)^2+q$$
$$= x^2+2px+p^2+q$$

$$\begin{cases} p=\frac{1}{2} \\ q=\frac{4}{4}-\frac{1}{4} \end{cases} \quad \begin{cases} p=1/2 \\ q=\frac{3}{4} \end{cases}$$

$$x+1 = A(x^2+x+1) + Bx(x-1) + C(x-1)$$

$$\Leftrightarrow x+1 = x^2(A+B) + x(A-B+C) + A-C$$

$$\begin{cases} A+B=0 \\ A-B+C=1 \\ A-C=1 \end{cases} \quad \begin{cases} C=-1/3 \\ B=2C \\ A=1+C \end{cases} \quad \begin{cases} C=-1/3 \\ B=-2/3 \\ A=2/3 \end{cases}$$

$$\frac{2}{3}\ln|x-1| - \frac{1}{3}\int \frac{2x+1}{x^2+x+1}\,dx$$

$$= \frac{2}{3}\ln|x-1| - \frac{1}{3}\ln(x^2+x+1) + c,\ c\in\mathbb{R} \text{ em int.}$$

$\boxed{12}$

a) $$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{1}{3}\int (3x^2)\times(1+x^3)^{-1/2}\,dx = \frac{1}{3}\times 2\times\sqrt{1+x^3}+c$$

$$= \frac{2}{3}\sqrt{1+x^3}+c,\ c\in\mathbb{R} \text{ em int}$$

b) $$\int \frac{1}{x^2\sqrt{1+x^2}}\,dx = \int \frac{\sec^2 t}{tg^2 t\,\sqrt{1+\cancel{tg^2 t}}}\,dt = \int \frac{\frac{1}{\cos t}}{\frac{\sin^2 t}{\cos^2 t}}\,dt = \int \frac{\cos t}{\sin^2 t}\,dt$$

$$= \int \cos t\,(\sin t)^{-2}\,dt$$

$$= \frac{\sin^{-1} t}{-1}+c$$

$$= -\csc t + c$$

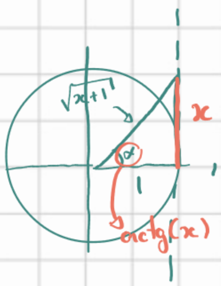
$x \longrightarrow t$

$x = g(t) = tg\,t$

$g'(t) = \sec^2 t > 0,\ t\in\,]0,\frac{\pi}{2}[$

g é dif. e inv. em $\longrightarrow$

$dx = \sec^2 t\,dt$



(*) !

$\sin(\text{arctg}(x)) = \frac{x}{\sqrt{x+1}}$

Voltando à variável inicial:

$t \longrightarrow x$

$t = \text{arctg}(x)$

$$= -\frac{1}{\sin(\text{arctg}\,t)} + c,\ c\in\mathbb{R} \text{ em int}$$

(*) $$= -\frac{1}{\frac{x}{\sqrt{x+1}}} + c$$

$$= -\frac{\sqrt{x+1}}{x} + c,\ c\in\mathbb{R} \text{ em int}$$

c) $$\int \frac{3x-1}{x^3+x}\,dx = \int \frac{3x-1}{x(x^2+1)}\,dx$$

$$\frac{3x-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\Leftrightarrow 3x-1 = A(x^2+1) + Bx^2 + Cx$$

$$\begin{cases} A+B=0 \\ C=3 \\ A=-1 \end{cases} \quad \begin{cases} B=1 \\ C=3 \\ A=-1 \end{cases}$$

$$= -\ln|x| + \int \frac{x+3}{x^2+1}\,dx = -\ln|x| + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + 3\int \frac{1}{x^2+1}\,dx$$

$$= -\ln|x| + \frac{1}{2}\ln(x^2+1) + 3\,\text{arctg}(x) + c,\ c\in\mathbb{R} \text{ em int.}$$

d) $$\int \frac{1}{e^{2x}+2}\,dx = \int \frac{\frac{1}{t}}{t^2+2}\,dt = \int \frac{1}{t(t^2+2)}\,dt = \frac{1}{2}\ln|t| - \frac{1}{4}\int \frac{2t}{t^2+2}\,dt$$

$$= \frac{1}{2}\ln|t| - \frac{1}{4}\ln(t^2+2)$$

$$= \frac{1}{2}x - \frac{1}{4}\ln(e^{2x}+2) + C,\ C \in \mathbb{R}$$

em int

$e^x = t$

$\Leftrightarrow x = \ln(t)$

$$\boxed{\begin{array}{l} x \to t \\ x = g(t) = \ln(t) \\ g'(t) = \frac{1}{t} > 0,\ t \in \mathbb{R}^+ \\ g \text{ é inv. e dif. em } \mathbb{R}^+ \\ dx = \frac{1}{t}\,dt \end{array}}$$

$1 = A(t^2+2) + Bt^2 + Ct$

$$\begin{cases} A + B = 0 \\ 2A = 1 \\ C = 0 \end{cases} \quad \begin{cases} B = -1/2 \\ A = 1/2 \\ C = 0 \end{cases}$$