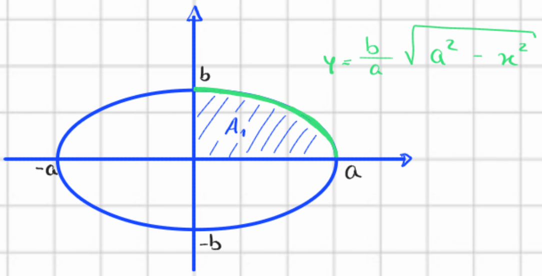
Aula 14

Elipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a, b \in \mathbb{R}^+$$

- Se $a = b$ temos uma circunferência.

$$x^2 + y^2 = R$$



Atendendo à simetria da figura, calculamos:

$$A_{\text{elipse}} = 4A_1 = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx = 4\int_0^{\frac{\pi}{2}} \frac{b}{\cancel{a}} \sqrt{a^2 - a^2 \sin^2 t} \times \cancel{a}\cos t\, dt$$

$$\boxed{\begin{array}{l} x \longrightarrow t \\ x = g(t) = a \sin t,\ t \in [0, \frac{\pi}{2}] \\ g'(t) = a\cos t,\ g' \text{ é contínua em } [0, \frac{\pi}{2}] \\ dx = a\cos t\, dt \end{array}}$$

Se $x = 0 \Rightarrow t = \arcsin(0) = 0$

Se $x = a \rightarrow t = \arcsin(1) = \frac{\pi}{2}$

$$= 4ab\int_0^{\frac{\pi}{2}} \cos^2 t\, dt$$

$$= 4ab\int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2}\cos(2t)\, dt$$

$$= 2ab\,[t]_0^{\pi/2} + ab\,[\sin(2t)]_0^{\pi/2}$$

$$= 2ab\left[\pi/2 - 0\right] + ab\left[\cancel{\sin(\pi)}^{0} - \cancel{\sin(0)}^{0}\right]$$

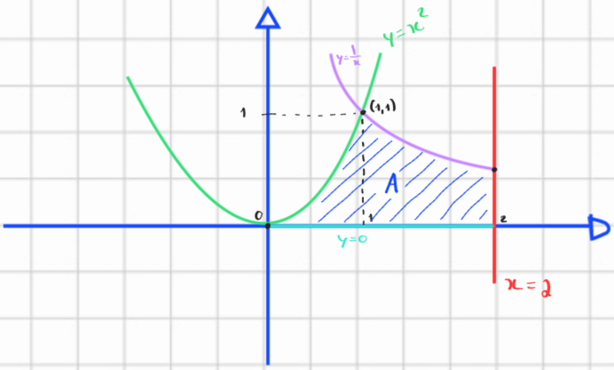
$$= ab\pi$$

Se $a = b$, temos: $A_{\text{círculo}} = \pi r^2$

$y = \frac{1}{x}$; $y = x^2$; $x = 2$; $y = 0$

$$\boxed{A = \int_0^2 \text{teto}(x) - \text{chão}(x)\, dx}$$

Fácil de lembrar!



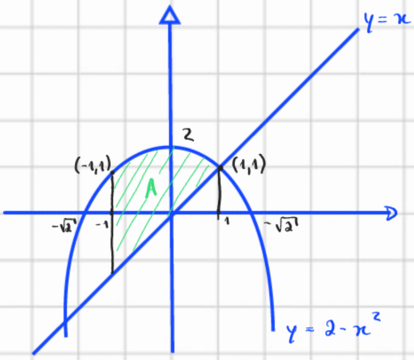
$$A = \int_0^1 (\underbrace{x^2}_{\text{teto}} - \underbrace{0}_{\text{chão}})\, dx + \int_1^2 \left(\frac{1}{x} + 0\right) dx$$

$$A = \int_0^1 x^2\, dx + \int_1^2 \frac{1}{x}\, dx$$

$$A = \left[\frac{x^3}{3}\right]_0^1 + \left[\ln x\right]_1^2$$

$$A = \frac{1}{3} + \ln 2 - \ln 1 = \frac{1}{3} + \ln 2$$

$$R = \{(x,y) \in \mathbb{R}^2 : x \in [-1,1] \wedge (x \leq y \leq 2-x^2)\}$$



$$2-x^2=0$$
$$\Leftrightarrow x^2 = 2$$
$$\Leftrightarrow x = -\sqrt{2} \vee x = \sqrt{2}$$

$$A = \int_{-1}^{1} (2-x^2) - x \, dx = \int_{-1}^{1} -x^2 - x + 2 \, dx = \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-1}^{1}$$

$$= -\frac{1}{3} - \cancel{\frac{1}{2}} + 2 - \left(\frac{1}{3} - \cancel{\frac{1}{2}} - 2\right)$$

$$= -\frac{2}{3} + \frac{12}{3} = \frac{10}{3} \simeq 3,\ldots$$

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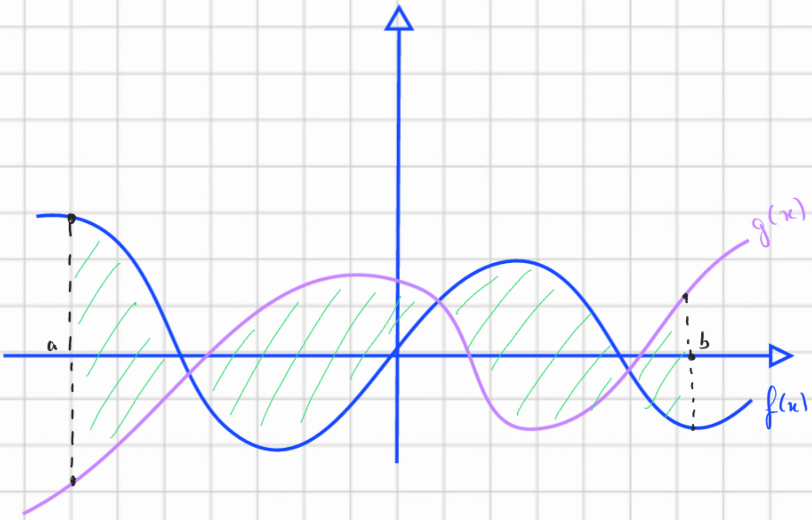
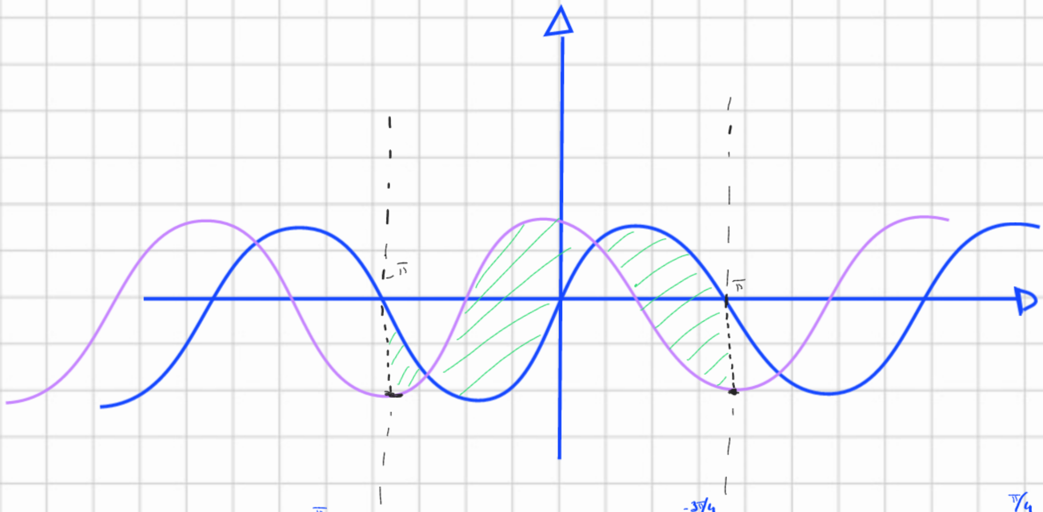
Calcular a área delimitada pelas curvas $y=f(x)$; $y=g(x)$ e $x \in [a,b]$



$$A = \int_{a}^{b} |f(x) - g(x)| \, dx$$

$x \in [-\pi, \pi]$

$f(x) = \sin x \qquad g(x) = \cos x$



$$A = \int_{-\pi}^{\pi} |\cos x - \sin x|\, dx = \int_{-\pi}^{-3\pi/4} (\sin x - \cos x)\, dx + \int_{-\frac{3\pi}{4}}^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\, dx$$

$(=) A = 4\sqrt{2}$

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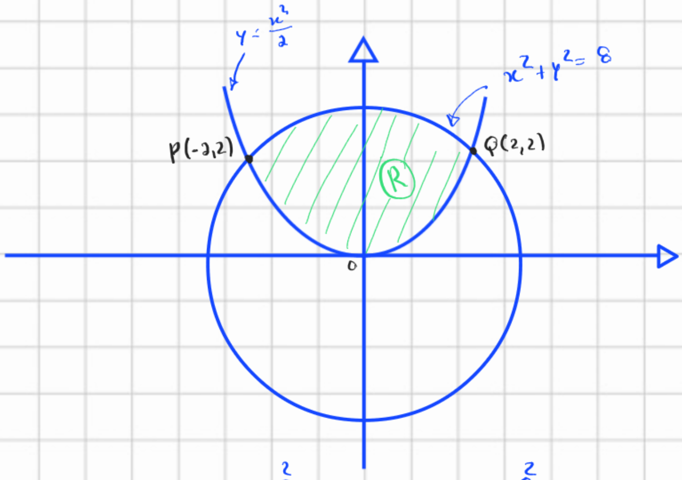

$$A = 2\int_0^2 \sqrt{8 - x^2}\, dx - \int_0^2 x^2\, dx \underset{(\dots)}{=} \frac{4}{3} + 2\pi$$