Folha 1

$\boxed{1}$

a) $\displaystyle\sum_{n=1}^{+\infty} \underbrace{n(n+1)}_{a_n \neq 0}(x-0)^n$

$c=0$

$$R = \lim_{n\to+\infty}\left|\frac{n(\cancel{n+1})}{(\cancel{n+1})(n+2)}\right| = \lim_{n\to+\infty}\left(\frac{1}{1+2/n}\right) = 1 \in \mathbb{R}^+$$

$I_c =]-1, 1[$

• $\underline{x=-1}$:

$$\sum_{n=1}^{+\infty} \underbrace{n(n+1)(-1)^n}_{u_n}$$

$$\lim_{n\to+\infty} u_n = \begin{cases} \lim_{n\to+\infty}(n^2+n), & \text{se } n \text{ for par} \\ \lim_{n\to+\infty} -(n^2+n), & \text{se } n \text{ for impar} \end{cases} \begin{cases} +\infty, & \text{se } n \text{ for par} \\ -\infty, & \text{se } n \text{ for impar} \end{cases}$$

Logo, como $\lim_{n\to+\infty} u_n \neq 0$ a série DIV

• $\underline{x=1}$:

$$\sum_{n=1}^{+\infty} \underbrace{n(n+1)}_{a_n}, \text{ DIV pois } \lim_{n\to+\infty}[n(n+1)] \neq 0$$

Assim, $D_c =]-1, 1[$, sendo abs. convergente em D_c

b) $$\sum_{n=1}^{+\infty} \frac{(2x)^n}{(n-1)!} = \sum_{n=1}^{+\infty} \frac{2^n}{(n-1)!} \times (x-0)^n$$

$c=0$

$$R = \lim_{n\to+\infty}\left|\frac{\frac{2^{\cancel{n}}}{(n-1)!}}{\frac{2^{\cancel{n}+1}}{n!}}\right| = \lim_{n\to+\infty}\left|\frac{n}{2}\right| = +\infty$$

Logo, $I_c = D_c =]-\infty, +\infty[= \mathbb{R}$, sendo abs. convergente em D_c

c) $$\sum_{n=1}^{+\infty} (-1)^n \frac{x^{n+1}}{n+1} = \sum_{n=2}^{+\infty} \frac{(-1)^{n-1}}{n}(x-0)^n$$

$c=0$

$$R = \lim_{n\to+\infty}\left|\frac{\frac{(-1)^{\cancel{n-1}}}{n}}{\frac{\cancel{(-1)^n}}{n+1}}\right| = \lim_{n\to+\infty}\left|-\frac{1+1/n}{n}\right| = 1 \in \mathbb{R}^+$$

$I_c =]-1, 1[$

• $x = -1$:

$$\sum_{n=2}^{+\infty} \frac{(-1)^{n-1}}{n} \times (-1)^n = \sum_{n=2}^{+\infty} -\frac{1}{n} = -\sum_{n=2}^{+\infty} \frac{1}{n}$$, DIV (Dirichlet $\alpha = 1 \leq 1$)

• $x = 1$:

$$\sum_{n=2}^{+\infty} \frac{1}{n} \times (-1)^{n-1}$$, Conv pelo critério de Leibniz
a dos módulos diverge por ser Dirichlet $\alpha = 1 \leq 1$

Assim, $D_c =]-1, 1]$, sendo abs. convergente em D_c menos para $x = 1$ que é simplesmente convergente

d) $$\sum_{n=1}^{+\infty} \frac{(2x-3)^n}{2n+4} = \sum_{n=1}^{+\infty} \underbrace{\frac{2^n}{2n+4}}_{a_n > 0} \times \left(x - \frac{3}{2}\right)^n$$

$c = \frac{3}{2}$

$$R = \lim_{n \to +\infty} \left| \frac{\frac{\cancel{2^n}}{2n+4}}{\frac{\cancel{2^n} \times 2}{2n+5}} \right| = \lim_{n \to +\infty} \left| \frac{1}{2} \times \frac{\cancel{2n}(1 + 4/2n)}{\cancel{2n}(1 + 5/2n)} \right| = \frac{1}{2}$$

$$\boxed{c - R = \frac{3}{2} - \frac{1}{2} = 1}$$

$$\sum_{n=1}^{+\infty} \frac{2^n}{2n+4} \times \left(1 - \frac{3}{2}\right)^n = \sum_{n=1}^{+\infty} \frac{2^n \times \left(-\frac{1}{2}\right)^n}{2n+4} = \sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+4}$$, converge (c. Leibniz)

A série dos módulos diverge pelo critério do limite com $\sum_{n=1}^{+\infty} \frac{1}{n}$ que diverge (Dirichlet $\alpha = 1 \leq 1$)

$$\lim_{n \to +\infty} \left| \frac{\frac{1}{2n+4}}{\frac{1}{n}} \right| = \lim_{n \to +\infty} \left| \frac{\cancel{n}}{2\cancel{n}(1 + 4/2n)} \right| = \frac{1}{2} \in \mathbb{R}^+$$

$$\boxed{c + R = \frac{3}{2} + \frac{1}{2} = 2}$$

$$\sum_{n=1}^{+\infty} \frac{2^n \times \left(2 - \frac{3}{2}\right)^n}{2n+4} = \sum_{n=1}^{+\infty} \frac{1}{2n+4}$$ DIV

Logo, $I_c =]1, 2[$ e $D_c = [1, 2[$, conv. absoluta $x \neq 1$

conv. simples

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a) $T_0^3(x^3+2x+1) = T_0^3(f(x))$, onde $f(x) = x^3+2x+1$

$$T_0^3(f(x)) = \sum_{k=0}^{3} \frac{f^{(k)}(0)}{k!}(x-0)^k = \sum_{k=0}^{3} \frac{f^{(k)}(0)}{k!} x^k$$
$$= f(0) + \frac{f'(0)\,x}{1} + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3$$

$f(x) = x^3+2x+1 \longrightarrow f(0) = 1$

$f'(x) = 3x^2+2 \longrightarrow f'(0) = 2$

$f''(x) = 6x \longrightarrow f''(0) = 0$

$f'''(x) = 6 \longrightarrow f'''(0) = 6$

Logo, $x^3+2x+1 \simeq T_0^3 = 1 + 2x + \frac{\cancel{6}x^3}{\cancel{6}} = 1+2x+x^3 = x^3+2x+1$

Como é um polinómio, não tem erro associado!

b) $T_\pi^3(\cos(x)) = T_x^3(f(x))$, onde $f(x) = \cos x$

$$T_\pi^3 = \sum_{m=0}^{3} \frac{f^{(k)}(\pi)}{k!}(x-\pi)^k = f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)}{2}(x-\pi)^2 + \frac{f'''(\pi)}{6}(x-\pi)^3$$

$f(x) = \cos(x) \longrightarrow f(\pi) = -1$

$f'(x) = -\sin(x) \longrightarrow f'(\pi) = 0$

$f''(x) = -\cos(x) \longrightarrow f''(\pi) = 1$

$f'''(x) = \sin(x) \longrightarrow f'''(\pi) = 0$

$$T_\pi^3 = -1 + \frac{(x-\pi)^2}{2}$$

f)

$T_1^m(\ln x)$, $f(x) = \ln(x)$

$$T_1^m(f(x)) = \sum_{k=0}^{m} \frac{f^{(k)}(1)}{k!}(x-1)^k = \frac{f(1)}{0!} + \frac{f'(1)(x-1)}{1!} + \frac{f''(1)}{2!}(x-1)^2 + \dots$$

$$f(x)=\ln(x) \longrightarrow f(1)=0!$$

$$f'(x)=\frac{1}{x} \longrightarrow f'(1)=0!=1$$

$$f''(x)=\frac{-1}{x^2} \longrightarrow f''(1)=-(1)!$$

$$f'''(x)=\frac{2}{x^3} \longrightarrow f'''(1)=(2)!$$

$$-(3)!$$

$$\vdots$$

$$f^{(k)}(x)=\frac{(-1)^{k+1}(k-1)!}{x^k}, \quad k>0$$

$$f^{(k)}(1)=(-1)^{k+1}(k-1)!, \quad k>0$$

Logo, $T_1^n(f(x)) = \cancel{f(1)}^{0} + \sum_{k=1}^{n} \frac{(-1)^{k+1}(k-1)!}{k!}(x-1)^k = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}(x-1)^k$

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$f(x)=sen(x)$

$$\left|R_\pi^5(sen(x))\right| = \left|\frac{f^{(5+1)}(\theta)}{(5+1)!}(x-\pi)^{5+1}\right| = \left|\frac{f^{(6)}(\theta)}{6!}(x-\pi)^6\right| = \left|-\frac{sen(\theta)}{\underbrace{\underbrace{6\times5}_{30}\times\underbrace{4\times3\times2}_{24}}_{720}}(x-\pi)^6\right|$$

$$\begin{array}{r} 30 \\ \times 24 \\ \hline 120 \\ +60 \\ \hline 720 \end{array}$$

$f(x)=sen(x)$
$f^{(1)}(x)=cos(x)$
$f^{(2)}(x)=-sen(x)$
$f^{(3)}(x)=-cos(x)$
$f^{(4)}(x)=sen(x)$
$f^{(5)}(x)=cos(x)$
$f^{(6)}(x)=-sen(x)$

$$=\frac{sen(\theta)(x-\pi)^6}{720}$$

$$\left|R_\pi^5(sen(3))\right| = \frac{\overbrace{sen(\theta)}^{\theta\in]\pi,5[\Rightarrow sen(\theta)<0}(3-\pi)^6}{720} < \frac{(3-\pi)^6}{720}$$

$\boxed{9}$

$$\ln(1+x)\leq x, \quad \overbrace{x>-1}^{1+x>0 \Leftrightarrow x>-1}$$

$$\ln(1+x)=T_0^1(\ln(1+x))+R_0^1(\ln(1+x))$$

$f(x)=\ln(1+x)$

$[\ln(1+x)]'=\frac{1}{1+x}$

$[\ln(1+x)]''=\frac{-1}{(1+x)^2}$

$$f(x)=\sum_{k=0}^{1}\frac{f^{(k)}(0)}{k!}(x)^k+\frac{f^{(1+1)}(\theta)}{(1+1)!}(x)^{1+1}$$

$$f(x)=\cancel{f(0)}^{0}+f'(0)x-\frac{x^2}{(1+\theta)^2\times2}$$

$$f(x)=x-\frac{x^2}{(1+\theta)^2\times2}$$

$f(x)+\frac{x^2}{(1+\theta)^2\times2}=x$, logo: $\ln(1+x)+\frac{x^2}{(1+\theta)^2\times2}=x$ e como $\frac{x^2}{(1+\theta)^2\times2}\geq0$, $x>-1 \wedge \theta\in]0,1[$

$\theta\in]0,1[$

$\Rightarrow \ln(1+x)\leq x, \; x>1$