Aula 02

Séries de Potências - Cálculo do Raio, Intervalo e Domínio de Convergência

$\hookrightarrow R$ $\hookrightarrow I_c$ $\hookrightarrow D_c$

• Exercícios Caderno Ⓐ do Facebook do Sr Esteves

①

$$\sum_{n=0}^{+\infty} \frac{1}{\sqrt{n+1}\, 3^n}(x-0)^n$$

$c = 0$

$$R = \lim_{n\to+\infty}\left|\frac{\frac{1}{\sqrt{n+1}\,\cancel{3^n}}}{\frac{1}{\sqrt{n+2}\,3^{\cancel{n}+1}}}\right| = \lim_{n\to+\infty}\left[\left(\frac{n+2}{n+1}\right)^{1/2}\times 3\right] = 3 \in \mathbb{R}^+$$

$c - R = -3$

• $x = -3$

$$\sum_{n=0}^{+\infty}\frac{1}{\sqrt{n+1}\,3^n}(-3)^n = \sum_{n=0}^{+\infty}\frac{1}{\sqrt{n+1}}\times(-1)^n$$

Critério de Leibniz:

- $u_n = \frac{1}{\sqrt{n+1}}$ é decrescente ✓
- $\lim_{n\to+\infty} u_n = \lim_{n\to+\infty}\frac{1}{\sqrt{n+1}} = 0$ ✓

$\hookrightarrow$ A série converge!

Estudando a série dos módulos: $\sum_{n=0}^{+\infty}\frac{1}{\sqrt{n+1}}$

$$L = \lim_{n\to+\infty}\frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} = 1 \in \mathbb{R}^+$$

e como $\sum_{n=1}^{+\infty}\frac{1}{n^{1/2}}$ (Dirichlet $\alpha = 1/2 < 1$) diverge

logo a série dos módulos diverge

• $x = 3$

$$\sum_{n=0}^{+\infty}\frac{1}{\sqrt{n+1}\,\cancel{3^n}}\times\cancel{3^n}$$ que diverge como já estudamos.

$I_c =]-3,3[$ e $D_c = [-3,3[$ onde a série converge absolutamente exceto em $x=-3$ onde é simples

②

$$\sum_{n=0}^{+\infty} 3^n(x+5)^n, \quad 3^n = a_n \neq 0$$

$C = -5$

$$R = \lim_{n\to+\infty}\left|\frac{\cancel{3^n}}{3^{\cancel{n}+1}}\right| = \frac{1}{3} \in \mathbb{R}^+$$

$$-5-\frac{1}{3}=-\frac{16}{3}$$

- $x=-\frac{16}{3}$

$$\sum_{n=0}^{+\infty} 3^n\left(-\frac{1}{3}\right)^n=\sum_{n=0}^{+\infty}(-1)^n$$

$$\lim_{n\to+\infty}(-1)^n\neq 0, \text{ logo DIV}$$

$$-5+\frac{1}{3}=-\frac{14}{3}$$

- $x=-\frac{14}{3}$

$$\sum_{n=0}^{+\infty} 3^n\left(\frac{1}{3}\right)^n=\sum_{n=0}^{+\infty}(1), \text{ DIV}$$

Logo, $I_c=\left]-\frac{16}{3},-\frac{14}{3}\right[=D_c$

(3) $\displaystyle\sum_{n=0}^{+\infty}\frac{2}{9^{n+1}\,n^2}(x-3)^n$, $\frac{2}{9^{n+1}\,n^2}\neq 0$, $c=3$

$$R=\lim_{n\to+\infty}\left|\frac{\frac{\cancel{2}}{\cancel{9^{n+1}}\,n^2}}{\frac{\cancel{2}}{9^{n+1+1}\,(n+1)^2}}\right|=9\times\lim_{n\to+\infty}\left(\frac{1}{1+\cancel{1/n}}\right)^{2}=9\in\mathbb{R}^+$$

$I_c=]-6,12[$

- $x=-6$

$$\sum_{n=0}^{+\infty}\frac{2}{9^{n+1}\,n^2}(-1)^n\times(9)^n=\frac{2}{9}\times\sum_{n=0}^{+\infty}\frac{(-1)^n}{n^2}, \text{ conv. absolut.}$$

- $x=12$

$$\sum_{n=0}^{+\infty}\frac{2}{9^{n+1}\,n^2}(\cancel{9^n})=\frac{2}{9}\sum_{n=1}^{+\infty}\frac{1}{n^2}, \text{ conv. absol.}$$

④

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n\,6^n} \times 3^n\left(x-\frac{2}{3}\right)^n = \sum_{n=1}^{+\infty} \frac{(-1)^n}{n \times 2^n}\left(x-\frac{2}{3}\right)^n$$

$$c = \frac{2}{3}$$

$$R = \lim_{n\to+\infty}\left|\frac{1}{\sqrt[n]{\left|\frac{(-1)^n}{n\times 2^n}\right|}}\right| = \lim_{n\to+\infty}\left|\frac{1}{\frac{1}{2}\times n^{-\frac{1}{n}}}\right| = 2\times \cancel{\lim_{n\to+\infty} n^{1/n}} = 2$$

$$I_c = \left]-\frac{4}{3}, \frac{8}{3}\right[$$

• $x = -4/3$

$$\sum_{n=1}^{+\infty} \frac{\cancel{(-1)^n}}{n\,\cancel{6^n}} \times \cancel{3^n}\,\cancel{(-2)^n} = \sum_{n=1}^{+\infty} \frac{1}{n^1} \quad \text{DIV}$$

• $x = 8/3$

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n\,\cancel{6^n}} \times \cancel{3^n}\,\cancel{(2^n)} \quad \text{CONV}$$

$$D_c = \left]-4/3, 8/3\right] \quad \text{simples}$$

⑤

$$\sum_{n=2}^{+\infty} \frac{1}{\ln n}\, x^{3n} = \sum_{n=2}^{+\infty} \frac{1}{\ln n} \times (z-0)^n$$

$z = x^3$ $\quad \boxed{c = 0}$

$\Leftrightarrow x = \sqrt{z}$

$$L = \lim_{n\to+\infty}\left|\frac{\frac{1}{\ln(n)}}{\frac{1}{\ln(n+1)}}\right| = 1 \in \mathbb{R}^+ \Rightarrow I_c =]-1,1[$$

• $x = -1$

$$\sum_{n=2}^{+\infty} \frac{1}{\ln n}(-1)^n \quad \text{conv. simples}$$

• $x = 1$

$$\sum_{n=2}^{+\infty} \frac{1}{\ln n} \quad \text{DIV}$$

$$D_{cz} = [-1, 1[\quad \text{conv. simples}$$

$$x = \sqrt[3]{z} \Rightarrow D_c = \left[\sqrt[3]{-1}, \sqrt[3]{1}\right] = [-1, 1]$$

- As mudanças de variável pode usar-se nas séries de potências para obter novas séries de potências

Exemplo

$$\boxed{\frac{1}{1-x} = \sum_{n=0}^{+\infty} x^n \, , \; x \in \,]-1,1[}$$

$$\frac{1}{1-\bullet} = \sum_{n=0}^{+\infty} \bullet^n \, , \; \bullet \in \,]-1,1[$$

Borrão 8 serve como "variável de variáveis" → "Metavariável"

$\bullet \leftarrow 2x$

$$\frac{1}{1-2x} = \sum_{n=0}^{+\infty} (2x)^n \, , \; 2x \in \,]-1,1[$$

$$= \sum_{n=0}^{+\infty} 2^n x^n \, , \; x \in \,\left]-\frac{1}{2}, \frac{1}{2}\right[$$

8 Vamos explorar isto mais para a frente