Aula 25

Exercícios:

Ⓐ $\begin{cases} 3y' - y = \cos t, \; t \geq 0 \\ y(0) = -1 \end{cases}$

$\mathcal{L}\{3y' - y\} = \mathcal{L}\{\cos t\}$

$3\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{\cos t\}$

$3[sY(s) - Y(0)] - Y(s) = \frac{s}{s^2+1}$

$3sY(s) - 3\overbrace{Y(0)}^{=-1} - Y(s) = \frac{s}{s^2+1}$

$[3s - 1]Y(s) = \frac{s}{s^2+1} - 3$

$Y(s) = \frac{-3s^2 + s - 3}{(s^2+1)(3s-1)}$

$Y(s) = \frac{-s^2 + 5/3 - 1}{\underbrace{(s^2+1)}_{\text{Raíze } \notin \mathbb{R}}(s - 1/3)}$

Separar!

$\frac{-s^2 + 5/3 - 1}{(s^2+1)(s-1/3)} = \frac{A}{s - 1/3} + \frac{Bs + C}{s^2+1}$

(...)

$Y(t) = \frac{-9/10}{s - 1/3} + \frac{(-1/10)s + 3/10}{s^2 + 1}$

$y(t) = \mathcal{L}^{-1}\{Y(t)\} = -\frac{9}{10} e^{t/3} - \frac{1}{10}\cos(t) + \frac{3}{10}\sin t, \; t \geq 0$

Nota: $\mathcal{L}\{y\} = Y(s)$

$\mathcal{L}\{y'\} = sY(s) + y(0)$

$\mathcal{L}\{y''\} = s^2 Y(s) + sy(0) + y'(0)$

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Ⓑ $\begin{cases} y'' + 2y' + 10y = 1 \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

$\mathcal{L}\{y'' + 2y' + 10y\} = \mathcal{L}\{1\}$

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 10\mathcal{L}\{y\} = \mathcal{L}\{1\}$$

$$s^2 Y(s) - s\cancelto{0}{y(0)} - \cancelto{0}{y'(0)} + 2sY(s) - 2\cancelto{0}{y(0)} + 10Y(s) = \frac{1}{s}$$

$$s^2 Y(s) + 2sY(s) + 10Y(s) = \frac{1}{s}$$

$$Y(s)(s^2 + 2s + 10) = 1/s$$

$$Y(s) = \frac{1}{(s^2+2s+10)s}$$

$$\frac{1}{(s^2+2s+10)s} = \frac{As+B}{s^2+2s+10} + \frac{C}{s}$$

$$\Leftrightarrow 1 = As^2 + Bs + Cs^2 + 2sC + 10C$$

$$\Leftrightarrow 1 = s^2(A+C) + s(B+2C) + 10C$$

$$\begin{cases} C = \frac{1}{10} \\ A = -\frac{1}{10} \\ B = -\frac{2}{10} \end{cases}$$

$$Y(s) = \frac{-\frac{1}{10}s - \frac{2}{10}}{s^2+2s+10} + \frac{\frac{1}{10}}{s}$$

$$= -\frac{1}{10}\,\frac{s+2}{s^2+2s+10} + \frac{1}{10} \times \frac{1}{s}$$

$$s^2+2s+10 = (s+p)^2 + q$$
$$= s^2 + 2ps + (p^2+q)$$

$$\begin{cases} p = 1 \\ p^2 + q = 10 \end{cases} \quad \begin{cases} p = 1 \\ q = 9 \end{cases}$$

$$\Downarrow$$

$$s^2+2s+10 = (s+1)^2 + 9$$

Logo: $y(s) = \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s+2}{\underbrace{s^2+2s+10}_{=(s+1)^2+9}}\right\}$

$$= \frac{1}{10}t^0 - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s+1)^2+9}\right\}$$

$$= \frac{1}{10} - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+3^2}\right\} - \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+9}\right\}$$

$a = -1$ $\quad a = -1$ $\quad a = -1$

$$= \frac{1}{10} - \frac{1}{10}\underbrace{e^{-t}}_{e^{at}}\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\} - \frac{1}{30}e^{-t}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}$$

$$= \frac{1}{10} - \frac{1}{10}e^{-t}\cos(3t) - \frac{1}{30}e^{-t}\sin(3t),\ t \geq 0$$

(c)

$$\begin{cases} y'' + 4y' + 5y = e^{-3t} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{e^{-3t}\}$$

$$s^2 Y(s) + s\cancelto{1}{y(0)} + \cancelto{0}{y'(0)} + 4sY(s) + 4\cancelto{1}{y(0)} + 5Y(s) = \frac{1}{s+3}$$

$$s^2 Y(s) + 4sY(s) + 5Y(s) = \frac{1}{s+3}$$

$$Y(s) = \frac{1}{(s+3)(s^2+4s+5)}$$

$$1 = A(s^2+4s+5) + B(s+3)$$

$$\Leftrightarrow 1 = As^2 + (4A+B)s + 5A + 3B$$

$$\vdots$$

<u>Solução</u>: $y(t) = \frac{1}{2} e^{-3t} + \frac{1}{2} e^{-2t} \cos t + \frac{5}{2} e^{-2t} \sin t, \ t \geq 0$