(...)

$\boxed{6}$

a) $x + yy' = 0$

$\Leftrightarrow y' = \frac{-x}{y}$

$\Leftrightarrow \int y \, dy = -\int x \, dx$

$\Leftrightarrow \frac{y^2}{2} + c_1 = -\frac{x^2}{2} + c_2 , \quad c_1, c_2 \in \mathbb{R}$

$\Leftrightarrow y^2 = -\cancel{2}\frac{x^2}{\cancel{2}} + \underbrace{2(c_2 - c_1)}_{= c,\, c \in \mathbb{R}}$

$\Leftrightarrow x^2 + y^2 = c , \quad c \in \mathbb{R}$

b) $xy' - y = 0$

$y' = \frac{y}{x}$

$\Leftrightarrow y' = \frac{\frac{1}{x}}{\frac{1}{y}}$

$\Leftrightarrow \int \frac{1}{y} dy = \int \frac{1}{x} dx$

$\Leftrightarrow \ln|y| + c_1 = \ln|x| + c_2 , \quad c_1, c_2 \in \mathbb{R}$

$\Leftrightarrow \ln|y| = \ln|x| + \underbrace{(c_2 - c_1)}_{= c,\, c \in \mathbb{R}}$

$\Leftrightarrow |y| = |x| \cdot \underbrace{e^c}_{= K,\, K \in \mathbb{R}^+}$

$\Leftrightarrow -y = |x| \cdot K \lor y = |x| \cdot K , \quad K \in \mathbb{R}^+$

$\Leftrightarrow y = |x| \cdot K , \quad K \in \mathbb{R} \setminus \{0\}$

$\Leftrightarrow y = -x \cdot K \lor y = x \cdot K$

$\Leftrightarrow y = xK , \quad K \in \mathbb{R} \setminus \{0\}$

c) $(t^2 - xt^2) \frac{dx}{dt} + x^2 = -t^2 x^2$ $\quad x'$

$\Leftrightarrow \frac{dx}{dt} = \frac{x^2(-t-1)}{t^2(1-x)}$

$\Leftrightarrow \frac{dx}{dt} = \frac{\frac{t+1}{t^2}}{\frac{1-x}{-x^2}}$

$\Leftrightarrow \int \frac{x-1}{x^2} dx = \int \frac{t+1}{t^2} dt$

$\Leftrightarrow \int \frac{1}{x} dx - \int x^{-2} dx = \int \frac{1}{t} dt + \int t^{-2} dt$

$$\Leftrightarrow \ln|x| + \frac{1}{x} + c_1 = \ln|t| - \frac{1}{t} + c_2$$

$$\Leftrightarrow \ln|x| + \frac{1}{x} = \ln|t| - \frac{1}{t} + \underbrace{(c_2 - c_1)}_{=c,\, c\in\mathbb{R}}$$

$$\Leftrightarrow |x| \cdot e^{1/x} = |t| \cdot e^{-1/t} \cdot \underbrace{e^{c}}_{=K,\, K\in\mathbb{R}^+}$$

$$\Leftrightarrow \frac{|x|}{|t|} = K \cdot e^{-1/t - 1/x}, \quad K\in\mathbb{R}^+$$

$$\Leftrightarrow \frac{x}{t} = K \cdot e^{-1/t - 1/x}, \quad K\in\mathbb{R}$$

d) $(x^2-1)y' + 2xy^2 = 0$

$$\Leftrightarrow y' = \frac{-2xy^2}{(x^2-1)}$$

$$\Leftrightarrow y' = \frac{-\frac{2x}{(x^2-1)}}{\frac{1}{y^2}}$$

$$\Leftrightarrow \int y^{-2}\,dy = -\int \frac{2x}{(x^2-1)}\,dx$$

$$\Leftrightarrow -\frac{1}{y} + c_1 = -\ln|x^2-1| + c_2$$

$$\Leftrightarrow \frac{1}{y} = \ln|x^2-1| + \underbrace{(-c_2+c_1)}_{=c\in\mathbb{R}}$$

$$\Leftrightarrow y = \frac{1}{\ln|x^2-1| + c}, \quad c\in\mathbb{R}$$

$\boxed{7}$ $\begin{cases} xy' + y = y^2 \\ y(1) = 1/2 \end{cases}$

$$y' = \frac{y^2-y}{x}$$

$$\Leftrightarrow y' = \frac{1/x}{\frac{1}{y^2-y}}$$

$$\Leftrightarrow \int \frac{1}{y(y-1)}\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$$

$$\Leftrightarrow 1 = A(y-1) + By$$

$$\begin{cases} A + B = 0 \\ -A = 1 \end{cases} \quad \begin{cases} B = 1 \\ A = -1 \end{cases}$$

$$\Rightarrow \frac{1}{y(y-1)} = -\frac{1}{y} + \frac{1}{y-1}$$

$$\Leftrightarrow -\int \frac{1}{y}\,dy + \int \frac{1}{y-1}\,dy = \ln|x| + c_2$$

$$\Leftrightarrow -\ln|y| + \ln|y-1| = \ln|x| + \underbrace{(c_2 - c_1)}_{=c\in\mathbb{R}}$$

$$\Leftrightarrow \ln\left|\frac{y-1}{y}\right| = \ln|x| + c$$

$$\Leftrightarrow \left|\frac{y-1}{y}\right| = |x| \cdot \underbrace{e^{c}}_{=K\in\mathbb{R}^+}$$

$$\Leftrightarrow \frac{y-1}{y} = xK, \quad K\in\mathbb{R}$$

Como $y(1) = 1/2 \Rightarrow \frac{1/2 - 1}{1/2} = 1K \Rightarrow K = \frac{-1/2}{1/2} \Rightarrow K = -1$

Assim: $\frac{y-1}{y} = -x \Leftrightarrow \frac{1}{y} = 1 + x \Leftrightarrow y = \frac{1}{1+x}$

(...)

$\boxed{8}$

a) $(x^2+y^2)\,y' = xy$

$\Leftrightarrow y' = \dfrac{xy}{x^2+y^2} = f(x,y)$

$f(\lambda x, \lambda y) = \dfrac{\lambda x \lambda y}{\lambda^2 x^2 + \lambda^2 y^2} = \dfrac{\cancel{\lambda^2}(xy)}{\cancel{\lambda^2}(x^2+y^2)} = \dfrac{xy}{x^2+y} = f(x,y)$

• Como $f(\lambda x, \lambda y) = f(x,y)$, logo a EDO é homogénia

• Seja $y = zx \Leftrightarrow z = \dfrac{y}{x}$

$(zx)' = \dfrac{x(zx)}{x^2+z^2x^2}$

$\Leftrightarrow z'x + z = \dfrac{\cancel{x^2}\,z}{\cancel{x^2}(1+z^2)}$

$\Leftrightarrow z' = \left(\dfrac{z}{(1+z^2)} - z\right) \times \dfrac{1}{x}$

$\Leftrightarrow z' = \dfrac{z - z(1+z^2)}{1+z^2} \times \dfrac{1}{x}$

$\Leftrightarrow z' = \dfrac{\frac{1}{x}}{\frac{1+z^2}{z-z(1+z^2)}}$

$\Leftrightarrow \displaystyle\int \frac{1+z^2}{\cancel{z}-\cancel{z}+z^3}\,dz = \int \frac{1}{x}\,dx$

$\Leftrightarrow \displaystyle\int \frac{1}{z^3}\,dz + \int \frac{1}{z}\,dz = \ln|x| + c_2$

$\Leftrightarrow -\dfrac{z^{-2}}{2} + \ln|z| + c_1 = \ln|x| + c_2$

$\Leftrightarrow -\dfrac{1}{2z^2} + \ln|z| = \ln|x| + \underbrace{(c_2 - c_1)}_{=c\in\mathbb{R}}$

$\boxed{z = \dfrac{y}{x}}$ $\downarrow$

... Deu errado :(

$\boxed{9}$

a) $y' = \underbrace{\dfrac{y}{x}(1+\ln y - \ln x)}_{f(x,y)}$, $x>0$

$f(\lambda x, \lambda y) = \dfrac{\cancel{\lambda} y}{\cancel{\lambda} x}(1+\ln \lambda y - \ln \lambda x) = \dfrac{y}{x}(1 + \cancel{\ln\lambda} + \ln y - \cancel{\ln\lambda} - \ln x)$

$f(\lambda x, \lambda y) = \dfrac{y}{x}(1+\ln y - \ln x) = f(x,y) \Rightarrow$ EDO é homogénia b)

$\boxed{y = zx}$

$(zx)' = \frac{z\cancel{x}}{\cancel{x}}(1 + \ln zx - \ln x)$, $x > 0$

$\Leftrightarrow z'x + \cancel{z} = \cancel{z} + z(\ln z + \cancel{\ln x} - \cancel{\ln x})$

$\Leftrightarrow z' = \frac{z \ln z}{x}$

$\Leftrightarrow z' = \frac{\frac{1}{x}}{\frac{1/z}{\ln z}}$

$\Leftrightarrow \int \frac{1/z}{\ln z} dz = \int \frac{1}{x} dx$

$\Leftrightarrow \ln|\ln z| = \ln|\underbrace{x}_{x>0}| + c$, $c \in \mathbb{R}$

$\Leftrightarrow \ln z = x \cdot \underbrace{e^c}_{=K \in \mathbb{R}^+}$

$\Leftrightarrow z = e^x \cdot e^K$

$\Leftrightarrow z = e^{Kx}$

$\boxed{z = \frac{y}{x}}$

$\frac{y}{x} = e^{Kx} \Leftrightarrow y = x e^{Kx}$, $K \in \mathbb{R}^+$, $x > 0$

$\boxed{10}$ $\frac{dy}{dx} = \frac{x + y - 3}{x - y - 1}$ y'

- Nem é de var. separáveis
- Nem é homogénia

$\begin{cases} x = z + h \\ y = w + K \end{cases}$, onde $\begin{cases} h + K - 3 = 0 \\ h - K - 1 = 0 \end{cases}$ $\begin{cases} 2K = 2 \\ h = 1 + K \end{cases}$ $\begin{cases} K = 1 \\ h = 2 \end{cases}$

$\Rightarrow \begin{cases} x = z + 2 \\ y = w + 1 \end{cases} \Rightarrow y' = w'$

Assim:

$w' = \frac{z + 2 + w + 1 - 3}{z + 2 - w - 1 - 1}$

$\Leftrightarrow w' = \underbrace{\frac{z + w}{z - w}}_{f(x,y)}$ $f(\lambda x, \lambda y) = \frac{\cancel{\lambda}(z+w)}{\cancel{\lambda}(z-w)} = \frac{z+w}{z-w} = f(x,y) \Rightarrow$ EDO é homogénia

$\boxed{w = v z}$ $\boxed{w' = \frac{dw}{dz}}$

$$v'z + v = \frac{z + vz}{z - vz}$$

$$\Leftrightarrow v'z = \frac{1+v}{1-v} - v$$

$$\Leftrightarrow v' = \frac{1 + \cancel{v} - \cancel{v} + v^2}{1-v} \times \frac{1}{z}$$

$$\Leftrightarrow v' = \frac{\frac{1}{z}}{\frac{1-v}{1+v^2}}$$

$$\Leftrightarrow \frac{dv}{dz} = \frac{1/z}{\frac{1-v}{1+v^2}}$$

$$\Leftrightarrow \int \frac{1-v}{1+v^2}\,dv = \int \frac{1}{z}\,dz$$

$$\Leftrightarrow \int \frac{1}{1+v^2}\,dv - \frac{1}{2}\int \frac{2v}{1+v^2}\,dv = \ln|z| + c_2,\ c_2 \in \mathbb{R}$$

$$\Leftrightarrow \operatorname{arctg}(v) - \frac{1}{2}\ln|\underbrace{1+v^2}_{>0}| = \ln|z| + c,\ c \in \mathbb{R}$$

$$\Leftrightarrow \operatorname{arctg}(v) - \frac{1}{2}\ln(1+v^2) = \ln|z| + c,\ c \in \mathbb{R}$$

$\boxed{v = \frac{w}{z}}$

$$\Rightarrow \operatorname{arctg}\left(\frac{w}{z}\right) - \frac{1}{2}\ln\left(1 + \frac{w^2}{z^2}\right) = \ln|z| + c$$

$$\begin{cases} x = z + 2 \\ y = w + 1 \end{cases} \Rightarrow \begin{cases} z = x - 2 \\ w = y - 1 \end{cases}$$

$$\Rightarrow \operatorname{arctg}\left(\frac{y-1}{x-2}\right) - \frac{1}{2}\ln\left(1 + \frac{(y-1)^2}{(x-2)^2}\right) = \ln|x-2| + c$$

b) $y' = \dfrac{y - x}{y - x + 2}$

- Não é de var. sep.
- Não é homogénia

$\boxed{z = y - x} \Rightarrow y = z + x \Rightarrow y' = z' + 1$

$$\rightarrow z' + 1 = \frac{z}{z+2} \Leftrightarrow z' = \frac{\cancel{z} - \cancel{z} - 2}{z+2} \Leftrightarrow z' = \underbrace{-2}_{P(x)} \cdot \underbrace{\frac{1}{z+2}}_{Q(z)}$$

$$\Leftrightarrow \int z + 2\,dz = \int -2\,dx$$

$$\Leftrightarrow \frac{z^2}{2} + 2z = -2x + c,\ c \in \mathbb{R}$$

$$\boxed{z = y - x}$$

$$\Rightarrow \frac{(y-x)^2}{2} + 2(y-x) = -2x + c$$

$$\Leftrightarrow (y-x)^2 = -\cancel{4x} - 4y + \cancel{4x} + c$$

$$\Leftrightarrow (y-x)^2 + 4y = c \,,\ c \in \mathbb{R}$$

$\boxed{11}$

a) $\underbrace{(2x + \operatorname{sen} y)}_{M(x,y)}\, dx + \underbrace{(x \cos y)}_{N(x,y)}\, dy = 0$

$$\begin{cases} M(x,y) = 2x + \operatorname{sen} y = \dfrac{dF}{dx} \\ N(x,y) = x \cos y = \dfrac{dF}{dy} \end{cases}$$

Nota: $(2x + \operatorname{sen} y)'_y = \cos y$

$(x \cos y)'_x = \cos y$ $\longmapsto \dfrac{d^2F}{dx\,dy} = \dfrac{d^2F}{dy\,dx}$

$$F(x,y) = \int 2x + \operatorname{sen} y \, dx = \frac{\cancel{2}x^2}{\cancel{2}} + \operatorname{sen} y \, x + K(y)$$

$$\Rightarrow \frac{dF}{dy} = (x^2 + \operatorname{sen} y \, x)'_y + \frac{dK}{dy}$$

$$\Leftrightarrow x \cancel{\cos y} = 0 + \cancel{\cos y}\, x + \frac{dK}{dy}$$

Como: $x \cos y = \dfrac{dF}{dy}$

$$\Leftrightarrow \frac{dK}{dy} = 0$$

$$\Leftrightarrow K(y) = \int 0 \, dy$$

$$\Leftrightarrow K(y) = K \,,\ K \in \mathbb{R}$$

Assim, como a solução geral é $F(x,y) = c$

$$\Rightarrow x^2 + \operatorname{sen} y \, x = c \,,\ c \in \mathbb{R}$$

b) $(2xy - x - e^y)\, dx = (x e^y + y - x^2)\, dy$

$$\Leftrightarrow (2xy - x - e^y)\, dx - (x e^y + y - x^2)\, dy = 0$$

$$\Leftrightarrow \underbrace{(2xy - x - e^y)}_{M(x,y)}\, dx + \underbrace{(x^2 - x e^y - y)}_{N(x,y)}\, dy = 0$$

!!!

$$\begin{cases} M(x,y) = 2xy - x - e^y = \dfrac{dF}{dx} \\ N(x,y) = x^2 - x e^y - y = \dfrac{dF}{dy} \end{cases}$$

$$F(x,y) = \int 2xy - x - e^y \, dx = \cancel{2}y \frac{x^2}{\cancel{2}} - \frac{x^2}{2} - e^y x + K(y)$$

$$\frac{dF}{dy} = x^2 - y x e^y + \frac{dK}{dy}$$

• Como: $\frac{dF}{dy} = x^2 - xe^y - y$

$\Rightarrow \cancel{x^2} - \cancel{xe^y} - y = \cancel{x^2} - \cancel{xe^y} + \frac{dK}{dy}$

$\Leftrightarrow \frac{dK}{dy} = -y$

$\Rightarrow K(y) = -\frac{y^2}{2} + c$

$\Rightarrow F(x,y) = yx^2 - \frac{x^2}{2} - e^y x - \frac{y^2}{2} + K, \ K \in \mathbb{R}$

Assim, como a solução é $F(x,y) = c$

$\Rightarrow yx^2 - \frac{x^2}{2} - e^y x - \frac{y^2}{2} = c, \ c \in \mathbb{R}$

c) $\underbrace{\left(\frac{y}{x} + 6x\right)}_{M(x,y)} dx + \underbrace{(\ln x - 2)}_{N(x,y)} dy = 0 \longrightarrow$ EDO Exata!

$$\begin{cases} M(x,y) = \frac{y}{x} + 6x = \frac{dF}{dx} \\ N(x,y) = \ln x - 2 = \frac{dF}{dy} \end{cases}$$

$F(x,y) = y\int \frac{1}{x}\, dx + 6\int x\, dx = y \ln|x| + 6\frac{x^2}{2} + K(y)$

$\frac{dF}{dy} = \ln|x| + \frac{dK}{dy}$

• Como: $\ln x - 2 = \frac{dF}{dy}$

$\Rightarrow \cancel{\ln x} - 2 = \cancel{\ln|x|} + \frac{dK}{dy}$

$\Rightarrow -2 = \frac{dK}{dy}$

$\Leftrightarrow K(y) = -2y + K, \ K \in \mathbb{R}$

$\Rightarrow F(x,y) = y \ln|x| + 6\frac{x^2}{2} - 2y + K, \ K \in \mathbb{R}$

Assim, como a solução é $F(x,y) = c$:

$\Rightarrow y \ln|x| + 6\frac{x^2}{2} - 2y = c, \ c \in \mathbb{R}$

$\Leftrightarrow y = \frac{c - 3x^2}{\ln|x| - 2}, \ c \in \mathbb{R}$

$\boxed{12}$

$e^x \sec y - \operatorname{tg} y + y' = 0, \ \mu(x,y) = e^{\beta x} \cos y$

$\Leftrightarrow (e^x \sec y - \operatorname{tg} y)\, dx + dy = 0$

$\Leftrightarrow \left(e^{\beta x} \cos y \cdot (e^x \sec y - \operatorname{tg} y)\right) dy + (e^{\beta x} \cos y)\, dy = 0$

(...)

$\boxed{13}$

a) $\underbrace{y}_{M(x,y)}\,dx + \underbrace{(y^2-x)}_{N(x,y)}\,dy = 0$

• Como $\dfrac{dM}{dy}(x,y) \neq \dfrac{dN}{dx}(x,y) \Rightarrow$ EDO não é Exata!

• Se $\mu(x,y) = y^{-2} \Rightarrow \begin{cases} M(x,y) = y^{-1} \\ N(x,y) = 1 - x\,y^{-2} \end{cases} \Rightarrow \begin{cases} \dfrac{dM}{dy}(x,y) = -y^{-2} \\ \dfrac{dN}{dx}(x,y) = -y^{-2} \end{cases}$ ✓ EDO Exata!

$\to (y^{-1})\,dx + (1 - x\,y^{-2})\,dy = 0$

(...)

b) $\underbrace{(2y - x^3)}_{M(x,y)}\,dx + \underbrace{x}_{N(x,y)}\,dy = 0 \to \dfrac{dM}{dy}(x,y) \neq \dfrac{dN}{dx}(x,y)$, Não é Exata!

Se $\mu(x,y) = x \Rightarrow \begin{cases} M(x,y) = (2yx - x^4) \\ N(x,y) = x^2 \end{cases} \quad \begin{cases} \dfrac{dM}{dy}(x,y) = 2x \\ \dfrac{dN}{dx}(x,y) = 2x \end{cases}$ ✓ EDO Exata!

$\to (2yx - x^4)\,dx + x^2\,dy = 0$

(...)

$\boxed{14}$

a) $y' + \underbrace{2}_{P(x)}y = \underbrace{\cos x}_{Q(x)}$

$\mu(x) = e^{2\int 1\,dx} = e^{2x}$

$\Rightarrow e^{2x} \cdot (y + 2y) = e^{2x}\cos x$

$\Leftrightarrow \left[e^{2x} \cdot y\right]' = e^{2x}\cos x$

$\Leftrightarrow e^{2x} \cdot y = \int \underbrace{e^{2x}}_{u}\underbrace{\cos x}_{v'}$ $\qquad u = e^{2x} \quad v = \sin x$; $\quad u' = 2e^{2x} \quad v' = \cos x$

$\Leftrightarrow e^{2x} \cdot y = e^{2x}\sin x - 2\int \underbrace{e^{2x}}_{a}\underbrace{\sin x}_{b'}\,dx$ $\qquad a = e^{2x} \quad b = -\cos x$; $\quad a' = 2e^{2x} \quad b' = \sin x$

$\Leftrightarrow e^{2x}y = e^{2x}\sin x + 2e^{2x}\cos x - 4\underbrace{\int e^{2x}\cos x\,dx}_{= e^{2x} \cdot y}$

$\Leftrightarrow 5\cancel{e^{2x}}\,y = \cancel{e^{2x}}\sin x + 2\cancel{e^{2x}}\cos x + K, \; K \in \mathbb{R}$

$\Leftrightarrow y = \dfrac{1}{5}\sin x + \dfrac{2}{5}\cos x + C\,e^{-2x}, \; C \in \mathbb{R}$

b) $x^3y' - y - 1 = 0$

$\Leftrightarrow y' + \left(-\dfrac{1}{x^3}\right)y = \dfrac{1}{x^3}$

$$e^{-\int x^{-3}dx} = e^{-\frac{x^{-2}}{-2}} = e^{\frac{x^{-2}}{2}}$$

$$\left(\frac{x^{-2}}{2}\right)' = \frac{1}{2}\times(-2)\times x^{-3} = -x^{-3}$$

$$\Rightarrow e^{\frac{x^{-2}}{2}}\cdot\left(y' + \left(-\frac{1}{x^3}\right)y\right) = e^{\frac{x^{-2}}{2}}\cdot\frac{1}{x^3}$$

$$\Rightarrow \left[e^{\frac{x^{-2}}{2}}\cdot y\right] = \int x^{-3} e^{\frac{x^{-2}}{2}} dx$$

$$\Leftrightarrow \cancel{e^{\frac{x^{-2}}{2}}}\cdot y = -\cancel{e^{\frac{x^{-2}}{2}}} + c$$

$$\Leftrightarrow y = -1 + c\,e^{-\frac{x^{-2}}{2}}$$

$$\Leftrightarrow y = -1 + C\,e^{-\frac{1}{2x^2}},\ C\in\mathbb{R}$$

c) $\frac{1}{x}y' - \frac{1}{x^2+1}y = \frac{\sqrt{x^2+1}}{x}$, $x\neq 0$

$$y' - \frac{x}{x^2+1}y = \sqrt{x^2+1}$$

$$\mu(x) = e^{-\frac{1}{2}\int\frac{2x}{x^2+1}dy} = e^{-\frac{1}{2}\ln(x^2+1)} = \cancel{e}^{\cancel{\ln}\left(\frac{1}{\sqrt{x^2+1}}\right)} = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{1}{\sqrt{x^2+1}}\cdot\left(y' - \frac{x}{x^2+1}y\right) = \frac{1}{\cancel{\sqrt{x^2+1}}}\times\cancel{\sqrt{x^2+1}}$$

$$\Leftrightarrow \left(\frac{1}{\sqrt{x^2+1}}\cdot y\right)' = 1$$

$$\Leftrightarrow \frac{1}{\sqrt{x^2+1}}\cdot y = x + c$$

$$\Leftrightarrow y = (x+c)(\sqrt{x^2+1}),\ c\in\mathbb{R}$$

$\boxed{15}$

$$x^2y' + 2xy = 1,\ x\in\,]0,+\infty[$$

$$\Leftrightarrow y' + 2x^{-1}y = x^{-2}$$

$$\Leftrightarrow \left[x^{2y}\cdot(y' + 2x^{-1}y)\right] = x^{-2}\cdot x^2 \qquad \mu(x) = e^{2\int x^{-1}dx} = e^{2\ln|x|_{>0}} = x^2$$

$$\Leftrightarrow x^2\cdot y = \int 1\,dx$$

$$\Leftrightarrow y = \frac{x}{x^2} + \frac{c}{x^2},\ C\in\mathbb{R}$$

$$\Leftrightarrow y = \frac{1}{x} + \frac{c}{x^2},\ C\in\mathbb{R}$$

<u>Quando $x\to+\infty$:</u> $y = \frac{1}{+\infty} + \frac{c}{+\infty} = 0 + 0 = 0$ c.q.m.

$\boxed{16}$

a) $x y' + y = y^2 \ln x \quad , \quad x > 0$

$\Leftrightarrow y' + x^{-1} y = y^{2} x^{-1} \ln x$

EDO de Bernoulli

$\Leftrightarrow \frac{y'}{y^2} + x^{-1} \frac{y}{y^2} = x^{-1} \ln x$

$\Leftrightarrow \frac{y'}{y^2} + x^{-1} \frac{1}{y} = x^{-1} \ln x$

$\boxed{z = \frac{1}{y}} \Rightarrow z' = -\frac{y'}{y^2} \quad , y \neq 0$

$\hookrightarrow -z' + x^{-1} z = x^{-1} \ln x$

$\Leftrightarrow z' + \left(-\frac{1}{x}\right) z = -\left(\frac{1}{x}\right) \ln x$ $\longrightarrow$ EDO Linear

$\mu(x) = e^{-\int \frac{1}{x} dx} = e^{-\ln |x|} \underset{x>0}{=} \frac{1}{x}$

$\Leftrightarrow \frac{1}{x} \cdot \left(z' - \frac{1}{x} z \right) = -\frac{1}{x} \cdot \frac{1}{x} \ln x$

$\Leftrightarrow \left(\frac{1}{x} \cdot z \right) = -\int \underbrace{\frac{1}{x^2}}_{u'} \cdot \underbrace{\ln x}_{v}$

$u = -x^{-1} \qquad v = \ln x$

$u' = x^{-2} \qquad v' = x^{-1}$

$\Leftrightarrow \frac{1}{x} \cdot z = -(-x^{-1} \ln x) + \int -x^{-1} \cdot x^{-1} dx$

$\Leftrightarrow \frac{1}{x} \cdot z = \frac{\ln x}{x} + \frac{1}{x} + c$

$\Leftrightarrow z = \ln x + 1 + cx \quad , \quad c \in \mathbb{R}$

$\boxed{z = \frac{1}{y}} \quad , y \neq 0$

$\hookrightarrow \frac{1}{y} = \ln x + 1 + c x$

$\Leftrightarrow y = \frac{1}{\ln x + 1 + cx} \quad , \quad c \in \mathbb{R} \quad , \quad x > 0 \quad , \quad y \neq 0$

Para $y = 0$: $x y' + 0 = 0 \Leftrightarrow \underset{x>0}{x y'} = 0 \Leftrightarrow y' = 0$ // $\longrightarrow$ Logo, $y = 0$ também é solução!

b) $y' - \frac{y}{2x} = 5 x^2 y^5 \quad , \quad x \neq 0$

$\hookrightarrow$ EDO de Bernoulli

$\Leftrightarrow \frac{y'}{y^5} - \frac{y}{2x} \times y^{-5} = 5x^2$

$\Leftrightarrow \frac{y'}{y^5} - \frac{1}{2x} \times \frac{1}{y^4} = 5x^2$

$\boxed{z = \frac{1}{y^4}} \quad , \boxed{y \neq 0} \longrightarrow$ Para $y = 0 \Rightarrow y' - 0 = 0 = 0$ // $\longrightarrow$ Logo: $y = 0$ é solução!

$\Rightarrow z' = \frac{-4y'}{y^5} \Rightarrow \frac{y'}{y^5} = \frac{z'}{-4}$

$\Rightarrow \frac{z'}{-4} - \frac{1}{2x} z = 5x^2$

$$\Rightarrow z' + \frac{2}{x} z = -20 x^2$$

$$\Leftrightarrow \left[x^2 \cdot \left(z' + \frac{2}{x} z\right)\right] = -20 x^2 \cdot x^2 \qquad \mu(x) = e^{2\int \frac{1}{x} dx} = e^{2 \ln|x|} = x^2$$

$$\Leftrightarrow x^2 \cdot z = \int -20 x^4 \, dx$$

$$\Leftrightarrow z = -20 \frac{x^5}{5} \times x^{-2} + C \times x^{-2}$$

$$\Leftrightarrow z = -4 x^3 + C \times x^{-2}$$

$$\boxed{z = \frac{1}{y^4}} \longrightarrow \frac{1}{y^4} = -4x^3 + \frac{C}{x^2} \Leftrightarrow y^4 = \frac{1}{-4x^3 + \frac{C}{x^2}}, \; C \in \mathbb{R}$$

$$\Leftrightarrow y^4 = \frac{x^2}{C - 4x^5}, \; C \in \mathbb{R}$$

e $y = 0$ também é solução!

17 ??? Não entendo! Coeficientes não são constantes.

18

$x^2 + 2y^2 = C, \; C > 0$

???

HVC:

$y_H = (\ldots)$

$y_p = c_1(x) \ldots + c_2(x) \ldots$

$$\begin{cases} c_1'(x) \ldots + c_2'(x) \ldots = 0 \\ c_1'(x)[\ldots]' + c_2'(x)[\ldots]' = 0 \\ c_1'(x)[\ldots]'' + c_2'(x)[\ldots]'' = b(x) \end{cases}$$

19

a) $y' + y = \operatorname{sen} x$

$\boxed{y' + y = 0}$ – (H)

$p(r) = r + 1 \qquad p(r) = 0 \Leftrightarrow r = -1$

$SFS = \{e^{-x}\} \longrightarrow y_H = c_1 e^{-x}, \; c_1 \in \mathbb{R}$

$\boxed{y' + y = \operatorname{sen} 1x}$ – (E)

MCI:

$z = 0 + 1i = i$, como z não é raiz característica:

$$y_p = x^0 (P_0 \cos x + Q_0 \sin x) e^{0x}$$
$$= P_0 \cos x + Q_0 \sin x$$

$$y_p' = -P_0 \sin x + Q_0 \cos x$$

Substituindo em (E):

$$-P_0 \sin x + Q_0 \cos x + P_0 \cos x + Q_0 \sin x = 0$$

$$\Leftrightarrow \cos x (Q_0 + P_0) + \sin x (-P_0 + Q_0) = \sin x$$

$$\begin{cases} Q_0 + P_0 = 0 \\ -P_0 + Q_0 = 1 \end{cases} \quad \begin{cases} Q_0 = 1/2 \\ P_0 = -1/2 \end{cases}$$

$$\Rightarrow y_p = -\frac{\cos x}{2} + \frac{\sin x}{2}$$

$$y_H = c_1 e^{-x}, \quad c_1 \in \mathbb{R}$$

$$y_E = y_p + y_H = c_1 e^{-x} - \frac{\cos x}{2} + \frac{\sin x}{2}, \quad c_1 \in \mathbb{R}$$

Outro método

↳ MVC: $y' + y = \operatorname{sen} x$ e $y_H = c_1 e^{-x}, \ c_1 \in \mathbb{R}$

$$y_p = c_1(x)\, e^{-x}$$

$$c_1'(x)\, e^{-x} = \operatorname{sen} x \Leftrightarrow c_1'(x) = e^{x} \operatorname{sen} x$$

$$\Leftrightarrow c_1(x) = \int \underbrace{e^{x}}_{u'} \underbrace{\operatorname{sen} x}_{v} \, dx$$

$u = e^x \qquad v = \operatorname{sen} x$

$u' = e^x \qquad v' = \cos x$

$$\Leftrightarrow c_1(x) = e^{x} \operatorname{sen} x - \int e^{x} \cos x \, dx$$

$$\Leftrightarrow c_1(x) = e^{x} \operatorname{sen} x - e^{x} \cos x - \underbrace{\int e^{x} \sin x \, dx}_{c_1(x)}$$

$w = \cos x$

$w' = -\sin x$

$$\Leftrightarrow 2c_1(x) = e^{x} \operatorname{sen} x - e^{x} \cos x$$

$$\Leftrightarrow c_1(x) = e^{x} \times \frac{\operatorname{sen} x - \cos x}{2}$$

$$\Rightarrow y_p = \cancel{e^{x}} \times \cancel{e^{-x}} \times \frac{\operatorname{sen} x - \cos x}{2}$$

Assim:

$$y_E = y_H + y_p = c_1 e^{-x} + \frac{\operatorname{sen} x}{2} - \frac{\cos x}{2}, \quad c_1 \in \mathbb{R}$$

b) $y'' - y + 2\cos x = 0$

$\boxed{y'' - y = 0}$ — (H)

$p(r) = 0 \Leftrightarrow r^2 - 1 = 0 \Leftrightarrow r = -1 \vee r = 1 \longrightarrow SFS = \{e^{-x}, e^{x}\}$

$$y_H = c_1 e^{-x} + c_2 e^{x}, \quad c_1, c_2 \in \mathbb{R}$$

$\boxed{y'' - y = -2\cos x}$ — (E)

$$b(x) = e^{0x} \times (-2\cos x)$$

$z = 0 + 1i = i$, não é raiz característica $(K=0)$

$$y_p = x^0 (P_0 \cos x + Q_0 \sin x) e^{0x}$$
$$= P_0 \cos x + Q_0 \sin x$$
$$y_p' = -P_0 \sin x + Q_0 \cos x$$
$$y_p'' = -P_0 \cos x - Q_0 \sin x$$

Substituindo em (E):

$$-P_0 \cos x - Q_0 \sin x - P_0 \cos x - Q_0 \sin x = -2\cos x$$

$$\Leftrightarrow \cos x(-P_0 - P_0) + \sin x(-Q_0 - Q_0) = -2\cos x$$

$$\begin{cases} -2P_0 = -2 \\ -2Q_0 = 0 \end{cases} \quad \begin{cases} P_0 = 1 \\ Q_0 = 0 \end{cases}$$

$$\Rightarrow y_P = \cos x$$

$$\Rightarrow y_E = y_H + y_P = c_1 e^{-x} + c_2 e^{x} + \cos x, \; c_1, c_2 \in \mathbb{R}$$

c)

$$y'' + y' = 2y + 3 - 6x$$

$$\Leftrightarrow \boxed{y'' + y' - 2y = -6x + 3} \; (E)$$

$$\boxed{y'' + y' - 2y = 0} \; (H)$$

$$p(r) = 0 \Leftrightarrow r^2 + r - 2 = 0 \Leftrightarrow r = \frac{-1 \pm \sqrt{\cancel{1 - 4 \times (-2)}}^{\,3}}{2}$$

$$\Leftrightarrow r = -2 \lor r = 1 \Rightarrow SFS = \{e^{-2x}, e^{x}\}$$

$$y_H = c_1 e^{-2x} + c_2 e^{x}, \; c_1, c_2 \in \mathbb{R}$$

$$b(x) = e^{0x}(\underbrace{-6x + 3}_{\text{pol. de grau } 1}), \; z = 0 + 0i = 0, \text{ como } z \text{ não raiz característica} \Rightarrow k = 0$$

$$y_P = x^0 \left((P_1 x + P_0)\cos(0x) + \cancel{(P_1 x + P_0)\operatorname{sen}(0x)}^{\,0} \right) e^{0x}$$
$$= P_1 x + P_0$$
$$y_P' = P_1$$
$$y_P'' = 0$$

Substituindo em (E):

$$0 + P_1 - 2P_1 x + 2P_0 = -6x + 3$$
$$\Leftrightarrow x(-2P_1) + (2P_0 + P_1) = -6x + 3$$

$$\begin{cases} -2P_1 = -6 \\ 2P_0 + P_1 = 3 \end{cases} \quad \begin{cases} P_1 = 3 \\ 2P_0 = 0 \end{cases} \quad \begin{cases} P_1 = 3 \\ P_0 = 0 \end{cases} \Rightarrow y_P = 3x$$

$$y_E = y_H + y_P = c_1 e^{-2x} + c_2 e^{x} + 3x, \; c_1, c_2 \in \mathbb{R}$$

d) $\boxed{y'' - 4y' + 4y = xe^{2x}}$ (E)

$\boxed{y'' - 4y' + 4y = 0}$ (H)

$p(r) = 0 \Leftrightarrow r^2 - 4r + 4 = 0 \Leftrightarrow (r-2)^2 = 0 \Leftrightarrow \underline{r = 2} \Rightarrow SFS = \{e^{2x}, xe^{2x}\}$
Por duplo!

$y_H = c_1 e^{2x} + c_2 x e^{2x}, \quad c_1, c_2 \in \mathbb{R}$

$b(x) = x e^{2x}$

$z = 2 + 0i = 2$, como 2 é raiz característica de multiplicidade $2 \Rightarrow K = 2$

$y_p = x^2 ((P_1 x + P_0)\cos(0x) + \cancel{(Q_1 x + Q_0)\sin(0x)})\, e^{2x}$

$= (P_1 x^3 + P_0 x^2) \cdot e^{2x}$

$y'_p = (3P_1 x^2 + 2P_0 x)\, e^{2x} + (P_1 x^3 + P_0 x^2) \cdot 2e^{2x}$

$y''_p = (6P_1 x + 2P_0 x)\, e^{2x} + (3P_1 x^2 + 2P_0 x) \cdot 2e^{2x} + (3P_1 x^2 + 2P_0 x)\, 2e^{2x} + (P_1 x^3 + P_0 x^2) \cdot 4e^{2x}$

Substituindo em (E):

$\cancel{e^{2x}}(6P_1 x + 2P_0 x + \cancel{6P_1 x^2} + \cancel{4P_0 x} + \cancel{6P_1 x^2} + \cancel{4P_0 x} + \cancel{4P_1 x^3} + \cancel{4P_0 x^2}$
$- 12\cancel{P_1 x^2} - \cancel{8P_0 x} - \cancel{8P_1 x^3} - \cancel{8P_0 x^2} + \cancel{4P_1 x^3} + \cancel{4P_0 x^2}) = x\cancel{e^{2x}}$

$\Leftrightarrow 6P_1 x + 2P_0 x = x$

Para $P_0 = 0 \Rightarrow P_1 = \frac{1}{6}$

$y_p = \frac{1}{6} x^3 e^{2x}$

Assim:

$y_E = y_H + y_p = c_1 e^{2x} + c_2 x e^{2x} + \frac{x^3}{6} e^{2x}$

$\Leftrightarrow y_p = \left(c_1 + c_2 x + \frac{x^3}{6}\right) e^{2x}, \quad c_1, c_2 \in \mathbb{R}$

e) $\boxed{y'' + y' = e^{-x}}$ (E)

$\boxed{y'' + y' = 0}$ (H)

$p(r) = 0 \Leftrightarrow r^2 + r = 0 \Leftrightarrow r(r+1) = 0 \Leftrightarrow r = 0 \lor r = -1 \Rightarrow SFS = \{e^{0x}, e^{-x}\}$
$= \{1, e^{-x}\}$

$y_H = c_1 + c_2 e^{-x}, \quad c_1, c_2 \in \mathbb{R}$

$b(x) = e^{-x} = e^{(-1)x} \times (1)$

$z = -1 + 0i = -1$, como -1 é raiz característica de multiplicidade $1 \Rightarrow \underline{K = 1}$

$y_p = x^1 (P_0 \cos(0x) + \cancel{Q_0 \sin(0x)})\, e^{(-1)x}$

$= P_0 x e^{-x}$

$$y'_p = P_0\left(e^{-x} + x \times (-1) \times e^{-x}\right)$$
$$= P_0 e^{-x}(1-x)$$
$$y''_p = P_0\left(-e^{-x}(1-x) + e^{-x}(-1)\right)$$
$$= P_0 e^{-x}(-1+x-1)$$
$$= P_0 e^{-x}(-2+x)$$

Substituindo em (E):

$$P_0 \cancel{e^{-x}}(-2+x) + P_0 \cancel{e^{-x}}(1-x) = \cancel{e^{-x}} \times 1$$

$$\Leftrightarrow P_0(\underbrace{-2+\cancel{x}+1-\cancel{x}}_{=-1}) = 1$$

$$\Leftrightarrow P_0 = -1$$

$$y_p = -x e^{-x} \Rightarrow y_E = y_H + y_P = c_1 + c_2 e^{-x} - x e^{-x}, \quad c_1, c_2 \in \mathbb{R}$$

$$\Leftrightarrow y_E = c_1 + (c_2 - x)e^{-x}, \quad c_1, c_2 \in \mathbb{R}$$

f) $\boxed{y'' + 4y = tg(2x)}$ – (E)

$\boxed{y'' + 4y = 0}$ – (H)

$$p(r) = 0 \Leftrightarrow r^2 + 4 = 0 \Leftrightarrow r^2 = -4 \Leftrightarrow r = \pm\sqrt{-4} \Leftrightarrow r = \pm 2i$$

$\hookrightarrow$ $SFS = \{e^{0x}\cos(2x), e^{0x}\sin(2x)\}$
$= \{\cos(2x), \sin(2x)\}$

$\hookrightarrow$ $y_H = c_1\cos(2x) + c_2\sin(2x), \quad c_1, c_2 \in \mathbb{R}$

$$b(x) = tg(2x) = e^{0x}\left(\sin(2x)\sec(2x)\right)$$

$z = 0 + 2i = 2i$, como z é raiz característica de multiplicidade $1 \Rightarrow K = 1$

$$y_p = x^1\left(P_0\cos(2x) + Q_0\sin(2x)\right)e^{0x}$$
$$= x P_0\cos(2x) + x Q_0 \sin(2x)$$

$$y'_p = P_0\left(\cos(2x) - 2x\sin(2x)\right) + Q_0\left(\sin(2x) + 2x\cos(2x)\right)$$
$$y''_p = P_0\left(-2\sin 2x - 2\left(\sin(2x) + 2x\cos 2x\right)\right) + Q_0\left(2\cos(2x) + 2\left(\cos(2x) - 2x\sin(2x)\right)\right)$$

Substituindo em (E):

$$P_0\left(-2\sin 2x - 2\left(\sin(2x) \cancel{+ 2x\cos 2x}\right)\right) + Q_0\left(2\cos(2x) + 2\left(\cos(2x) \cancel{- 2x\sin(2x)}\right)\right)$$
$$\cancel{+ 4x P_0\cos(2x)} + \cancel{4x Q_0 \sin(2x)} = tg(2x)$$
$$\Leftrightarrow -4P_0\sin 2x + 4Q_0\cos 2x = tg(2x)$$

(...) Não consigo acabar :(

Resolvido pelo MVI:

$$y'' + 4y = \operatorname{tg} 2x \quad e \quad y_H = c_1 \cos 2x + c_2 \sin 2x \,, \; c_1, c_2 \in \mathbb{R}$$

$$y_p = c_1(x) \cos 2x + c_2(x) \sin 2x$$

Cumprindo-se:

$$\begin{cases} c_1'(x) \cos 2x + c_2'(x) \sin 2x = 0 \\ c_1'(x)(\cos 2x)' + c_2'(x)(\sin 2x)' = \operatorname{tg} 2x \end{cases}$$

$$\begin{bmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{bmatrix} \begin{bmatrix} 0 \\ \operatorname{tg} 2x \end{bmatrix}$$

$$c_1'(x) = \frac{\begin{vmatrix} 0 & \sin 2x \\ \operatorname{tg} 2x & 2\cos 2x \end{vmatrix}}{\begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}} = \frac{-\sin 2x \operatorname{tg} 2x}{2\cos^2 2x + 2\sin^2 2x} = \frac{-\sin 2x \operatorname{tg} 2x}{(2 - \cancel{2\sin^2 2x}) + \cancel{2\sin^2 2x}}$$

$$= \frac{-\sin^2 2x}{2\cos 2x}$$

$$c_2'(x) = \frac{\begin{vmatrix} \cos 2x & 0 \\ -2\sin 2x & \operatorname{tg} 2x \end{vmatrix}}{2} = \frac{\cos 2x \operatorname{tg} 2x}{2} = \frac{\sin 2x}{2}$$

$$c_1(x) = -\frac{1}{2} \times \int \frac{\sin^2 2x}{\cos 2x}\, dx = (\ldots) = \frac{\ln|\tan 2x + \sec 2x|}{4} - \frac{\sin 2x}{4}$$

$$c_2(x) = \frac{1}{2} \int \sin(2x) = -\frac{1}{4} \cos(2x)$$

$(\ldots)$