Sistemas de Numeração e Códigos

2^n	n
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1024	10
⋮	

1

a) $1101_2 = 2^3 + 2^2 + 1 = 8 + 4 + 1 = 13_{10}$

b) $37_9 = 3 \times 9 + 7 = 27 + 7 = 34_{10}$

c) $0.0110_2 = 2^{-2} \times 2^{-3} = 2^{-5} = \frac{1}{32}_{10}$

d) $0{,}16_7 = 7^{-1} \times 6 \times 7^{-2} = \frac{1}{7} \times \frac{6}{49} = \frac{6}{343}_{10}$

2

a) $11210_{(2i)} = (2i)^4 + (2i)^3 + 2(2i)^2 + (2i)$

$= 16\cancel{i^4} + 8 \times i^2 \times i + 2 \times (4i^2) + 2i$

$= 16 - 8i - 8 + 2i$

$= -6i + 8$

3

a) $5A_{16} = 132_b$

$\Leftrightarrow 5 \times 16 + 10 = b^2 + 3b + 2$

$\Leftrightarrow b^2 + 3b - 88 = 0$

$\Leftrightarrow b = 8 \vee b = -11$

b)

$20_{10} = 110_c \Leftrightarrow 20 = c^2 + c$

$\Leftrightarrow c^2 + c - 20 = 0$

$\Leftrightarrow c = 4 \vee c = -5$

4

$\sqrt{41_b} = 5_b \Leftrightarrow \sqrt{4b+1} = 5$

$\Leftrightarrow 4b + 1 = 25$

$\Leftrightarrow b = \frac{24}{4}$

$\Leftrightarrow b = 6$

$\boxed{7}$

a) $011111_2 = 2^4 + 2^3 + 2^2 + 2 + 1 = 16 + 8 + 4 + 2 + 1 = 31_{10}$

$$\begin{array}{r|l}
31 & 5 \\
30 & 6 \;|\; 5 \\
1 & 5 \;|\; 1 \;|\; 5 \\
 & 1 \;|\; 0 \;|\; 0 \\
 & \quad\;\, 1
\end{array}$$

$N_2 = 011111_2 = 31_{10} = 111_5$

$P_2 = 100000_2 = 32_{10} = 112_5$

b)

$0\,011111_2 = 010000_{Gray}$ $\qquad$ $0\,100000_2 = 110000_{Gray}$

010000 $\qquad$ 110000

$\boxed{9}$

a)

$-2^{5-1} \leq x \leq 2^{5-1} - 1$

$\Leftrightarrow -16 \leq x \leq 15$

$$\begin{array}{r}
1 = 1\;0\;0\;0 \\
1\;1\;0\;1\;0 \\
+\;1\;1\;1\;0\;0 \\
\hline
1\;0\;1\;1\;0
\end{array}$$

$= -2^4 + 2^2 + 2$
$= -10$

$-2^4 + 2^3 + 2 = -6$

$-2^4 + 2^3 + 2^2 = + \; -4$

-10

0

b) Agora com 8 bits

$$\begin{array}{r}
1 = 0\;1\;1\;1\;0\;0\;0 \\
1\;1\;1\;1\;1\;0\;1\;0 \\
+\;1\;1\;1\;1\;1\;1\;0\;0 \\
\hline
1\;1\;1\;1\;0\;1\;1\;0
\end{array}$$

$= -10$

$\boxed{10}$

a) $127 = 0111\;1111_{2's}$

$31 = 11111_2 = 011111_{2's} = 0001\;1111_{2's}$

$$\begin{array}{r|l}
31 & 2 \\
30 & 15 \;|\; 2 \\
1 & 14 \;|\; 7 \;|\; 2 \\
 & 1 \;|\; 6 \;|\; 3 \;|\; 2 \\
 & \;\; 1 \;|\; 2 \;|\; 1 \;|\; 2 \\
 & \quad\; 1 \;|\; 1 \;|\; 0 \;|\; 0 \\
 & \qquad\;\; 1
\end{array}$$

$$\begin{array}{r}
0\;1\;1\;1\;1\;1\;1\;1 \\
-\;0\;0\;0\;1\;1\;1\;1\;1 \\
\hline
\end{array}
\rightarrow
\begin{array}{r}
1 = 1\;1\;1\;1\;1\;1\;1\;1 \\
0\;1\;1\;1\;1\;1\;1\;1 \\
+\;1\;1\;1\;0\;0\;0\;0\;0 \\
\hline
0\;1\;1\;0\;0\;0\;0\;0
\end{array}$$

b)

31 6
63 7

$64 \times 2 = 64 + 64$

$64_{10} = 0010 \quad 0000_{2's}$

$$
\begin{array}{r}
0\,\cancel{1}\;1\,0\,0\,0\;0\,0\,0\,0 \\
0\,1\,0\,0\;0\,0\,0\,0 \\
+\;0\,1\,0\,0\;0\,0\,0\,0 \\
\hline
1\,0\,0\,0\;0\,0\,0\,0
\end{array}
$$

→ Overflow, necessitavamos de 9 bits para fazer a operação

c)

$-64 + 127 = 127 - 64$

$127 = 0111 \quad 1111_{2's}$

$64 = 0010 \quad 0000_{2's}$

$$
\begin{array}{r}
0111 \quad 1111 \\
-\;0010 \quad 0000 \\
\hline
\end{array}
\;\longrightarrow\;
\begin{array}{r}
1 = 1\,1\,1\,1 \quad 1111 \\
0111 \quad 1111 \\
+\;1101 \quad 1111 \\
\hline
0101 \quad 1111
\end{array}
$$

Algebra de Boole

$\boxed{1}$

$$
\begin{cases}
\overline{A} + AB = 0 \\
AC = AB \\
AB + A\overline{C} + CD = \overline{C}D
\end{cases}
\Leftrightarrow
\begin{cases}
\overline{A} + AB = 0 \\
AC = AB \\
A + CD = \overline{C}D
\end{cases}
\Leftrightarrow (A = 1 \wedge B = 0) \wedge (A = 0 \vee C = B) \wedge \left[(A = 0 \wedge D = 0) \vee (A = 1 \wedge C = 0 \wedge D = 1)\right]
$$

$\Leftrightarrow \; A = 1 \wedge B = 0 \wedge C = B \wedge A = 1 \wedge C = 0 \wedge D = 1$

$\Leftrightarrow \; A = 1 \wedge B = 0 \wedge C = 0 \wedge D = 1$

A, B - Verdadeiros e B, C - Falsos

$\boxed{9}$

$$y = \overline{\overline{x_1 \overline{x_2} + x_3 + \overline{x_1}\,\overline{x_3}\,x_4 + x_2 \overline{x_3} x_4}}$$

$$= \overline{\left(\overline{x_1 \overline{x_2}}\right) \cdot \overline{x_3} \cdot \left(\overline{\overline{x_1}\,\overline{x_3}\,x_4}\right) \cdot \left(\overline{x_2 \overline{x_3} x_4}\right)}$$

$\boxed{10}$

a) $M(x,y,z) = \underset{1\,1}{xy} + \underset{1\,1}{xz} + \underset{1\,1}{yz}$

	x	y	z	M
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

b)

$$M^D = (x+y)\cdot(x+z)\cdot(y+z)$$
$$= x + (y\cdot z)\cdot(y+z)$$
$$= x\cdot(y+z) + (y\cdot z)$$
$$= xy + xz + yz$$

c)

$1^a FC = SOP = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

$2^a FC = POS = \underline{(x+y+z)\cdot(x+y+\bar{z})\cdot(x+\bar{y}+z)}\cdot(\bar{x}+y+z)$

$3^a FC = \overline{\overline{SOP}} = \underline{(\overline{\bar{x}yz})\cdot(\overline{x\bar{y}z})\cdot(\overline{xy\bar{z}})\cdot(\overline{xyz})}$

$4^a FC = \overline{POS} = (\overline{x+y+z}) + (\overline{x+y+\bar{z}}) + (\overline{x+\bar{y}+z}) + (\overline{\bar{x}+y+z})$

$\boxed{14}$

wz \ xy	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

x, z, w, y

a) 5 Implicantes primos

b)

$w \oplus x = \bar{w}x + w\bar{x}$

$\overline{w \oplus x} = \overline{\bar{w}x + w\bar{x}}$
$= (w+\bar{x})\cdot(\bar{w}+x)$
$= wx + \bar{x}\bar{w}$

$$F = \bar{x}y\bar{w} + x\bar{w}z + \bar{x}wz + xyw$$
$$= \bar{x}y\bar{w} + xyw + z(x\bar{w} + \bar{x}w)$$
$$= y(\bar{x}\bar{w} + xw) + z(x \oplus w)$$
$$= y(\overline{w \oplus x}) + z(x \oplus w)$$

15

cd \ ab	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	0	0
10	0	1	1	0

a) 1

b) $MSOP = \bar{a}\bar{c}\bar{d} + b\bar{c}d + \bar{a}cd + bc\bar{d}$

$MPOS = (\bar{a}+c+d)\cdot(b+c+\bar{d})\cdot(\bar{a}+\bar{c}+\bar{d})\cdot(b+\bar{c}+d)$

Verdadeira!

16

A

C \ AB	00	01	11	10
0	1			1
1	1	1	1	

B

a) IPE = 2
IPNE = 2

b)

$F = \bar{B}\bar{C} + BC + \bar{A}C$

$= \bar{A}C + \overline{(C \oplus B)}$

Depende da escolha do IPNE

ou

$F = \bar{B}\bar{C} + BC + \bar{A}\bar{B}$

$= \bar{A}\bar{B} + \overline{(C \oplus B)}$

$C \oplus B = \bar{C}B + C\bar{B}$

$\overline{(C \oplus B)} = (C+\bar{B})\cdot(\bar{C}+B)$
$= CB + \bar{B}\bar{C}$

$\bar{C}D + C\bar{D}$

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$$f(A,B,C,D) = \underbrace{(A+B)}_{A+B=1}\cdot\underbrace{\bar{C}}_{0} + \underbrace{A}_{1}\cdot\underbrace{(C\oplus D)}_{C\oplus D=1} + \underbrace{AB\bar{C}D}_{1101}$$

$\Rightarrow A \neq 0 \vee B \neq 0$ $\qquad \Rightarrow C \neq D$

A

CD \ AB	00	01	11	10
00		1	1	1
01		1	1	1
11				
10			1	1

C D B

b) $B\bar{C}$, $A\bar{C}$ e $A\bar{D}$

c) $MSOP: B\bar{C} + A\bar{C} + A\bar{D}$

d) $MPOS: (A+B)\cdot(A+\bar{C})\cdot(\bar{C}+\bar{D})$

$\boxed{18}$

a) $f(A,B,C,D) = \bar{A}\cdot(\bar{B}+\bar{C}) + \bar{C}\bar{B} + \overline{(C+B)}\,D$

$$= \underset{0}{\bar{A}}\cdot\underset{0}{\bar{B}} + \underset{0}{\bar{A}}\underset{0}{\bar{C}} + \underset{0}{\bar{C}}\underset{0}{\bar{B}} + \underset{0}{\bar{C}}\underset{0}{\bar{B}}\underset{1}{D}$$

b)

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	1	0	0	0
10	1	0	0	0

c)

$F(A,B,C,D) = \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$

d)

$F^D = (\bar{A}+\bar{B})\cdot(\bar{A}+\bar{C})\cdot(\bar{B}+\bar{C})$

$= \left[\bar{A} + (\bar{B}\cdot\bar{C})\right]\cdot(\bar{B}+\bar{C})$

$= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}\cancel{\bar{B}} + \bar{B}\bar{C}\cancel{\bar{C}}$

$= \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{C}\bar{B} = F$

e)

$F' = \bar{F} = (\underset{0}{A}+\underset{0}{B})\cdot(\underset{0}{A}+\underset{0}{C})\cdot(\underset{0}{C}+\underset{0}{B})$, é a mínima soma de produtos,

Pelo Mapa de Karnaugh, verificamos que o resultado da função apenas é zero quando A e B são "0", quando A e C são "0" e quando C e B são "0", logo F' é a mínima soma de produtos