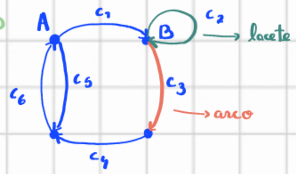
Aula 20

Grafo de Euler (1736)

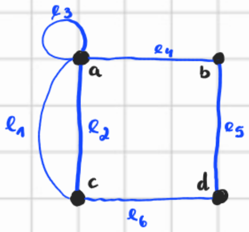
- Sendo n o número de vértices de grau ímpar:
  - → Se $n = 0$, o grafo contém pelo menos um circuito euleriano
  - → Se $n = 2$, há pelo menos um caminho euleriano, não circuito
  - → Se $n > 2$, não há nem caminho nem circuito euleriano

Multi digrafo



Conceitos fundamentais

G



→ **Multigrafo:**

vértice, arestas, função de incidência

$G = (V, E, \psi)$

$V = \{a, b, c, d\}$, $|V| = \nu$ → ordem de G

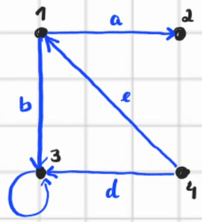
$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, $|E| = \epsilon$ → dimensão de G

$\psi : E \longrightarrow \{A \subseteq V : 1 \leq |A| \leq 2\}$

$\psi(e_3) = \{a\}$, $\psi(e_4) = \{a, b\}$, ...

— // —

$\vec{J}$



→ (multi) **Digrafo:**

$\vec{J} = (V_{\vec{J}}, E_{\vec{J}}, \psi_{\vec{J}})$

não precisa de $\psi_{\vec{J}}$

$V_{\vec{J}} = \{1, 2, 3, 4\}$

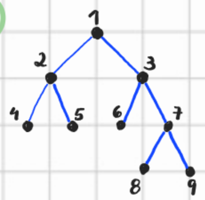
$E_{\vec{J}} = \{(1,2), (1,3), (3,3), (4,3), (4,1)\}$

cauda do arco 1 → 2   cabeça do arco 1 → 2

— // —

"Tree"

T



Vamos estudar os grafos simples!

→ **Grafos simples:**

$T = (V, E)$

$V = \{1, \dots, 9\}$, $|V| = 9$

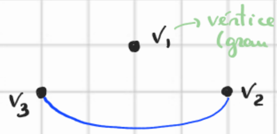
$E = \{12, 13, 24, 25, 36, 37, 78, 79\} = 8$

Dimensão 8 e ordem 9

notação mais leve → $\{1, 2\}$

podemos utilizar as 2 notações

H

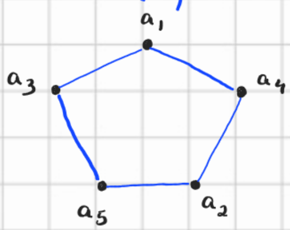


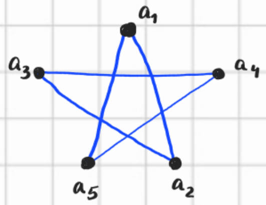
$H = (V, E)$
$|V| = 3$
$|E| = 1$

Vamos estudar apenos os grafos simples!

Grafo complementar $\left[(G^c)^c = G\right]$

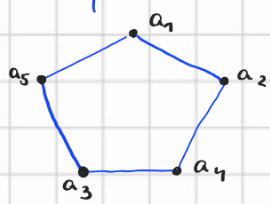
$G = (V, E)$ ⟶ $G^{c} = (V, E^c)$ ← complementar





$\equiv$

Isomorfo a G^c

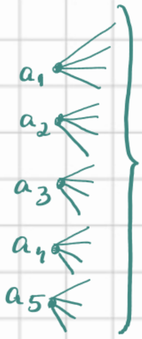


$E = \{a_1 a_4, a_1 a_3, a_3 a_5, a_5 a_2, a_2 a_4\}$

$E^c = \{a_1 a_2, a_1 a_5, a_3 a_4, a_3 a_2, a_4 a_5\}$

$0 \leq |E^c| \leq \binom{5}{2} = 10$

$0 \leq |E| \leq (?) = \binom{5}{2} = 10$



$= \frac{5 \times 4}{2} = 10 = \binom{5}{2}$

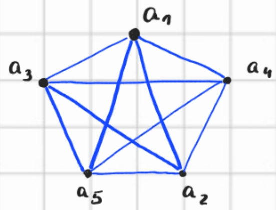
↳ repetidos!

Grafo completo de ordem n

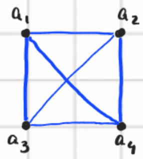
$K_n = (V, E)$, $d(v_i) = n - 1$
$v_i \in V$
$i = 1, 2, 3, \ldots, n$

$|E| = \binom{n}{2}$ ← número máximo de ligações

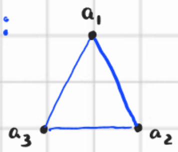
K_5:



K_4:



K_3:



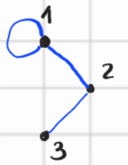
K_2:

K_1:

Vizinhança e grau dos vértices

→ $N(v) = \{$ conjunto dos vértices adjacentes a $v \}$

G

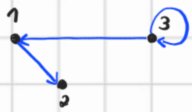


→ Multigrafo: $G = (V, E)$

$N(1) = \{1, 2\}$, $N(2) = \{1, 3\}$
$N(3) = \{2\}$

Com orientação!

$\vec{J}$



⇒

$N^-(v)$ → vizinhanças de entrada de v
$N^+(v)$ → vizinhanças de saída de v
$\Big\}$ $N(v) = N^-(v) \cup N^+(v)$

$N^-(1) = \{3\}$, $N^+(1) = \{2\} \Rightarrow N(1) = N^-(1) \cup N^+(1) = \{2, 3\}$

$N^-(2) = \{1\}$, $N^+(2) = \{\ \} \Rightarrow N(2) = \{1\}$

$N^-(3) = \{3\}$, $N^+(3) = \{1, 3\} \Rightarrow N(3) = \{1, 3\}$

—//—

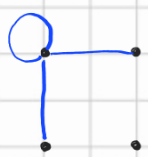
$G = (V, E)$, grau de $v \in V$, $\underbrace{d(v)}$ número de arestas incidentes em v (lacete conta duas vezes)

$\Delta(G) = \max\{d(v) \mid v \in V\}$
↳ maior grau

$\delta(G) = \min\{d(v) \mid v \in V\}$
↳ menor grau

ex:

G_1 :



$\Delta(G_1) = 4$
$\delta(G_1) = 0$

—//—

$\vec{G} = (V, E)$

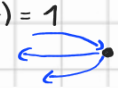
$d(v) = \underbrace{d^-(v)} + \underbrace{d^+(v)}$

$d^+(v) = d^+_{\vec{G}}(v)$ → semigrau de saída de v (nº de arcos com cauda em v)

ex: $d^+(v) = 2$

↳ $d^-(v) = d^-_{\vec{G}}(v)$ → semigrau de entrada de v (nº de arcos com cabeça em v)

↳ ex: $d^-(v) = 1$



⇒ $\boxed{d(v) = 3}$

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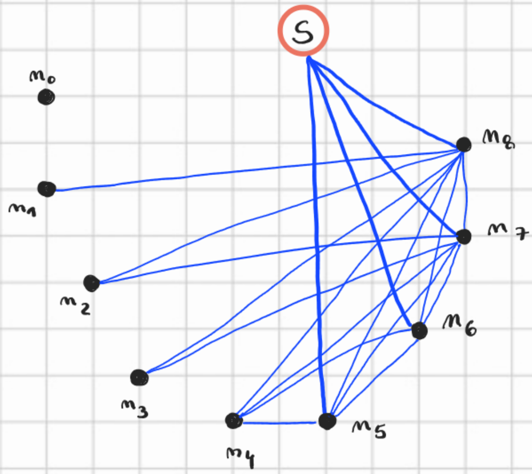
Exemplo (dos Slides):

S: Senhor Silva



$G = (V, E)$, $V = \{S, m_0, m_1, \dots, m_8\}$, $|V| = 10$

m_j deu j apertos de mão

$d(m_8) = 8, \dots \; d(m_j) = j$, $j = 0, 1, \dots, 8$

S deu 4 apertos de mão

Casais:

$m_0 - m_8$

$m_1 - m_7$

$m_2 - m_6$

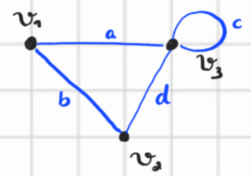
$m_3 - m_5$

$m_4 - S$

Representação de grafos por matrizes

Matriz de incidência: $V \times E \longrightarrow \mathbb{R}$

G



$$M_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 \end{bmatrix} \end{array}$$

$V = \{v_1, v_2, v_3, v_4\}$

$E = \{a, b, c, d\}$

Representação de digrafos por matrizes

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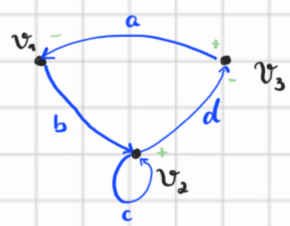
Matriz de incidência: $\boxed{V \times E \longrightarrow \mathbb{R}}$

$\vec{G}$



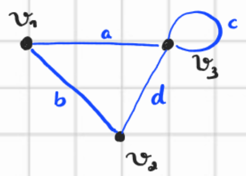
$$M_{\vec{G}} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & 0 \\ 0 & -1 & 2 & +1 \\ +1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

$d(v_3) = \underbrace{d^-(v_3)}_{=1} + \underbrace{d^+(v_3)}_{=1} = 1 + 1 = 2$

Matriz de adjacência

Matriz de adjacência : $V \times V \longrightarrow \mathbb{R}$

G

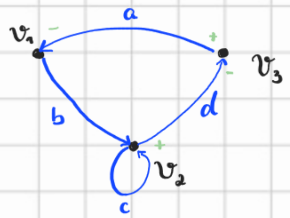


$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \end{array} = A_G^T$$

$??$

Matriz de adjacência : $\boxed{V \times V \longrightarrow \mathbb{R}}$

$\vec{G}$



$$A_{\vec{G}} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \neq A_{\vec{G}}^T$$