Folha 3

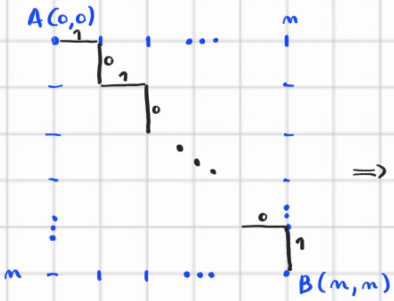
1

230 deputados $\begin{cases} 164 \ ♂ \\ 66 \ ♀ \end{cases}$

$$C = \binom{164}{5} \times \binom{66}{5}$$

2

a)

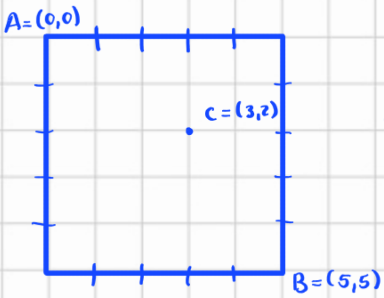


Consideramos o 0 : "andar para baixo"
1 : "andar para o lado direito"

$\Rightarrow$ Seq. binárias de comprimento $2m$ com $K = m$ uns e m zeros $\to m' - 1 = m \Rightarrow m' = m+1$

$$\left(\!\!\binom{m+1}{m}\!\!\right) = \binom{m+1+m-1}{m} = \binom{2m}{m}$$

b)



• De $\underline{A}$ a $\underline{C}$: Seq. binárias de comprimento 5 com 3 uns e 2 zeros $\Rightarrow \left(\!\!\binom{3}{3}\!\!\right) = \binom{3+3-1}{3} = \frac{5!}{3!\,2!} = \frac{5\times4}{2} = 10$

$\hookrightarrow m'-1 = 2 \Rightarrow m' = 3$

• De $\underline{C}$ a $\underline{B}$: Seq. binárias de comprimento 5 com 2 uns e 3 zeros $\Rightarrow \left(\!\!\binom{4}{2}\!\!\right) = \binom{4+2-1}{2} = \frac{5!}{2!\,3!} = 10$

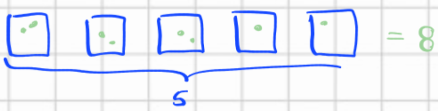
$\hookrightarrow m' = 4$

Logo : $C = 10 \times 10 = 100$

3

8 presentes
5 crianças

a)



$$x_1 + x_2 + x_3 + x_4 + x_5 = 8$$

Número de seq. binárias com $K = \underline{8}$ uns e $m-1 = \underline{5} - 1 = 4$ zero

$$\left(\!\!\binom{5}{8}\!\!\right) = \binom{5+8-1}{8} = \binom{12}{8} = \frac{12!}{8!\,4!} = \frac{\cancel{12} \times 11 \times 10 \times 9}{\cancel{4 \times 3} \times 2} = 11 \times 5 \times 9$$

b)

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$$

6

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Número de seq. binárias com $K = \underline{5}$ uns e $m - 1 = \underline{12} - 1 = 11$ zeros e que comecem por 101 e terminem em 0001

$1\ 0\ 1\ \boxed{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}\ 0\ 0\ 0\ 1$

Número de seq. binários com $K = 5-3 = \underline{2}$ uns e $n-1 = \underline{8}-1 = 7$ zeros

$$\left(\!\binom{8}{2}\!\right) = \binom{8+2-1}{2} = \binom{9}{2} = \frac{9!}{2!\,7!} = \frac{9\times 8}{2} = 9\times 4 = 36$$

$\boxed{8}$

4 laranjas iguais e 6 maçãs diferentes

$\downarrow$

$c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5$

- Colocar 4 laranjas iguais:

$\rightarrow$ Seq. binários com $K = \underline{4}$ uns e $n-1 = \underline{5}-1 = 4$ zeros

$$P_1 = \left(\!\binom{5}{4}\!\right) = \binom{8}{4} = \frac{8!}{4!\,4!} = \frac{8\times 7\times \cancel{6}\times 5}{4\times \cancel{3\times 2}} = 2\times 7\times 5 = 70$$

- Colocar 6 maçãs diferentes:

$$P_2 = 5\times 5\times 5\times 5\times 5 = 5^6$$

- Logo:

$$P = P_1 \times P_2 = 70\times 5^6 = 1093750$$

$\boxed{9}$

57 canções (considerar ordem) $*$

$$A^s(57,20) = \frac{57!}{(57-20)!} = \frac{57\times 56\times \dots \times 38\times \cancel{37!}}{\cancel{37!}} = 57\times 56\times\dots\times 38$$

ou

$$\binom{57}{20}\times \underbrace{20!}_{*}$$

$\boxed{12}$

a)

$$3^n = (2+1)^n = \sum_{K=0}^{n} \binom{n}{K} 2^K \times \cancel{1^{n-K}}^{\,1}$$
$$= \sum_{K=0}^{n} \binom{n}{K} 2^K$$

$$(a+b)^n = \sum_{K=0}^{n} \binom{n}{K} a^K b^{n-K}$$

$\boxed{13}$

a)

$$\left(x^2 + \frac{y}{x} + 2z\right)^6 = \sum_{n_1+n_2+n_3=6} \binom{6}{n_1, n_2, n_3} (x^2)^{n_1} \left(\frac{y}{x}\right)^{n_2} (2z)^{n_3}$$

- Para determinar o coeficiente $x y^3 z$: $n_3 = 1$, $n_2 = 3$, $n_1 = 2$

Assim:

$$(x^2)^{m_1}\left(\frac{y}{x}\right)^{m_2}(2z)^{m_3} = x^4 \times x^{-3} \times y^3 \times 2 \times z$$
$$= 2 \times x\, y^3\, z$$

Logo o coeficiente é $2 \times \binom{6}{2,3,1} = \cancel{2} \times \frac{6!}{\cancel{2!}\,3!\,1!} = 6 \times 5 \times 4 = 120$ //

c)
$$\sum_{k=0}^{n} \binom{n}{k} = 32 = 2^5 \to n = 5$$

$$(x^3 + \sqrt{x})^n = \sum_{k=0}^{n} \binom{n}{k} x^{3k} (\sqrt{x})^{n-k}$$

$$x^{3k} \times x^{\frac{n-k}{2}} = x^{\frac{6k+n-k}{2}} = x^{\frac{5k+n}{2}}$$

$$\frac{5k+5}{2} = 10 \Leftrightarrow 5k = 15 \Leftrightarrow k = \frac{15}{5} \Leftrightarrow k = 3$$

Logo: para $n = 5$ e $k = 3$:

$$\underbrace{\binom{5}{3}}_{\frac{5!}{3!\,2!} = \frac{5 \times 4}{2} = 10} \times x^{10}$$

$\frac{5!}{3!\,2!} = \frac{5 \times 4}{2} = 10$ //

— // — (+)

8

4 L iguais
6 M dif. } 10

$\square \circ \square \circ \square \circ \square \circ \square$

$$\binom{5+4-1}{4} = \binom{8}{4}$$

Pensar ao contrário! ← Por uma caixa em qualquer maçã

$5 \times 5 \times 5 \times 5 \times 5 = 5^6$

$M_1 \; M_2 \; M_3 \; M_4 \; M_5 \; M_6$

c_1, c_2, c_3, c_4, c_5

Donde, $\binom{8}{4} \times 5^6$

10

a)

$$\binom{12+45}{\underbrace{1,1,1,\ldots,1}_{12\text{ uns}},45} \quad \_\_ \; S_1 \; \_\_ \; S_2 \; \_\_ \_\_ \_\_ \_\_ \ldots \; S_{12}$$

ou

$$\binom{57}{12} \times 12! \quad S_1 \_\_ \_\_ \_\_ S_2 \_\_ \_\_ \_\_ S_3 \_\_ \_\_ \_\_ S_4 \ldots \_\_ \_\_ S_{12}$$

$45 - 33 = 12$

b)

distribuir 12 bolos por 13 caixas

$$\overbrace{c_1 \quad c_2 \quad c_3 \quad \cdots \quad c_{13}}$$

$$\left(\!\!\binom{13}{12}\!\!\right) = \binom{12+13-1}{12} = \binom{24}{12}, \text{ donde}$$

$$\cancel{\binom{24}{12} \times 12!}$$

ou

$$\binom{12+12}{\underbrace{1,1,\ldots,1}_{11 \text{ uns}},12}$$

7

a) $x_1 + x_2 + \ldots + x_m = r, \quad x_i \in \mathbb{N} = \{0,1,\ldots\}$

$i = 1, 2, \ldots m, \; m \geq 1$

$$f : Q \longrightarrow S$$

$$x \longmapsto f(x) = (\underbrace{b_1, b_2}, \ldots, b_{m-1+r})$$

$$(r_1, 0, r_2, 0, \ldots, 0, r_{m-1}, 0, r_m)$$

$Q = \left\{ \begin{array}{l} \text{soluções inteiras} \\ \text{da equação} \\ x_1 + x_2 + \ldots + x_m = r, \\ x_i \in \mathbb{N}, m \geq 1, \\ i = 1, 2, \ldots, m, r \in \mathbb{R} \end{array} \right\}$

$x = (\underbrace{x_1}, \underbrace{x_2}, \ldots, \underbrace{x_m})$

$r_1 + r_2 + \ldots + r_m = r$

r_i nº de uns em x_i

$S = \left\{ \begin{array}{l} \text{seq. binária de comp. } m-1+r, \\ \text{c/ } m-1 \text{ zeros e } r \text{ uns} \end{array} \right\}$

Existe $f^{-1} : S \longrightarrow Q$

Logo, f é bijetiva

$$|Q| = |S| = \underbrace{\binom{m-1+r}{r}} = \binom{m-1+r}{m-1}$$

$$\left(\!\!\binom{3}{11}\!\!\right) \longleftarrow \text{comb. com repetição}$$

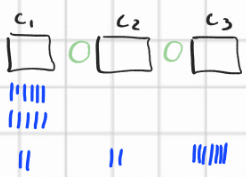
$m = 3, \quad r = 11, \quad m - 1 + r = 13$

ex:

$Q^* : x_1 + x_2 + x_3 = 11$

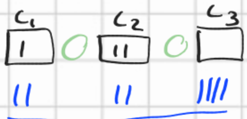
$11 + 0 + 0 = \underbrace{111\ldots1}_{11 \text{ uns} \to r_1}00$

$2 + 2 + 7 \; \Vert \; 110110\underbrace{11\ldots1}_{7 \text{ uns} \to r_3}$



$$|Q^*| = \binom{13}{11} = \binom{13}{2}$$

b) $x_1 + x_2 + x_3 = 11$, com $x_1 \geq 1$ e $x_2 \geq 2$



$$\underbrace{\quad}_{\text{cmp. } 8+2=10}$$

$$\binom{8+3-1}{8} = \binom{10}{8} = \binom{10}{2}$$

<u>ou</u>

$$x_1 + x_2 + x_3 = 11 \quad , \quad \underbrace{x_1, x_2, x_3 \geq 0}_{\in \mathbb{N}}$$

$$\underbrace{x_1 - 1}_{y_1} \geq 0 \quad e \quad \underbrace{x_2 - 2}_{y_2} \geq 0$$

$$x_1 = y_1 + y \qquad\qquad x_2 = y_2 + 2$$

$$(y_1 + 1) + (y_2 + 2) + y_3 = 11 \Leftrightarrow y_1 + y_2 + y_3 = 8$$

$\boxed{11}$

a) PARALELEPÍPEDO

3 P's, 1 R, 3 E's, 1 D, 2 A's, 2 L's, 1 I, 1 O

$$\underbrace{n_1}_{3} + \underbrace{n_2}_{3} + \underbrace{n_3}_{2} + \underbrace{n_4}_{2} + \underbrace{n_5}_{1} + \underbrace{n_6}_{1} + \underbrace{n_7}_{1} + \underbrace{n_8}_{1} = \overbrace{14}^{n}$$

$$\binom{n}{u_1, u_2, \dots u_8} = \binom{14}{3,3,2,2,1,1,1,1} = \frac{14!}{3!3!2!2!}$$

b) $\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{11} + \underbrace{\underline{P}\ \underline{P}\ \underline{P}}_{1} = 12$

Total $-$ nº de seq. em que aparecem 3 P's seguidos $= \dfrac{14!}{3!3!2!2!} - \left(\dfrac{12}{3!\,2!\,2!}\right)$