Exame - Analítico (2022)

1

a)

$$A = T + R$$

$$A = (25{,}3 \pm 0{,}2) + (10{,}0 \pm 0{,}1) = 35{,}3 \pm 0{,}3 \text{ cm}$$

b)

$$D = T - S$$

$$D = (25{,}3 \pm 0{,}2) - (5{,}0 \pm 0{,}2) = 20{,}3 \pm 0{,}4 \text{ cm}$$

c)

$$r = \frac{25{,}3}{10{,}0} = 2{,}53$$

sem o erro associado

$$\frac{\Delta r}{r} = \frac{\Delta T}{T} + \frac{\Delta R}{R} \Leftrightarrow \frac{\Delta r}{r} = \frac{0{,}2}{25{,}3} + \frac{0{,}1}{10{,}0}$$

sem os erros associados

$$\Leftrightarrow \frac{\Delta r}{r} = 0{,}00791 + 0{,}01$$

$$\Leftrightarrow \frac{\Delta r}{r} = 0{,}01791$$

$$\Leftrightarrow \Delta r = 2{,}53 \times 0{,}01791$$

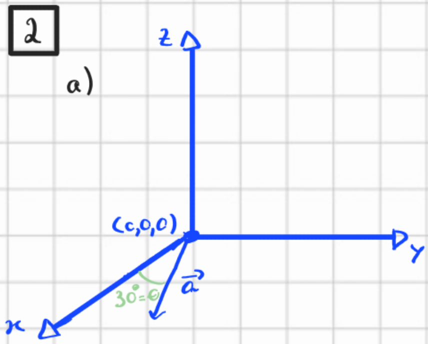
$$\Leftrightarrow \Delta r = 0{,}0453$$

erro arredondado por excesso

Logo: $r = 2{,}53 \pm 0{,}05$ cm

2

a)



$\|\vec{a}\| = 3$ cm

$\vec{a} = (x_A, y_A, 0)$

$30° = \frac{\pi}{6}$ rad

$$\cos\left(\frac{\pi}{6}\right) = \frac{x_A}{\|\vec{a}\|} \Leftrightarrow x_A = \frac{3\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} \times 3$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{y_A}{\|\vec{a}\|} \Leftrightarrow y_A = \frac{3}{2}$$

$$\vec{a} = \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}, 0\right)$$

b)

$\vec{b} = (1, 0, -1)$

$$a \cdot b = \frac{3\sqrt{2}}{2} + 0 + 0 = \frac{3\sqrt{3}}{2}$$

$\|\vec{a}\| = 3$

$\|\vec{b}\| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$a \cdot b = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos(\vec{a} \wedge \vec{b})$$

$$\Leftrightarrow \frac{3\sqrt{3}}{2} = 3\sqrt{2} \cos(\vec{a} \wedge \vec{b})$$

$$\Leftrightarrow \frac{\cancel{3}\sqrt{3}}{2 \times \cancel{3}\sqrt{2}} = \cos(\vec{a} \wedge \vec{b}) \Leftrightarrow \frac{\sqrt{6}}{4} = \cos(\vec{a} \wedge \vec{b}) \Leftrightarrow \vec{a} \wedge \vec{b} \simeq 52{,}24°$$

c)

Seja $\vec{c}$, um vetor prependicular a $\vec{a}$ e a $\vec{b}$.

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3\sqrt{3}}{2} & 3/2 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i}\left(-\frac{3}{2}\right) - \hat{j}\left(-\frac{3\sqrt{3}}{2}\right) + \hat{k}\left(-3/2\right)$$

$$= \left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}, -3/2\right)$$

$\boxed{3}$

Queremos calcular: $$\left| \left(\int_{x_i}^{x_{i+1}} f(x)\,dx\right)_{\text{EXATO}} - \left(\int_{x_i}^{x_{i+1}} f(x)\,dx\right)_{\text{ap. rectangulos}} \right| = \left| \left(\int_{x_i}^{x_{i+1}} f(x)\,dx\right)_{\text{EXATO}} - \left(f(x_{i+1})\,\delta t\right) \right|$$

$$\left(\int_{x_i}^{x_{i+1}} f(x)\,dx\right)_{\text{EXATO}} = ?$$

<u>Função $f(x)$ pela série de tayler em volta de x_{i+1}:</u>

$$f(x) = f(x_{i+1}) + \left.\frac{df}{dx}\right|_{x_{i+1}} (x - x_{i+1}) + \frac{1}{2} \left.\frac{d^2f}{dt^2}\right|_{x_{i+1}} (x - x_{i+1})^2 + o(x - x_{i+1})^3$$

<u>Substituindo no integral:</u>

$$\int_{x_i}^{x_{i+1}} \left(f(x_{i+1}) + \left.\frac{df}{dx}\right|_{x_{i+1}} (x - x_{i+1}) + \frac{1}{2} \left.\frac{d^2f}{dx^2}\right|_{x_{i+1}} (x - x_{i+1})^2 + o(x - x_{i+1})^3 \right)$$

$$= \cancel{\int_{x_i}^{x_{i+1}} f(x_{i+1})\,dx}^{\,0} + \int_{x_i}^{x_{i+1}} \left(\underbrace{\left.\frac{df}{dx}\right|_{x_{i+1}}}_{} (x - x_{i+1}) \right) + \frac{1}{2} \int_{x_i}^{x_{i+1}} \left.\frac{d^2f}{dx^2}\right|_{x_{i+1}} (x - x_i)^2\,dx + \int o(x - x_{i+1})^3\,dx$$

$$\int f'(x_{i+1})\,(x - x_{i+1})$$