Deriving the Schrödinger Equation from Compression Principles

1) Postulates (what we assume)

1. **State of knowledge** about a particle (mass m) at time t is a probability density (x,t) on configuration space and a phase field S(x,t) that generates a current v = S/m.

2. **Dynamics = optimal compression** of the state's spatial structure subject to ordinary physical constraints. "Compression" is made concrete as minimizing the **Fisher information content** (a measure of spatial roughness/description length) of .

3. We must still respect **probability flow** (continuity) and ordinary **energetics** (potential V(x,t)).

4. A single constant with units of action, call it ħ, sets the scale of the information–action tradeoff (it will drop out as the Lagrange multiplier and calibrates to experiment).

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2) The compression functional

Use a Lagrangian density that balances: - a "coding cost" (Fisher information) that penalizes unnecessary structure in , and - standard kinetic/potential terms expressed in the hydrodynamic (Madelung) variables (,S).

Define the action

$$\mathcal{A}[\rho, S] = \int dt \int d^3x \left\{ \rho \left(\partial_t S + \tfrac{(\nabla S)^2}{2m} + V \right) + \frac{\hbar^2}{8m} \frac{|\nabla \rho|^2}{\rho} \right\}.$$

Interpretation: - The first term enforces energy accounting and probability flow. - The second term is the Fisher information density, i.e., the **compression penalty**; minimizing it prefers smoother, more compressible probability fields unless the constraints demand structure.

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3) Variation → Euler–Lagrange equations

- Variation w.r.t. S gives the **continuity equation**:

$$\partial_t \rho + \nabla \cdot \left(\rho \, \frac{\nabla S}{m} \right) = 0.$$

- Variation w.r.t.   gives the **quantum Hamilton–Jacobi (QHJ) equation**:

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0.$$

The extra term

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$

is the **quantum potential**, arising directly from the Fisher information penalty.

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4) Package (,S) into a wavefunction

Define the complex field

$$\psi(x,t) = \sqrt{\rho(x,t)}\, e^{iS(x,t)/\hbar}.$$

This transforms the continuity + QHJ equations into

$$i\hbar\, \partial_t \psi = \Big(-\frac{\hbar^2}{2m}\nabla^2 + V \Big)\, \psi,$$

i.e., the **time-dependent Schrödinger equation**.

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5) Interpretation

- **Quantum dynamics emerges as the unique fixed point** where:
  - Probability flow and energy bookkeeping are respected, and
  - The state description is optimally compressible (minimizing Fisher information).
- The quantum term is the shadow of the compression drive.
- ħ is the tradeoff scale, fixed empirically.

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6) Sanity checks

- **Classical limit**: ħ $\to$ 0 eliminates the compression penalty, recovering classical Hamilton–Jacobi mechanics.
- **Interference**: arises naturally from multi-path compression encoded in the phase.
- **Uniqueness**: Fisher information is essentially the only local, Galilean-invariant compression functional.

7) Bottom line

Schrödinger's equation is not arbitrary—it is the **equation of motion that best compresses probabilistic descriptions of reality under physical constraints**.