Compression Fold of the Strong Force

1) Setup — gauge field and notation

- Gauge field: $A_\mu(x) = A^a_\mu(x)T^a$ with T^a the SU(3) generators (trace normalized $\mathrm{Tr}\, T^aT^b = \frac{1}{2}\delta^{ab}$).
- Field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$ (components $F^a_{\mu\nu}$).
- Covariant derivative in adjoint / matrix form: $D_\mu = \partial_\mu + g[A_\mu, \,\cdot\,]$.
- Inner product on Lie-algebra-valued fields: $\langle X, Y\rangle = \int d^4x\, \mathrm{Tr}(X_\mu(x)Y^\mu(x))$.

We consider a probability density functional $\rho[A]$ over gauge-field configurations A. Compression will be expressed as a Fisher-information penalty on ρ.

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2) Candidate global compression functional (functional-Fisher + energy)

Define the total functional:

$$\mathcal{S}[\rho] = \int \mathcal{D}A\ \rho[A]\ \mathcal{E}[A] + \frac{\lambda}{2}\, I_F[\rho]$$

where

- $\mathcal{E}[A]$ is the physical energy functional (gauge-invariant),

$$\mathcal{E}[A] = \int d^4x\ \mathrm{Tr}(\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}).$$

- $I_F[\rho]$ is the functional Fisher information:

$$I_F[\rho] = \int \mathcal{D}A\ \rho[A] \int d^4x\ \mathrm{Tr}\Big(\frac{\delta \ln\rho[A]}{\delta A_\mu(x)}\frac{\delta \ln\rho[A]}{\delta A^\mu(x)}\Big).$$

- $\lambda > 0$ is the compression tradeoff constant.

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3) Stationary condition (variation in ρ)

Variation yields the stationary condition (schematic form):

$$\mathcal{E}[A] - \lambda\, \frac{1}{\rho^{1/2}[A]} \int d^4x\ \mathrm{Tr}\Big(\frac{\delta^2\rho^{1/2}[A]}{\delta A_\mu(x)\,\delta A^\mu(x)}\Big) = \mu,$$

which implies

$$\rho[A] \propto \exp\Big(-\frac{1}{\lambda}\,\mathcal{E}_{\mathrm{eff}}[A]\Big).$$

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4) Saddle-point / semiclassical limit → Yang–Mills

In the small-λ limit, $\rho[A]$ peaks around A^* minimizing $\mathcal{E}[A]$:

$$D_\mu F^{\mu\nu}[A^*] = 0,$$

i.e. the classical Yang–Mills equations.
Finite λ adds Fisher-information corrections $Q[\rho, A]$.

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5) Local ansatz — field-level model

Introduce a compressibility field $\varphi(x)$:

$$\mathcal{L}(A, \varphi) = \mathrm{Tr}\big(\tfrac{1}{2} F_{\mu\nu} F^{\mu\nu}\big) + \frac{\kappa}{2}\, \mathrm{Tr}\big((D_\mu \varphi)(D^\mu \varphi)\big) + V(\varphi).$$

Here φ is Lie-algebra valued and acts as a local order parameter of compression.

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6) Interpretations

- **Confinement:** compression favors color-singlet combinations and disfavors isolated color charges.
- **Asymptotic freedom:** at short distances the Fisher penalty is negligible → quarks behave nearly free.
- **Gluon self-interaction:** nonlinear commutators in $F_{\mu\nu}$ remain; compression biases preferred configurations.

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7) Next steps

1. **Lattice toy:** add Fisher penalty to Wilson action; simulate.
2. **Local model:** simulate $\mathcal{L}(A, \varphi)$ with varying potentials $V(\varphi)$.
3. **Analytic checks:** expand for small λ; compare corrections with known loop effects.
4. **Relate** λ to QCD scale Λ_{QCD}.

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8) Summary

- Defined a compression functional for SU(3) gauge fields.
- Variation recovers Yang–Mills in the saddle-point limit.
- Fisher terms give quantum-like corrections.
- Local ansatz couples compression field φ to the gauge field; could model confinement transitions.
- Suggests the strong force as the next fold in the Genesis Pattern.