The `fractalof()` Operator — Formal Definition

This document defines the **`fractalof()`** operator, the core mathematical mechanism of the Theory of Absolutely Everything [https://doi.org/10.5281/zenodo.17042178].

In plain words:
- Reality (and consciousness) evolves by continually finding the **simplest, most coherent description** of itself.
- The function `fractalof()` is the operator that applies this principle recursively.
- Mathematically, it is expressed as a **flow that minimizes a compression functional**.
- The stable outcomes of this flow are the **attractors** we experience as physical laws, conscious perceptions, and shared meaning.

---

1. State & Notation

- **State space:** $C4$, the fundamental complex Hilbert space.
- **State vector:** $|\psi\rangle \in C4$, with normalization $\langle\psi|\psi\rangle = 1$.
- **Density:** $\rho(x) = |\psi(x)|^2$, probability density on domain X.
- **Reference frame:** B, basis/language used to express the state.
- **Coherence model:** $\phi \in C4$, representing an interlocutor or reference attractor.
- **Complexity proxy:** $\widetilde{K}(\cdot)$, a computable proxy (e.g. MDL score, entropy).
- **Tradeoffs:** $\lambda, \gamma \geq 0$.

---

2. Compression Objective

The universal functional minimized by consciousness is

$$J[\psi;\phi] \;=\; S[\rho] \;+\; \lambda\, R_{\mathrm{coh}}[\psi,\phi] \;+\; \gamma\, I_F[\rho],$$

with the following proxy terms:

- **Entropy / description length:**

$$S[\rho] = \int_X \rho(x)\, \log\rho(x)\, dx.$$

Intuition: this term pushes the state toward **simplicity** — preferring patterns that can be described concisely.

- **Coherence (Love) prior:**

$$R_{\mathrm{coh}}[\psi, \phi] = -\log\left(\langle \psi | \phi \rangle + \epsilon\right), \quad \epsilon > 0.$$

Intuition: this term pulls the state toward **connection** — aligning with others and reinforcing shared meaning (love as coherence).

- **Fisher information penalty:**

$$I_F[\rho] = \int_X \frac{|\nabla \rho(x)|^2}{\rho(x)}\, dx.$$

Intuition: this term enforces **smoothness** — avoiding sharp oscillations and ensuring stability across scales.

---

3. Functional Gradient

The variational derivative of J with respect to ψ^* is

$$\frac{\delta J}{\delta \psi^*}(x) = (1 + \log \rho(x))\psi(x) - \lambda\, \frac{\phi(x)}{\langle \psi | \phi \rangle + \epsilon} + \gamma\, \mathcal{F}[\rho](x)\, \psi(x),$$

where the Fisher operator is

$$\mathcal{F}[\rho](x) = -\nabla \cdot \left(\frac{2\nabla \rho}{\rho} \right) - \frac{|\nabla \rho|^2}{\rho^2}.$$

---

4. Gradient Flow

Consciousness evolves by projected gradient descent on J:

$$\partial_t \psi = -\left(\frac{\delta J}{\delta \psi^*} - \langle \psi | \tfrac{\delta J}{\delta \psi^*} \rangle \psi \right) + \eta(t),$$

- The projection enforces normalization $\langle \psi | \psi \rangle = 1$.
- $\eta(t)$ is optional stochastic noise (exploration / creativity).
- Fixed points satisfy stationarity: $\delta J / \delta \psi^* \parallel \psi$.

---

5. The `fractalof()` Operator

Define the **beta function**:

$$\beta(\psi) = -\left(\frac{\delta J}{\delta \psi^*} - \langle \psi | \tfrac{\delta J}{\delta \psi^*} \rangle \psi\right).$$

Then the recursive flow is

$$\frac{d\psi}{dt} = \beta(\psi), \qquad \psi(0) = \psi_0.$$

The **`fractalof()` operator** is the asymptotic map:

$$fractalof(\psi_0) = \lim_{t \to \infty} \Phi_t(\psi_0),$$

where Φ_t is the flow generated by β.
Fixed points satisfy $\beta(\psi) = 0$, yielding idempotency:

$$fractalof(fractalof(\psi)) = fractalof(\psi).$$

---

6. Multi-Scale Extension

With scale parameter s (coarse $\to$ fine), define scale-dependent objective J_s.
The renormalization flow is

$$\frac{d\psi_s}{ds} = -\nabla_\psi \widetilde{K}_s(\psi_s),$$

whose fixed points are **scale-stable attractors** (percepts persisting across levels).

---

7. Minimal Summary (for citation)

$$fractalof(\psi) = \lim_{t \to \infty} \psi(t), \qquad \partial_t \psi = -\Big(\tfrac{\delta J}{\delta \psi^*} - \langle \psi | \tfrac{\delta J}{\delta \psi^*} \rangle \psi\Big).$$

$$J[\psi; \phi] = \int \rho \log \rho + \lambda\big(-\log(\langle \psi | \phi \rangle + \epsilon)\big) + \gamma \int \tfrac{|\nabla \rho|^2}{\rho}.$$
