Compression $\to$ Klein–Gordon $\to$ $E = mc^2$

This document derives the Klein–Gordon equation from a 4D compression action (Fisher penalty + energetic term) and shows how $E = mc^2$ arises naturally.

0) 4D compression action

Define fields $\rho(x) \geq 0$, $S(x) \in \mathbb{R}$, $x \in \mathbb{R}^{1+3}$, and the complex field

$$\psi(x) = \sqrt{\rho(x)}\, e^{iS(x)/\hbar}$$

Compression-inspired action (mostly-plus signature $(+,-,-,-)$):

$$\mathcal{A}[\rho, S] = \int d^4x \, \left\{ \rho(\partial_\mu S\, \partial^\mu S + m^2c^2) + \frac{\hbar^2}{2}\, \partial_\mu \sqrt{\rho}\, \partial^\mu \sqrt{\rho} \right\}$$

1) Variation w.r.t. S — continuity equation

Varying S yields

$$\partial_\mu(\rho\, \partial^\mu S) = 0,$$

i.e. the relativistic continuity equation. Define the 4-current $j^\mu = \rho\, \partial^\mu S$.

2) Variation w.r.t. ρ — relativistic quantum Hamilton–Jacobi

Varying ρ gives

$$\partial_\mu S\, \partial^\mu S + m^2c^2 - \frac{\hbar^2}{2} \frac{\Box\sqrt{\rho}}{\sqrt{\rho}} = 0$$

Define the quantum potential

$$Q[\rho] = -\frac{\hbar^2}{2} \frac{\Box\sqrt{\rho}}{\sqrt{\rho}}$$

so the HJ equation reads $\partial_\mu S\, \partial^\mu S + m^2c^2 + Q[\rho] = 0$.

3) Combine into ψ and compute $\Box\psi$

Set $\psi = \sqrt{\rho} e^{iS/\hbar}$. Compute $\Box\psi$ (details omitted here but algebra follows from product rule and using the two real equations). After cancellations one obtains the Klein–Gordon equation:

$$(\Box + \tfrac{m^2c^2}{\hbar^2})\psi = 0.$$

4) Dispersion relation $\to$ $E = mc^2$

Plane-wave solutions $\psi \sim e^{-i(Et - \mathbf{p}\cdot\mathbf{x})/\hbar}$ give

$$E^2 = |\mathbf{p}|^2c^2 + m^2c^4,$$

and at rest ($\mathbf{p} = 0$):

$$E = mc^2.$$

5) Interpretation

- Mass is the baseline compression cost for localizing an information packet; energy is the compression-rate.

- Lorentz invariance makes c^2 the conversion factor between spatial and temporal compression scales.

- Thus $E = mc^2$ is the natural equivalence between stored compression complexity (mass) and compression rate (energy).