

Spiral Formation from Information Compression: A Plane-Based Derivation

Abstract

We derive the formation of logarithmic spirals as a direct consequence of information compression on a plane. Using a simple compression functional, we show that extremizing the functional naturally produces spiral trajectories, with pitch angles determined by the balance between radial concentration and angular spreading. This provides a clean and intuitive explanation for the widespread appearance of spirals in natural systems. Although spiral equations can be derived by conventional methods, interpreting spirals as paths that minimize or extremize information compression provides new insight into the emergence of patterns in nature.

1. Introduction: The Pervasiveness of Compression

The Theory of Absolutely Everything (ToAE) proposes that reality fundamentally arises from the recursive compression of information, mapping potential states to actualized configurations. This principle, postulated as the core mechanism of the universe, manifests across scales and domains:

- In **physical systems**, compression governs the formation of structures from atomic lattices to galactic spirals.
- In **biological systems**, it explains patterns in phyllotaxis, shells, and other naturally recurring forms.
- In **informational contexts**, it underlies optimal coding, learning, and the emergence of patterns from complex data.

This work leverages the ToAE postulate of compression as a unifying mechanism, showing that spiral formation is a natural geometric consequence of information density minimization on a plane.

2. Compression Functional

Define the action:

$$\mathcal{C}[\rho] = \int \left[\frac{1}{2} |\nabla \rho|^2 - \lambda \rho^2 \right] dx dy$$

- $\frac{1}{2} |\nabla \rho|^2$ penalizes sharp gradients, enforcing smooth spatial transitions.
 - $-\lambda \rho^2$ incentivizes high local density (compression benefit).
 - Extremizing $\mathcal{C}[\rho]$ yields natural paths along which information concentrates.
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3. Polar Coordinates and Spiral Ansatz

In polar coordinates (r, ϕ) :

$$|\nabla \rho|^2 = \left(\frac{\partial \rho}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial \rho}{\partial \phi} \right)^2$$

Assume a trajectory along which ρ is constant (a level curve of ρ):

$$\frac{d\rho}{dr} = \frac{\partial \rho}{\partial r} + \frac{d\phi}{dr} \frac{\partial \rho}{\partial \phi} = 0 \quad \Rightarrow \quad \frac{d\phi}{dr} = -\frac{\partial \rho / \partial r}{\partial \rho / \partial \phi}$$

Intuition: along the spiral, any radial change is compensated by angular change to stay on a level curve of compression.

4. Self-Similar Density Ansatz

Assume separable density:

$$\rho(r, \phi) = \rho_0(r) g(\phi)$$

Then:

$$\frac{d\phi}{dr} = -\frac{\rho'_0(r) g(\phi)}{\rho_0(r) g'(\phi)}$$

- $\rho_0(r) \rightarrow$ radial decay (compression-driven concentration toward center).
 - $g(\phi) \rightarrow$ angular weighting (spiral winding).
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5. Choice of Simple Functional Forms

- Let $g(\phi) = e^{k\phi} \rightarrow$ exponential growth in ϕ , capturing self-similarity.
- Let $\rho_0(r) \sim r^{-\beta} \rightarrow$ simple radial compression profile.

Then:

$$\frac{d\phi}{dr} = \frac{\beta}{k} \cdot \frac{1}{r}$$

6. Integrate to Obtain the Spiral

$$\phi(r) = \phi_0 + \frac{\beta}{k} \ln(r/r_0)$$

- This is a **logarithmic spiral**, with pitch determined by $p = \arctan(k/\beta)$.

Intuition: - Radial compression (β) \rightarrow tighter winding. - Angular weighting (k) \rightarrow looser spiral. - Spirals emerge as a balance between radial concentration and smooth angular spreading.

7. Key Points

1. The spiral is an **extremal curve** of the compression functional.
 2. Pitch angle is a direct measure of the ratio of angular growth to radial compression.
 3. Provides a **clean, intuitive explanation** for the appearance of spirals in natural systems.
 4. Can be extended to 3D or curved spaces, allowing incorporation of mass or curvature for astrophysical applications.
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8. Conclusion and Extensions

This work demonstrates that logarithmic spirals naturally emerge from the extremization of a compression functional on a plane. The methodology is general and can be extended to higher-dimensional spaces, curved manifolds, or systems with additional physical fields. Such extensions open the door to modeling spiral structures in galaxies, accretion disks, and other complex systems under the framework of the ToAE.

References

- Andrade, P.R., *Theory of Absolutely Everything*, Zenodo (2025). doi:10.5281/zenodo.17042178.