Electrostatic Potential Energy

In physics, the conservation of energy is one of the fundamental principles that states that the total energy of an isolated system remains constant over time. This principle applies to a wide variety of physical situations, including the movement of objects under the action of conservative forces. In particular, when a conservative force acts on an object, the work done by this force becomes a change in the otential energy of the object.

$$dU = -\boldsymbol{F}dl$$

Where $\boldsymbol{F}$ is the conservative force.

A conservative force is one that, by doing work on an object, does not change the total amount of mechanical energy in the object. This is because the total energy of an object in a conservative field can be expressed as the sum of its kinetic energy and its potential energy. Kinetic energy is the energy associated with the motion of the object, while potential energy is the energy stored in the object due to its position in the conservative field.

$$\Delta U + \Delta K = 0$$

Potential energy is a form of energy that an object possesses due to its position in a conservative field. In other words, it is the energy that an object has due to its location in space in relation to other objects or systems. Potential energy can be defined as the amount of work that can be done by releasing the energy stored in an object in a conservative field. For example, in a gravitational field, the potential energy of an object is defined as the energy that the object has due to its position in the gravitational field.

Electrostatic potential energy refers to the energy that a charged object has due to its position in an electric field. Electrostatic potential energy can be defined as the amount of work that can be done in moving an electric charge from one location to another in the presence of an electric field. In other words, it is the energy that an electric charge has due to its location in space in relation to other charged objects or systems.

$$dU = -q_0\boldsymbol{E}.dl$$

When work is done on an object by a conservative force, this force transfers energy from the object to the conservative field, which decreases the object's potential energy and increases its kinetic energy. In other words, the work done by a conservative force is converted into a change in the potential energy of the object.

For example, in the case of gravitational potential energy, when an object is lifted to a certain height, work is done against the earth's gravitational force. This work becomes an increase in the object's gravitational potential energy, since energy has been stored in the object due to its position in the gravitational field. When the object is released and falls freely, gravitational potential energy is converted to kinetic energy, since the object gains speed as it falls.

This process can be better understood through an example. Consider a ball on top of a mountain that is released, starting toroll down the mountainside. As the ball descends, the gravitational potential energy of the system decreases and the kinetic energy of the ball increases. The gravitational force acting on the ball is conservative, and the work done by this force becomes a change in the potential energy of the system.

In the case of electrostatic potential energy, when an electric charge moves in the presence of an electric field, work is done against the electrostatic force. This work becomes an increase in the electrostatic potential energy of the charge, since energy has been stored in the charge due to its position in the electric field. When the charge is released, the electrostatic potential energy is converted to the kinetic energy of the charge, since the charge gains speed as it moves in the electric field. It is important to note that conservation of energy holds at all times, which means that the total energy of the system, made up of kinetic energy and potential energy, remains constant.

In short, work done by a conservative force decreases the potential energy of an object in a conservative field, either gravitational or electrostatic. Potential energy is a form of energy that is stored in an object due to its position in the conservative field, and can be converted to kinetic energy by releasing the object. Conservation of energy is a fundamental principle in physics and is true at all times, which means that the total energy of a system remains constant. Understanding these concepts is fundamental to understanding a wide variety of physical phenomena and is essential for the study of physics.

Potential difference (Voltage)

When a particle with charge q_0 is subjected to an electric field E, this charge will experience a force F.

$$\boldsymbol{F} = \boldsymbol{E}q_0$$

This force can move the particle q_0 from an initial point A to another final point B. Such that during this journey it experiences a variation of electrostatic potential energy.

$$\Delta U = U_b - U_a = \int_a^b dU = -\int_a^b q_0 \boldsymbol{E}.dl$$

If we consider this variation of energy per unit charge, we can define the potential difference

$$\Delta V = \frac{\Delta U}{q_0} = -\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{E}.dl$$

Where a and b are points in space and the displacement is finite for q_0

The function V that is described by the above equation is called electric potential or simply potential. Since a potential difference can be defined, this means that an electric potential can be defined at each point in the system in such a way that:

$$\Delta V = V_b - V_a = -\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{E}.dl$$

Its measure by the $\frac{\bar{\boldsymbol{J}}}{\boldsymbol{C}} = \boldsymbol{V}$, where is defined as the amount of work required to move a unit charge from one point to another in an electrical circuit. For example, if a battery has a potential difference of 9 volts, it means that the battery can do 9 joules of work for every coulomb of charge that flows through the circuit.

The value of the electric potential function at any point is normally determined by arbitrarily choosing V to be zero at a suitable point. For example, in the expression of gravitational potential energy, a height is chosen as reference $h = 0$. If it is the case of the potential function, it is better to take as reference an infinite distance to the origin of the electric field.

This potential function is very important in engineering since it establishes the principle of what electricity is and since it is normally measured in volts, the electric potential function is known as Voltage.

This electric field can be created by various types of sources that can be point or distributed surfaces and this expression reflects important characteristics about the behavior of charges against electric fields.

Given an electric field, if we drop a positive test charge at a point in that field, that is, we leave it free, this charge will accelerate in the direction of the electric field along the line of that field. This will cause the kinetic energy to increase, but its potential energy to decrease.

Taking into account the behavior of charges against electric fields, we can say that a positive point charge will always go from a potential of greater value to a potential of lower value within an electric field, this because its potential energy tends to decrease.

Thus the charge moves towards a region of lower electrostatic potential energy in the same way that a falling body loses potential energy.

On the other hand, if we leave a negatively charged body free, the effect will be the opposite, that is, instead of moving from a point of higher potential to a point of lower potential, it will move from a point of lower potential to one of higher potential. , looking for a region of higher potential energy, due to the nature of the negative charge going against the direction of the electric field.

But this raises an interesting question. What happens if a positive point charge is being moved from a region of lower potential energy to a region of higher potential energy, given an electric field. This means that there is an external force that produces this work against the proper work of the electric field. It means, therefore, that a potential energy is being included in the body that makes it go to a region of greater potential energy, contrary to the natural form of movement when it is left free.

In summary, an electric field generates in a body an acceleration and at the same time a decrease in potential energy that is expressed by means of the electric potential function (Voltage). The voltage at a certain point of the electric field determines if it is an area of greater electric potential energy or if it is a region of lower electric potential energy and this has several practical applications such as electric circuits.

Voltage at point charges

We know that a positive point charge produces an outgoing radial electric field which is given by

$$\boldsymbol{E} = \frac{kq}{r^2}\hat{\boldsymbol{r}}$$

We also know that a negative point charge produces an incoming radial electric field.

Regardless of the net charge of the point mass, we can establish that said particle will produce a different field intensity along r. That. is, in the environment close to said point mass, the electric field intensity will be greater than in an increasingly distant environment, until its electric field intensity is zero.

If we place a test charge in this electric field generated by the point charge, the test charge will undergo an acceleration due to the electric force.

At the same time that it accelerates, its potential energy decreases, so that a function is established that measures this decrease in the potential energy of the test charge along the field, this function is known as the electric potential function. or voltage which is defined as:

$$dV = -\boldsymbol{E}.dl$$

The voltage will decrease as the analysis point moves away from the source charge that generates the field, this is easily verifiable, because the electric field in areas far from the charge that generates it will decrease because the electric field intensity is inversely proportional to the square of the distance, that is to say that at a reasonably distant point the electric field will be 0.

It is for this reason that at a far point in the field it can be established that the voltage is 0 (since the intensity of the field is very low or null). And taking the definition into account, we can see that the voltage will also decrease in relation to the distance.

Taking into account the mathematical definition we can say that:

$$dV = -Edl = -\frac{kq}{r^2}dr$$

Because in a r = ∞ the electric field will be equal to 0, then the voltage at a distant point will be equal to 0, therefore, the voltage specified as a function of distance due to an electric field generated by a point charge will be expressed as

$$V = \frac{kq}{r}$$

Where the potential will be positive or negative depending on the charge of q.We also know that the work generated by a force can be defined mathematically as:

$$W = \int_A^B \boldsymbol{F}.dl$$

Finally, the question arises, what happens if we want to analyze the voltage at a point in space, but this space is being affected by several point charges. For this we will use the superposition of effects, that is, the voltage at a point in space will be equal to the sum of the voltage produced by each of the point charges at that point of analysis.