

# QRAND

A multi-platform and multi-protocol quantum random number generator for arbitrary probability distributions

Pedro Rivero – April 30, 2021



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  - *Sycamore protocol*
  - *BCMV protocol*
- QRAND

# The need for *random numbers*\*

## Applications

- Information/communications security (e.g. *crypto-systems, IoT*)
- Computation (e.g. *algorithms, physical simulations, Monte Carlo*)
- Financial systems (e.g. *transactions, credit card chips*)
- Statistical sampling (e.g. *scientific experiments, social studies*)
- Testing (e.g. *randomized benchmarking*)
- Legal and trust sensitive processes (e.g. *jury, lotteries, Kleroterion*)

\**Randomly generated numbers*

# Building blocks of a crypto-system

## Migration to quantum safe solutions

- These can all be performed quantumly:

### HIDING MECHANISM

Algorithm



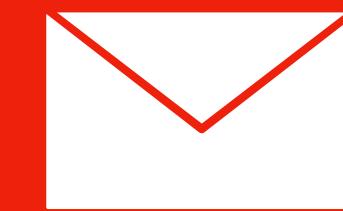
### UNPREDICTABLE SECRET

Random key generation



### SECRET DISTRIBUTION

Key exchange  
(BB84)



# What we mean by random

## A slippery concept

- The attribute of being random applies more correctly to a sequence of numbers:  
It is strictly related to **lack/impossibility of predictability**.
- The impossibility of **compression** can be considered as the defining trait of randomness (*Chaitin & Kolmogorov*).
- **Shannon Entropy (H)** → Measure of disinformation/uncertainty

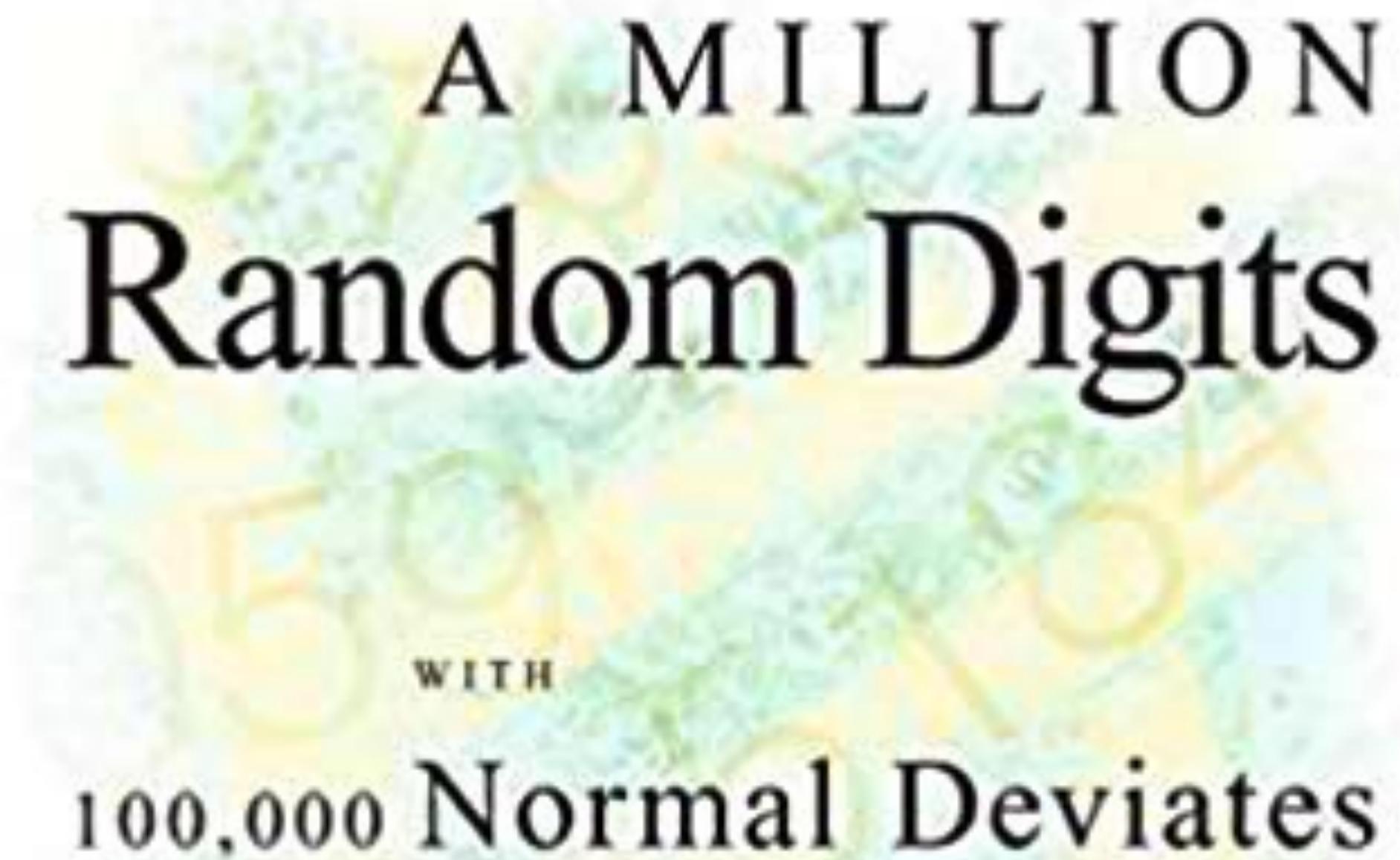
*The minimum number of yes/no questions that need to be answered about something to be able to guess it correctly with 100% confidence.*

- Bit strings which are not random have limited entropy and can therefore be described with **fewer bits than the string itself**.

# Randomness quality

## Output tests

- It is nearly impossible to establish whether a source is truly random.
- We can analyze the general properties of output strings (e.g. weight)
- A passing output will always pass (the tests): an adversary might *predict* or *fabricate* the output.
- Numbers are not inherently random *per se*:  $R(NG) \neq (RN)G$



# A MILLION Random Digits

WITH  
100,000 Normal Deviates



TDB

★★★★★ Not really random

Reviewed in the United States on September 26, 2012

I bought two copies of this book. I find that the first copy perfectly predicts what the numbers will be in the second copy. I feel cheated.

141 people found this helpful

# Random number generation

## Properties

- **Uniformity** → all sequences have the same probability of occurring
- **Scalability** → any test applicable to a sequence must apply to subseq.
- **Consistency** → the behavior must be consistent across all outputs
- **Forward unpredictability** → the next output should not be predictable
- **Backward unpredictability** → previous outputs should not be predictable

# Random number generation

## The challenge

- Generating random numbers is hard → you need to get your hands dirty and start experimenting with physical hardware.

### RANDOM PROCESS

To conveniently sample  
from for getting entropy

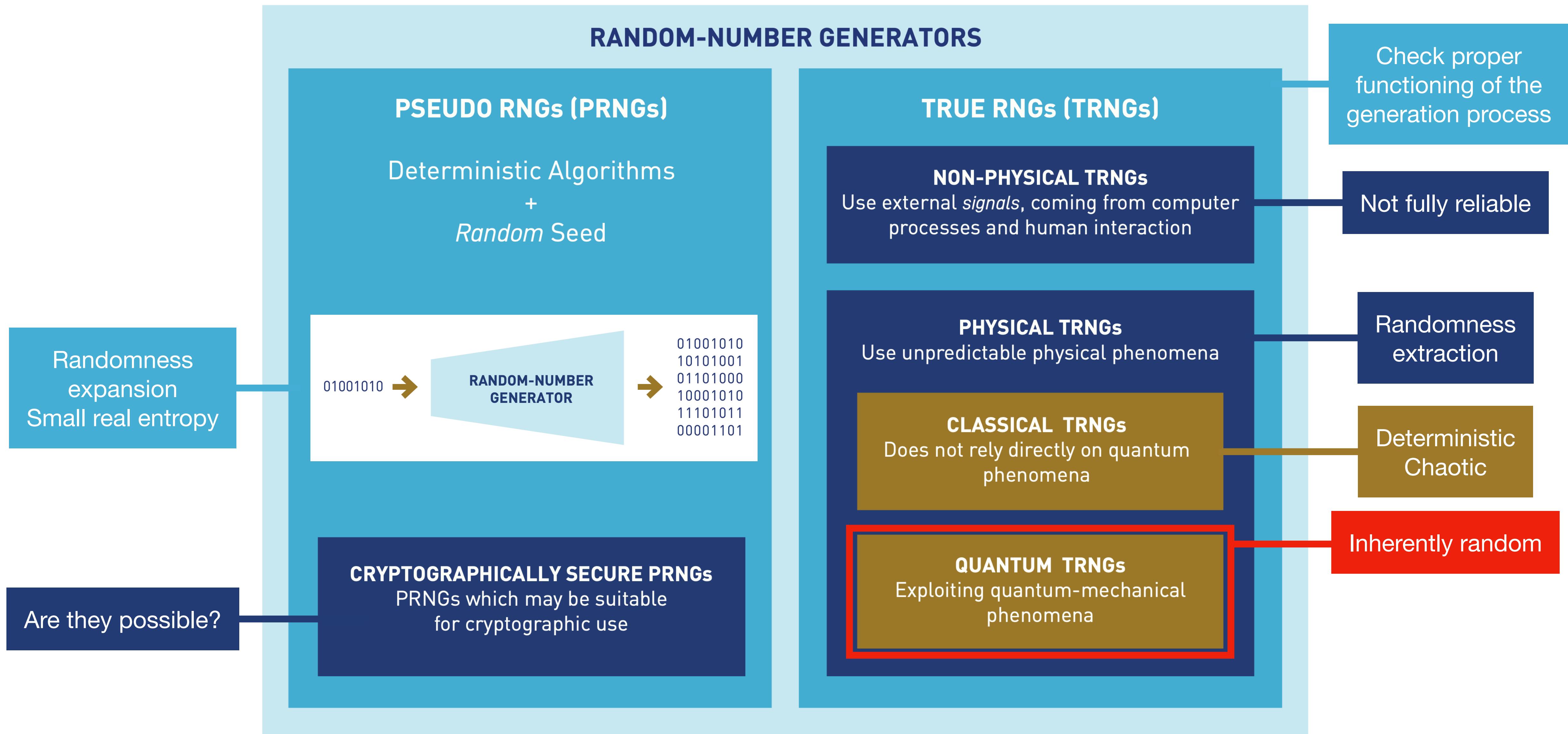
### NO SECURITY BY OBSCURITY

Transparent entropy  
sources

### RANDOM NUMBER DELIVERY

Keep the entropy pure  
(no leaks)

# Random number generation

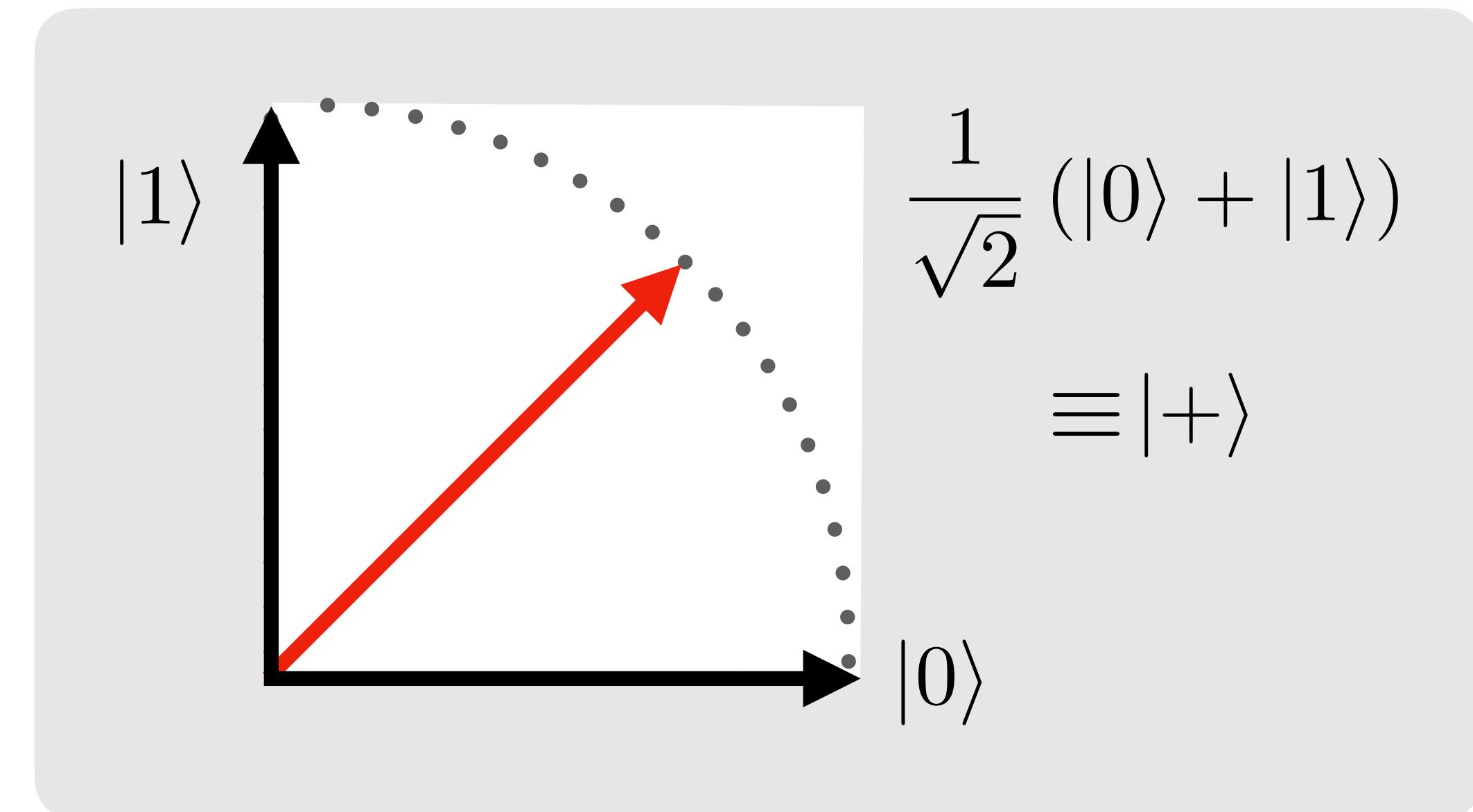


# Quantum random number generators

## Quantum mechanics primer

Bits:  $\{0, 1\}$   $\rightarrow$  Qubits:  $\{|0\rangle, |1\rangle\}$

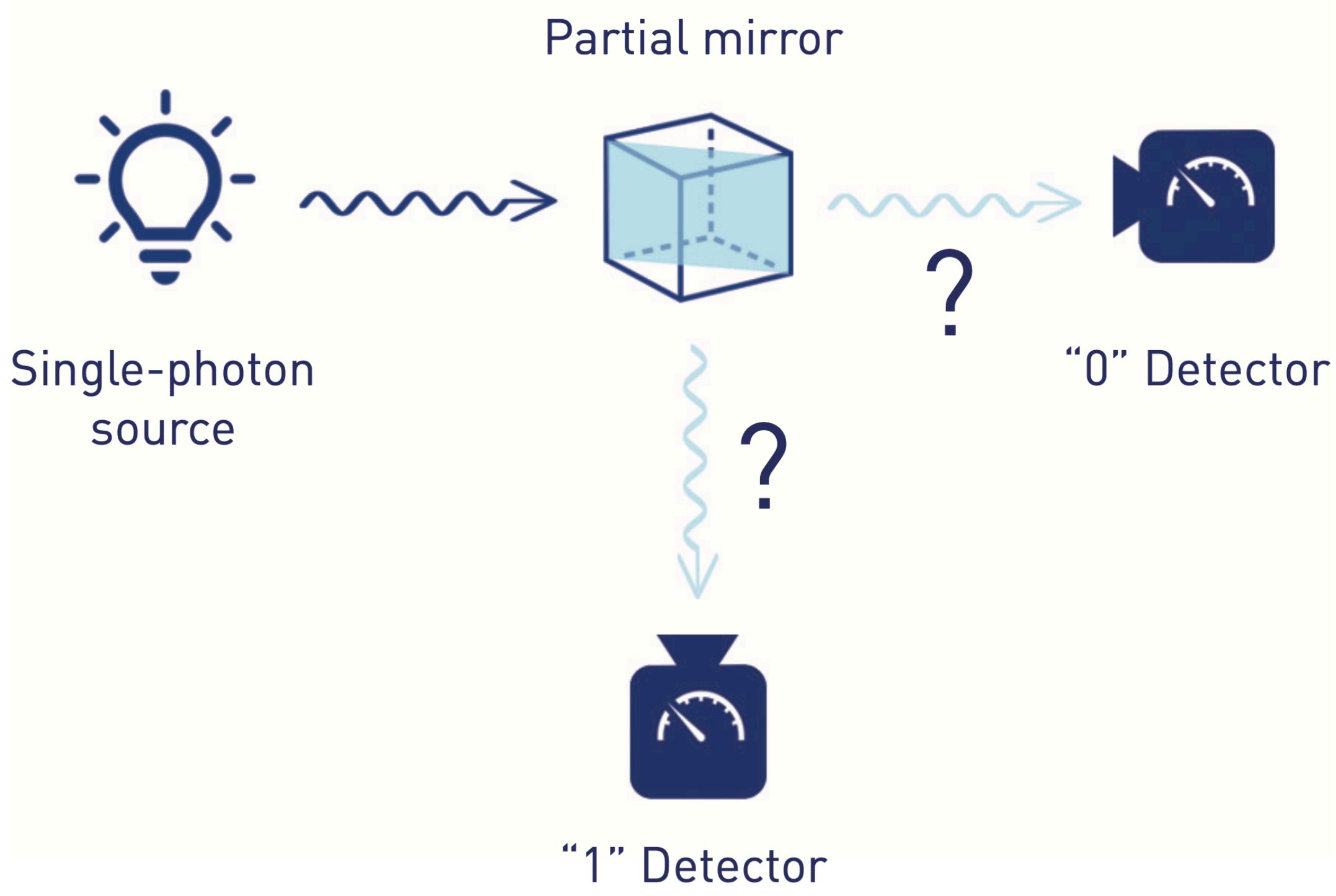
- **Qubits**  $\rightarrow$  quantum system with two possible (distinguishable) states
- **Superposition**  $\rightarrow$  a quantum system can be in a state which is a mix of classically distinguishable states.
- **Inherently random measurements**
- **Entanglement**  $\rightarrow$  classically inexplicable correlations between subsystems



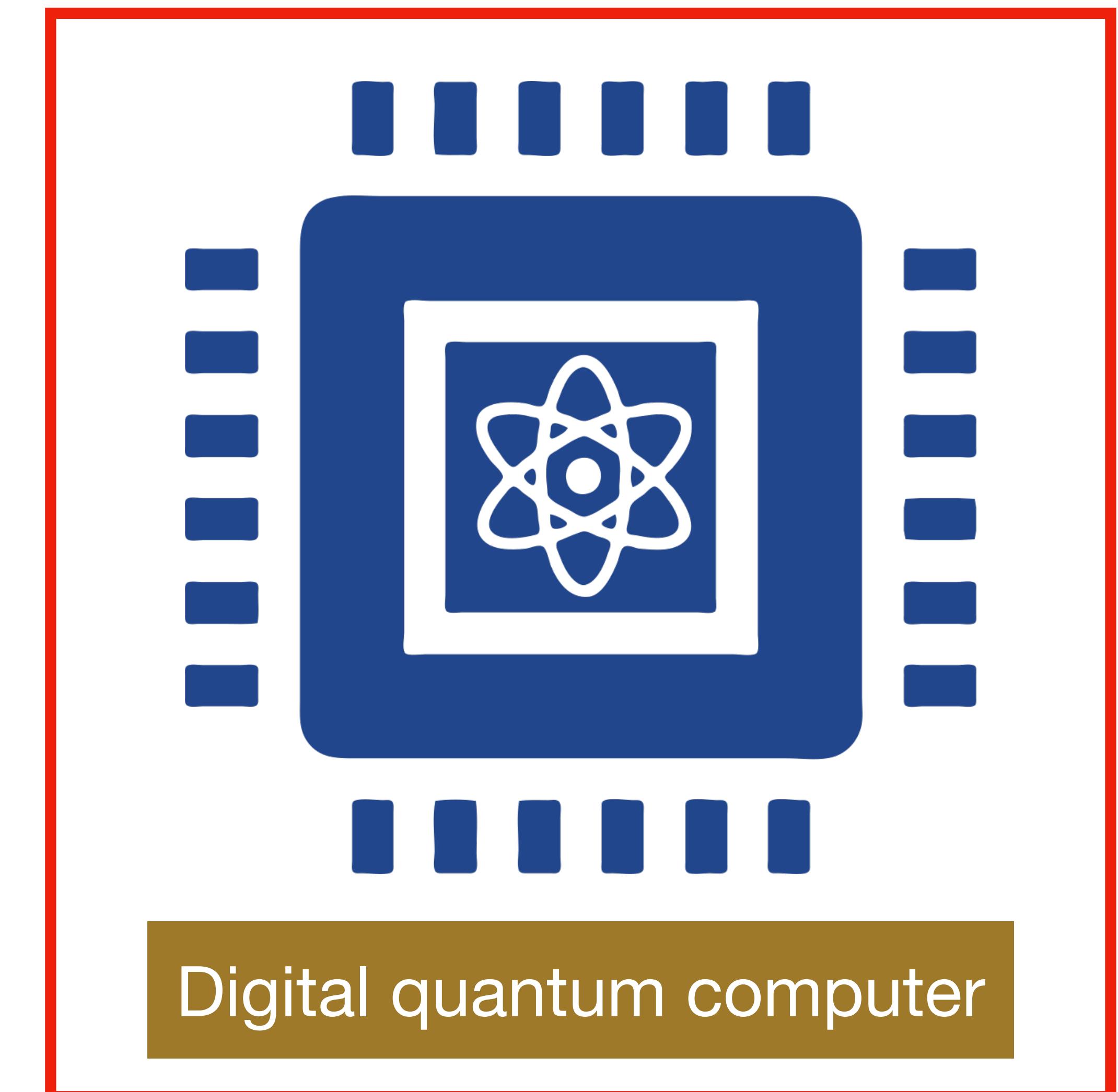
$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

# Quantum random number generators

## Practical realizations

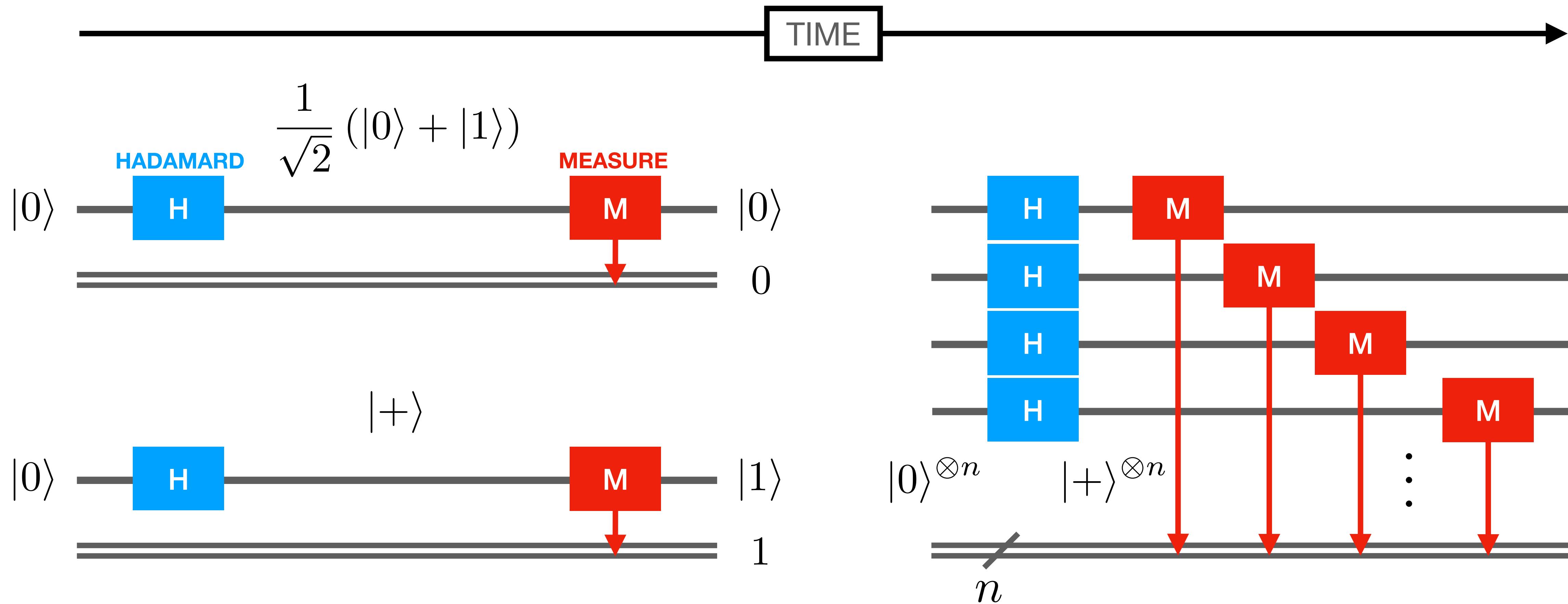


Analog quantum hardware



# Hadamard Protocol

The naive approach using quantum circuits



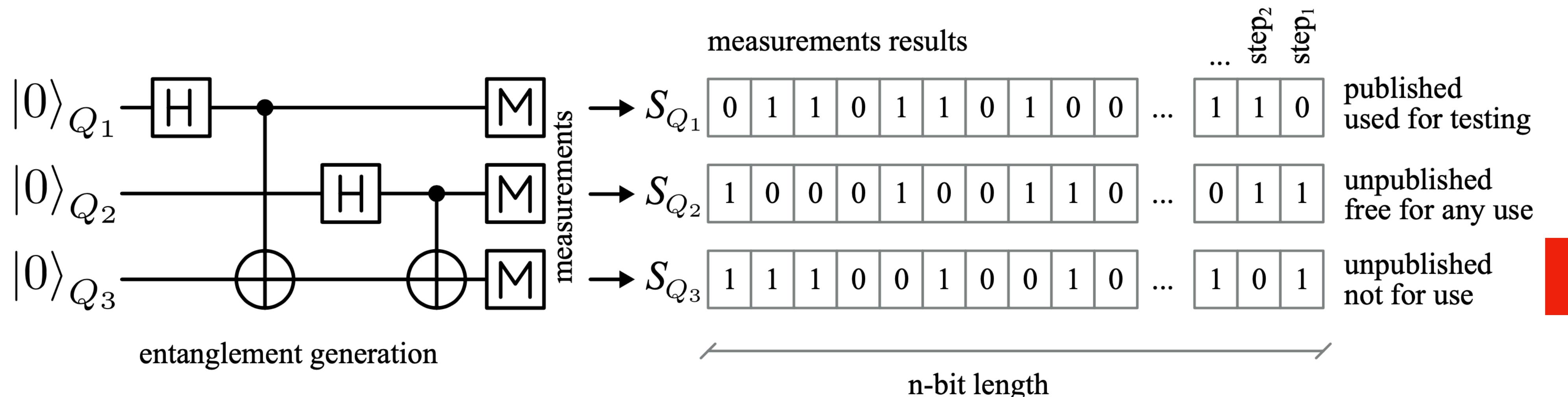
# Entanglement Protocol

## Validation outsourcing

- Subsequence validation → Exponential overhead

$$|\psi_{Q_1 Q_2 Q_3}\rangle = \frac{1}{2} (|000\rangle + |011\rangle + |101\rangle + |110\rangle)$$

TWO BITS OF ENTROPY



# Entanglement Protocol

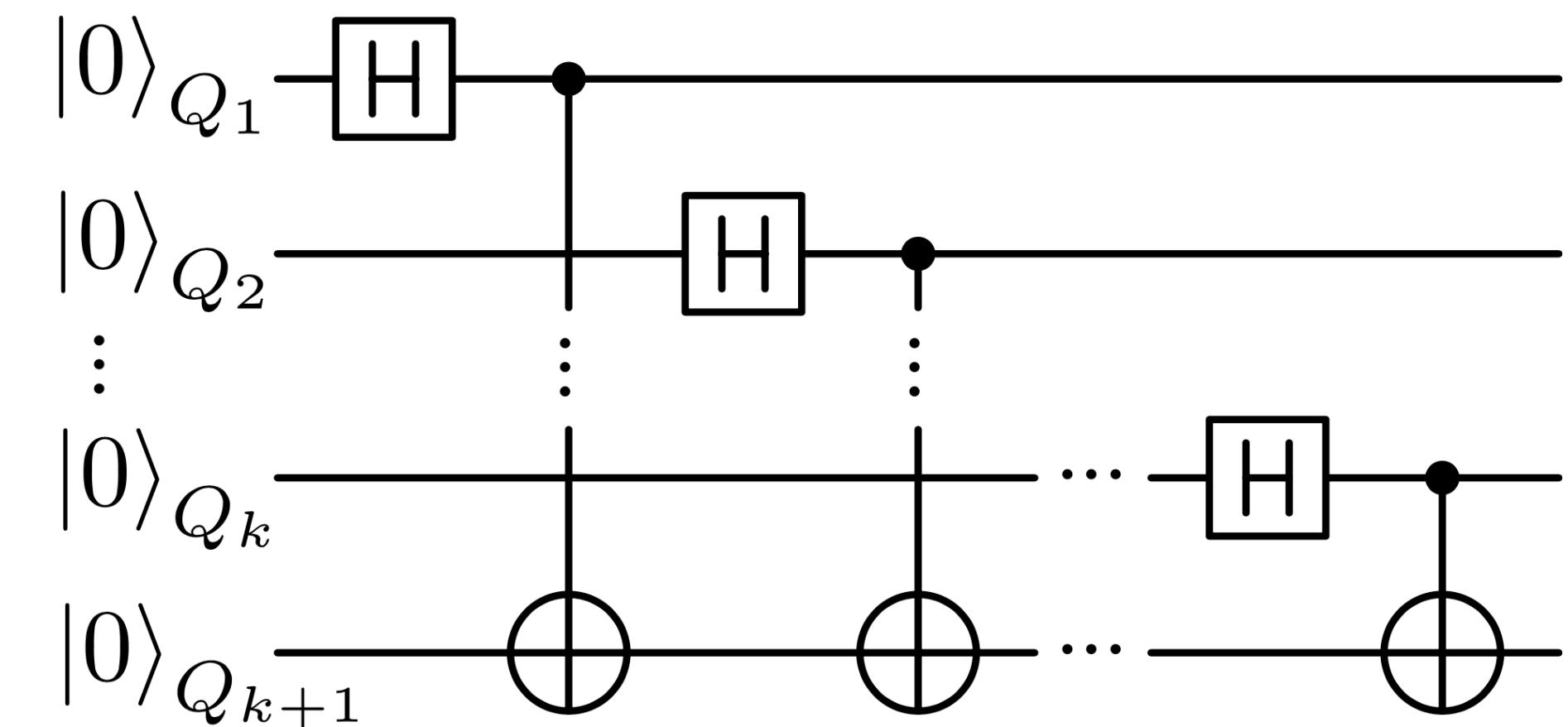
## Error correction and scaling

- Parity bit

|           |  |     |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
|-----------|--|-----|-----|-----|-----|-----|-----|-----|-----|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--|---|---|---|-----|-----|-----|
| $S_{Q_1}$ | <table border="1"><tr><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td><td>XOR</td></tr></table> | 0   | 1   | 1   | 0   | 1   | 1   | 0   | 1   | 0 | 0 | XOR | ... | <table border="1"><tr><td>1</td><td>1</td><td>0</td></tr><tr><td>XOR</td><td>XOR</td><td>XOR</td></tr></table> | 1 | 1 | 0 | XOR | XOR | XOR |
| 0         | 1  | 1   | 0   | 1   | 1   | 0   | 1   | 0   | 0   |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| XOR       | XOR  | XOR | XOR | XOR | XOR | XOR | XOR | XOR | XOR |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| 1         | 1  | 0   |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| XOR       | XOR  | XOR |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| $S_{Q_2}$ | <table border="1"><tr><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td></tr></table>                     | 1   | 0   | 0   | 0   | 1   | 0   | 0   | 1   | 1 | 0 | =   | =   | =   | =   | =   | =   | =   | =   | =   | =   | ... | <table border="1"><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>=</td><td>=</td><td>=</td></tr></table>       | 0 | 1 | 1 | =   | =   | =   |
| 1         | 0  | 0   | 0   | 1   | 0   | 0   | 1   | 1   | 0   |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| =         | =  | =   | =   | =   | =   | =   | =   | =   | =   |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| 0         | 1  | 1   |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| =         | =  | =   |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| $S_{Q_3}$ | <table border="1"><tr><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td></tr><tr><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td><td>=</td></tr></table>                     | 1   | 1   | 1   | 0   | 0   | 1   | 0   | 0   | 1 | 0 | =   | =   | =   | =   | =   | =   | =   | =   | =   | =   | ... | <table border="1"><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>=</td><td>=</td><td>=</td></tr></table>       | 1 | 0 | 1 | =   | =   | =   |
| 1         | 1  | 1   | 0   | 0   | 1   | 0   | 0   | 1   | 0   |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| =         | =  | =   | =   | =   | =   | =   | =   | =   | =   |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| 1         | 0  | 1   |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |
| =         | =  | =   |     |     |     |     |     |     |     |   |   |     |     |     |     |     |     |     |     |     |     |     |  |   |   |   |     |     |     |

PURIFY ENTROPY

1 SEQUENCE TO VALIDATE

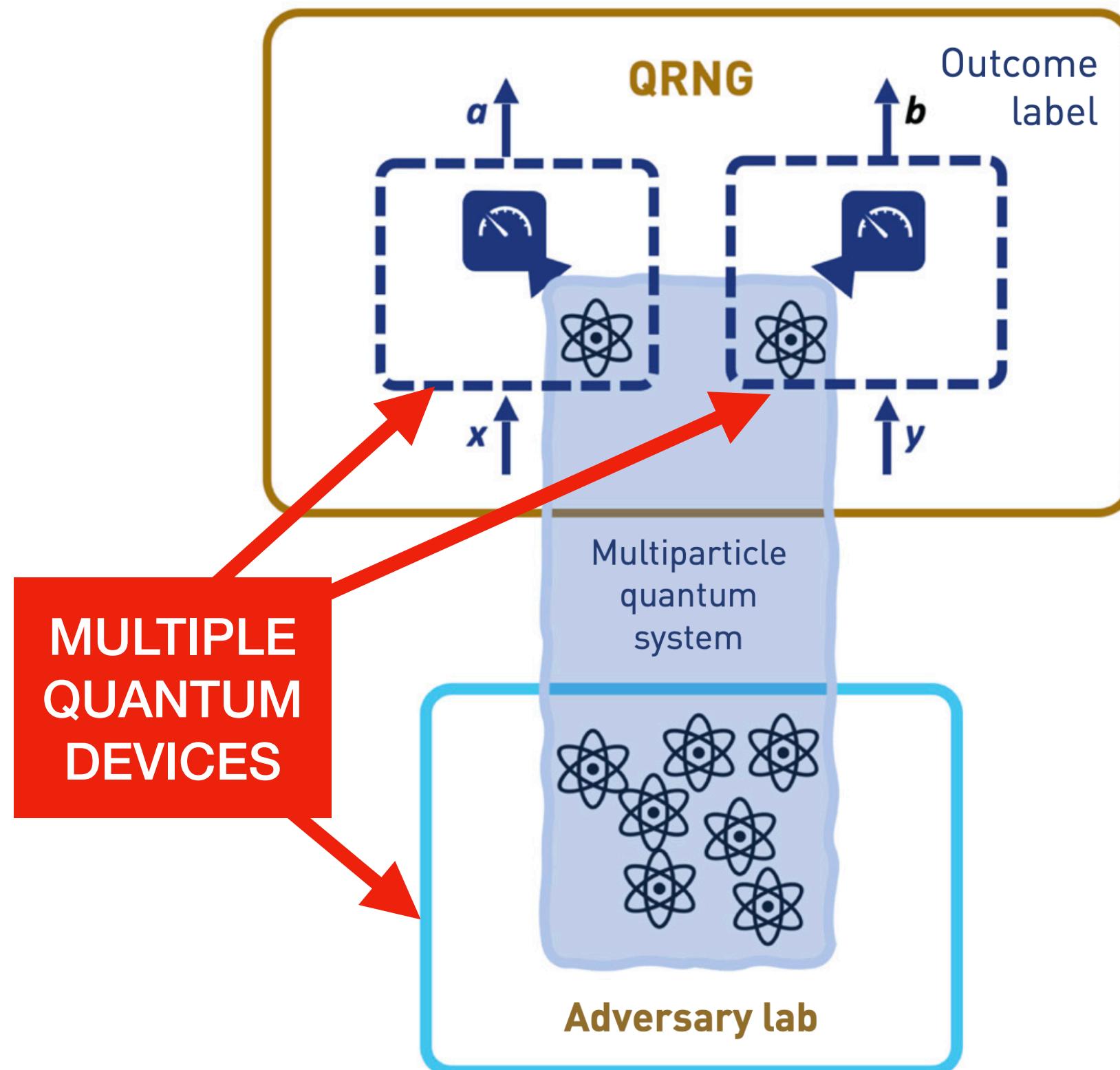


K-1 USABLE SQUENCES

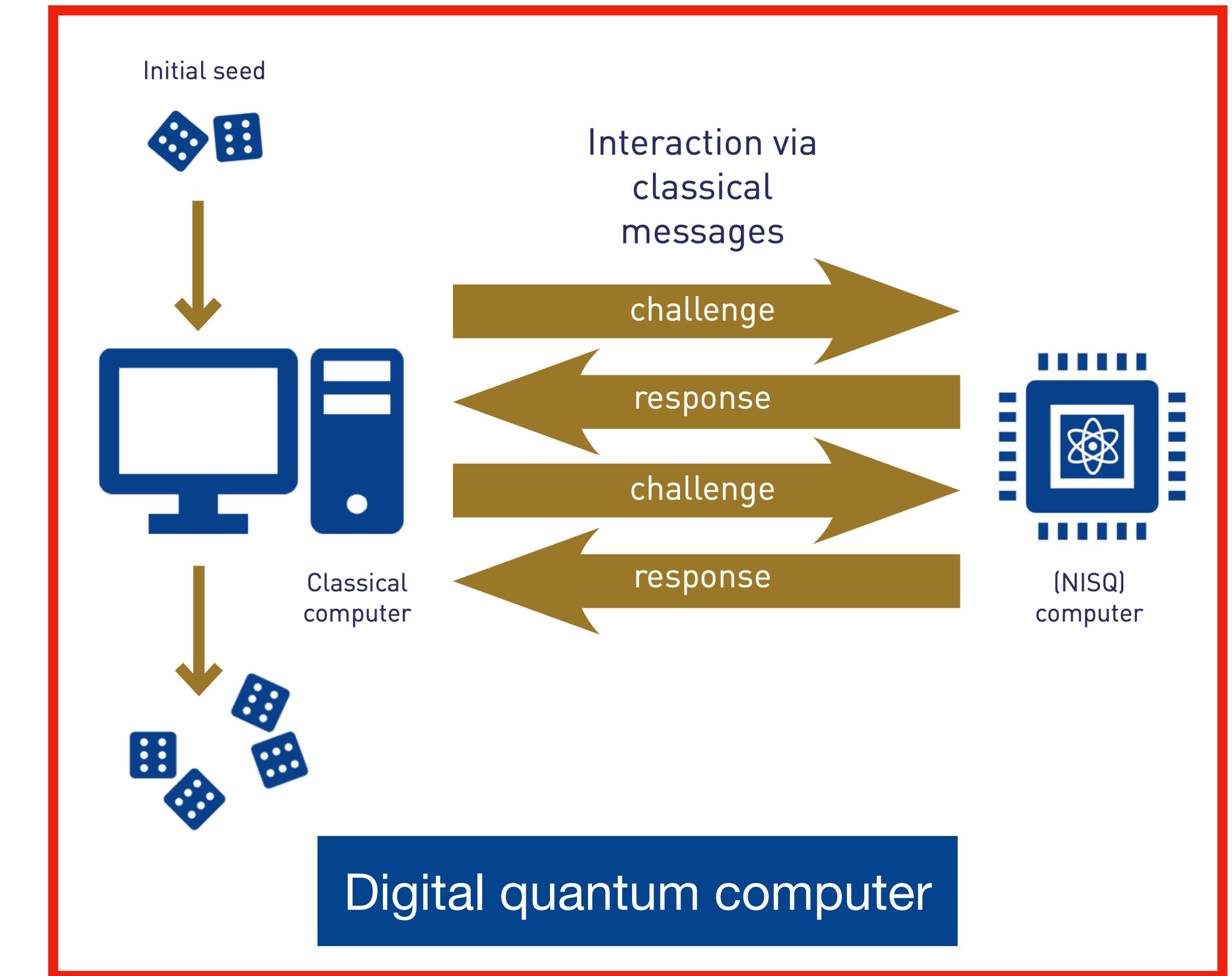
PARITY

# Verification strategies

## Device independency



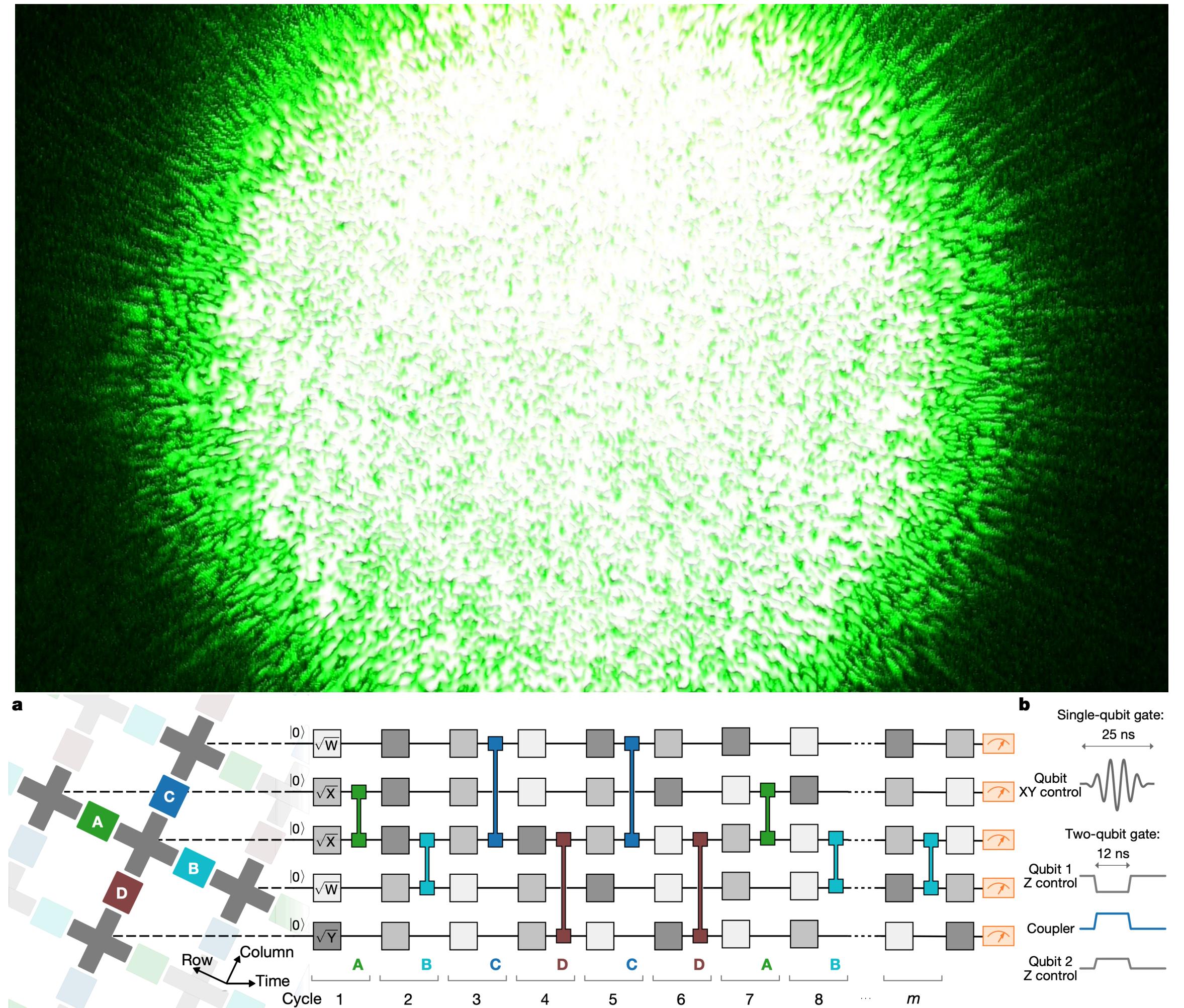
Locality proofs



# Sycamore Protocol

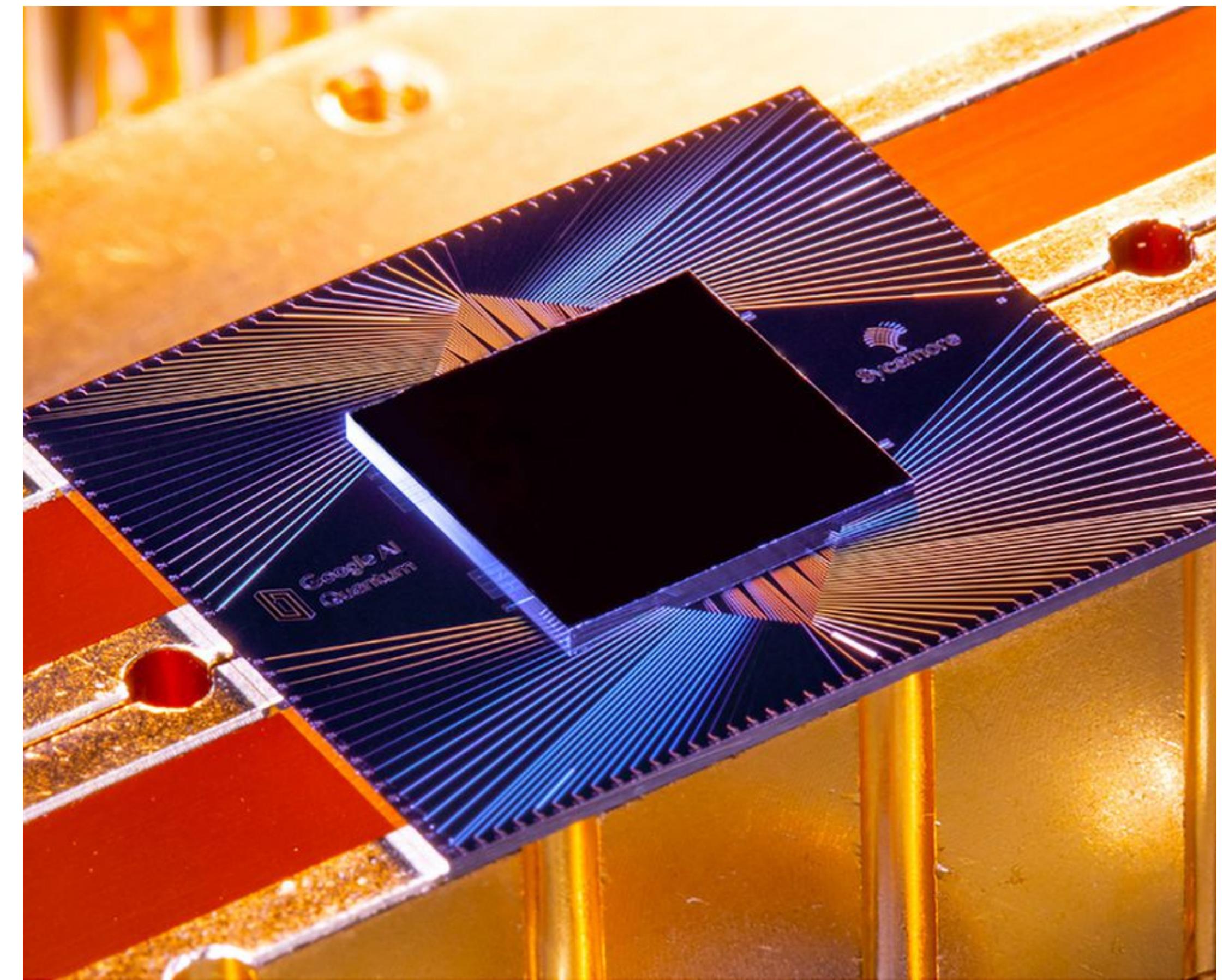
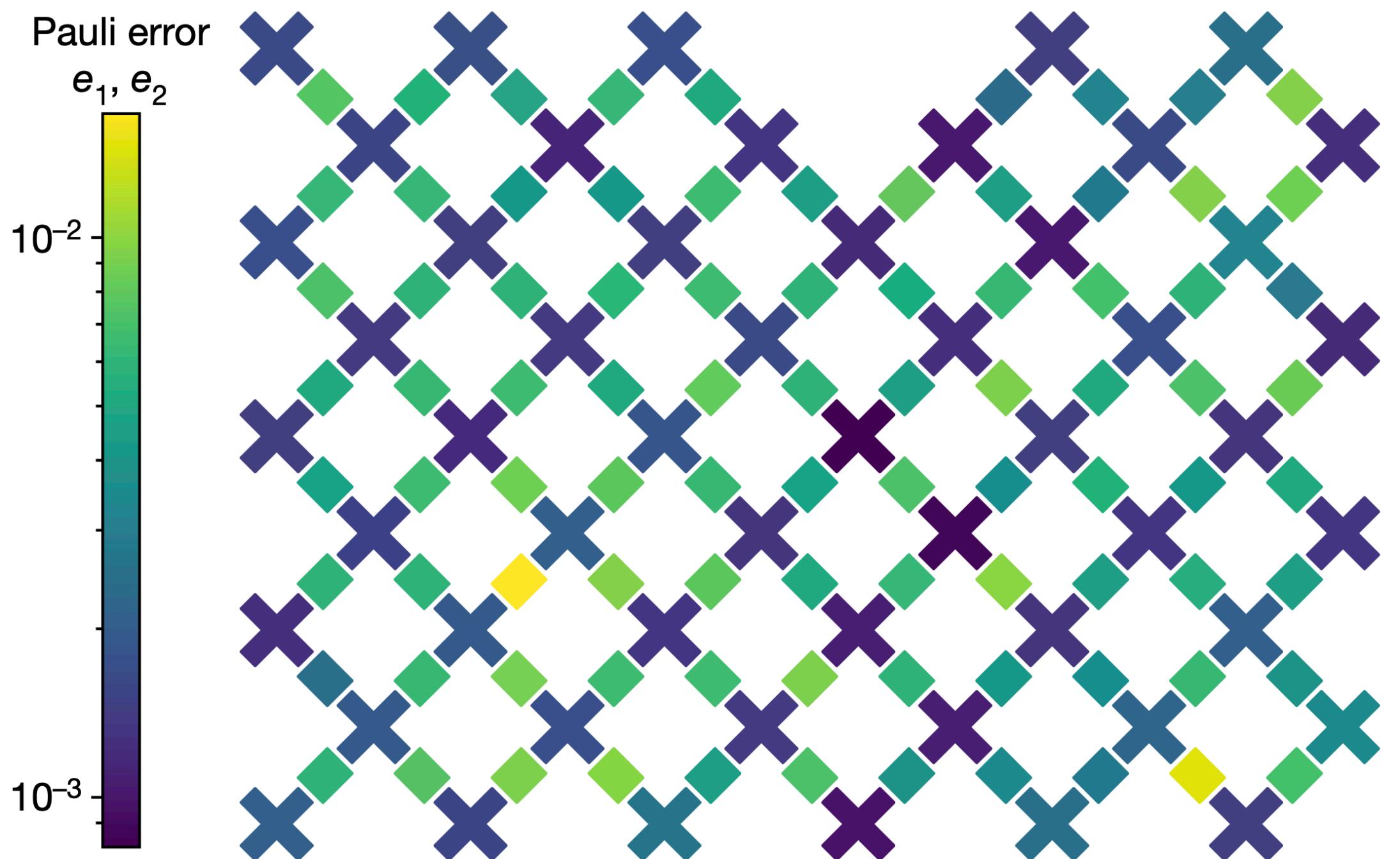
Scott Aaronson

- Process:
  1. *Run a randomly chosen (known) circuit.*
  2. *Make a "small" number of measurements.*
  3. *Simulate results classically and see if they agree.*
- Computational hardness assumption.



# Sycamore Protocol

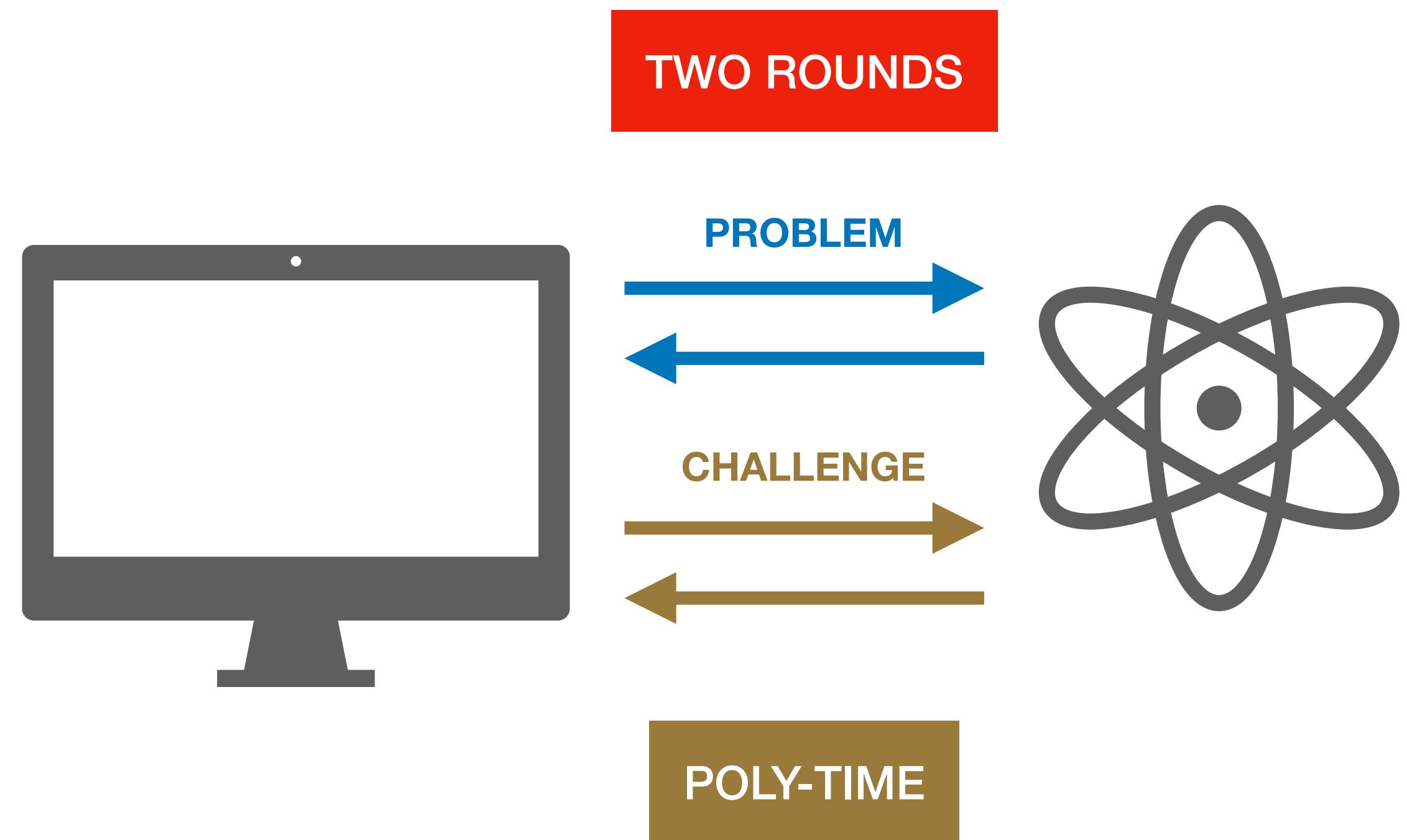
Google's quantum supremacy claim



# BCMV Protocol

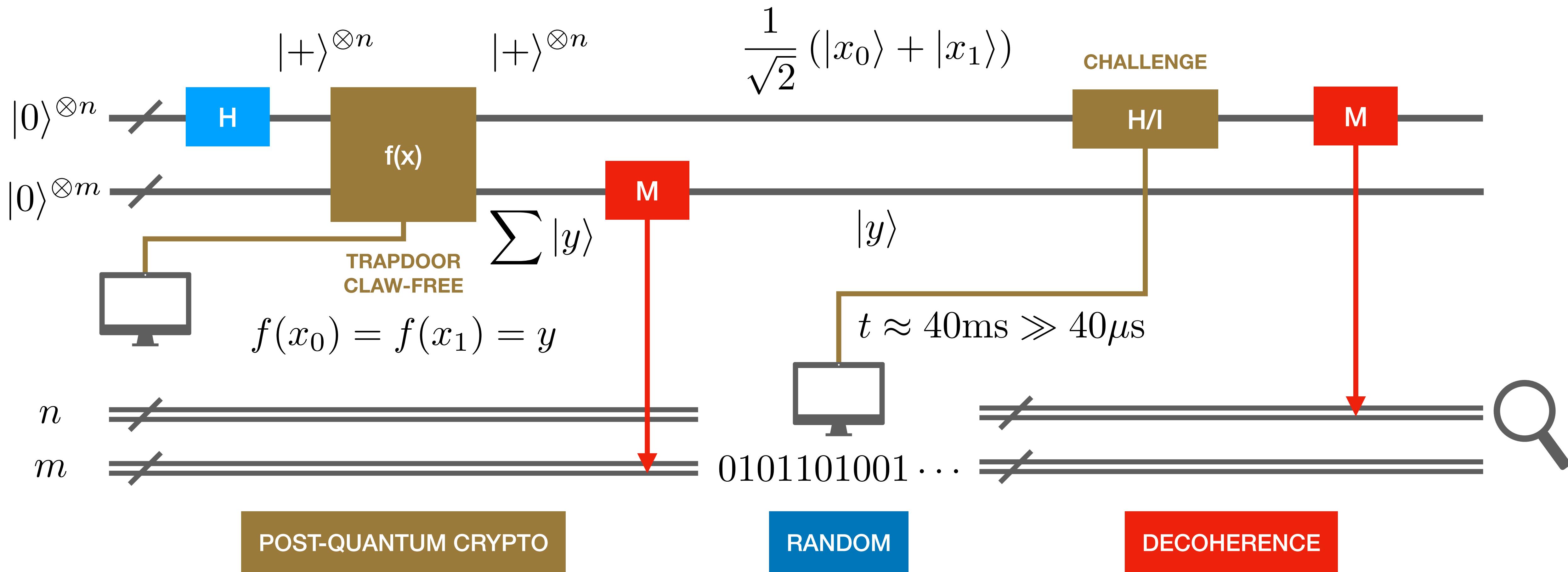
## Post-quantum cryptography approach

- Previous approach relied on the classical exponential overhead → *NISQ devices*
- **Post-quantum crypto** (not to be confused with quantum crypto)
- Two rounds:
  - *Problem*
  - *Challenge*



# BCMWV Protocol

Post-quantum cryptography approach



# QRAND pypi

## Free and Open Source Software

- Multiprotocol → *Hadam, Entang, ...*
- Multiplatform → *qiskit, cirq, q#*
- Output formats → *bitstring, int, float, complex, hex, base64...*
- Entropy validation suite
- NumPy interface → non-uniform probability distributions
- Efficiency sampling → *Backend Select. + Multithreading + Caching*

```
from qrand import QiskitBitGenerator
from qiskit import IBMQ

provider = IBMQ.load_account()
bitgen = QiskitBitGenerator(provider)

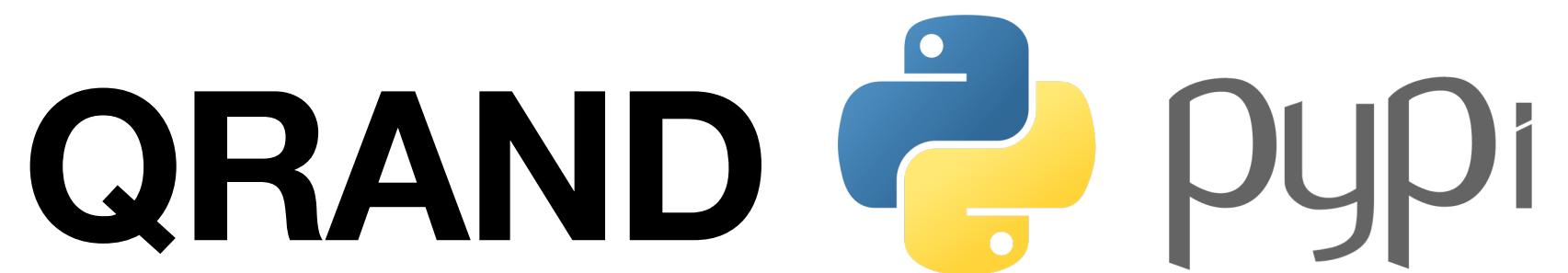
s = bitgen.random_bitstring(2)
i = bitgen.random_uint(4)

from qrand import Qrng
qrng = Qrng(bitgen)

x = qrng.get_random_double(0, 0.1)
z = qrng.get_random_complex_polar()

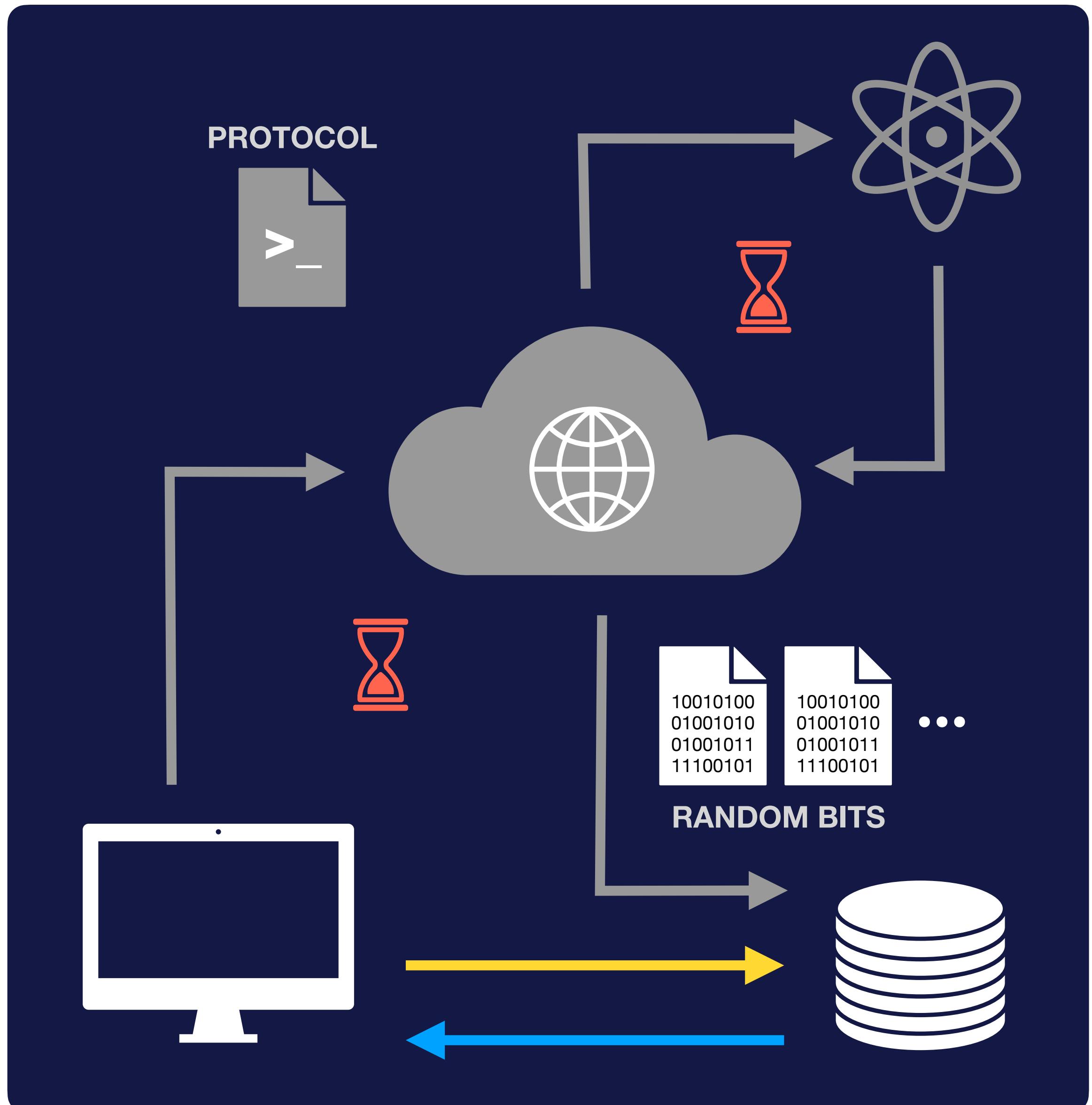
from numpy.random import Generator
gen = Generator(bitgen)

l = gen.logistic()
p = gen.poisson()
```



Free and Open Source Software

- Multiprotocol → *Hadam, Entang, ...*
- Multiplatform → *qiskit, cirq, q#*
- Output formats → *bitstring, int, float, complex, hex, base64...*
- Entropy validation suite
- NumPy interface → non-uniform probability distributions
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# Unitary

May 14–30<sup>th</sup>

# HACK

The Unitary Fund is proud to host our first quantum open source hackathon with SWAG and BOUNTIES on May 14–30th!

[hack2021.unitary.fund](https://hack2021.unitary.fund)

Digital swag for  
all participants

>\$2k USD in  
bounties

Random drawings  
for swag mail

# References

- (1) Piani et al, *Quantum Random-Number Generators: Practical Considerations and Use Cases*. **EvolutionQ**.
- (2) Abellán, *Randomness and quantum entropy w/ Carlos Abellán from Quside. Quantum Barcelona*.
- (3) Jacak et al, *Quantum random number generators with entanglement for public randomness testing*. **Nature Research**.
- (4) Arute et al, *Quantum supremacy using a programmable superconducting processor*. **Nature**.
- (5) Brakerski et al, *A Cryptographic Test of Quantumness and Certifiable Randomness from a Single Quantum Device*. **IEEE 59th Annual FOCS Symp.**

**Thanks**  **Unitary  
Fund**

Pedro Rivero

**ILLINOIS  
TECH**

CHICAGO  
**QUANTUM  
EXCHANGE**

Argonne  
NATIONAL LABORATORY 

# Migration to quantum safe solutions

Why it is important to act now

**COMPLIANCE  
WITH REGULATIONS**

NIST standards

**LONG  
MIGRATION TIME**

Crypto-agile  
methodologies

**PRODUCTS WITH  
LONG LIFETIMES**

Car industry, self-serving

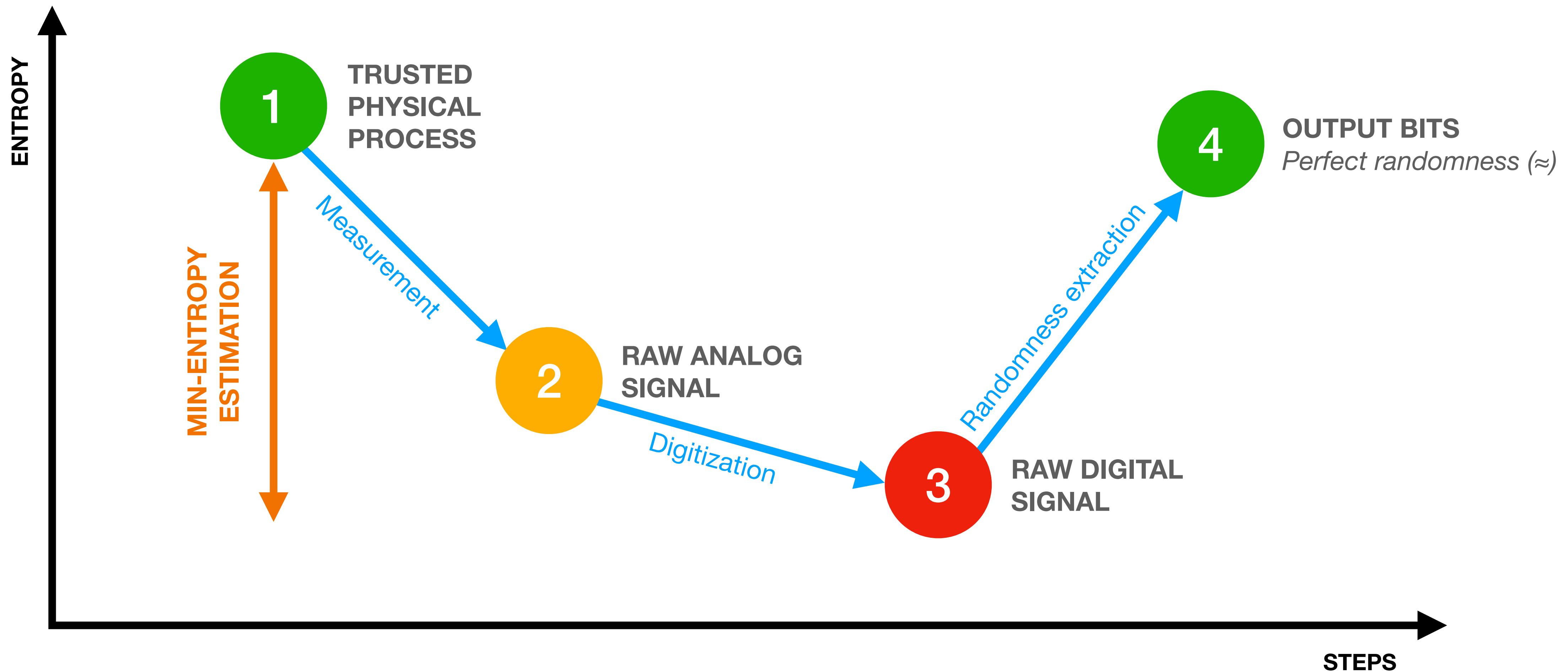
**STORE NOW  
DECRYPT LATER**

Information can be  
sensitive for extended  
periods of time

| Exemplar bit string produced   | String probability assuming 0 and 1 equally likely and bits are independent | Fraction of zeros | Displays Pattern? | Is the source random?  |
|--|---|-------------------|-------------------|--|
| 0 0 0 0 0 0 0 0<br><b>H = 0</b>  | $2^{-10}$   | 100%              | Yes               | It is reasonable to suspect it is not; it might well be that the source outputs only 0s  |
| 0 1 0 1 0 1 0 1<br><b>H = 1</b>  | $2^{-10}$   | 50%               | Yes               | It is reasonable to suspect it is not; it might well be that the source output simply switches between 0 and 1   |
| 0 0 1 0 0 0 0 0 1<br>$H = - \sum p(x) \log p(x)$<br><b>H ≈ 7.2 &lt; 10</b> | $2^{-10}$   | 80%               | No                | Potentially yes, but it appears but it appears to be biased; it may indicate that 0 and 1 are not equally likely – that is, the assumption of 0 and 1 being actually likely may not hold |
| 1 0 1 1 0 1 0 0 1 0<br><b>H = 10</b>                                       | $2^{-10}$   | 50%               | No                | Potentially yes, and it appears also unbiased; 0 and 1 may really be equally likely  |

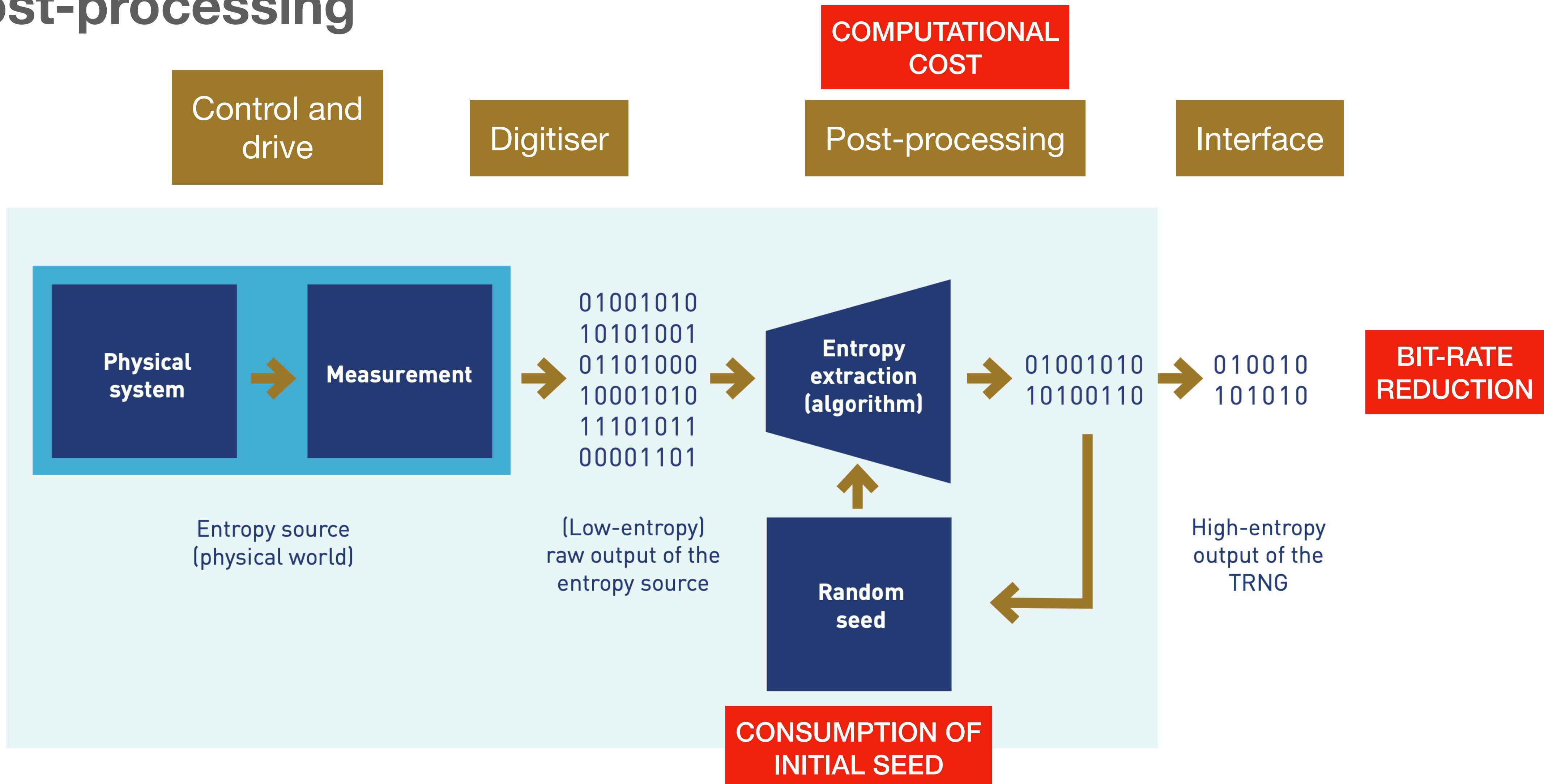
# Randomness extraction

Post-processing: randomness metrology



# Randomness extraction

## Post-processing



# Quantum random number generators

## Trade-offs of quantum RNGs

### ENTROPY SOURCE

Fundamentally random

### ENTROPY QUALITY

High from the start

### CERTIFICATION

Can validate the underlying process

### SECURITY

Built-in with device independent techniques

### SPEED

High for its quality  
Slower on digital quantum computers

### SIZE

Variable, from very small to room size

### VERSATILITY

Easily scalable, specially with universal quantum computers

### SUSTAINABILITY

Better for universal quantum computers