# Tarefa 02 - Métodos Numéricos II

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↳ Desenvolva as fórmulas Fechada e Aberta para um polinômio de substituição de grau 4.

▪ Polinômio de Substituição de Grau 4:

$$g(s) = \sum_{k=0}^{4} \binom{s}{k} . \Delta^k f_0$$

$$k=0 \begin{cases} \dfrac{s!}{0!(s-0)!} = 1 \\ \Delta^0 f_0 = f_0 \end{cases}$$

$$k=1 \begin{cases} \dfrac{s!}{1!(s-1)!} = s \\ \Delta^1 f_0 = f_1 - f_0 \end{cases}$$

$$k=2 \begin{cases} \dfrac{s!}{2!(s-2)!} = \dfrac{s.(s-1)}{2} \\ \Delta^2 f_0 = f_2 - 2f_1 + f_0 \end{cases}$$

$$k=3 \begin{cases} \dfrac{s!}{3!(s-3)!} = \dfrac{s.(s-1).(s-2)}{6} \\ \Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0 \end{cases}$$

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$$k=4\begin{cases}\dfrac{s!}{4!(s-4)!} = \dfrac{s.(s-1).(s-2).(s-3)}{24}\\ \Delta^4 f_0 = f_4 - 4f_3 + 6f_2 - 4f_1 + f_0\end{cases}$$

$$g(s) = f_0 + s.(f_1 - f_0) + \frac{s.(s-1)}{2}.(f_2 - 2f_1 + f_0) +$$

$$+ \frac{s.(s-1).(s-2)}{6}.(f_3 - 3f_2 + 3f_1 - f_0) +$$

$$+ \frac{s.(s-1).(s-2).(s-3)}{24}.(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

>>> Isolando os pontos

$$g(s) = f_0.\left[1 - s + \frac{1}{2}.s.(s-1) - \frac{1}{6}s.(s-1).(s-2) + \frac{1}{24}.s.(s-1).(s-2).(s-3)\right]$$

$$+ f_1.\left[s - 2.\frac{1}{2}.s.(s-1) + 3.\frac{1}{6}.s.(s-1)(s-2) - 4.\frac{1}{24}.s(s-1).(s-2).(s-3)\right]$$

$$+ f_2.\left[\frac{1}{2}.s.(s-1) - 3.\frac{1}{6}.s.(s-1)(s-2) + 6.\frac{1}{24}.s.(s-1).(s-2).(s-3)\right]$$

$$+ f_3.\left[\frac{1}{6}.s.(s-1).(s-2) - 4.\frac{1}{24}.s.(s-1).(s-2).(s-3)\right]$$

$$+ f_4.\left[\frac{1}{24}.s.(s-1).(s-2).(s-3)\right]$$

>>> Multiplicações Auxiliares

- $s.(s-1) = s^2 - s$
- $s.(s-1).(s-2) = s^3 - 3s^2 + 2s$
- $s.(s-1).(s-2).(s-3) = s^4 - 6s^3 + 11s^2 - 6s$

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>>> CALCULANDO OS COEFICIENTES DE CADA PONTO.

$\cdot f_0: \dfrac{24-24s+12.(s^2-s)-4.(s^3-3s+2s)+s^4-6s^3+11s^2-6s}{24}$

$=\dfrac{24-24s-12s-8s-6s+12s^2+12s^2+11s^2-4s^3-6s^3+s^4}{24}=$

$=(24-50s+35s^2-10s^3+s^4).\dfrac{1}{24}$

$=1-\dfrac{25s}{12}+\dfrac{35s^2}{24}-\dfrac{5s^3}{12}+\dfrac{s^4}{24}$

$\cdot f_1: \dfrac{24s-24.(s^2-s)+12.(s^3-3s+2s)-4.(s^4-6s^3+11s^2-6s)}{24}=$

$=\dfrac{24s+24s+24s+24s-24s^2-36s^2-44s^2+12s^3+24s^3-4s^4}{24}=$

$=4s-\dfrac{13s^2}{3}+\dfrac{3s^3}{2}-\dfrac{s^4}{6}$

$\cdot f_2: \dfrac{12(s^2-s)-12.(s^3-3s^2+2s)+6.(s^4-6s^3+11s^2-6s)}{24}=$

$=\dfrac{-12s-24s-36s+12s^2+36s^2+66s^2-12s^3-36s^3+6s^4}{24}=$

$=-3s+\dfrac{57s^2}{12}-2s^3+\dfrac{s^4}{4}$

$\cdot f_3: \dfrac{4.(s^3-3s^2+2s)-4.(s^4-6s^3+11s^2-6s)}{24}=$

$=\dfrac{8s+24s-12s^2-44s^2+4s^3+24s^2-4s^4}{24}=$

$=\dfrac{4s}{3}-\dfrac{7s^2}{3}+\dfrac{7s^3}{6}-\dfrac{s^4}{6}$

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- $f_4: -\frac{s}{4} + \frac{11s^2}{24} - \frac{s^3}{4} + \frac{s^4}{24}$

>>> POLINÔMIO FINAL

$$g(s) = f(0).\left(1 - \frac{25s}{12} + \frac{35s^2}{24} - \frac{5s^3}{12} + \frac{s^4}{24}\right) +$$
$$+ f(1).\left(4s - \frac{13s^2}{3} + \frac{3s^3}{2} - \frac{s^4}{6}\right) +$$
$$+ f(2).\left(-3s + \frac{57s^2}{12} - 2s^3 + \frac{s^4}{4}\right) +$$
$$+ f(3).\left(\frac{4s}{3} - \frac{7s^2}{3} + \frac{7s^3}{6} - \frac{s^4}{6}\right) +$$
$$+ f(4).\left(-\frac{s}{4} + \frac{11s^2}{24} - \frac{s^3}{4} + \frac{s^4}{24}\right)$$

- ABORDAGEM FECHADA

↳ $x_i$ e $x_f$ entram na equação

$s = 0 \rightarrow x_i$
$s = 1 \rightarrow x_i + h$
$s = 2 \rightarrow x_i + 2h$
$s = 3 \rightarrow x_i + 3h$
$s = 4 \rightarrow x_i + 4h$ ou $x_f$

$x(s) = x_i + s.h$

$x'(s) = h$

$h = \frac{\Delta x}{4}$

$h = \frac{|x_f - x_i|}{4}$

- $\int_{x_i}^{x_f} f(x)\,dx \approx h.\int_0^4 g(s)\,ds$

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>>> Dividindo os Componentes de G(s) para Integral.

• Em f(0):

$$f_0 \cdot \int_0^4 1 - \frac{25s}{12} + \frac{35s^2}{24} - \frac{5s^3}{12} + \frac{s^4}{24}\, ds$$

$$\left( s\Big|_0^4 - \frac{25s^2}{24}\Big|_0^4 + \frac{35s^3}{72}\Big|_0^4 - \frac{5s^4}{48}\Big|_0^4 + \frac{s^5}{120}\right) \cdot f_0 =$$

$$= \left(4 - \frac{\overset{100}{40\cancel{0}}}{\cancel{24}_6} + \frac{\overset{280}{224\cancel{0}}}{\cancel{72}_9} - \frac{\overset{80}{128\cancel{0}}}{\cancel{48}_3} + \frac{\overset{128}{10\cancel{24}}}{\cancel{120}_{15}}\right) f_0 =$$

$$= f_0(0) \cdot \left(\frac{14}{45}\right)$$

• Em f(1)

$$f_1 \cdot \int_0^4 4s - \frac{13s^2}{3} + \frac{3s^3}{2} - \frac{s^4}{6}\, ds =$$

$$= f_1 \cdot \left(2s^2\Big|_0^4 - \frac{13s^3}{9}\Big|_0^4 + \frac{3s^4}{8}\Big|_0^4 - \frac{s^5}{30}\Big|_0^4\right)$$

$$= f_1 \cdot \left(32 - \frac{832}{9} + \frac{768}{8} - \frac{1024}{30}\right)$$

$$= f_1 \cdot \left(\frac{64}{45}\right)$$

Em f(2):

$$f_2 \cdot \int_0^4 -3s + \frac{57s^2}{12} - 2s^3 + \frac{s^4}{4}\, ds =$$

$$= f_2 \cdot \left(\frac{-3s^2}{2}\Big|_0^4 + \frac{57s^3}{36}\Big|_0^4 - \frac{\cancel{2}^{1}s^4}{\cancel{4}_2}\Big|_0^4 + \frac{s^5}{20}\Big|_0^4\right) =$$

$$= f_2 \cdot \left(-\frac{\overset{24}{4\cancel{8}}}{\cancel{2}} + \frac{\overset{304}{364\cancel{8}}}{\cancel{36}_3} - 128 + \frac{\overset{256}{10\cancel{24}}}{\cancel{20}_5}\right) = \left(\frac{8}{15}\right) \cdot f_2$$

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• Em $f(3)$:

$$f_3 . \int_0^4 \frac{4s}{3} - \frac{7s^2}{3} + \frac{7s^3}{6} - \frac{s^4}{6} \, ds =$$

$$= f(3) . \left( \frac{2s^2}{3}\Big|_0^4 - \frac{7s^3}{9}\Big|_0^4 + \frac{7s^4}{24}\Big|_0^4 - \frac{s^5}{30}\Big|_0^4 \right)$$

$$= f(3) . \left( \frac{3840 - 17920 + 26880 - 12288}{360} \right)$$

$$= f(3) . \left( \frac{512}{360} \right) = f_3 . \left( \frac{64}{45} \right)$$

Em $f(4)$:

$$f_4 . \int_0^4 -\frac{s}{4} + \frac{11s^2}{24} - \frac{s^3}{4} + \frac{s^4}{24} \, ds =$$

$$= f_4 . \left( -\frac{s^2}{8}\Big|_0^4 + \frac{11s^3}{72}\Big|_0^4 - \frac{s^4}{16}\Big|_0^4 + \frac{s^5}{120}\Big|_0^4 \right)$$

$$= f_4 . \left( -2 + \frac{\cancel{704}\ 88}{\cancel{72}\ 9} - 16 + \frac{\cancel{1024}\ 128}{\cancel{120}\ 15} \right)$$

$$= f_4 . \left( \frac{14}{45} \right)$$

>>> Somando tudo $\qquad x(s) = x_i + s.h$

$$\int_{x_i}^{x_f} f(x)\,dx \approx h . \left[ f(0) . \left(\frac{14}{45}\right) + f(1) . \left(\frac{64}{45}\right) + f(2) \left(\frac{24}{45}\right) + f(3) . \left(\frac{64}{45}\right) + f(4) . \left(\frac{14}{45}\right) \right] =$$

$$= \frac{h . 2}{45} . \left[ f(0) . 7 + f(1) . 32 + f(2) . 12 + f(3) . 32 + 7 . f(4) \right] \quad \text{com } h = \frac{|x_f - x_i|}{4}$$

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- **Abordagem Aberta:**

↳ $x_i$ e $x_f$ não entram na equação.

$s=-1 \to x_i$
$s=0 \to x_i+h$
$s=1 \to x_i+2h$
$s=2 \to x_i+3h$
$s=3 \to x_i+4h$
$s=4 \to x_i+5h$
$s=5 \to x_i+6h$
↳ $x_f$

$$h = \frac{|x_f - x_i|}{6}$$

$$x(s) = x_i + h + s.h$$
$$x'(s) = h$$

$$\int_{x_i}^{x_f} f(x)\,dx \approx \int_{-1}^{5} g(s)\,ds$$

>>> Dividindo os componentes de $g(s)$ p/ integral

Em $f(0)$:

$$f(0).\left( s\Big|_{-1}^{5} - \frac{25s^2}{24}\Big|_{-1}^{5} + \frac{35s^3}{72}\Big|_{-1}^{5} - \frac{5s^4}{48}\Big|_{-1}^{5} + \frac{s^5}{120}\Big|_{-1}^{5} \right) =$$

$$= f(0).\left(6 - \frac{600}{24} + \frac{4410}{72} - \frac{3120}{48} + \frac{3126}{120}\right) =$$

$$= f(0).\left(\frac{33}{10}\right)$$

Em $f(1)$:

$$f(1).\left( 2s^2\Big|_{-1}^{5} - \frac{13s^3}{9}\Big|_{-1}^{5} + \frac{3s^4}{8}\Big|_{-1}^{5} - \frac{s^5}{30}\Big|_{-1}^{5} \right) =$$

$$= f(1).\left(48 - 182 + 234 - \frac{521}{5}\right) =$$

$$= f(1).\left(\frac{-21}{5}\right)$$

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Em f(2):

$$f(2).\left(-\frac{3s^2}{2}\Big|_{-1}^{5}+\frac{57s^3}{36}\Big|_{-1}^{5}-\frac{s^4}{2}\Big|_{-1}^{5}+\frac{s^5}{20}\Big|_{-1}^{5}\right)=$$

$$=f(2).\left(-36+\frac{399}{2}-312+\frac{1563}{10}\right)=$$

$$=f(2).\left(\frac{78}{10}\right)=f(2).\left(\frac{39}{5}\right)$$

Em f(3):

$$f(3).\left(\frac{2s^2}{3}\Big|_{-1}^{5}-\frac{7s^3}{9}\Big|_{-1}^{5}+\frac{7s^4}{24}\Big|_{-1}^{5}-\frac{s^5}{30}\Big|_{-1}^{5}\right)=$$

$$=f(3).\left(16-98+182-\frac{521}{5}\right)=$$

$$=f(3).\left(\frac{-21}{5}\right)$$

Em f(4):

$$f(4).\left(-\frac{s^2}{8}\Big|_{-1}^{5}+\frac{11s^3}{72}\Big|_{-1}^{5}-\frac{s^4}{16}\Big|_{-1}^{5}+\frac{s^5}{120}\Big|_{-1}^{5}\right)=$$

$$=f(4).\left(-3+\frac{77}{4}-39+\frac{521}{20}\right)=$$

$$=f(4).\left(\frac{66}{20}\right)=f(4).\left(\frac{33}{10}\right)$$

>>> SOMANDO TUDO

$$x(s)=x_i+h+s.h$$

$$h=\frac{|x_f-x_i|}{6}$$

$$\int_{x_i}^{x_f} f(x)\,dx \approx h.\int_{-1}^{5} g(s)\,ds$$

$$\approx h.\left[f(0).\left(\frac{33}{10}\right)-\left(\frac{21}{5}\right).f(1)+\left(\frac{39}{5}\right).f(2)-\left(\frac{21}{5}\right).f(3)+\left(\frac{33}{10}\right).f(4)\right]$$