

TAREFA 5 - MÉTODOS NUMÉRICOS II

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↳ Desenvolva a Quadratura de Gauss - Legendre com 4 pontos

1) Polinômio de Legendre de grau 4 ($P_4(\alpha)$)

$$P_4(\alpha) = \frac{1}{2^4 4!} \cdot \frac{d^4}{d\alpha^4} [(\alpha^2 - 1)^4] = \frac{1}{8} (35\alpha^4 - 30\alpha^2 + 3)$$

2) Raízes de $P_4(\alpha)$

$$\frac{1}{8} (35\alpha^4 - 30\alpha^2 + 3) = 0 \Rightarrow 35\alpha^4 - 30\alpha^2 + 3 = 0$$

usando $\alpha^2 = x$, temos:

$$35x^2 - 30x + 3 = 0 \rightarrow \Delta = 30^2 - 4 \cdot 35 \cdot 3$$

$$x = \frac{30 \pm 4\sqrt{30}}{70} \quad \Delta = 900 - 420 = 480$$

$$\text{Logo teremos: } \alpha_1 = -\sqrt{\frac{30 + 4\sqrt{30}}{70}}$$

$$\alpha_2 = -\sqrt{\frac{30 - 4\sqrt{30}}{70}}$$

$$\alpha_3 = \sqrt{\frac{30 - 4\sqrt{30}}{70}}$$

$$\alpha_4 = \sqrt{\frac{30 + 4\sqrt{30}}{70}}$$

3) Cálculo de $x(\alpha_1)$, $x(\alpha_2)$, $x(\alpha_3)$ e $x(\alpha_4)$:

$$x(\alpha_k) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2} \cdot \alpha_k, \text{ Logo}$$

$$x(\alpha_1) = \frac{x_i + x_f}{2} - \frac{x_f - x_i}{2} \cdot \sqrt{\frac{30 + 4\sqrt{30}}{70}}$$

$$x(\alpha_2) = \frac{x_i + x_f}{2} - \frac{x_f - x_i}{2} \cdot \sqrt{\frac{30 - 4\sqrt{30}}{70}}$$

$$x(\alpha_3) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2} \cdot \sqrt{\frac{30 - 4\sqrt{30}}{70}}$$

$$x(\alpha_4) = \frac{x_i + x_f}{2} + \frac{x_f - x_i}{2} \cdot \sqrt{\frac{30 + 4\sqrt{30}}{70}}$$

4) Cálculo dos pesos w_1, w_2, w_3 e w_4

Note que: $w_1 = w_4$ e $w_2 = w_3$.

$$\rightarrow L_1 = L_4 = \frac{(\alpha - \alpha_2) \cdot (\alpha - \alpha_3) \cdot (\alpha - \alpha_4)}{(\alpha_1 - \alpha_2) \cdot (\alpha_1 - \alpha_3) \cdot (\alpha_1 - \alpha_4)} \quad \begin{matrix} \nearrow \star_1 \\ \searrow \star_2 \end{matrix}$$

$$\star_1 = \left(\alpha + \sqrt{\frac{30 - 4\sqrt{30}}{70}} \right) \cdot \left(\alpha - \sqrt{\frac{30 - 4\sqrt{30}}{70}} \right) \cdot \left(\alpha - \sqrt{\frac{30 + 4\sqrt{30}}{70}} \right)$$

$$\star_1 = \alpha^3 - \alpha^2 \cdot \sqrt{\frac{30 + 4\sqrt{30}}{70}} - \alpha \cdot \left(\frac{30 - 4\sqrt{30}}{70} \right) + \left(\frac{30 - 4\sqrt{30}}{70} \right) \cdot \sqrt{\frac{30 + 4\sqrt{30}}{70}}$$

$$\star_2 = \left(-\sqrt{\frac{30 + 4\sqrt{30}}{70}} + \sqrt{\frac{30 - 4\sqrt{30}}{70}} \right) \cdot \left(-\sqrt{\frac{30 + 4\sqrt{30}}{70}} - \sqrt{\frac{30 - 4\sqrt{30}}{70}} \right) \cdot \left(-\sqrt{\frac{30 + 4\sqrt{30}}{70}} - \sqrt{\frac{30 + 4\sqrt{30}}{70}} \right)$$

$$\star_2 = \frac{-16\sqrt{30}}{70} \cdot \sqrt{\frac{30+4\sqrt{30}}{70}}$$

$$W_3 = W_4 = \frac{-70}{16\sqrt{30} \cdot \sqrt{\frac{30+4\sqrt{30}}{70}}} \int_{-1}^1 \left(\alpha^3 - \alpha^2 \cdot \sqrt{\frac{30+4\sqrt{30}}{70}} - \alpha \cdot \left(\frac{30-4\sqrt{30}}{70} \right) + \left(\frac{30-4\sqrt{30}}{70} \right) \cdot \sqrt{\frac{30+4\sqrt{30}}{70}} \right) d\alpha$$

$$W_3 = W_4 = \frac{-70}{16\sqrt{30} \cdot \sqrt{\frac{30+4\sqrt{30}}{70}}} \left[0 - \frac{2}{3} \cdot \sqrt{\frac{30+4\sqrt{30}}{70}} - 0 + 2 \cdot \left(\frac{30-4\sqrt{30}}{70} \right) \cdot \sqrt{\frac{30+4\sqrt{30}}{70}} \right]$$

$$W_3 = W_4 = 0,3478548451374538$$

$$\rightarrow L_2 = L_3 = \frac{(\alpha - \alpha_1) \cdot (\alpha - \alpha_3) \cdot (\alpha - \alpha_4)}{(\alpha_2 - \alpha_1) \cdot (\alpha_2 - \alpha_3) \cdot (\alpha_2 - \alpha_4)} > \star_1$$

$$> \star_2$$

$$\star_1 = \left(\alpha + \sqrt{\frac{30+4\sqrt{30}}{70}} \right) \cdot \left(\alpha - \sqrt{\frac{30-4\sqrt{30}}{70}} \right) \cdot \left(\alpha - \sqrt{\frac{30+4\sqrt{30}}{70}} \right)$$

$$\star_1 = \alpha^3 - \alpha^2 \cdot \sqrt{\frac{30-4\sqrt{30}}{70}} - \alpha \cdot \left(\frac{30+4\sqrt{30}}{70} \right) + \left(\frac{30+4\sqrt{30}}{70} \right) \cdot \sqrt{\frac{30-4\sqrt{30}}{70}}$$

$$\star_2 = \left(-\sqrt{\frac{30-4\sqrt{30}}{70}} + \sqrt{\frac{30+4\sqrt{30}}{70}} \right) \cdot \left(-\sqrt{\frac{30-4\sqrt{30}}{70}} - \sqrt{\frac{30-4\sqrt{30}}{70}} \right) \cdot \left(-\sqrt{\frac{30-4\sqrt{30}}{70}} - \sqrt{\frac{30+4\sqrt{30}}{70}} \right)$$

$$\star_2 = \frac{16\sqrt{30}}{70} \cdot \sqrt{\frac{30-4\sqrt{30}}{70}}$$

$$W_2 = W_3 = \frac{70}{16\sqrt{30} \cdot \sqrt{\frac{30-4\sqrt{30}}{70}}} \int_{-1}^1 \alpha^3 - \alpha^2 \cdot \sqrt{\frac{30-4\sqrt{30}}{70}} - \alpha \cdot \left(\frac{30+4\sqrt{30}}{70} \right) + \left(\frac{30+4\sqrt{30}}{70} \right) \cdot \sqrt{\frac{30-4\sqrt{30}}{70}} d\alpha$$

$$W_2 = W_3 = \frac{70}{16\sqrt{30} \cdot \sqrt{\frac{30-4\sqrt{30}}{70}}} \left[0 - \frac{2}{3} \cdot \sqrt{\frac{30-4\sqrt{30}}{70}} - 0 + \left(\frac{30+4\sqrt{30}}{70} \right) \cdot \sqrt{\frac{30-4\sqrt{30}}{70}} \right]$$

$$W_2 = W_3 = 0,6521451548625461$$

5) A Integral

$$I = \int_{x_i}^{x_f} f(x) dx \approx \frac{x_f - x_i}{2} \left[f(x(\alpha_1)) \cdot W_1 + f(x(\alpha_2)) \cdot W_2 + f(x(\alpha_3)) \cdot W_3 + f(x(\alpha_4)) \cdot W_4 \right]$$