Tarefa 10 - Métodos Numéricos II

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- Calcular os Autovalores e Autovetores

$$\rightarrow \begin{bmatrix} 5 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} = M_1$$

$$\begin{vmatrix} 5-\lambda & 2 & 1 \\ 2 & 3-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$\rightarrow (5-\lambda).(3-\lambda).(2-\lambda)+4.(2-\lambda)-(5-\lambda)-(3-\lambda) =$$

$$= -\lambda^3 + 10\lambda^2 - 25\lambda + 18 = 0$$

$$\lambda_1 = 4-\sqrt{7} \quad \lambda_2 = 2 \quad \lambda_3 = 4+\sqrt{7}$$

p/ $\lambda = 4-\sqrt{7}$

$$\begin{bmatrix} 1+\sqrt{7} & 2 & 1 \\ 2 & -1+\sqrt{7} & 1 \\ 1 & 1 & -2+\sqrt{7} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(1+\sqrt{7}).X_1 + 2.X_2 + X_3 = 0$$
$$2.X_1 + (-1+\sqrt{7}).X_2 + X_3 = 0$$
$$X_1 + X_2 + (-2+\sqrt{7}).X_3 = 0$$

$$(X_1, X_2, X_3) = \left(\frac{3-\sqrt{7}}{2}, \frac{1-\sqrt{7}}{2}, 1\right)$$

p/ $\lambda = 2$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3X_1 + 2X_2 + X_3 = 0$$
$$2X_1 + X_2 + X_3 = 0$$
$$X_1 + X_2 = 0$$

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$$(X_1, X_2, X_3) = (1, -1, -1)$$

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$p/ \lambda = 4 + \sqrt{7}$ $\begin{bmatrix} 1-\sqrt{7} & 2 & 1 \\ 2 & -1-\sqrt{7} & 1 \\ 1 & 1 & -2-\sqrt{7} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$(1-\sqrt{7})x_1 + 2x_2 + x_3 = 0$

$2 \cdot x_1 + (-1-\sqrt{7})x_2 + x_3 = 0$ $(x_1, x_2, x_3) = \left(\frac{3+\sqrt{7}}{2}, \frac{1+\sqrt{7}}{2}, 1\right)$

$x_1 + x_2 + (-2-\sqrt{7})x_3 = 0$

$\rightarrow \begin{bmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}$

$\begin{vmatrix} \frac{1}{3}-\lambda & -2/3 & -2/3 \\ -2/3 & \frac{1}{3}-\lambda & -2/3 \\ -2/3 & -2/3 & \frac{1}{3}-\lambda \end{vmatrix} = 0$

$\Delta \left(\frac{1}{3}-\lambda\right)^3 - \frac{16}{27} - \frac{12}{9} \cdot \left(\frac{1}{3}-\lambda\right) =$

$= -\lambda^3 + \lambda^2 + \lambda - 1 = 0$

$\lambda_1 = -1$ $\lambda_2 = 1$

$p/ \lambda = -1$ $\begin{bmatrix} 4/3 & -2/3 & -2/3 \\ -2/3 & 4/3 & -2/3 \\ -2/3 & -2/3 & 4/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$4/3 \cdot x_1 - 2/3 \cdot x_2 - 2/3 \cdot x_3 = 0$

$-2/3 \cdot x_1 + 4/3 \cdot x_2 - 2/3 \cdot x_3 = 0$ $(x_1, x_2, x_3) = (1, 1, 1)$

$-2/3 \cdot x_1 - 2/3 \cdot x_2 + 4/3 \cdot x_3 = 0$

$p/ \lambda = 1$ $\begin{bmatrix} -2/3 & -2/3 & -2/3 \\ -2/3 & -2/3 & -2/3 \\ -2/3 & -2/3 & -2/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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$-2/3 \, x_1 - 2/3 \cdot x_2 - 2/3 \cdot x_3 = 0$ $(x_1, x_2, x_3) = (1, -1, 0)$

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$$\begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} = M3$$

$$\begin{vmatrix} \frac{2}{3}-\lambda & -1/3 & -1/3 \\ -1/3 & \frac{2}{3}-\lambda & -1/3 \\ -1/3 & -1/3 & \frac{2}{3}-\lambda \end{vmatrix} = 0$$

$$\left(\frac{2}{3}-\lambda\right)^3 - \frac{2}{27} - 3\cdot\left(\frac{1}{9}\left(\frac{2}{3}-\lambda\right)\right) =$$

$$= -\lambda^3 + 2\lambda^2 - \lambda = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = 1$$

p/ $\lambda = 0$

$$\begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{2}{3}\cdot x_1 - x_2/3 - x_3/3 = 0$$
$$-x_1/3 + \frac{2}{3}x_2 - x_3/3 = 0 \qquad (x_1, x_2, x_3) = (1, 1, 1)$$
$$-x_1/3 - x_2/3 + \frac{2}{3}\cdot x_3 = 0$$

p/ $\lambda = 1$

$$\begin{bmatrix} -1/3 & -1/3 & -1/3 \\ -1/3 & -1/3 & -1/3 \\ -1/3 & -1/3 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

VETOR POSSÍVEL

$$-x_1/3 - x_2/3 - x_3/3 = 0 \qquad (x_1, x_2, x_3) = (1, -1, 0)$$

$$\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} = M4$$

$$\begin{vmatrix} \frac{1}{3}-\lambda & 1/3 & 1/3 \\ 1/3 & \frac{1}{3}-\lambda & 1/3 \\ 1/3 & 1/3 & \frac{1}{3}-\lambda \end{vmatrix} = 0$$

$$\left(\frac{1}{3}-\lambda\right)^3 + \frac{2}{27} - 3\cdot\left(\frac{1}{9}\cdot\left(\frac{1}{3}-\lambda\right)\right) =$$

$$= -\lambda^3 + \lambda^2 = 0$$

$$\lambda_1 = 0 \qquad \lambda_2 = 1$$

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p/ $\lambda = 0$ $\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

VETOR POSSÍVEL

$x_1/3 + x_2/3 + x_3/3 = 0$ $(x_1, x_2, x_3) = (1, -1, 0)$

p/ $\lambda = 1$ $\begin{bmatrix} -2/3 & 1/3 & 1/3 \\ 1/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & -2/3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$-2/3\, x_1 + x_2/3 + x_3/3 = 0$ $(x_1, x_2, x_3) = (1, 1, 1)$

$x_1/3 - 2/3 \cdot x_2 + x_3/3 = 0$

$x_1/3 + x_2/3 - 2/3 \cdot x_3 = 0$