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TAREFA 06 - MÉTODOS NUMÉRICOS II

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1) POLINÔMIOS (n=4)

-> Hermite: $16x^4 - 48x^2 + 12$ (H_4)

-> Laguerre: $\frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$ (L_4)

-> Chebyshev: $8x^4 - 8x^2 + 1$ (T_4)

2) Raízes

-> $H_4 = 0$ $\quad 16x^4 - 48x^2 + 12 = 0$

Fazendo:

$x^2 = y$, temos $\quad 16y^2 - 48y + 12 = 0 \quad \Delta = 96$

$\div 4$

$4y^2 - 12y + 3 = 0$

$$y = \frac{12 \pm \sqrt{96}}{8} = \frac{3 \pm \sqrt{6}}{2}$$

Logo,

$$x_1 = -\sqrt{\frac{3+\sqrt{6}}{2}}$$

$$x_2 = -\sqrt{\frac{3-\sqrt{6}}{2}}$$

$$x_3 = \sqrt{\frac{3-\sqrt{6}}{2}}$$

$$x_4 = \sqrt{\frac{3+\sqrt{6}}{2}}$$

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→ $L_4 = 0 \quad \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$

$x_1 = 0,3225476442$
$x_2 = 1,7457611561$
$x_3 = 4,5366201401$
$x_4 = 9,3950710297$

★ Raízes Aproximadas pelo método de Ponto Falso com 10^{-6} de erro e máximo de 500 iteraç.

→ $T_4 = 0 \quad 8x^4 - 8x^2 + 1 = 0$

fazendo $y = x^2$, temos $8y^2 - 8y + 1 = 0 \quad \Delta = 32$

$$y = \frac{8 \pm 4\sqrt{2}}{16} = \frac{2 \pm \sqrt{2}}{4}$$

Logo, $x_1 = -\sqrt{\frac{2+\sqrt{2}}{4}}$

$x_2 = -\sqrt{\frac{2-\sqrt{2}}{4}}$

$x_3 = \sqrt{\frac{2-\sqrt{2}}{4}}$

$x_4 = \sqrt{\frac{2+\sqrt{2}}{4}}$

3) Pesos

→ $H_4(x_k) \quad w_k = \frac{2^3 \cdot 4! \sqrt{\pi}}{4^2 \cdot [H_3(x_k)]^2}$

$w_1 = w_4 = 0,0813128354$

$w_2 = w_3 = 0,8049140900$

→ $L_4(x_k)$ $\quad w_k = \dfrac{x_k}{5^2 \cdot [L_5(x_k)]^2}$

$w_1 = 0,6031541043$
$w_2 = 0,3574186924$
$w_3 = 0,0388879085$
$w_4 = 0,0005392947$

→ $T_4(x_k)$ $\quad w_k = \pi/4$

$w_1 = w_2 = w_3 = w_4 = \dfrac{\pi}{4}$

4) A Tabela

	$\|-\|_4(x_k)$
$x_1 = -\sqrt{\dfrac{3+\sqrt{6}}{2}}$	$w_1 = 0,0813128354$
$x_2 = -\sqrt{\dfrac{3-\sqrt{6}}{2}}$	$w_2 = 0,8049140900$
$x_3 = \sqrt{\dfrac{3-\sqrt{6}}{2}}$	$w_3 = 0,8049140900$
$x_4 = \sqrt{\dfrac{3+\sqrt{6}}{2}}$	$w_4 = 0,0813128354$

$L_4(x_k)$

$x_1 = 0,3225476442$	$w_1 = 0,6031541043$
$x_2 = 1,7457611561$	$w_2 = 0,3574186924$
$x_3 = 4,5366201401$	$w_3 = 0,0388879085$
$x_4 = 9,3950710297$	$w_4 = 0,0005392947$

$T_4(x_i)$

$$x_1 = -\sqrt{\frac{2+\sqrt{2}}{4}} \qquad w_1 = \frac{\pi}{4}$$

$$x_2 = -\sqrt{\frac{2-\sqrt{2}}{4}} \qquad w_2 = \frac{\pi}{4}$$

$$x_3 = \sqrt{\frac{2-\sqrt{2}}{4}} \qquad w_3 = \frac{\pi}{4}$$

$$x_4 = \sqrt{\frac{2+\sqrt{2}}{4}} \qquad w_4 = \frac{\pi}{4}$$