

TAREFA 08 - Métodos Numéricos II

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1. Mudança de Variável 1.

$$x(\alpha, \beta) = 40\alpha \cos(\beta)$$

$$y(\alpha, \beta) = 40\alpha \sin(\beta)$$

↳ substituindo em $f(x, y) = 0,2(x^2 - y^2)$

$$f(\alpha, \beta) = 0,2 \cdot [(40\alpha \cos(\beta))^2 - (40\alpha \sin(\beta))^2]$$

$$= 320\alpha^2 \cos^2(\beta) - 320\alpha^2 \sin^2(\beta)$$

Logo: $h(\alpha, \beta) = \frac{df(\alpha, \beta)}{d\alpha} = 640\alpha \cos^2(\beta) - 640\alpha \sin^2(\beta)$

$$p(\alpha, \beta) = \frac{df(\alpha, \beta)}{d\beta} = -1280\alpha^2 \sin(\beta) \cdot \cos(\beta)$$

2. Mudança de Variável 2.

$$A = \left[\int_0^1 \left(\int_0^{2\pi} \sqrt{(h(\alpha, \beta))^2 + (p(\alpha, \beta))^2 + 1} d\beta \right) d\alpha \right]$$

$$\alpha(r, s) = \frac{0+1}{2} + \frac{(1-0)r}{2} = \frac{1}{2} \cdot (1+r)$$

$$\beta(r, s) = \frac{0+2\pi}{2} + \frac{(2\pi-0)s}{2} = \pi \cdot (1+s)$$

$$J = \begin{bmatrix} \frac{d\alpha}{dr} & \frac{d\alpha}{ds} \\ \frac{d\beta}{dr} & \frac{d\beta}{ds} \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & \pi \end{bmatrix} \Rightarrow \pi/2 = |J|$$

$$A \approx \frac{\pi}{2} \int_{-1}^1 \int_{-1}^1 \left(\sqrt{\left(h\left(\frac{1}{2}(r+1)\right), (\pi(s+1)) \right)^2 + \left(p\left(\frac{1}{2}(r+1)\right), (\pi(s+1)) \right)^2} + 1 \right) [dr] ds$$

3. Resultado do Gauss-Legendre 3 pontos

→ Através do código ANEXO

$$A = 1700.871760 \quad \text{erro} \approx 10^{-6}$$