

Comparing Theoretical and Simulation Models

Pedro Cortes dos Santos

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Simulations

The following code includes the lambda, number of exponentials and the number of simulations we'll run. We generate a numeric vector named “means”, which contains the result of the simulations. The following simulations generate the mean.

```
set.seed(1000)

lambda <- 0.2

n <- 40

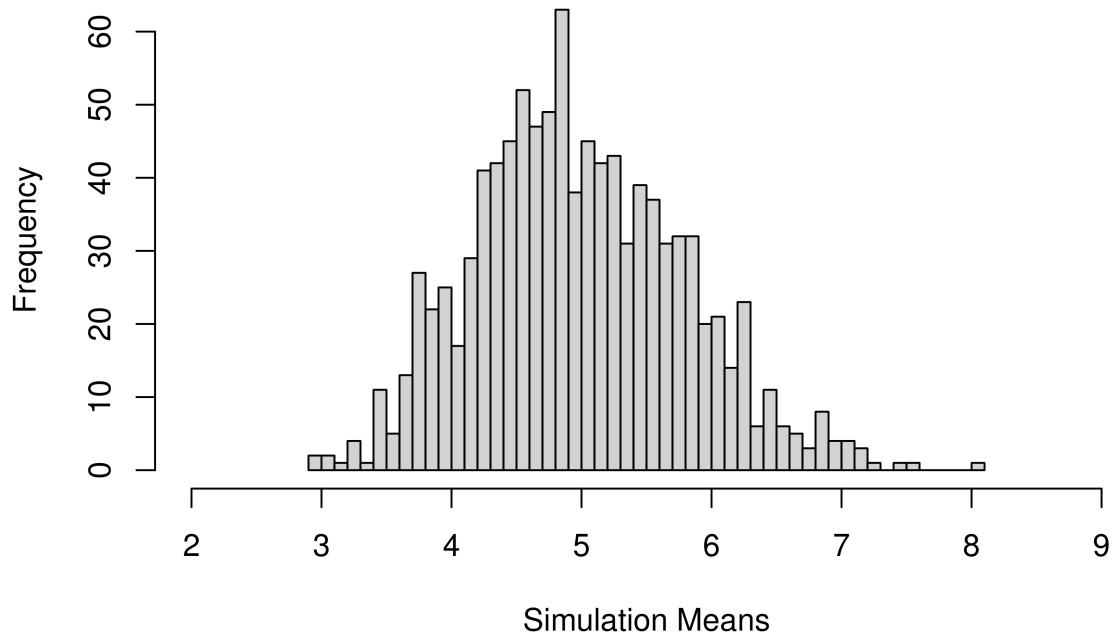
sims <- 1000

running_sim <- replicate(sims, rexp(n,lambda))

means_expsim <- apply(running_sim,2, mean)
```

We'll also generate a histogram of the simulated mean values.

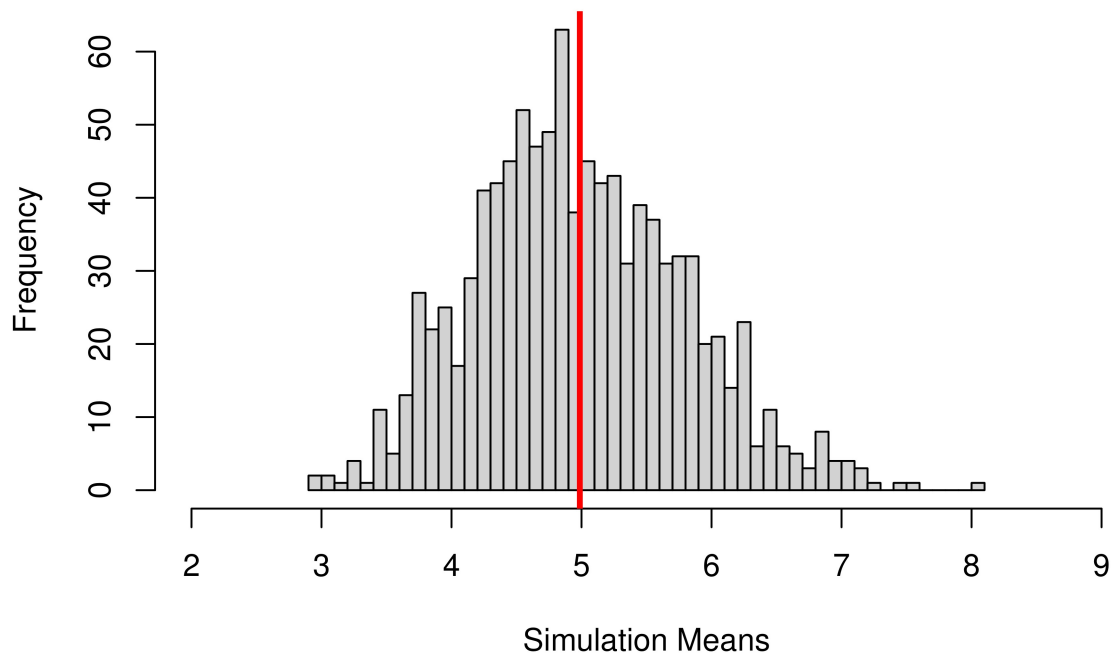
Means of Exponential Function Simulation



Sample Mean x Theoretical Mean

The exponential distribution mean is $1/\lambda$. Considering that for this simulation λ is 0.2, the theoretical mean should be 5.

Sample Mean x Theoretical Mean



```
mean(means_expsim)
```

```
## [1] 4.986963
```

The sample mean is approximately 4.98, very close to the theoretical mean of 5.

Sample Variance x Theoretical Variance

The standard deviation of the exponential distribution is $(1/\lambda)/\sqrt{n}$. So we'll have to compare this equation to our previous simulations.

```
print(paste("Theoretical Variance: ", round( (1/lambda)^2/n, 3)))
```

```
## [1] "Theoretical Variance: 0.625"
```

```
print(paste("Sample Variance: ", round( var(means_expsim),3)))
```

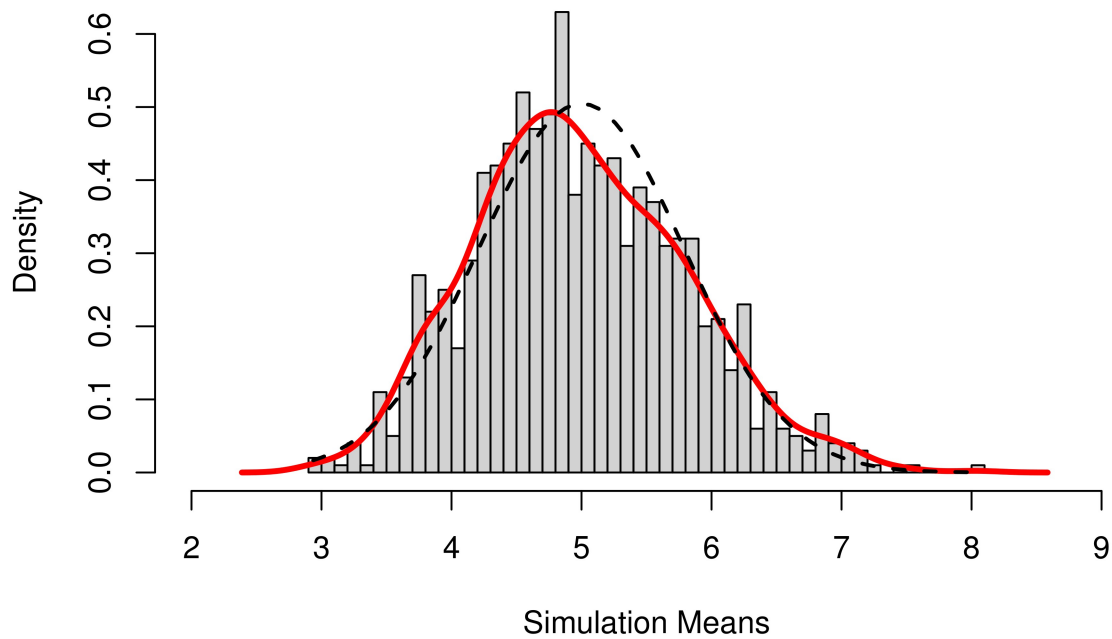
```
## [1] "Sample Variance: 0.654"
```

Considering the previous results, we can say the sample variance is pretty close to the calculated variance.

Distribution

The following investigates whether the exponential distribution is approximately normal. Due to the Central Limit Theorem, the means of the sample simulations should follow a normal distribution.

Means of Exponential Function Simulation



Conclusion

As shown above, the distribution of means of the simulated exponential distributions follows a normal distribution due to the Central Limit Theorem. We can also observe that as the number of samples increase, the distribution (red line) gets even closer to the standard normal distribution (represented by the dotted line).