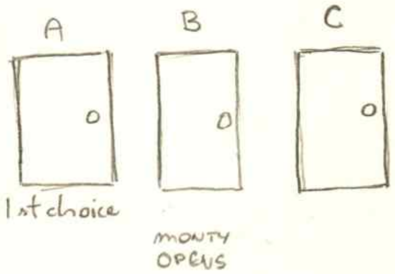
MONTY HALL - PROBLEM III



DO WE SWITCH?

2 PROBABILITIES

$P(prize = A \mid opens = B)$ - probability of winning **without switching**

$P(prize = C \mid opens = B)$ = probability of winning **with switching**

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

PROBABILITY OF WINNING w/ SWITCHING

$$P(prize = C \mid opens = B) = \frac{P(opens = B \mid prize = C) \cdot P(prize = C)}{P(opens = B)}$$

First, the simple ones:

PRIOR

$P(A) = P(prize = C) = \frac{1}{3}$ WITH NO INFORMATION, THERE'S AN EQUAL PROBABILITY OF THE CAR BEING BEHIND EACH DOOR.

NORMALIZING CONSTANT

$P(B) = P(opens = B) = \frac{1}{2}$ AFTER CHOOSING DOOR A, MONTY CAN EITHER OPEN DOOR B OR DOOR C.

Now the difficult one...

LIKELIHOOD

$P(B \mid A) = P(opens = B \mid prize = C) = 1$! SINCE WE CHOSE DOOR A AND WE'RE CONDITIONING THIS PROBABILITY ON prize = C, MONTY ONLY HAS ONE DOOR TO OPEN, B!

POSTERIOR

$$\frac{P(B \mid A) \cdot P(A)}{P(B)} = \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

OUR ORIGINAL PROBABILITY OF WINNING WAS $\frac{1}{3}$, SO BY SWITCHING WE DOUBLE OUR WINNING CHANCE!

SINCE THE TWO PROBABILITES ARE COMPLIMENTARY, THE PROBABILITY OF WINNING NOT SWITCHIN DOORS = 1 - PROBABILITY OF WINNING WHEN SWITCHING

$$P(prize = A \mid opens = B) = 1 - P(prize = C \mid opens = B) = \frac{1}{3}$$