Lab 1

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Data from the statement

March

- People that vote for QQ in Orejilla: 36
- People that vote for QTUI in Orejilla: 18
- Total voters: 36 + 18 = 54

September

- People that vote for QQ in Orejilla: 37
- People that vote for QTUI in Orejilla: 17
- Total voters: 37 + 17 = 54

From the census of the two elections we know that one person leave/move out from Orejilla and another one arrive/move in between the elections.

```
!diff census_march16.csv  census_september16.csv
```

```
5c5
< Rogelia,Cornejo
---
> Daisy,Palomero
```

$\rho = 0$, Nobody change his/her vote

We noticed that QQ won a vote and QTUI lost one between both elections. If nobody change his/her vote it means the person who arrived to Orejilla votes for QQ ($p_{QQ\_in1} = 1, p_{QQ\_in2} = 1$) and the one who left votes for QTUI ($p_{QQ\_out1} = 0, p_{QQ\_out2} = 0$). As long as we already know the vote of one person in each elections, then the probability that a random person of Orejilla (the ones that remains in the town) votes for QQ in the elections, either March and September, is $p_{QQO1} = p_{QQO2} = \frac{36}{53}$.

$\rho = 0.01$

Now, we cannot affirm that the person who move in vote for a specific party, neigther with the people that left. However, let's see what happend if we compute the probability that a person that remains in the town change his/her point of view.

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$$Bi(p|n,x) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$p_{ch} = Bi(p=0.01|n=53, x>0) = 1 - Bi(p=0.01|n=53, x=0) =$$
$$= 1 - \binom{53}{0} \cdot 0.01^0 \cdot (1-0.01)^{(53-0)} = 1 - 1 \cdot 1 \cdot 0.99^{53} = 0.41296321806251557$$

```
from scipy import stats
```

```
1 - stats.binom.pmf(0, 53, 0.01)
```

```
0.41296321806251557
```

So, from the previous calculus we know that the probability that in Orejilla nobody change his/her vote is higher (>15%) that the probability that *at least* one voter change its vote. Therefore, now we can estimate the probabilities of vote as total probabilities:

- Person who move out:
$$p_{QTUI\_out2} = P(OUT_2 = QTUI|\overline{p_{ch}}) \cdot \overline{p_{ch}} + P(OUT_2 = QTUI|p_{ch}) \cdot p_{ch}) =$$
$$= P(QTUI\_out2 \cap \rho = 0) \cdot \overline{p_{ch}} + p_{QTUIO2} \cdot p_{ch} =$$
$$= 1 \cdot 0.5870367819374844 + \frac{17}{54} \cdot 0.41296321806251557 =$$
$$0.7170437209571653$$

$$p_{QTUI\_out1} = p_{QTUI\_out2} \cap \overline{\rho} = 0.7098732837475936$$

- Person who move in:
$$p_{QQ\_in1} = \{0.6, 0.7\}$$

$$p_{QQ\_in2} = P(IN_2 = QTUI|\overline{p_{ch}}) \cdot \overline{p_{ch}} + P(IN_2 = QQ|p_{ch}) \cdot p_{ch}) =$$

$$= P(IN_2 \cap \rho = 0) \cdot \overline{p_{ch}} + p_{QQ\_in1} \cdot \overline{\rho} \cdot p_{ch} =$$

$$= 1 \cdot 0.5870367819374844 + \{0.6, 0.7\} \cdot 0.01 \cdot 0.9962428978738637 =$$
$$\{0.8348147127749938, 0.8761110345812453\}$$

- Random person of Orejilla:
$$p_{QQO1} = P(O_1 = QQ|\overline{p_{ch}}) \cdot \overline{p_{ch}} + P(O_1 = QQ|p_{ch}) \cdot p_{ch} =$$
$$= P(O_1 \cap \overline{p_{ch}}) \cdot \overline{p_{ch}} + +P(O_1 = QQ|p_{ch}|OUT_1 = QQ) \cdot p_{QQ\_out1} +$$
$$+P(O_1 = QQ|p_{ch}|OUT_1 = QTUI) \cdot p_{QTUI\_out1}) \cdot p_{ch} =$$
$$= \frac{36}{53} \cdot 0.5870367819374844 + (\frac{35}{53} \cdot 0.2901267162524064 +$$
$$\frac{36}{53} \cdot 0.7098732837475936) \cdot 0.9962428978738637 = 0.6769846856152923$$

-

$$p_{QQO2} = P(O_2 = QQ|\overline{p_{ch}}) \cdot \overline{p_{ch}} + P(O_2 = QQ|p_{ch}) \cdot p_{ch} =$$
$$= P(O_2 \cap \overline{p_{ch}}) \cdot \overline{p_{ch}} +$$
$$+P(O_2 = QQ|p_{ch}|IN_2 = QQ) \cdot p_{QQ\_in2} +$$
$$+P(O_2 = QQ|p_{ch}|IN_2 = QTUI) \cdot p_{QTUI\_in2}) \cdot p_{ch} =$$
$$= \frac{35}{53} \cdot 0.5870367819374844 +$$
$$+(\frac{35}{53} \cdot (1 - \rho = 0.01) \cdot \{0.8348147127749938, 0.8761110345812453\} +$$
$$+\frac{36}{53} \cdot (1 - \{0.8348147127749938, 0.8761110345812453\})) \cdot$$
$$\cdot 0.41296321806251557 = \{0.6593878061923263, 0.6589534153691414\}$$

```
print("\np_{QTUI_out1}\n")
print(1*0.5870367819374844 + 17/54*0.41296321806251557)
print("\np_{QTUI_out2}\n")
print(0.99*(1*0.5870367819374844 + 17/54*0.41296321806251557))
print("\np_{QQ\_in2}\n")
for i in [0.6, 0.7]:
    print(1*0.5870367819374844 + i*0.41296321806251557)
print("\np_{QQO1}\n")
print(36/53*0.5870367819374844 + (35/53*0.2901267162524064 + 36/53*0.70987328374
print("\np_{QQO2}\n")
for i in [0.8348147127749938, 0.8761110345812453]:
    print(35/53*0.5870367819374844 + ((35/53*0.99)*i + 36/53*(1-i))*0.4129632180
```

```
p_{QTUI_out1}

0.7170437209571653

p_{QTUI_out2}

0.7098732837475936

p_{QQ\_in2}

0.8348147127749938
0.8761110345812453

p_{QQO1}

0.6769846856152923

p_{QQO2}

0.6593878061923263
0.6589534153691414
```

We can notice that for $\rho = 0.01$ the probabilities change from $\rho = 0$. However, the probability in Orejilla (p_{QQO}) is almost the same and the probability of the person that move in is close to the previous case. With the person that move out the probability changes more but is still likely that votes for QTUI.

$\rho = 0.1$

Let's follow the same reasoning as for $\rho = 0.01$:

$$Bi(p|n,x) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$P_{ch}(p=0.1|n=53, x>0) = 1 - P_{ch}(p=0.1|n=53, x=0) =$$

$$= 1 - \binom{53}{0} \cdot 0.1^0 \cdot (1-0.9)^{(53-0)} = 1 - 1 \cdot 1 \cdot 0.9^{53} = 0.9962428978738637$$

```
stats.binom.pmf(0, 53, 0.1)
```

```
0.003757102126136367
```

And now, we can do the same computations as in the previous case:

$$p_{QTUI_out1} = 0.3173891255308713$$
$$p_{QTUI_out2} = 0.3142152342755626$$
$$p_{QQ\_in1} = \{0.6, 0.7\}$$
$$p_{QQ\_in2} = \{0.6015028408504545, 0.7011271306378408\}$$
$$p_{QQO1} = 0.6663545622203195$$
$$p_{QQO2} = \{0.6282952773256015, 0.6198683912840194\}$$

```
print("\np_{QTUI_out1}\n")
print(1*0.003757102126136367 + 17/54*0.9962428978738637)
print("\np_{QTUI_out2}\n")
print(0.99*(1*0.003757102126136367 + 17/54*0.9962428978738637))
print("\np_{QQ\_in2}\n")
for i in [0.6, 0.7]:
    print(1*0.003757102126136367 + i*0.9962428978738637)
print("\np_{QQO1}\n")
print(36/53*0.003757102126136367 + (35/53*0.6857847657244374 + 36/53*0.314215234
print("\np_{QQO2}\n")
for i in [0.6015028408504545, 0.7011271306378408]:
    print(35/53*0.003757102126136367 + ((35/53*0.9)*i + 36/53*(1-i))*0.996242897
```

```
p_{QTUI_out1}

0.3173891255308713

p_{QTUI_out2}

0.3142152342755626
```

```
p_{QQ\_in2}

0.6015028408504545
0.7011271306378408

p_{QQO1}

0.6663545622203195

p_{QQO2}

0.6282952773256015
0.6198683912840194
```

With $\rho = 0.1$ the probabilites of changes are really high what gives rise that the probabilities of the voters in each census is close to the probability of the rest of Spain.

Extra credit

To maximize our results are going to find a ρ such as:

- The person who move out always vote for QTUI.
- The person who move in always vote for QQ.
- The proability that a random person in Orejilla votes for QQ is $\frac{36}{53}$ in the first and in the second election.

$$P[(O_1 = \frac{36}{53} \cap O_2 = \frac{36}{53})|(p_{QTUI_out1} = 1 \cap p_{QQ_in1} = 1)] = 0 \iff$$
$$\iff P[Bi(n = 53|p = \frac{36}{53}, k = 36) \cap$$
$$\cap Bi(n = 53|p = \frac{36}{53} \cdot (1-\rho) + \frac{17}{53} \cdot \rho, k = 36)] = 0 \iff$$
$$\iff [\binom{53}{36} \cdot \frac{36}{53}^{36} \cdot \frac{17}{53}^{17}] \cdot [\binom{53}{36} \cdot (\frac{36}{53} \cdot (1-\rho) + \frac{17}{53} \cdot \rho)^{36} \cdot$$
$$\cdot (1 - \frac{36}{53} \cdot (1-\rho) - \frac{17}{53} \cdot \rho)^{17}] = 0 \iff$$
$$\iff [\binom{53}{36} \cdot \frac{36}{53}^{36} \cdot \frac{17}{53}^{17}] \cdot [\binom{53}{36} \cdot (\frac{36}{53} - \frac{19}{53} \cdot \rho)^{36} \cdot (\frac{17}{53} + \frac{19}{53} \cdot \rho)^{17}] = 0$$

From the experiment in the cell bellow it seems that for $\rho = 1$ we have value that maximize our results.

```
# Binomial for the first election
p_qqo1 = stats.binom.pmf(36, 53, 36/53)
print(p_qqo1)

# Binomial for the second election
```

```
import numpy as np

for rho in np.linspace(0, 1, 10):
    prob = 36/53*(1-rho)+17/53*rho
    num = p_qqo1*stats.binom.pmf(36, 53, prob)
    print(str(num) +"\t- rho="+str(rho)+" -\t"+ str(36/53))
```

```
0.11674040610069855
0.013628322416556015    - rho=0.0 -     0.6792452830188679
0.011319976416336636    - rho=0.1111111111111111 -      0.6792452830188679
0.006634560274891454    - rho=0.2222222222222222 -      0.6792452830188679
0.002800277866577579    - rho=0.3333333333333333 -      0.6792452830188679
0.000856113642327576    - rho=0.4444444444444444 -      0.6792452830188679
0.00018828843530643085  - rho=0.5555555555555556 -      0.6792452830188679
2.9218220468750783e-05  - rho=0.6666666666666666 -      0.6792452830188679
3.093794708037156e-06   - rho=0.7777777777777777 -      0.6792452830188679
2.1241689962443424e-07  - rho=0.8888888888888888 -      0.6792452830188679
8.774540666434788e-09   - rho=1.0 -     0.6792452830188679
```