Optimization and Algorithms

Single Target Tracking Using Sensor Readings

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Abstract—**This paper presents three distinct approaches for single target tracking using sensor data. One uses only sensors' readings and discretizes the space with a grid. The second also discretizes the space but adds a target's dynamics term to improve on the estimation. The third drops the discretized space assumption and also uses a target's dynamics model. Several simulations were performed that suggest that the second approach is suitable for a posteriori] use and the third approach can be used in real time applications, due to their corresponding computational weight.**

I. Introduction

Target tracking is the problem of inferring a moving target's position based on signals generated from a set sensors. The target is modelled as a random walker, where each time it chooses a direction with certain probability distribution. There are many applications with a similar problem, e.g. radar and sonar-based systems, surveillance and habitat monitoring using distributed wireless sensors, collision avoidance modules envisioned for modern transportation systems, and mobile robot localisation and navigation in static and dynamically changing environments [1]. Three different approaches to solve this problem are presented, two of them assume a discrete space (grid) while the third approach assumes a continuous space. The remainder of the paper is organised as follows. Sections II and III describe each approach in detail and the optimisation problem associated with each stage, as well as the algorithms used to solve them. In IV the paper presents and compares the results from each approach.

II. Grid based approach

In these grid based approaches the space is defined as a uniformly discretized grid on a rectangle with $\mathbf{x}_{min}$ and $\mathbf{x}_{max}$ bounding the bottom left and upper right corners respectively. The sensors' positions $\mathbf{r}_i$ are randomly defined on the rectangle with uniform distribution. Each sensor returns a signal defined as

$$y_i(t) = \frac{1}{1+\lambda\left\|\mathbf{r}_i - \mathbf{p}(t)\right\|^2} + \mathcal{N}(0,\sigma^2) \tag{1}$$

Where $\mathbf{p}$ is the target's position and λ is fixed so that,

$$\frac{1}{(1+\lambda\left\|\mathbf{x}_{min} - \mathbf{x}_{max}\right\|^2)} = 0.1 \tag{2}$$

A. Sensors only approach

The problem was solved by minimizing at each time the following cost function

$$\underset{\mathbf{x}\in\text{grid}}{\text{minimize}} \quad \sum_{i=1}^{N}\left(y_i - \frac{1}{1+\lambda\left\|\mathbf{r}_i - \mathbf{x}\right\|^2}\right)^2 \tag{3}$$

where N is the number of sensors. In order to solve it, the cost function was calculated for each point in the grid and the best one was chosen as the solution. This was done for each time step. This shows that the complexity is $O(nT)$ (where n is number of points in the grid and T is the number of steps of the target).

B. Optimization with target dynamics

In this approach the position on the grid is encoded as $\mathbf{x} \in \{0,1\}^M$, such that $\sum_{i=0}^{M} x_i = 1$ where M is the number of points in the grid. As such, each position is a vector of the form:

$$\mathbf{x} \in \left\{ \begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}, \cdots, \begin{bmatrix}0\\\vdots\\0\\1\\0\\\vdots\\0\end{bmatrix}, \cdots, \begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix} \right\}$$

This approach keeps the former term of the sensors' readings multiplied by a weighting factor plus a new term that integrates temporal adjacency. The second term uses the information that the target only moves to adjacent grid points, i.e. the probability of the next grid point not being adjacent to the current one is 0.

The optimization problem is formulated as shown in (4).

$$\begin{aligned}\underset{\mathbf{x}(1),\ldots,\mathbf{x}(T)\in\{0,1\}^M}{\text{minimize}} \quad & \tfrac{1}{2\sigma^2}\textstyle\sum_{t=1}^{T}\left\|\mathbf{y}(t) - \mathbf{H}\mathbf{x}(t)\right\|^2 + \\ & + \textstyle\sum_{t=2}^{T}\phi(\mathbf{x(t)}, \mathbf{x(t-1)})\end{aligned} \tag{4}$$

$$\mathbf{H} = \begin{bmatrix} \frac{1}{1+\lambda\left\|\mathbf{r}_1 - \mathbf{p}_1\right\|^2} & \cdot\cdot & \frac{1}{1+\lambda\left\|\mathbf{r}_1 - \mathbf{p}_M\right\|^2} \\ \vdots & \cdots & \vdots \\ \frac{1}{1+\lambda\left\|\mathbf{r}_N - \mathbf{p}_1\right\|^2} & \cdots & \frac{1}{1+\lambda\left\|\mathbf{r}_N - \mathbf{p}_M\right\|^2} \end{bmatrix}$$

Where $\mathbf{p}_i$ is a point in the grid and $\phi(\mathbf{x}(t), \mathbf{x}(t-1)) = -\log(\mathbb{P}(\mathbf{x}(t)|\mathbf{x}(t-1)))$. A dynamic programming approach was used to solve the problem. The cost function was decomposed in a sum of $\Psi_t(\mathbf{u}, \mathbf{v}) = \frac{1}{2\sigma^2}\left\|\mathbf{y}(t) - \mathbf{H}\mathbf{u}\right\|^2 + \phi(\mathbf{v}, \mathbf{u})$ functions, $\mathbf{u}$ and $\mathbf{v}$ belong to the same space as $\mathbf{x}$. Then the cost function can be written as

$$\sum_{t=2}^{T} \Psi_t(\mathbf{x}(t-1), \mathbf{x}(t)) + \frac{1}{2\sigma^2}\left\|\mathbf{y}(T) - \mathbf{H}\mathbf{x}(T))\right\|^2 \quad (5)$$

Hence a dynamic programing equation can be written. Let $q(i,j)$ be minimum of (5) passing through position i until time j, where $i \in \{1, ..., M\}$ and $j \in \{1, .., T\}$. Then

$$q(i, T) = \left\|\mathbf{y}(T) - \mathbf{H}\mathbf{x}_i\right\|^2 \quad (6)$$

$$q(i,j) = min\{q(1, j+1) + \Psi_j(\mathbf{x}_i, \mathbf{x}_1), ..., q(M, j+1) + \Psi_j(\mathbf{x}_i, \mathbf{x}_M)\} \quad (7)$$

As q is build, was stored the index from where comes the minimum in (7).Then a traceback is done in order to find the optimal path.

Without the use of dynamic programing the complexity would give $O(n^T)$ because it is needed to find a path in the grid which minimize the cost function. Since dynamic programing was used the complexity is $O(n^2 T)$.

III. Online approach

The final approach drops the discretized space assumption and allows the target to move freely in the continuous space in the up, down, right or left directions. The dynamics of the target's movement is modeled as the expected value of the next position knowing the previous position $\mathbb{E}(x_{t+1}|x_t)$, according to some distribution. This value is given by

$$\mathbb{E}(\mathbf{x}(\mathbf{t+1})|\mathbf{x}(\mathbf{t})) = \mathbf{x}(\mathbf{t}) + s \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix}^{\top} \begin{bmatrix} P_{up} \\ P_{down} \\ P_{left} \\ P_{right} \end{bmatrix} \quad (8)$$

where $\mathbf{x}(\mathbf{t})$ expresses the position $\mathbf{x}$ of the target at time t, P_* represents the *a priori* probabilities of the target moving in either of the allowed directions and s is the step size of the target.

The target's dynamic will contribute, along with the sensors' readings, to the cost function

$$\underset{\mathbf{x} \in \mathbb{R}^2}{minimize} \quad \frac{\eta}{2}\left\|\mathbf{x} - \mathbb{E}(\mathbf{x}(\mathbf{t+1})|\mathbf{x}(\mathbf{t}))\right\|^2 + + \sum_{i=1}^{N}\left(y_i - \frac{1}{1+\lambda\|\mathbf{r}_i - \mathbf{x}\|^2}\right)^2 \quad (9)$$

where η is the weight of the target dynamics' contribution and the variables in the second term are the same as explained in the previous sections with the exception of $\mathbf{x}$ belonging to a continuous space. The parameter η should be adjusted depending on the quality of the sensors' readings. Readings with high noise will require a higher η to improve the results, and *vice versa*. Since the optimization problem expressed in (9) is non convex, it's possible that the result of this optimization is not the global minimum. Therefore, in order to improve efficiency and the quality of the results, the minimization is initialized with the estimation of the previous position. Since the target's position between times should not differ greatly, this procedure increases the chances of finding a solution close to the previous position.

IV. Results

This section presents the results of each approach in two distinct paths with 20 steps. Each path was generated randomly based on probabilities for each direction. In the generation of one of the paths each direction had equal probability. The other had a probability of 90% to the left and 10% distributed between the rest. Both paths are illustrated on Figures 1 and 2. The step size of the target on the generated path is of 10 in a 200 by 200 space. The generated grid to this space is 20 by 20, which means the distance between points is exactly the size of the target's step.

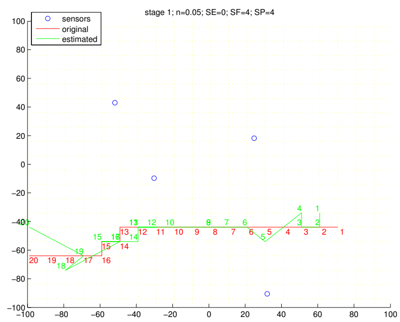

Fig. 1. Path with 90% probability for the left direction. Yellow lines represent the grid matrix. The target's true path and estimate are represented by the red and green line, respectively.

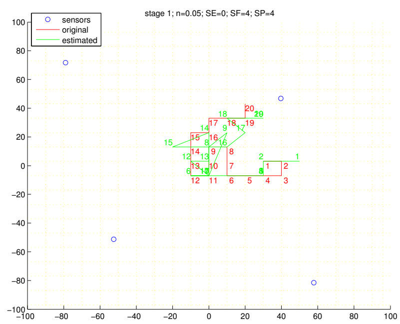

Fig. 2. Path with equal probability for each direction.

The generated signals are polluted with Gaussian noise with different standard deviations. Two kinds of noises were analyzed: one with periodic noise spikes and the other without. Data with noise spikes allow to observe how the different approaches react to conditions more similar to what is expected from real sensors. Figure 3 and 4 show the errors (euclidean distance from estimate to real point) of the three approaches (including variation of η for online approach) for signal noise of 0.01 standard deviation without and with spikes, respectively.

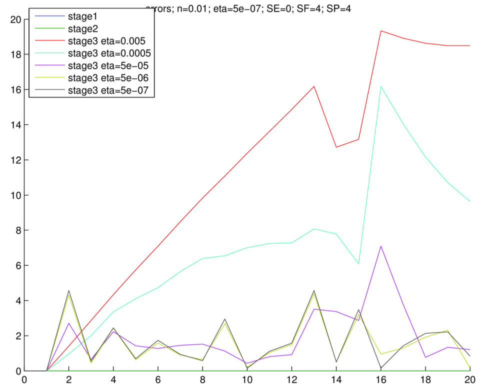

Fig. 3. Errors for different approaches for noise with 0.01 standard deviation.

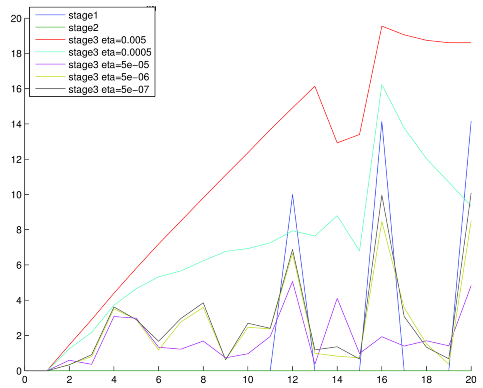

Fig. 4. Errors for different approaches for noise with 0.01 standard deviation and noise spikes every fourth reading.

The results of Figures 3 and 4 reveal that the second approach estimates the path with no error with and without noise spikes. The first approach estimates correctly without noise spikes presents errors specially around noise spikes. The online approach presents errors in both cases, however it shows more resilience to noise spikes than approach 1, for some η (this becomes clearer for greater noise). It's also clear that, for an $\eta \to 0$ (reducing the influence of the target's dynamics term), the performance of online approach is close to that of the first approach, though never being the same even for $\eta = 0$, since the minimization of the former is performed over a continuous space (with no guarantees of convergence to global minima) while the later always returns the global minima.

Figures 5 and 6 present the results for 0.05 standard deviation noise. The second approach's results are once

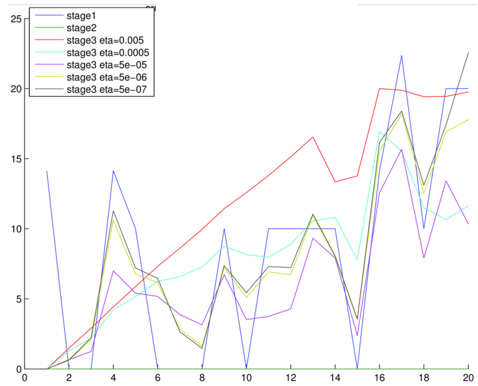

Fig. 5. Errors for different approaches for noise with 0.05 standard deviation.

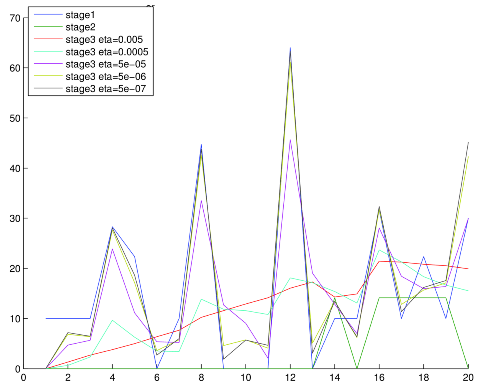

Fig. 6. Errors for different approaches for noise with 0.05 standard deviation and noise spikes every fourth reading.

again the best, having minimal error. The error, overall, has increased has would be expected with an increased noise. It should be noted the superior performance of the online approach on the presence of noise spikes relative to that of the first approach. While the later presents significant error, the former has reasonable error for higher η. In fact, for very high η (> 1), the results are similar to the result for $\eta = 0.005$. This is because the target's dynamics for the online approach suits very well the target's true path (the path is mostly a straight line to the left and that direction as a probability of 90%). Once again, it's clear that the results of the online approach become similar to those of the first approach for very small η.

Both previous cases suggest that the online approach is more prone to the accumulation of errors for future estimates for higher η. This is to be expected since for a big η the target's dynamics has substantial influence in the estimate and that dynamic is the expected value of the next position summed to the current estimate. If both expected value and current estimate have errors this propagates for future estimations.

Results for the random direction path are presented on Figures 7 and 8. These results correspond to a noise with 0.05 standard deviation.

From the analysis of Figures 7 and 8, it's clear that the

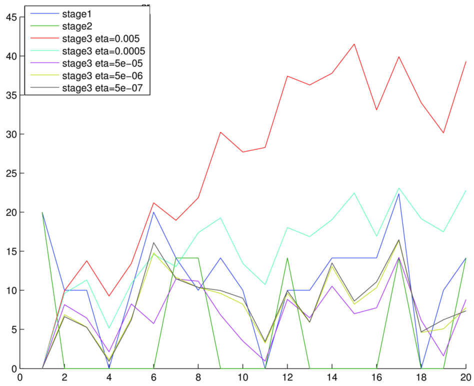

Fig. 7. Errors for different approaches for noise with 0.05 standard deviation. The path is generated for directions with equal probability.

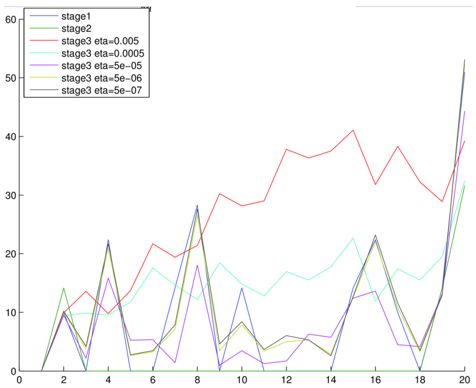

Fig. 8. Errors for different approaches for noise with 0.05 standard deviation and noise spikes every fourth reading. The path is generated for directions with equal probability.

performance of the first and third approaches are worse on this path, while the second approach maintained the same level of effectiveness. It should also be noted that even with noise spikes, the online approach doesn't perform much better than the first approach. This is specially true for high η since the accumulation of error is quite significant. A deeper look on why this is reveals that for equiprobable directions the expected value of the next position relative to the current is 0, i.e. the target's dynamics best estimate is that the target is not moving.

Further conclusions can be drawn from these results and other experiments not presented. The first relates to the grid in the first two approaches. The amount and position of grid points should be suited to the path's points, otherwise, since the estimates will always be grid points, a good estimation is unlikely. Ideally, a very large number of grid points should be used to approximate as much as possible to the continuous case. This, however, would have a very high computational price, specially for the second approach. In the second approach, an assumption was made concerning the temporal-spatial adjacency, specifically that in each time step the target could only move from one grid point to another adjacent to it. However, this is assuming a target that moves always at the same speed and that the distance between grid points matches exactly the step size of the target's movement, which might not happen in a real world scenario for all targets.

The accuracy of the target's dynamics in both the second and online approach is very influential. In the presented experiments these dynamics (i.e. the probabilities for each direction) matched exactly the generated behaviour of the target. This, however, might not be possible in a real world situation, where the movement dynamics are but an approximation to what actually happens. If the dynamics (i.e. the distributions for the directions) differed greatly the results would be significantly worse, as further simulations demonstrated.

So far the analysis has been focused in effectiveness of the different approaches in correctly estimating the target's path. However efficiency is also very important. Not surprisingly the performance of the three approaches varied very little on problems of the same size, having a mean of 2.8921s for the first approach, 34.3004s for the second and 0.4272s for the last for problems with a 20 step path, a 200 by 200 space and a 20 by 20 grid. The online approach is by far the fastest and, among the three presented, the only usable in a real-time system - hence it's name.

V. Conclusions

This work presented three distinct approaches to estimate a target's path from sensors' readings. Two of them used knowledge about the target's dynamics. The results showed that the first approach (based on sensors' readings alone) performed well on low noise environments but is not resilient to increased noise or noise spikes (as one might expect in a real world environment). The second approach adds a temporal adjacency component with the target's dynamics term (target is assumed to only move to adjacent grid points and moves every time step), which performs better on higher noise environments and even noise spikes. It has the drawback that it's much more computationally expensive to run an estimation. This approach was implemented using dynamic programming to reduce the complexity of the problem. At last, the online approach uses both the sensors' readings and target's dynamics, but the later is just the expected value which means that for random distributions of the target's dynamics this approach does not yield good results. If the distribution is heavily biased to one direction, the results are better and more resilient to noise spikes. This approach has the advantage of being the fastest of the three and the most suitable for a real time application, while the second approach would be more useful to perform estimations *a posteriori*.

References

[1] Shahrokh Farahmand, Georgios B Giannakis, Geert Leus and Zhi Tian. *Tracking target signals strengths on a grid using sparsity.* EURASIP Journal on Advances in Signal Processing 2014 2011