TABLE II
PERFORMANCE OF HYBRID CODERS

	b_c bits/pel	Nonadaptive SNR (dB)	Nonadaptive NMSE (%)	Line Adaptive SNR (dB)	Line Adaptive NMSE (%)	Overhead bits/pel
	4	34.03	0.0395	38.91	0.0129	0.035
	3	27.96	0.1601	34.23	0.0377	0.035
$I = 2$	2	22.33	0.5848	28.90	0.1287	0.035
	1.5	21.59	0.6943	25.93	0.2554	0.018
	1	16.97	0.2008	23.25	0.4728	0.018
	4	33.66	0.0431	38.28	0.0149	0.070
	3	31.61	0.0691	36.15	0.0243	0.070
$I = 4$	2	26.64	0.2170	31.92	0.0643	0.053
	1.5	21.64	0.6852	28.57	0.1392	0.053
	1	20.58	0.8741	25.23	0.2998	0.035
	4	31.85	0.0653	35.24	0.0299	0.141
	3	30.12	0.0972	35.03	0.0314	0.141
$I = 8$	2	25.20	0.3021	32.89	0.0514	0.123
	1.5	24.56	0.3502	30.94	0.0805	0.088
	1	20.42	0.9081	27.41	0.1817	0.070
	4	29.82	0.1041	31.98	0.0634	0.281
	3	29.75	0.1060	32.57	0.0553	0.264
$I = 16$	2	28.34	0.1464	32.82	0.0523	0.229
	1.5	23.57	0.4397	31.48	0.0712	0.193
	1	22.15	0.6100	28.37	0.1457	0.141

An exploratory simulation was also performed to determine the improvement resulting by adapting bit allocation among the I quantizers on a line-to-line basis, keeping the average number of quantizer bits per coefficient equal for each line. The bit allocation was determined from the quantized sample variances σ_i^{*2}, $i = 1, 2, \cdots, I$, by the methods of [2], [12]–[14]. This complex scheme yielded only 0.5 dB improvement in SNR for the source image of Fig. 4 with $r = 1$ and $I = 4$ compared to LAQ.

V. CONCLUSION

The experimental results indicate that the feedforward LAQ provides an effective means to increase the performance of the hybrid coding system. Even though this makes the hardware implementation somewhat more complex by requiring additional memory and separate transmission of variance information, the improvement may justify the use of LAQ in some applications.

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Diversity ALOHA—A Random Access Scheme for Satellite Communications

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***Abstract*—A generalization of the slotted ALOHA random access scheme is considered in which a user transmits multiple copies of the same packet. The multiple copies can be either transmitted simultaneously on different frequency channels (frequency diversity) or they may be transmitted on a single high-speed channel but spaced apart by random time intervals (time diversity). In frequency diversity, two schemes employing channel selections with and without replacements**

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have been considered. In time diversity, two schemes employing a fixed number of copies or a random number of copies for each packet have been considered. In frequency diversity, activity factor–throughput tradeoffs and in time diversity, delay–throughput tradeoffs for various diversity orders have been compared. It is found that under light traffic, multiple transmission gives better delay performance. If the probability that a packet fails a certain number or more times is specified not to exceed some time limit (realistic requirement for satellite systems having large round trip propagation delay), then usually multiple transmission gives higher throughput.

I. INTRODUCTION

An attractive feature of slotted ALOHA multiple access [1]–[11] is that little coordination among users is required. However, due to occasional collision among packets, on the average a packet may have to be transmitted more than once before it is received correctly. This will introduce large packet delay in satellite slotted ALOHA systems, where each round trip propagation delay is about 270 ms.

This paper considers a generalization of the slotted ALOHA scheme in which, whenever a user generates a packet, he transmits k copies of the same packet. It is assumed that there exists some arrangement which allows a receiver to reject all but one correctly received copy of any packet. The multiple copies can be transmitted either simultaneously on different frequency channels, which we call frequency diversity, or they may be transmitted on a single high-speed channel but spaced apart by random time intervals; we call this time diversity.

In Section II of this paper we have considered two frequency diversity schemes where the channels are selected with and without replacement, respectively. We have defined an activity factor which is the average number of times a packet must be transmitted to be successful. The activity factor–throughput tradeoff has been studied for various orders of diversity k. The performance of the system for various k has also been compared using another criterion where it is stipulated that the probability that a packet fails a certain number of times should not exceed a specified limit. In Section III, the delay–throughput characteristics of two time-diversity schemes have been considered.

For each type of diversity, two types of schemes are suggested. For all the schemes, the delay–throughput tradeoff has been studied for various orders of diversity k. The performance of the schemes for various k has also been compared using another criterion, where it is stipulated that the probability that a packet fails a certain number of times should not exceed a specified limit.

II. FREQUENCY DIVERSITY

A. Scheme 1 (Channel Selection with Replacement)

Suppose there are l frequency channels, each having the same bandwidth, and that users transmit packets of constant length. The duration τ of a packet is the same in all the channels. We assume this time τ to be also the slot duration. Whenever a user generates a packet (this may be a new or a previously collided one) he makes k choices and, in each choice, selects one out of the l channels at random and with equal probability. A channel may be selected more than once. Hence, the user will actually choose k or fewer distinct channels. A copy of the packet to be transmitted will be transmitted on all the chosen channels. If, however, a particular channel is chosen more than once, then only once will a copy be transmitted on that channel. If a copy of the packet is received correctly on at least one of the channels, then a positive acknowledgment is sent to the user. This positive acknowledgment, however, need not be an explicit one. The transmitting user may monitor the downlink and find out for himself whether at least one copy transmission was successful. If all copies of the packet collide with other packet copies, then the user will not get a positive acknowledgment and he will retransmit copies of the packet after some random rescheduling delay. The rescheduling delay has to be random to prevent recurring collisions. In subsequent analysis we assume the total packet generation (new and retransmitted) to be Poisson. This requires that the average rescheduling delay must be large compared to the packet duration. In other words, sufficient time must elapse for conditions on the channel to change significantly before a retransmission is effected. In the following discussion we say that a packet succeeds if any copy succeeds. If more than one copy of a given packet succeeds, then that counts as only one *packet success*.

Suppose that, in the steady state, the probability of success for a packet is P_s, and furthermore, that at the time a particular user u_1 transmits copies of a packet, r other users also transmit. We will assume an infinite user population for tractability of analysis. Let the probability that there are m (out of the possible l) channels not chosen when the r other users each make their k selections be denoted by $P_0(m; k, r, l)$. The user u_1 will succeed if at least one of his k choices is one of these empty channels. The probability that none of the choices made by user u_1 are empty channels is $[1 - (m/l)]^k$, and the probability that at least one choice is an empty channel is $1 - [1 - (m/l)]^k$. Thus, the probability that a certain ready user u_1 succeeds in a time slot, given that r other users are also transmitting in that time slot, is given by

$$P_{s/r} = \sum_{m=0}^{l} P_0(m; k, r, l)\{1 - [1 - (m/l)]^k\}. \tag{1}$$

We assume that the packets (including new and retransmitted ones) are generated according to a Poisson point process with rate Λ. When a particular user u_1 is ready in a certain time slot, the probability that r other users are also ready in that time slot is $[(\Lambda\tau)^r/r!]e^{-\Lambda\tau}$. Therefore, the probability of success for the user u_1 is

$$P_s = \sum_{r=0}^{\infty} P_{s/r} e^{-\Lambda\tau}(\Lambda\tau)^r/r! \tag{2}$$

Recall that $P_0(m; k, r, l)$ is the probability that exactly m out of the l channels will remain empty after each of r users have made k choices for a channel. To determine $p_0(m; k, r, l)$, consider a similar problem in which $\boldsymbol{a}$ balls are to be placed in $\boldsymbol{b}$ cells. Each of the $\boldsymbol{a}$ balls are placed in one of the $\boldsymbol{b}$ cells at random and with equal probability. Suppose $P_m(a, b)$ represents the probability that exactly m cells will remain empty. This is a standard combinatorial problem [12] whose solution is given by

$$P_m(a, b) = \binom{b}{m} \sum_{\nu=0}^{b-m} (-1)^\nu \binom{b-m}{\nu} \left(1 - \frac{m+\nu}{b}\right)^a. \tag{3}$$

It is clear that $P_0(m; k, r, l) = P_m(kr, l)$. So

$$P_0(m; k, r, l) = \binom{l}{m} \sum_{\nu=0}^{l-m} (-1)^\nu \binom{l-m}{\nu} \left(1 - \frac{m+\nu}{l}\right)^{kr}. \tag{4}$$

Using (1), (2), and (4) we get

$$P_s = \sum_{r=0}^{\infty} \frac{(\Lambda\tau)^r}{r!} e^{-\Lambda\tau} \sum_{m=0}^{l} \{1 - [1 - (m/l)]^k\} \binom{l}{m} \cdot \sum_{\nu=0}^{l-m} (-1)^\nu \binom{l-m}{\nu} \left(1 - \frac{m+\nu}{l}\right)^{kr}.$$

On simplification we get

$$P_s = \sum_{m=1}^{l} \left\{1 - \left(1 - \frac{m}{l}\right)^k\right\} \binom{l}{m} \sum_{\nu=0}^{l-m} (-1)^\nu \binom{l-m}{\nu} \cdot \exp\left[-\Lambda\tau \left\{1 - \left(1 - \frac{m+\nu}{l}\right)^k\right\}\right]. \tag{5}$$

The total traffic (including new and retransmitted packets) generated by the user population is $\Lambda\tau$. We define the normalized total traffic as

$$G \triangleq (\Lambda\tau)/l. \tag{6}$$

Then (5) becomes

$$P_s = \sum_{m=1}^{l} \left\{1 - \left(1 - \frac{m}{l}\right)^k\right\} \binom{l}{m} \sum_{\nu=0}^{l-m} (-1)^\nu \binom{l-m}{\nu} \cdot \exp\left[-Gl \left\{1 - \left(1 - \frac{m+\nu}{l}\right)^k\right\}\right]. \tag{7}$$

For the special case of $k = 1$, we get

$$P_s = \sum_{m=1}^{l} \frac{m}{l} \binom{l}{m} \sum_{\nu=0}^{l-m} (-1)^\nu \binom{l-m}{\nu} \cdot \exp\left[-Gl \left(\frac{m+\nu}{l}\right)\right].$$

After simplification we get

$$P_s = \exp(-G). \tag{8}$$

This is the standard slotted ALOHA relation.

We define the normalized throughput S as the average number of *packet successes* per time slot divided by the number of channels l. Packet successes are counted according to our earlier definition. We divide by l so that $S = 1$ is the maximum possible throughput (for perfect scheduling). The normalized throughput represents the bandwidth utilization of the system. It is clear that

$$S = GP_s. \tag{9}$$

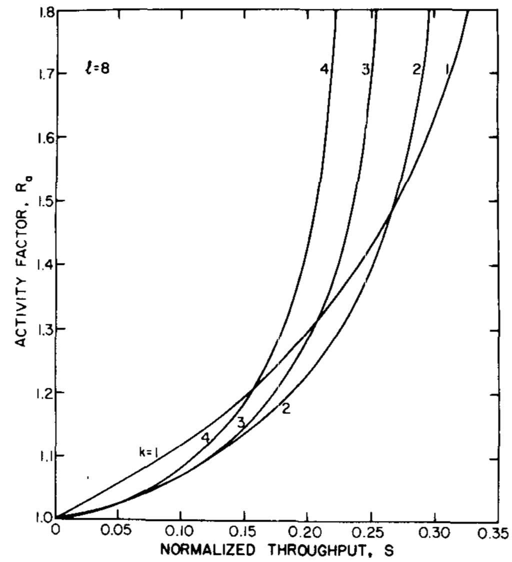

Fig. 1. Normalized throughput-activity factor tradeoff. Frequency diversity, scheme 1, $l = 8$, various k.

The activity factor R_a may be defined as the average number of times a packet must be transmitted to be successful. Then

$$R_a = G/S = 1/P_s. \tag{10}$$

This factor is a measure of delay. For any given normalized total traffic G, we can calculate S and the corresponding R_a using (7), (9), and (10). In Figs. 1 and 2 the activity factor R_a has been plotted against normalized throughput S for various l and k. In the steady state, S must also be equal to the normalized input traffic. It can be seen that for light input traffic, larger values of k give better delay performance while for heavy input traffic, smaller values of k give better delay performance. A value of $k = 2$ gives a robust scheme in the sense that delay performance is quite good over a large range of input traffic.

Let us consider another type of performance criterion. Suppose it is stipulated that the probability that a packet fails n or more times should not be more than β. We wish to find out the maximum normalized throughput S the system can have under the above condition. This is a reasonable requirement for satellite slotted ALOHA systems where the round-trip propagation delay is large. Now the probability of success of a packet in each try is P_s. With independent tries, the random variable N representing the number of times a packet must be transmitted to be successful will have a geometric distribution. According to the properties of this distribution,

$$P(N = n) = P_s(1 - P_s)^{n-1} \tag{11}$$

and

$$P(N > n) = (1 - P_s)^n. \tag{12}$$

So the requirement is $(1 - P_s)^n \leqslant \beta$ or equivalently $P_s \geqslant 1 - \beta^{1/n}$. Now, since P_s is a probability,

$$1 \geqslant P_s \geqslant 1 - \beta^{1/n}. \tag{13}$$

From (7), (9), and (10) we can determine the maximum possible normalized throughput $S_{\max}$ consistent with the above

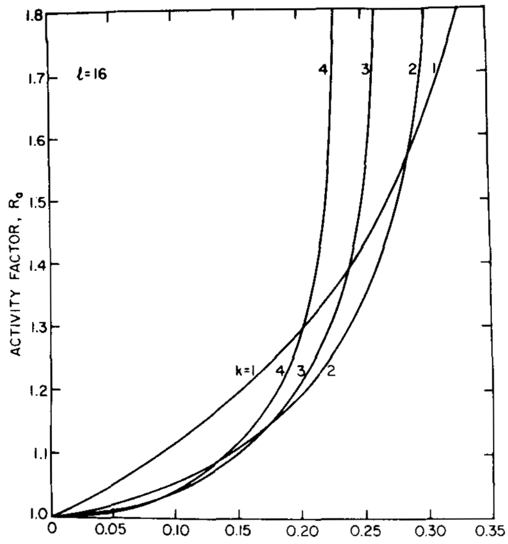

Fig. 2. Normalized throughput-activity factor tradeoff. Frequency diversity, scheme 1, $l = 16$, various k.

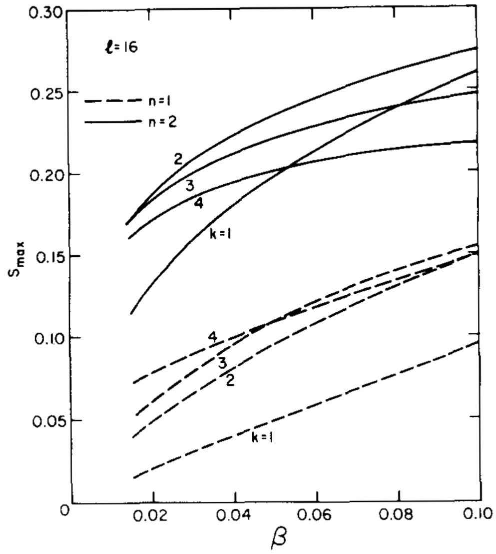

Fig. 3. Maximum attainable normalized throughput S_{max} when it is given that the probability that a packet fails n or more times has to be less than β. Frequency diversity, scheme 1, $l = 16$, various n, k.

condition. In Fig. 3, S_{max} has been plotted as a function of β for various n and k and $l = 16$. It is seen that for both $n = 1$ and 2 the $k = 2$ scheme gives larger throughput compared to the standard slotted ALOHA ($k = 1$) scheme. For $n = 1$, the $k = 3$ scheme gives even better throughput performance.

Before concluding this section, let us compare the performance of the frequency diversity scheme employing l channels and k replications per transmitted packet with a system employing a single channel and l times the bandwidth, but with no replications. Since the bandwidth is l-fold, the packet length will be reduced to τ/l. If we assume the same total packet generation rate Λ as the diversity scheme, then the total traffic will be $\Lambda\tau/l$. So the probability of success for a transmitted packet will be

$$P_s = \exp(-\Lambda\tau/l) = \exp(-G). \tag{14}$$

The throughput S and activity factor R_a will be given by

$$S = GP_s \tag{15}$$

$$R_a = G/S - 1/P_s. \tag{16}$$

Equations (14)-(16) are exactly the same as (8)-(10), which indicates that the $R_a - S$ characteristic of the single high-speed channel is exactly the same as that of the diversity scheme with $k = 1$ as shown in Figs. 1 and 2.

Next consider packet delay. The packet delay has three parts: packet transmission delay, round trip propagation delay, and packet rescheduling delay. Assume the round trip propagation delay to be T and the average rescheduling delay for the frequency diversity scheme and the single channel scheme to be $\bar{R}_1$ and $\bar{R}_2$, respectively. A packet will succeed in R_a attempts on the average. So, on the average, the packet transmission delay and the round trip propagation delay will occur R_a times, while rescheduling delay will occur $(R_a - 1)$ times. So the delays D_1 and D_2 in a frequency diversity scheme and a single channel scheme with the same R_a and will be given by

$$D_1 = R_a\tau + R_aT + (R_a - 1)\bar{R}_1 \tag{17}$$

$$D_2 = R_a\tau/l + R_aT + (R_a - 1)\bar{R}_2. \tag{18}$$

For a satellite system with large round trip propagation delay,

$$T \gg \tau, \bar{R}_1, \text{ and } \bar{R}_2. \tag{19}$$

So the delays D_1 and D_2 will both be approximately equal to R_aT. We have shown before that the $R_a - S$ characteristic of the single high-speed channel is exactly the same as that of the multichannel diversity scheme with $k = 1$. Now we may further conclude that the delay-throughput characteristic of the single high-speed channel is also approximately the same as that of the multichannel scheme with $k = 1$. This latter conclusion is true, however, only for satellite systems.

B. Scheme 2 (Channel Selection Without Replacement)

Suppose there are l frequency channels as in scheme 1, each having the same bandwidth. Whenever a user has a packet ready to be transmitted, he chooses k out of the l channels at random. However, unlike scheme 1, k channels chosen by a user in this scheme must be distinct. As in the last scheme, we assume the packet generation (new plus retransmitted) to be Poisson with rate Λ. Each time a packet is generated, a copy is transmitted on k of the l channels. The probability that a copy will be transmitted on a particular channel is k/l. Hence, it follows from the properties of the Poisson process that the copy transmission on any particular channel also will be Poisson with rate $\Lambda k/l$. Thus, the traffic on each channel will be $\Lambda k\tau/l = kG$, where $G \triangleq \Lambda\tau/l$ as defined previously. It follows that the probability of success of a copy on any channel will be

$$P_s' = \exp(-kG). \tag{20}$$

The probability that a copy will collide on any particular channel is $1 - P_s'$. The probability that all k copies collide is $(1 - P_s')^k$. Therefore, the probability of success of a packet is given by

$$P_s = 1 - (1 - P_s')^k. \tag{21}$$

As in scheme 1, the normalized throughput is given by

$$S = GP_s \tag{22}$$

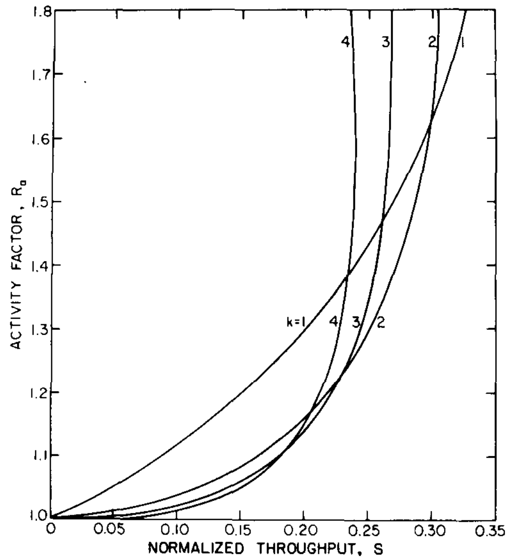

Fig. 4. Normalized throughput-activity factor tradeoff. Frequency diversity, scheme 2, various k.

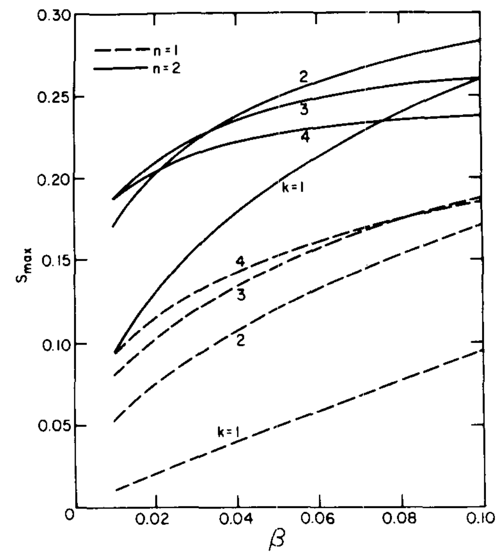

Fig. 5. Maximum attainable normalized throughput $S_{\max}$ when it is given that the probability that a packet fails n or more times has to be less than β. Frequency diversity, scheme 2, various n, k.

and the activity factor

$$R_a = 1/P_s. \tag{23}$$

Using (20)–(23), a relationship between normalized throughput S and activity factor R_a can be obtained:

$$S = -(1/(kR_a)) \ln [1 - (1 - 1/R_a)^{1/k}]. \tag{24}$$

It is to be noted that, in contrast with scheme 1, the relationships (20)–(24) do not depend on the total number of channel l. In Fig. 4, R_a has been plotted as a function of S for various k.

Now let us consider the other type of performance criterion, where it is stipulated that the probability that a packet fails n or more times should not be more than β. As shown in (13), the above restriction will limit P_s in the interval

$$1 \geq P_s \geq 1 - \beta^{1/n}. \tag{25}$$

From (23) and (24), S can be expressed as a function of P_s as

$$S = (P_s/k) \ln [1 - (1 - P_s)^{1/k}]. \tag{26}$$

The maximum allowable normalized throughput $S_{\max}$ compatible with the condition (25) can be determined from (26). In Fig. 5, $S_{\max}$ has been plotted as a function of β for various n and k. For both $n = 1$ and 2, the $k = 2$ scheme gives larger throughput than the standard slotted ALOHA ($k = 1$) scheme. For $n = 1$, the $k = 3$ and $k = 4$ schemes give throughputs even larger than the $k = 2$ scheme.

III. TIME DIVERSITY

A. Scheme 1 (Deterministic Packet Transmission)

In this scheme we consider a single high-speed satellite channel. Suppose the round trip propagation delay is T. Usually, T will be much larger than the packet duration τ. After transmitting a packet, a user waits for a time T to learn if the transmission is successful. If it is not, he retransmits the packet after some random rescheduling delay R. The average rescheduling delay $\bar{R}$ should be large (typically 5–10 times) compared to the packet duration τ, so that successive packet transmissions can be assumed to be independent. For satellite systems, $\bar{R}$ is usually small compared to the round trip propagation delay T, although it is large compared to τ.

Now suppose each time a packet is ready, k replications of it ($k \geq 1$) are transmitted. In order that the transmissions of copies can be assumed to be independent of one another, these transmissions are spaced by random rescheduling delays R. After the kth replication is transmitted, the user waits to find out whether any of them succeeded. If none of them have succeeded, then after a random rescheduling delay R, he again transmits k replications of the packet with random rescheduling delays R from copy to copy.

We assume that the packet generation by the user population (considering both new and retransmitted) is Poisson with rate Λ. We define the total traffic G as the total number of packets (new and retransmitted) generated on the average per time slot. Then $G = \Lambda\tau$. Also, we define S as the average number of packets that succeed per time slot. It is to be noted that in Section II we defined normalized G and S by dividing total traffic and throughput by the number of frequency channels. In this section we need not do that, since in time diversity there is only one frequency channel.

For each packet, k replications are transmitted. Since they are transmitted independently (this is guaranteed by having a large average value of the random variable R), we can assume that the transmission of replications is also Poisson with rate Λk. Thus, the average number of packet copies generated per time slot is $\Lambda k\tau = \Lambda G$ and the probability that a particular transmitted copy succeeds is given by

$$P_s' = \exp(-kG). \tag{27}$$

The probability that at least one of the k replications of a packet will be successful is given by

$$P_s = 1 - (1 - P_s')^k. \tag{28}$$

The throughput is then

$$S = GP_s. \tag{29}$$

We now consider derivation of an expression for delay. In general, each user attempts a number of times to transmit a packet and, in each attempt, transmits k replications of the packet. We define the delay D to be the time difference between the instant when a user transmits the first replication in the first attempt and the instant when the user knows for the first time that a packet transmission has been successful (there may be more than one success for the same packet, but in considering delay, only the first one is important). Suppose $\bar{D}$ represents the expect delay.

During the first attempt of the user, the following events may take place.

Event 1: At least one copy of the first attempt succeeds and the first successful copy is the mth one, where $1 \leqslant m \leqslant k$. The probability of this event is $(1-P_s')^{m-1}P_s'$ and the expected delay, given this event has occurred, is $T+(m-1)\bar{R}$.

Event 2: All the k copies of the first attempt fail. The probability of this event is $(1-P_s')^k$. If event 2 occurs, then after an expected time $T+(k-1)\bar{R}$, the user knows that all the copies of the first attempt have failed. After knowing this, the user waits for a random time R and then starts the second attempt. Since the probability of success in a certain attempt is not influenced by the outcome of previous attempts, the expected remaining delay at the beginning of the second attempt is the same as that at the beginning of the first attempt, which is $\bar{D}$. So the expected total delay under the event 2 is $T+(k-1)\bar{R}+\bar{R}+\bar{D}$.

Events 1 and 2 are the only possible events during the first attempt. Therefore,

$$\bar{D}=\sum_{m=1}^{k}(1-P_s')^{m-1}P_s'[T+(m-1)\bar{R}] + (1-P_s')[T+(k-1)\bar{R}+\bar{R}+\bar{D}].$$

After some algebraic manipulation, we find

$$\bar{D}=\frac{T}{1-(1-P_s')^k}+\frac{\bar{R}(1-P_s')}{P_s'}. \tag{30}$$

Normalizing with respect to the round trip propagation delay, we get the normalized delay as

$$\bar{D}=\frac{1}{1-(1-P_s')^k}+\frac{(\bar{R}/T)(1-P_s')}{P_s'}. \tag{31}$$

For any total traffic G, we can determine the throughput S and the corresponding normalized delay $\bar{D}_n$ using (27)-(31). In Figs. 6 and 7, $\bar{D}_n$ has been plotted as a function of S for various values of the ratio $\bar{R}/T$ and k. In Fig. 6 we have also shown the delay obtained by simulation. In simulation we have assumed the number of users in the system to be 100 and round trip propagation delay T equal to 100 slots. In the steady state the throughput S is also equal to the input traffic. So we see from Figs. 6 and 7 that for light input traffic, the $k > 1$ schemes outperform the standard slotted ALOHA ($k = 1$) scheme, but for heavy input traffic the $k = 1$ scheme is better. On the whole, the $k = 2$ scheme is a robust one since it gives quite good throughput-delay performance over a wide range of input traffic. It is clear from Figs. 6 and 7 that the superiority of the $k > 1$ schemes for light input traffic is enhanced for a smaller $\bar{R}/T$ ratio.

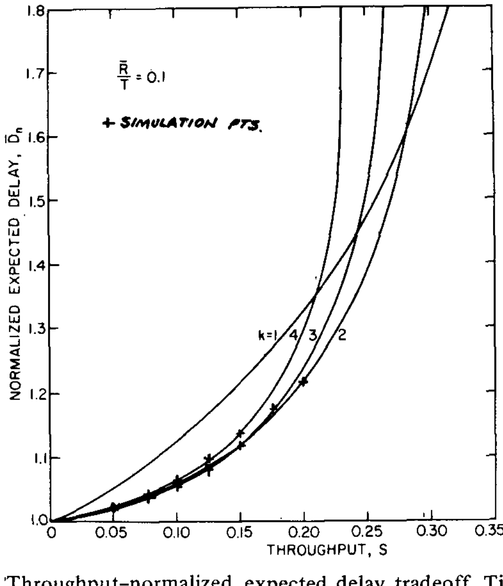

Fig. 6. Throughput-normalized expected delay tradeoff. Time diversity, scheme 1, $\bar{R}/T = 0.1$, various k.

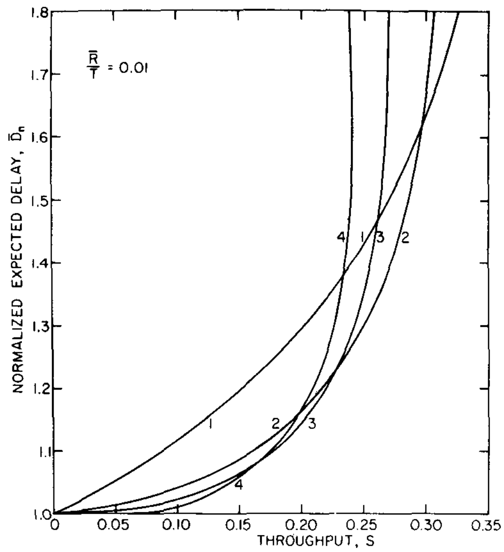

Fig. 7. Throughput-normalized expected delay tradeoff. Time diversity, scheme 1, $\bar{R}/T = 0.01$, various k.

If we consider the other type of performance criterion, in which it is stipulated that the probability that a user's n or more consecutive attempts (each attempt consists of sending k replications of the packet) for packet transmission fails must be less than β, then the S_{max} versus β curves will be the same as that for frequency diversity scheme 2 shown in Fig. 5, because (27)-(29) for P_s and S in this scheme are the same as (26), (21), and (22), respectively for frequency diversity scheme 2.

B. Scheme 2 (Probabilistic Packet Transmission)

In scheme 1 it was assumed that in each attempt a user transmits k replications of the same packet, the successive transmissions being separated by random intervals R. Now we assume that after transmitting the first replication of the packet, the user does not transmit the second one with certainty but rather with a probability p. Also, for simplicity we confine ourselves to the case $k = 2$. It is clear that for $p = 0$

and 1, this scheme is identical to time diversity scheme 1 with $k = 1$ and 2, respectively.

During each attempt a user will transmit two replications with probability p or one replication with probability $(1 - p)$. So the average number of replications transmitted during each try is

$$k_{av} = 2p + (1 - p) \equiv 1 + p. \tag{32}$$

We assume that the packet generation by the user population is Poisson with rate Λ. Since successive transmissions of replications are assumed to be independent, it follows that replications transmitted on the channel also follow a Poisson point process with rate $\Lambda k_{av} = \Lambda(1 + p)$. Thus, the channel traffic is given by $\Lambda(1 + p)\tau = G(1 + p)$ (where $G = \Lambda\tau$) and the probability of success of a copy transmitted on the channel is

$$P_s' = \exp[-G(1 + p)]. \tag{33}$$

Let P_s represent the probability that on a particular attempt a packet succeeds (that is, at least one copy succeeds). If the user decides to transmit one replication of the packet, the probability of success will be P_s, but if he decides to transmit two replications, then the probability of success will be $1 - (1 - P_s')^2$. Thus, the probability of packet success is $P_s = (1 - p)P_s' + p[1 - (1 - P_s')^2]$ or

$$P_s = (1 + p)P_s' = p(P_s')^2. \tag{34}$$

The throughput S is given by

$$S = GP_s. \tag{35}$$

We now determine an expression for delay. Denote the expected delay as $\bar{D}$. During the first attempt of the user the following events may take place.

Event 1: The first replication will succeed. The probability of this event is P_s' and the expected delay, given this event has occurred, is T.

Event 2: The user decides not to transmit the second replication and the first replication fails. The probability of this event is $(1 - p)(1 - P_s')$ and the expected delay given this event has occurred is $T + \bar{R} + \bar{D}$.

Event 3: The user decides to transmit the second replication, the first replication fails, and the second replication succeeds. The probability of this event is $p(1 - P_s')P_s'$ and the expected delay, given this event has occurred, is $T + \bar{R}$.

Event 4: The user decides to transmit the second replication and both the replications fail. The probability of this event is $p(1 - P_s')^2$ and the expected delay, given this event has occurred, is $T + \bar{R} + \bar{R} + \bar{D}$. Since these are all the possible events, the expected delay is given by

$$\bar{D} = P_s'T + (1 - p)(1 - P_s')[T + \bar{R} + \bar{D}]$$
$$+ P(1 - P_s')P_s'[T + \bar{R}] + P(1 - P_s')^2[T + \bar{R} + \bar{R} + D].$$

After some algebraic manipulations and normalization with respect to T, we get the normalized expected delay as

$$\bar{D}_n = \frac{1}{P_s'\{1 + p(1 - P_s')\}} + \frac{(\bar{R}/T)(1 - P_s')}{P_s'}. \tag{36}$$

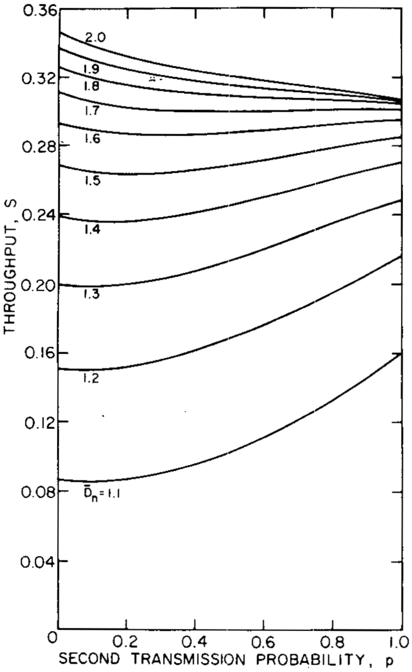

Fig. 8. Throughput S as a function of second transmission probability p for specified normalized expected delay $\bar{D}_n$. Time diversity, scheme 2, $\bar{R}/T = 0.01$, various $\bar{D}_n$.

Using (33)-(36), the throughput S can be determined for any specified normalized expected delay $\bar{D}_n$. S will be a function of p and we should choose that p which maximizes S. In Fig. 8, S has been plotted as a function of p for different specified normalized delays $\bar{D}_n$ and with $\bar{R}/T = 0.01$. It is found that the maximum of S is always at one of the endpoints, i.e., either at $p = 0$ or at $p = 1$. Thus, we find that for $\bar{R}/T = 0.01$, the deterministic policy is always better than probabilistic policy. This conclusion has been found to be true over a wide range of values of $\bar{R}/T$.

IV. CONCLUSIONS

In this paper we have investigated the possibility of improvement in the performance of a satellite slotted ALOHA system on transmitting multiple copies of each generated packet. It is found that multiple transmission gives better delay performance if the throughput is somewhat below its maximum. Also, if the probability that a packet fails a certain number of times or more is not to exceed a certain specified limit, then the multiple transmissions usually give greater throughput.

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