Fairness-Based Superframe Design and Resource Allocation for Dynamic Rate Adaptation in DVB-RCS2 Satellite Systems

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Abstract—In this letter, we consider a satellite communication system where a gateway and multiple return channel satellite terminals (RCSTs) exchange information via a satellite based on the DVB-RCS2 standard. Transmission rate adaptation is used to deal with dynamic condition of return channels from the RCSTs. A joint superframe design and resource allocation problem is formulated to maximize the Jain's faireness index. To overcome the high computational complexity to obtain the optimal solution of the problem, we decompose the joint problem into two-level hierarchical problems and propose iterative algorithms to solve them. Simulation results show that the optimal solution of two-level hierarchical problems is very close to that of the joint problem, and that the proposed algorithms have low computational complexity at the price of reduced fairness.

Index terms — Satellite communication system, DVB-RCS2, superframe design, resource allocation, iterative algorithm.

I. INTRODUCTION

DVB-RCS2 standard provides return link specification of satellite communication [1], [2]. In a DVB-RCS2 satellite system, a gateway and multiple return channel satellite terminals (RCSTs) exchange information via a satellite. In the forward link, the gateway sends forward link signalling (FLS) messages to the RCSTs in a time-division multiplexing manner. The FLS messages have information of superframe structure, resource allocation, transmission parameters such as modulation and coding scheme, etc. Based on the information, the RCSTs transmit bursts in superframes using multi-frequency time division multiple access.

Dynamic rate adaptation (DRA) for the return link is supported in the DVB-RCS2 standard, i.e., the transmission rates of RCSTs can be different by adjusting the modulation, channel coding, and symbol rate [3]-[5]. The gateway assigns a DRA scheme to the RCSTs based on their channel condition and power headroom. When the channel condition of an RCST gets worse, the gateway may attempt to control power to keep the current DRA scheme if the RCST has available power headroom. Otherwise, the gateway changes the DRA scheme of the RCST into that with a less transmission rate. In [3], performance of DRA was analyzed and compared to that of adaptive coding and modulation schemes with a single symbol rate. In [4], DRA was applied to a multi-beam satellite system taking into account system imperfections. In

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[5], a superframe design algorithm is proposed to maximize the throughput and max-min fairness index. However, because the RCSTs with different symbol rates use different channel bandwidths (BWs), the gateway needs to carefully allocate time slots jointly considering the superframe design with non-uniform channels. To the best of our knowledge, the previous works on DVB-RCS2 satellite systems have not investigated joint superframe design and resource allocation for fairness maximization.

In this letter, we investigate a DVB-RCS2 satellite system supporting DRA. An optimization problem is formulated to jointly find superframe structure and resource allocation which maximize the Jain's faireness index. We decompose the problem into two-level hierarchical problems using a property of fairness measure. Iterative algorithms are proposed to solve the two-level hierarchical problems with low computational complexity.

Notation: $\mathbb{Z}_{\geq 0}$ and $\mathbb{R}_{\geq 0}$ denote the spaces of non-negative integers and real numbers, respectively. $\lfloor x \rfloor$ and $\lceil x \rceil$ are the floor and ceiling functions, respectively. $\operatorname{lcm}(x,y)$ stands for the least common multiple of x and y. $\mathbb{E}[\cdot]$ denotes the expectation operator.

II. SYSTEM MODEL

Consider a DVB-RCS2-based satellite communication system consisting of a gateway, a satellite, and multiple RCSTs. Return link transmission from the RCSTs takes place over superframes, each with BW BW_{SF} and duration T_{SF} . A superframe is divided into multiple channels on the frequency domain, each consisting of time slots. The gateway designs a superframe structure and allocates time slots to the RCSTs to meet resource requests from the RCSTs. Based on information of superframe structure and resource allocation in FLS messages, each RCST transmits traffic bursts on dedicated time slots. Each traffic burst is transmitted within one time slot of which duration is larger than the burst length to avoid interburst interference [1].

We assume that our system supports K different DRA schemes, each with the symbol rate of R_k , $k \in \{1, 2, \dots, K\}$. The channel BW for DRA scheme k is given by [6]

$$BW_k = R_k(1+\alpha) + GB_k \tag{1}$$

where α is the roll-off factor and GB_k is a guard band. It is assume that L_k RCSTs transmit bursts using DRA scheme k over m_k channel(s) in a superframe where $\sum_{k=1}^K BW_k m_k \leq BW_{SF}$. The time slot duration for DRA scheme k is given by

$$T_{TS,k} = \frac{B_k + GS_k}{R_k} \tag{2}$$

where B_k is the burst length in symbols and GS_k is the number of guard symbols. The number of time slots per channel for DRA scheme k is calculated as

$$N_k = \left| \frac{T_{SF}}{T_{TS,k}} \right|. \tag{3}$$

Let $x_{k,l} \in \mathbb{Z}_{\geq 0}$ denote the number of time slots allocated to RCST $l, l \in \{1, 2, \dots, L_k\}$, with DRA scheme $k, k \in$ $\{1, 2, \dots, K\}$, then the time slot allocation vector for L_k RCSTs with DRA scheme k is expressed as

$$\mathbf{x}_{k} = [x_{k,1}, x_{k,2}, \cdots, x_{k,L_{k}}]^{T}.$$
 (4)

The time slot allocation vector for all RCSTs is given by

$$\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_K^T]^T. \tag{5}$$

Let $C_{k,l} \in \mathbb{R}_{\geq 0}$ denote the amount of resource request of RCST $l, l \in \{1, 2, \dots, L_k\}$, with DRA scheme $k, k \in$ $\{1, 2, \dots, K\}$. Then the number of required time slots for the resource request is given by

$$x_{k,l}^{req} = \left[\frac{C_{k,l}}{D_k} \right] \tag{6}$$

where D_k is the payload size of a traffic burst. To consider fairness based on resource requests, we normalize the number of allocated time slots by the number of required time slots as

$$\hat{x}_{k,l} = \frac{x_{k,l}}{x_{k,l}^{req}}. (7)$$

The Jain's fairness index for normalized numbers of time slots is given by [7]

$$J(\mathbf{x}) = \frac{(\sum_{k=1}^{K} \sum_{l=1}^{L_k} \hat{x}_{k,l})^2}{K(\sum_{k=1}^{K} L_k) \sum_{k=1}^{K} \sum_{l=1}^{L_k} \hat{x}_{l-1}^2}.$$
 (8)

III. JAIN'S FAIRNESS INDEX MAXIMIZATION

In this section, we formulate an optimization problem to find superframe structure and time slot allocation which maximize the Jain's fairenss index. Two-level hierarchical decomposition is employed and iterative algorithms are proposed to solve decomposed problems with low computational complexity.

A. Problem Formulation

Let $\mathbf{m} = [m_1, m_2, \cdots, m_K]^T$ be a vector of channel numbers in the superframe, then a joint superframe design and time slot allocation problem to maximize the Jain's fairness index can be formulated as

maximize
$$J(\mathbf{x})$$
 (9a)

maximize
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 (9a)
subject to $\hat{x}_{k,l} = \frac{x_{k,l}}{x_{k,l}^{req}}, \quad \forall k, l,$ (9b)

$$\sum_{l=1}^{L_k} x_{k,l} = m_k N_k, \quad \forall k,$$
 (9c)

$$\sum_{k=1}^{K} BW_k m_k \le BW_{SF},\tag{9d}$$

$$x_{k,l} \in \mathbb{Z}_{>0}, \quad \forall k, l,$$
 (9e)

$$m_k \in \mathbb{Z}_{>0}, \quad \forall k.$$
 (9f)

Problem (9) is an integer problem with non-concave objective function (8), which is difficult to solve. The optimal solution can be obtained by an exhaustive search, but the computational complexity to search all possible superframe structure and time slot allocation is significantly high. To reduce the computational complexity, we use Theorem 1 to decompose problem (9) into two-level hierarchical problems.

Theorem 1: The Jain's fairness index maximization problem (9) can be decomposed in two levels: 1) superframe structure design and 2) time slot allocation for each DRA scheme.

Proof: According to the axiom of irrelevance of partition for fairness measures [7], problem (9) can be solved hierarchically in two levels. At the first level, the total number of time slots in the superframe are divided into K time slot chunks, y_1, y_2, \dots, y_K , such that $\sum_{k=1}^K y_k = \sum_{k=1}^K \sum_{l=1}^{L_k} x_{k,l}$, which maximizes the Jain's fairness index for the chunks. And at the second level, each time slot chunk y_k is fairly allocated to the corresponding RCSTs. Let $y_k = \sum_{l=1}^{L_k} x_{k,l}, \ k \in \{1,2,\cdots,K\}$, be the time slot chunk for DRA scheme k. Then, the first level determines the time slot chuck for each DRA schemes, which is equivalent to a superframe design problem because the number of channels for DRA schemes can be calculated as $m_k = y_k/N_k$. And at the second level, for each DRA scheme $k, k \in \{1, 2, \dots, K\}$, y_k time slots are assigned to L_k RCSTs, which is time slot allocation for each DRA scheme.

B. Two-Level Hierarchical Decomposition

We normalize the time slot chunk for DRA scheme $k, k \in$ $\{1, 2, \dots, K\}$, by the number of required time slots, i.e.,

$$\hat{y}_k = \frac{y_k}{y_k^{req}} \tag{10}$$

where $y_k^{req} = \sum_{l=1}^{L_k} x_{k,l}^{req}$. The Jain's fairness index for the normalized numbers of time slot aggregation is given by

$$J(\mathbf{y}) = \frac{(\sum_{k=1}^{K} \hat{y}_k)^2}{K \sum_{k=1}^{K} \hat{y}_k^2}$$
(11)

where $\mathbf{y} = [y_1, y_2, \dots, y_K]^T$. A superframe design problem to maximize Jain's fairness index (11) is formulated as

$$\begin{array}{ll}
\text{maximize} & J(\mathbf{y}) \\
\end{array} \tag{12a}$$

subject to
$$y_k = m_k N_k$$
, $\forall k$, (12b)

$$\sum_{k=1}^{K} BW_k m_k \le BW_{SF},\tag{12c}$$

$$\hat{y}_k = \frac{y_k}{v_k^{req}}, \quad \forall k, \tag{12d}$$

$$m_k, y_k \in \mathbb{Z}_{>0}, \quad \forall k.$$
 (12e)

According to Theorem 1, given the superframe structure m*, the time slot allocation for each DRA scheme becomes independent from that for other DRA schemes. The Jain's fairness index for the normalized numbers of allocated time slots for L_k RCSTs with DRA scheme k is given by

$$J(\mathbf{x}_k) = \frac{(\sum_{l=1}^{L_k} \hat{x}_{k,l})^2}{L_k \sum_{l=1}^{L_k} \hat{x}_{k,l}^2}.$$
 (13)

The time slot allocation problem to maximize Jain's fairness index (13) is formulated as

maximize
$$J(\mathbf{x}_k)$$
 (14a)

subject to
$$\hat{x}_{k,l} = \frac{x_{k,l}}{x_{k,l}^{req}}, \quad \forall k, l,$$
 (14b)

$$\sum_{l=1}^{L_k} x_{k,l} = m_k^* N_k, \quad \forall k,$$
 (14c)

$$x_{k,l} \in \mathbb{Z}_{\geq 0}, \quad \forall k, l.$$
 (14d)

Although two-level hierarchical problems (12) and (14) are still not easy to solve due to non-concavity of the objective functions, the computational complexity to solve the problems is much lower than that for problem (9). To compute the complexity to solve problems (12) and (14), we consider a special case when $BW_i/BW_j=2^a$ and $BW_{SF}/BW_i=b$, for all $i,j\in\{1,\cdots,K\},\ i\geq j,\ a,b\in\mathbb{Z}_{\geq 0}$. In this case, the complexity of an exhaustive search for problem (12) is given by $O(BW_{SF}^{K-1}/\prod_{k=2}^K BW_k)$ and that for problem (14) is given by $O(\sum_{k=1}^K N_k^{L_k-1} m_k^{L_k-1})$. Then, the complexity to solve problems (12) and (14) is given by $O(BW_{SF}^{K-1}/\prod_{k=2}^K BW_k+\sum_{k=1}^K N_k^{L_k-1} m_k^{L_k-1})$, while the complexity to solve problem (9) is given by $O(\sum_{k=1}^K N_k^{L_k-1} m_k^{L_k-1} BW_{SF}^{K-1}/\prod_{k=2}^K BW_k)$.

C. Heuristic Iterative Algorithm

According to the definition of Jain's fairness index, the objective function of problem (12) is maximized when the normalized numbers of time slot aggregation are all equal, i.e., $\hat{y}_1 = \hat{y}_2 = \cdots = \hat{y}_K$. If \hat{y}_k is larger than the others, it means that more channels are assigned to DRA scheme k than those for other DRA schemes considering the resource requests. Using this property, we propose a superframe design algorithm which iteratively transfers channels of DRA scheme k_{max} to DRA scheme k_{min} where $k_{max} = \arg\max_k \hat{y}_k$ and $k_{min} = \arg\min_k \hat{y}_k$ in order to make \hat{y}_k 's comparable.

We set an initial superframe structure **m** satisfying constraint (12c) and $J_{max} = 0$. For each DRA scheme k, we calculate the channel BW BW_k , the time slot duration $T_{TS,k}$, and the number of time slots per channel N_k using (1), (2), and (3), respectively, and obtain the aggregated number of required time slots y_k^{req} by summing $x_{k,l}^{req}$ over l. And the normalized number of aggregated time slots for each DRA scheme k, \hat{y}_k , is obtained from (10) and constraint (12b). In order to increase the Jain's fairness index, we transfers channels of DRA scheme k_{max} to DRA scheme k_{min} . Because the BWs of one channel for DRA schemes k_{max} and k_{min} are different, the minimum BW of channels that can be transferred from DRA scheme k_{max} to k_{min} should be the least common multiple of $BW_{k_{max}}$ and $BW_{k_{min}}$ in order to satisfy constraint (12c). The minimum numbers of channels to be updated for DRA schemes k_{max} and k_{min} are given by $\Delta_{k_{max}} = \text{lcm}(BW_{k_{max}}, BW_{k_{min}})/BW_{k_{max}}$ and $\Delta_{k_{min}} = \text{lcm}(BW_{k_{max}}, BW_{k_{min}})/BW_{k_{min}}$, respectively. Then, $m_{k_{max}}$ decreases by $\beta \Delta_{k_{max}}$ while $m_{k_{min}}$ increases by $\beta \Delta_{k_{min}}$ where β is a weighting factor for updates of channel numbers. By using the updated **m**, we calculate the normalized number of aggregated time slots for each DRA scheme k, \hat{y}_k

Algorithm 1 Proposed algorithms for superframe design and time slot allocation

```
Input: BW_{SF}, T_{SF}, R_k, GB_k, B_k, GS_k, L_k, C_{k,l}, D_k, \beta, and \gamma.
Output: \mathbf{m}^* and \mathbf{x}^*.
   1: Initialize \mathbf{m}^{(0)} = [m_1^{(0)}, m_2^{(0)}, \cdots, m_K^{(0)}]^T satisfying (12c)
          and J_{max} = 0.
   2: Calculate BW_k, T_{TS,k}, and N_k using (1), (2), and (3),
          respectively, for all k.
   3: Calculate x_{k,l}^{req} using (6) for all k and l.
   4: Calculate y_k^{req} = \sum_{l=1}^{L_k} x_{k,l}^{req} for all k.

5: Calculate \hat{y}_k^{(0)} = m_k^{(0)} N_k / y_k^{req} for all k.

6: for i = 0 to i_{max}^{SF} - 1 do
               Obtain k_{max} = \arg\max_{k} \hat{y}_{k}^{(i)} and k_{min} = \arg\min_{k} \hat{y}_{k}^{(i)}.
               Calculate \Delta_{k_{max}} = \text{lcm}(BW_{k_{max}}, BW_{k_{min}})/BW_{k_{max}} and \Delta_{k_{min}} = \text{lcm}(BW_{k_{max}}, BW_{k_{min}})/BW_{k_{min}}.

Calculate m_{k_{max}}^{(i+1)} = m_{k_{max}}^{(i)} - \beta \Delta_{k_{max}} and m_{k_{min}}^{(i+1)} = m_{k_{max}}^{(i)}
               m_{k_{min}}^{(i)} + \beta \Delta_{k_{min}}.

Calculate \hat{y}_k^{(i+1)} = m_k^{(i+1)} N_k / y_k^{req} for all k.

Calculate J(\mathbf{y}^{(i+1)}) using (11).
 10:
               if J(y^{(i+1)}) > J_{max} then
 12:
                     Update J_{max} = J(\mathbf{y}^{(i+1)}).
 13:
 14:
                     Update \mathbf{m}^* = \mathbf{m}^{(i+1)}.
 15:
                     break
 16:
 17:
               end if
 18: end for
        for k = 1 to K do

Initialize \mathbf{x}_{k}^{(0)} = [x_{k,1}^{(0)}, x_{k,2}^{(0)}, \cdots, x_{k,L_k}^{(0)}]^T satisfying (14c)
 19:
              and J_{max} = 0.
Calculate \hat{x}_{k,l}^{(0)} = x_{k,l}^{(0)}/x_{k,l}^{req} for all l.
for i = 0 to i_{max}^{TS} - 1 do
21:
22:
                    Obtain l_{max} = arg max<sub>l</sub> \hat{x}_{k,l}^{(i)} and l_{min} = arg min<sub>l</sub> \hat{x}_{k,l}^{(i)}. Calculate x_{k,l_{max}}^{(i+1)} = x_{k,l_{max}}^{(i)} - \gamma and x_{k,l_{min}}^{(i+1)} = x_{k,l_{min}}^{(i)} + \gamma. Calculate \hat{x}_{k,l}^{(i+1)} = x_{k,l}^{(i+1)} / x_{k,l}^{req} for all l. Calculate J(\mathbf{x}_{k}^{(i+1)}) using (13).
23:
24:
25:
 26:
                    if J(\mathbf{x}_{k}^{(i+1)}) > J_{max} then
27:
                          Update J_{max} = J(\mathbf{x}_k^{(i+1)})
 28:
 29:
                           Update \mathbf{x}_k^* = \mathbf{x}_k^{(i+1)}.
 30:
 31:
32:
                     end if
33:
               end for
```

and the Jain's fairness index J(y). If $J(y) > J_{max}$, we update $J_{max} = J(y)$, otherwise, iteration ends returning \mathbf{m}^* .

Similar to the superframe design algorithm, we first set an initial time slot allocation vector \mathbf{x}_k satisfying constraint (14c) and $J_{max} = 0$. The normalized number of time slots for L_k RCSTs, $\hat{x}_{k,l}$, is given from constraint (14b). In order to maximize the Jain's fairness index, we transfer γ time slots of RCST l_{max} to RCST l_{min} where $l_{max} = \arg \max_l \hat{x}_{k,l}$ and $l_{min} = \arg \min_l \hat{x}_{k,l}$. Then, $x_{k,l_{max}}$ and $x_{k,l_{min}}$ respectively decreases and increases by γ . By using the updated \mathbf{x}_k , we

34: end for

TABLE I DEFINITION OF DRA SCHEMES [1], [2]

DRA scheme index	1	2	3	4	5
Symbol rate [ksps]	128	256	512	1,024	2,048
Waveform ID	13	13	13	14	18
Burst length [symbols]	1,616	1,616	1,616	1,616	1,616
Payload size [bytes]	123	123	123	188	355
Modulation	QPSK	QPSK	QPSK	QPSK	8PSK
Code rate	1/3	1/3	1/3	1/2	2/3
Channel BW [kHz]	156.25	312.5	625	1,250	2,500

calculate the normalized number of time slots for L_k RCSTs, $\hat{x}_{k,l}$ and the Jain's fairness index $J(\mathbf{x}_k)$. If $J(\mathbf{x}_k) > J_{max}$, we update $J_{max} = J(\mathbf{x}_k)$, otherwise, iteration ends returning \mathbf{x}_k^* .

Let τ_{SF} and τ_{TS} be the numbers of iterations for superframe design and time slot allocation, respectively. Then, the computational complexity of the proposed algorithms is calculated as $O(\tau_{SF}K + \tau_{TS} \sum_{k=1}^{K} L_k)$ where the first and second terms come from lines 11 and 26, respectively, in Algorithm 1.

IV. SIMULATION RESULTS AND DISCUSSION

Suppose that the superframe duration $T_{SF}=1$ sec, the roll-off factor $\alpha=0.2$, and the number of DRA schemes K=5. Five DRA schemes are defined as shown in Table I where the guard band is approximately 18% of the channel BW and the number of guard symbols are set to 4 for all DRA schemes [1], [2]. Assume that the number of RCSTs for each DRA scheme, L_k , follows Poisson distribution with mean λ and the amount of resource requests from the RCSTs, $C_{k,l}$, is generated by unifrom distribution in $[0, C_{max}]$. For Fig. 1, the Jain's fairness index is averaged over 10,000 independent realizations with $\lambda=30$ and $C_{max}=1$ kbytes and, for Fig. 2, the simulation results are obtained with a specific realization.

Fig. 1 shows the average Jain's fairness index versus the superframe BW with $\beta = 1$ and $\gamma = 1$. For comparison, the dynamic superframe design (DSD) proposed in [5], random superframe design (RSD), and uniform time slot allocation are considered. It is shown that the differences between optimal solutions of problem (9) and the two-level hierarchical problems are very small to be ignored, which verifies the axiom of irrelevance of partition for fairness measures. It is also shown that the iterative algorithms have higher fairness than the DSD with uniform time slot allocation, because the DSD only considers fair superframe design which does not guarantee fair resource allocation to the RCSTs.

Fig. 2 shows the objective functions (12a) and (14a) versus the number of iterations with $BW_{SF}=20$ MHz. It is shown that as the iteration goes on, the Jain's fairness index increases until saturated at a certain value. When we choose β and γ , there exists a trade-off between accuracy and computational complexity of the algorithms. Large values of β and γ can be selected for delay-sensitive systems while small values are suitable for delay-tolerant systems requiring high fairness.

In conclusions, we consider a DVB-RCS2 satellite system consisting of a gateway, a satellite, and multiple RCSTs. We formulate a superframe design and resource allocation problem to maximize Jain's fairness index. To solve the problem, two-level hierarchical decomposition is employed and iterative al-

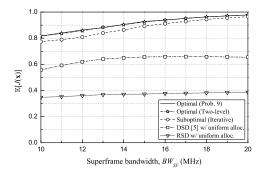


Fig. 1. Average Jain's fairness index versus superframe BW.

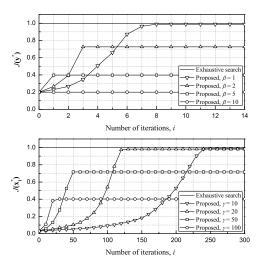


Fig. 2. Objective functions (12a) and (14a) versus number of iterations i.

gorithms with low-complexity are proposed. Simulation results show that the optimal solution of the two-level hierarchical problems is very close to that of the joint superframe design and resource allocation problem. It is also shown that the iterative algorithms have much less computational complexity than the exhaustive search at the price of reduced fairness.

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