Mechanization and Overhaul of Feature Featherweight Java with Coq

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August 08, 2017

Objetivo

Mecanizar Feature Featherweight Java

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Por que mecanizar?

- Aplicações de criticamente seguras: Bugs são inaceitáveis
  - Controladores de Avião
  - Equipamentos Médicos
  - Carros
- Bugs encontrados tardiamente são caros
- A medida que o software e o hardware crescem dificulta encontrar bugs através de testes.

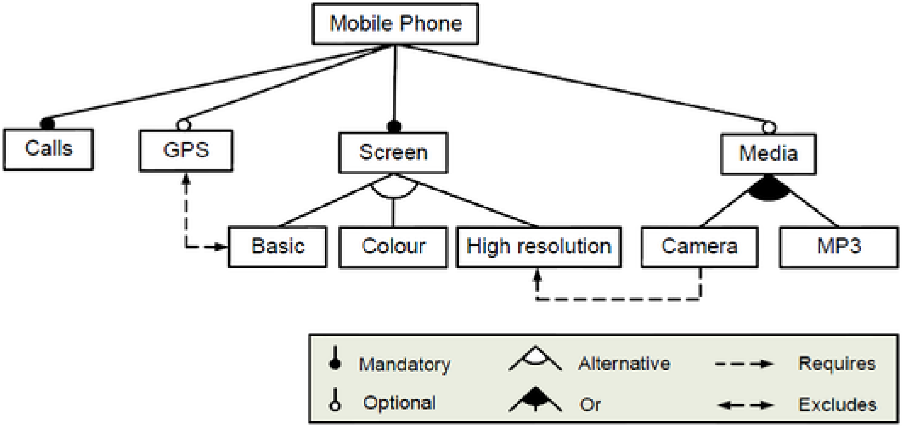
O que é feature?

Feature

'A feature is a unit of functionality of a software system that satisfies a requirement, represents a design decision, and provides a potential configuration option.' - S. Apel & K. Kastner

Feature Oriented Programming

Software Product Line



$\mathtt{CD}$ ::= *class declarations:*

$\mathtt{class\ C\ extends\ D\ \{\bar{C}\ \bar{f};\ K\ \bar{M}\}}$

$\mathtt{K}$ ::= *constructor declarations:*

$\mathtt{C(\bar{C}\ \bar{f})\{super(\bar{f});\ this.\bar{f}=\bar{f};\}}$

$\mathtt{KD}$ ::= *constructor refinements:*

$\mathtt{refines\ C(\bar{E}\ \bar{h},\ \bar{C}\ \bar{f})\{original(\bar{f});\ this.\bar{f}=\bar{f};\}}$

$\mathtt{M}$ ::= *method declarations:*

$\mathtt{C\ m\ (\bar{C}\ \bar{x})\ \{return\ e;\}}$

```
e ::=            expressions:
      x
      e.f
      e.m(ē)
      new C(ē)
      (C)e

v ::=            values:
      new C(ē)
```

$$\frac{}{\texttt{C <: C}} \qquad \frac{\texttt{C <: D} \qquad \texttt{D <: E}}{\texttt{C <: E}} \qquad \frac{\texttt{class C extends D \{ ... \}}}{\texttt{C <: D}}$$

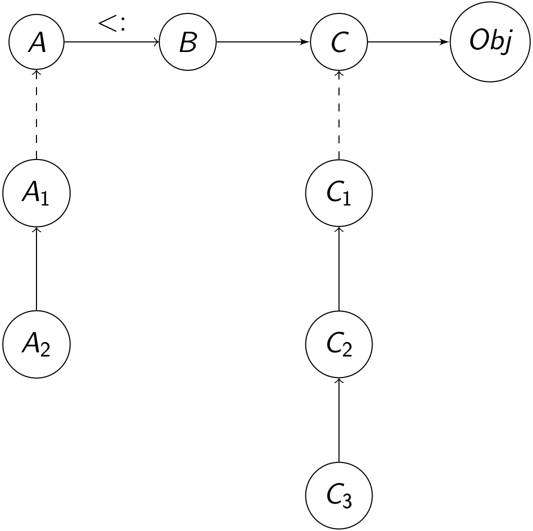
Featherweight Java

Relação de Subtipo

$$A \xrightarrow{<:} B \longrightarrow C \longrightarrow Obj$$

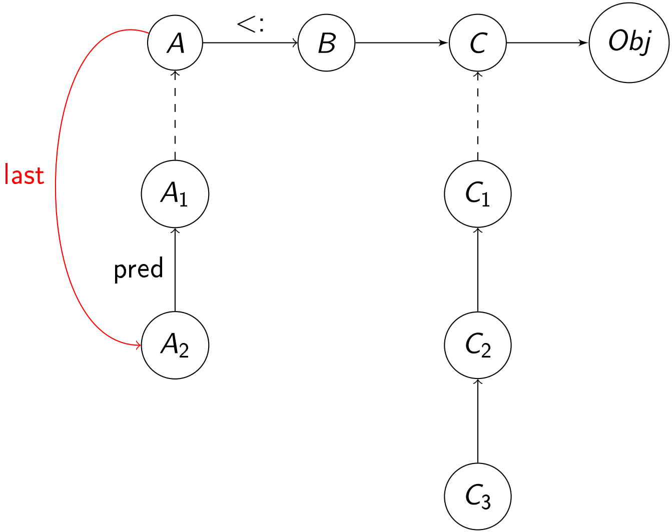
Featherweight Java

Relação de Subtipo



Featherweight Java

Relação de Subtipo



Sintaxe Abstrata

R ::= *refinement names:*

`C@feat`

CR ::= *class refinements:*

$\texttt{refines class R}\ \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{KD}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\}$

KD ::= *constructor refinements:*

$\texttt{refines C}(\bar{\texttt{E}}\ \bar{\texttt{h}},\ \bar{\texttt{C}}\ \bar{\texttt{f}})\{\texttt{original}(\bar{\texttt{f}});\ \texttt{this}.\bar{\texttt{f}}=\bar{\texttt{f}};\}$

MR ::= *method refinements:*

$\texttt{refines C m}\ (\bar{\texttt{R}}\ \bar{\texttt{x}})\ \{\texttt{return e};\}$

Field Lookup

$$\textit{fields}\ \texttt{Object} = \bullet$$

$$\frac{\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\} \qquad \neg \textit{last}\ \texttt{C}}{\textit{fields}\ \texttt{C} = \textit{fields}\ \texttt{D},\ \bar{\texttt{C}}\ \bar{\texttt{f}}}$$

$$\frac{\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\}}{\textit{fields}\ \texttt{C} = \textit{fields}\ \texttt{D},\ \bar{\texttt{C}}\ \bar{\texttt{f}},\ \textit{fields}_R\ (\textit{last}\ \texttt{C})}$$

Field Lookup (Refinement)

$$\frac{\texttt{refines R \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; KR }\bar{\texttt{M}}\ \bar{\texttt{M}}\texttt{R\}} \qquad \neg \mathit{pred}\ \texttt{R}}{\mathit{fields}_R\ \texttt{R}\ =\ \bar{\texttt{C}}\ \bar{\texttt{f}}}$$

$$\frac{\texttt{refines R \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; KR }\bar{\texttt{M}}\ \bar{\texttt{M}}\texttt{R\}}}{\mathit{fields}_R\ \texttt{R} = \mathit{fields}_R\ (\mathit{pred}\ \texttt{P}), \bar{\texttt{C}}\ \bar{\texttt{f}}}$$

Method Type Lookup

$$\frac{\texttt{class C extends D \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; K}\ \bar{\texttt{M}}\texttt{\}} \qquad \texttt{B m (}\bar{\texttt{B}}\ \bar{\texttt{x}}\texttt{) \{return e;\}} \in \bar{\texttt{M}} \qquad \neg \mathit{mtype}_R\ (\texttt{m}, \mathit{last}\ \texttt{C})}{\mathit{mtype}\ (\texttt{m}, \texttt{C}) \ = \ \bar{\texttt{B}} \ \rightarrow \ \texttt{B}}$$

$$\frac{\texttt{class C extends D \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; K}\ \bar{\texttt{M}}\texttt{\}} \qquad \texttt{m} \notin \bar{\texttt{M}} \qquad \neg \mathit{mtype}_R\ (\texttt{m}, \mathit{last}\ \texttt{C})}{\mathit{mtype}\ (\texttt{m}, \texttt{C}) \ = \ \mathit{mtype}\ (\texttt{m}, \texttt{D})}$$

$$\frac{\texttt{class C extends D \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; K}\ \bar{\texttt{M}}\texttt{\}}}{\mathit{mtype}\ (\texttt{m}, \texttt{C}) \ = \ \mathit{mtype}_R\ (\texttt{m}, \mathit{last}\ \texttt{C})}$$

Method Type Lookup (Refinement)

$$\frac{\texttt{refines class R } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{KR}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\} \qquad \texttt{B m } (\bar{\texttt{B}}\ \bar{\texttt{x}})\ \{\texttt{return e;}\} \in \bar{\texttt{M}}}{\mathit{mtype}_R\,(\texttt{m}, \texttt{R}) = \bar{\texttt{B}} \rightarrow \texttt{B}}$$

$$\frac{\begin{array}{c}\texttt{refines class R } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{KR}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\} \qquad \texttt{m} \notin \bar{\texttt{M}} \\ \texttt{refines B m } (\bar{\texttt{B}}\ \bar{\texttt{x}})\ \{\texttt{return e;}\} \in \overline{\texttt{MR}}\end{array}}{\mathit{mtype}_R\,(\texttt{m}, \texttt{R}) = \bar{\texttt{B}} \rightarrow \texttt{B}}$$

$$\frac{\begin{array}{c}\texttt{refines class R } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{KR}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\} \\ \texttt{m} \notin \bar{\texttt{M}} \quad \texttt{m} \notin \overline{\texttt{MR}}\end{array}}{\mathit{mtype}_R\,(\texttt{m}, \texttt{R}) = \mathit{mtype}_R\,(\texttt{m}, \mathit{pred}\ \texttt{P})}$$

Method Body Lookup

$$\frac{\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\} \qquad \texttt{B m } (\bar{\texttt{B}}\ \bar{\texttt{x}})\ \{\texttt{return e;}\} \in \bar{\texttt{M}} \qquad \neg \mathit{mbody}_R\ (\texttt{m}, \mathit{last}\ \texttt{C})}{\mathit{mbody}\ (\texttt{m}, \texttt{C}) \ = \ \bar{\texttt{x}}.\texttt{e}}$$

$$\frac{\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\} \qquad \texttt{m} \notin \bar{\texttt{M}} \qquad \neg \mathit{mbody}_R\ (\texttt{m}, \mathit{last}\ \texttt{C})}{\mathit{mbody}\ (\texttt{m}, \texttt{C}) \ = \ \mathit{mbody}\ (\texttt{m}, \texttt{D})}$$

$$\frac{\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\}}{\mathit{mbody}\ (\texttt{m}, \texttt{C}) \ = \ \mathit{mbody}_R\ (\texttt{m}, \mathit{last}\ \texttt{C})}$$

Method Body Lookup (Refinement)

$$\frac{\texttt{refines class R \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; KR}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\texttt{\}} \qquad \texttt{B m (}\bar{\texttt{B}}\ \bar{\texttt{x}}\texttt{) \{return e;\}} \in \bar{\texttt{M}}}{mbody_R\,(\texttt{m},\texttt{R}) \;=\; \bar{\texttt{x}}.\texttt{e}}$$

$$\frac{\begin{array}{c}\texttt{refines class R \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; KR}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\texttt{\}} \qquad \texttt{m} \notin \bar{\texttt{M}} \\ \texttt{refines B m (}\bar{\texttt{B}}\ \bar{\texttt{x}}\texttt{) \{return e;\}} \in \overline{\texttt{MR}}\end{array}}{mbody_R\,(\texttt{m},\texttt{R}) \;=\; \bar{\texttt{x}}.\texttt{e}}$$

$$\frac{\begin{array}{c}\texttt{refines class R \{}\bar{\texttt{C}}\ \bar{\texttt{f}}\texttt{; KR}\ \bar{\texttt{M}}\ \overline{\texttt{MR}}\texttt{\}} \\ \texttt{m} \notin \bar{\texttt{M}} \quad \texttt{m} \notin \overline{\texttt{MR}}\end{array}}{mbody_R\,(\texttt{m},\texttt{R}) \;=\; mbody_R\,(\texttt{m}, last\ \texttt{P})}$$

$$E ::= \Box \mid E.f_i \mid E.m(\overline{e}) \mid e.m(\overline{e_l}, E, \overline{e_r}) \mid (C)\ E \mid new\ \ C(\overline{e_l}, E, \overline{e_r})$$

Table: Evaluation Context

Lemmas

Lemma (Typed method has body)

If $mtype(m, C) = B \to \overline{B}$
then $\exists \overline{x}\ \exists e$ *such that* $mbody(m, C) = \overline{x}.e$

Lemmas

Lemma (Typed method has body - Refinement)

If $mtype_R(m, R) = \mathtt{B} \rightarrow \mathtt{Bs}$
then $\exists \overline{x}\ \exists e$ *such that* $mbody_R(m, R) = \overline{x}.e$

Lemma (Body method has type - Refinement)

If $mbody_R(m, R) = \overline{x}.e$
then $\exists \overline{B}\ \exists B$ *such that* $mtype_R(m, R) = \mathtt{B} \rightarrow \mathtt{Bs}$

Lemmas

Lemma (Subtype respects method types)

If $\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\}$
then $mtype(m, C) = mtype(m, D)$

Lemmas

Lemma (Refinement respects method types)

If $\texttt{class C extends D } \{\bar{\texttt{C}}\ \bar{\texttt{f}};\ \texttt{K}\ \bar{\texttt{M}}\}$
then $\forall feat,\ mtype_R(m, C@feat) = mtype(m, D)$

Lemmas

Lemma (A1.4 - Method Body is Typable)

If $mtype\ (m, C) = \overline{D} \rightarrow D$
and $mbody\ (\mathtt{m}, \mathtt{C}) = \overline{x}.e$
then $\exists C <: D, \exists C_0 <: D_0,\ this : D, \overline{x} : \overline{D} \vdash e : C_0$

Lemmas

Lemma (Method Body is Typable - Refinement)

If $mtype_R$ $(\mathtt{m}, \mathtt{C@feat}) = \overline{D} \rightarrow D$
and $mbody_R$ $(\mathtt{m}, \mathtt{C@feat}) = \overline{x}.e$
then $\exists C <: D, \exists C_0 <: D_0,$ $this : D, \overline{x} : \overline{D} \vdash e : C_0$

Theorems

Theorem (Preservation)

If $\Gamma \vdash e : C$ *and* $e \rightarrow e'$, *then* $\Gamma \vdash e' : C'$ *for some* $C' <: C$.

Theorems

Theorem (Progress)

Suppose e is closed, well-typed normal form.
Then either (1) e is a value, or (2) for some evaluation context E, we can express e as $e = E[(C)(newD(\overline{e}))]$, with $D \not<: C$.

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ACM Transactions on Programming Languages and Systems (TOPLAS) 2001.

Apel, Sven, Christian Kästner, and Christian Lengauer.
Feature Featherweight Java: A calculus for feature-oriented programming and stepwise refinement
Proceedings of the 7th international conference on Generative programming and component engineering. ACM, 2008.

Class Name

$$\frac{\mathtt{R} = \mathtt{C@feat}}{\mathit{class\_name}\ \mathtt{R} = \mathtt{C}}$$

Refinements of a class

$$\frac{\mathit{filter}\ (\lambda R \cdot \mathit{class\_name}\ \mathtt{R} == \mathtt{C})\ \mathrm{RT} = \bar{\mathtt{R}}}{\mathit{refinements\_of}\ \mathtt{C} = \bar{\mathtt{R}}}$$

Predecessor

$$\frac{\begin{array}{c}\mathit{refinements\_of}\ (\mathit{class\_name}\ \mathtt{R}) = \bar{\mathtt{R}} \\ \mathit{index}\ \mathtt{R}\ \bar{\mathtt{R}} \ = \ n \qquad \mathit{get}\ (n-1)\ \bar{\mathtt{R}} \ = \ \mathtt{P}\end{array}}{\mathit{pred}\ \mathtt{R} \ = \mathtt{P}}$$

Last

$$\frac{\mathit{refinements\_of}\ \mathtt{C} = \bar{\mathtt{R}} \qquad \mathit{tail}\ \bar{\mathtt{R}} \ = \ \mathtt{R}}{\mathit{last}\ \mathtt{C} \ = \mathtt{R}}$$

FJ Typing Rules

$$\Gamma \vdash x : \Gamma(x) \qquad \text{(T-Var)}$$

$$\frac{\Gamma \vdash e_0 : C_0 \qquad \mathit{fields}\ (C_0) = \overline{C}\ \overline{f}}{\Gamma \vdash e_0.f_i : C_i} \qquad \text{(T-Field)}$$

$$\frac{\Gamma \vdash e_0 : C_0 \qquad \mathit{mtypes}\ (m,\ C_0) = \overline{D} \to C \qquad \Gamma \vdash \overline{e} : \overline{C} \qquad \overline{C}\ <:\ \overline{D}}{\Gamma \vdash e_0.m(\overline{e}) : C} \qquad \text{(T-Invk)}$$

$$\frac{\mathit{fields}(C) = \overline{D}\ \overline{f} \qquad \Gamma \vdash \overline{e} : \overline{C} \qquad \overline{C}\ <:\ \overline{D}}{\Gamma \vdash \mathit{new}\ C(\overline{e}) : C} \qquad \text{(T-New)}$$

$$\frac{\Gamma \vdash e_0 : D \qquad D\ <:\ C}{\Gamma \vdash (C)\ e_0 : C} \qquad \text{(T-UCast)}$$

$$\frac{\Gamma \vdash e_0 : D \qquad C\ <:\ D \qquad C \neq D}{\Gamma \vdash (C)\ e_0 : C} \qquad \text{(T-DCast)}$$

$$\frac{\Gamma \vdash e_0 : D \qquad C\ \not<:\ D \qquad D\ \not<:\ C \qquad \mathit{stupid\ warning}}{\Gamma \vdash (C)\ e_0 : C} \qquad \text{(T-SCast)}$$

FJ Computation Rules

$$\frac{fields\ (C) = \overline{C\, f}}{(new\ \ C(\bar{e})).f_i \rightarrow e_i} \qquad \text{(R-Field)}$$

$$\frac{mbody\ (m, C) = \overline{x}.e_0}{(new\ \ C\ (\overline{e})).m\ (\overline{d}) \rightarrow [\overline{d}/\overline{x}, new\ \ C\ (\bar{e})/this]e_0} \qquad \text{(R-Invk)}$$

$$\frac{C <: D}{(D)(new\ \ C\ (\overline{e})) \rightarrow new\ \ C\ (\overline{e})} \qquad \text{(R-Cast)}$$

FJ Congruence Rules

$$\frac{e_0 \to e_0'}{e_0.f \to e_0'.f} \quad \text{(RC-Field)}$$

$$\frac{e_0 \to e_0'}{e_0.m\ (\overline{e}) \to e_0'.m\ (\overline{e})} \quad \text{(RC-Invk-Recv)}$$

$$\frac{e_i \to e_i'}{e_0.m\ (\ldots, e_i, \ldots) \to e_0'.m\ (\ldots, e_i, \ldots)} \quad \text{(RC-Invk-Arg)}$$

$$\frac{e_i \to e_i'}{new\ \ C\ (\ldots, e_i, \ldots) \to new\ \ C\ (\ldots, e_i', \ldots)} \quad \text{(RC-New-Arg)}$$

$$\frac{e_0 \to e_0'}{(C)e_0 \to (C)e_0'} \quad \text{(RC-Cast)}$$

$$\frac{\mathit{mtype}\ (\mathtt{m}, \mathtt{D}) = \bar{\mathtt{D}} \to \mathtt{D}\ \mathit{implies}\ \bar{\mathtt{C}} = \bar{\mathtt{D}}\ \mathit{and}\ \mathtt{C}_0 = \mathtt{D}}{\mathit{override}\ \mathtt{m}\ \mathtt{D}\ \bar{\mathtt{C}}\ \mathtt{C}_0}$$

$$\frac{\mathtt{class\ C\ extends\ D\ \{}\bar{\mathtt{C}}\ \bar{\mathtt{f}}\mathtt{;\ K}\ \bar{\mathtt{M}}\mathtt{\}} \qquad \mathtt{C}_0\ \mathtt{m}\ (\bar{\mathtt{C}}\ \bar{\mathtt{x}})\ \mathtt{\{return\ e;\}} \in \bar{\mathtt{M}} \qquad \neg\ \mathit{pred}\ \mathtt{R} \qquad \mathtt{R} = \mathtt{C@feat}}{\mathit{override}_R\ \mathtt{m}\ \mathtt{R}\ \bar{\mathtt{C}}\ \mathtt{C}_0}$$

$$\frac{\mathtt{refines\ class\ P\ \{}\bar{\mathtt{C}}\ \bar{\mathtt{f}}\mathtt{;\ KR}\ \bar{\mathtt{M}}\ \overline{\mathtt{MR}}\mathtt{\}} \qquad \mathtt{C}_0\ \mathtt{m}\ (\bar{\mathtt{C}}\ \bar{\mathtt{x}})\ \mathtt{\{return\ e;\}} \in \bar{\mathtt{M}} \qquad \mathit{pred}\ \mathtt{R} = \mathtt{P}}{\mathit{override}_R\ \mathtt{m}\ \mathtt{R}\ \bar{\mathtt{C}}\ \mathtt{C}_0}$$

$$\frac{\mathtt{refines\ class\ P\ \{}\bar{\mathtt{C}}\ \bar{\mathtt{f}}\mathtt{;\ KR}\ \bar{\mathtt{M}}\ \overline{\mathtt{MR}}\mathtt{\}} \qquad \mathtt{m} \notin \bar{\mathtt{M}} \qquad \mathit{pred}\ \mathtt{R} = \mathtt{P} \qquad \mathit{override}_R\ \mathtt{m}\ \mathtt{P}\ \bar{\mathtt{C}}\ \mathtt{C}_0}{\mathit{override}_R\ \mathtt{m}\ \mathtt{R}\ \bar{\mathtt{C}}\ \mathtt{C}_0}$$

$$\frac{\mathit{pred}\ \mathtt{R}\ =\ \mathtt{S} \qquad \neg\ \mathit{mtype}_R\ (\mathtt{m},\mathtt{S})}{\mathit{introduce}\ \mathtt{m}\ \ \mathtt{R}}$$

$$\frac{\begin{array}{c}\neg\ \mathit{pred}\ \mathtt{R} \qquad \mathtt{R}\ =\ \mathtt{C@feat} \\ \mathtt{class\ C\ extends\ D\ \{\bar{C}\ \bar{f};\ K\ \bar{M}\}} \qquad \mathtt{m} \notin \bar{\mathtt{M}}\end{array}}{\mathit{introduce}\ \mathtt{m}\ \ \mathtt{R}}$$

$$\frac{\begin{array}{c}\overline{\mathtt{x}} : \overline{\mathtt{C}}, \mathtt{this} : \mathtt{C} \vdash \mathtt{t}_0 : \mathtt{E}_0 \qquad \mathtt{E}_0 <: \mathtt{C}_0 \\ CT(\mathtt{C}) = \mathtt{class\ C\ extends\ D}\ \{\ldots\} \qquad \mathit{override}(\mathtt{m}, \mathtt{D}, \overline{\mathtt{C}} \to \mathtt{C})\end{array}}{\mathtt{C}_0\ \mathtt{m}\ (\overline{\mathtt{C}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \mathtt{OK\ in\ C}}$$

$$\frac{\begin{array}{c}\overline{\mathtt{x}} : \overline{\mathtt{C}}, \mathtt{this} : \mathtt{C} \vdash \mathtt{t}_0 : \mathtt{E}_0 \qquad \mathtt{E}_0 <: \mathtt{C}_0 \qquad \mathtt{R} = \mathtt{C@feat} \\ CT(\mathtt{C}) = \mathtt{class\ C\ extends\ D}\ \{\ldots\} \qquad RT(\mathtt{R}) = \mathtt{refines\ R}\ \{\ldots\ \overline{\mathtt{M}}\ \ldots\} \\ \mathit{override}(\mathtt{m}, \mathtt{D}, \overline{\mathtt{C}} \to \mathtt{C}) \qquad \mathit{introduce}\ \mathtt{m}\ \mathtt{R} \qquad \mathtt{m} \in \overline{\mathtt{M}}\end{array}}{\mathtt{C}_0\ \mathtt{m}\ (\overline{\mathtt{C}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \mathtt{OK\ in\ R}}$$

$$\frac{\begin{array}{c}\overline{\mathtt{x}} : \overline{\mathtt{C}}, \mathtt{this} : \mathtt{C} \vdash \mathtt{t}_0 : \mathtt{E}_0 \qquad \mathtt{E}_0 <: \mathtt{C}_0 \qquad \mathtt{R} = \mathtt{C@feat} \\ RT(\mathtt{R}) = \mathtt{refines\ R}\ \{\ldots\ \overline{\mathtt{M}},\ \overline{\mathtt{MR}}\ \ldots\} \qquad \mathtt{m} \notin \overline{\mathtt{M}} \qquad \mathtt{m} \in \overline{\mathtt{MR}} \\ \mathit{override}_R(\mathtt{m}, \mathtt{R}, \overline{\mathtt{C}} \to \mathtt{C}) \qquad \mathit{introduce}\ \mathtt{m}\ \mathtt{R}\end{array}}{\mathtt{refines}\ \mathtt{C}_0\ \mathtt{m}\ (\overline{\mathtt{C}}\ \overline{\mathtt{x}})\{\mathtt{return}\ \mathtt{t}_0;\}\ \mathtt{OK\ in\ R}}$$

$$\frac{\texttt{K} \;=\; \texttt{C}\ (\overline{\texttt{D}}\ \overline{\texttt{g}},\ \overline{\texttt{C}}\ \overline{\texttt{f}})\{\texttt{super}(\overline{\texttt{g}});\ \texttt{this}.\overline{\texttt{f}} = \overline{\texttt{f}}\} \qquad \mathit{fields}(\texttt{D}) \;=\; \overline{\texttt{D}}\ \overline{\texttt{g}} \qquad \overline{\texttt{M}}\ \texttt{OK in C}}{\texttt{class C extends D}\ \{\overline{\texttt{C}}\ \overline{\texttt{f}};\ \texttt{K}\ \overline{\texttt{M}}\}\ \texttt{OK}}$$

$$\frac{\overline{\texttt{M}}\ \texttt{OK in R} \qquad \overline{\texttt{MR}}\ \texttt{OK in R}}{\texttt{refines class R}\ \{\overline{\texttt{C}}\ \overline{\texttt{f}};\ \texttt{KR}\ \overline{\texttt{M}}\ \overline{\texttt{MR}}\}\ \texttt{OK}}$$