Mechanization and Overhaul of Feature Featherweight Java with Coq

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Mecanização

• Mecanizar significa provar teoremas com a assistência do computador.

Mecanização

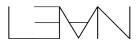
- Mecanizar significa provar teoremas com a assistência do computador.
 - Automaticamente;

Mecanização

- Mecanizar significa provar teoremas com a assistência do computador.
 - Automaticamente;
 - Iterativamente

Provadores Iterativos de Teoremas















Por que mecanizar?

- Aplicações de segurança critica: Bugs são inaceitáveis
 - Controladores de Avião
 - Equipamentos Médicos
 - Carros

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- Aplicações de segurança critica: Bugs são inaceitáveis
 - Controladores de Avião
 - Equipamentos Médicos
 - Carros
- A medida que o software e o hardware crescem dificulta encontrar bugs através de testes.
- Bugs encontrados tardiamente são caros. 1

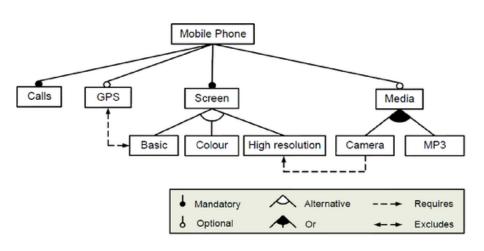
O que é feature?

Feature

'A feature is a unit of functionality of a software system that satisfies a requirement, represents a design decision, and provides a potential configuration option.' - S. Apel & K. Kastner

Feature Oriented Programming

Software Product Line



Objetivo '

Mecanizar Feature Featherweight Java

Sintaxe Abstrata

```
\begin{array}{lll} \text{CD} ::= & \textit{class declarations:} \\ & \text{class C extends D } \{ \overline{\textbf{C}} \ \overline{\textbf{f}} \, ; \ \textbf{K} \ \overline{\textbf{M}} \} \\ \\ \text{K} ::= & \textit{constructor declarations:} \\ & \text{C}(\overline{\textbf{C}} \ \overline{\textbf{f}}) \{ \text{super}(\overline{\textbf{f}}) \, ; \ \text{this.} \overline{\textbf{f}} = \overline{\textbf{f}} \, ; \} \\ \\ \text{M} ::= & \textit{method declarations:} \\ & \text{C m } (\overline{\textbf{C}} \ \overline{\textbf{x}}) \ \{ \text{return e} \, ; \} \end{array}
```

Sintaxe Abstrata

```
e ::= expressions:
    x
    e.f
    e.m(ē)
    new C(ē)
    (C)e

v ::= values:
    new C(ē)
```

Relação de Subtipo

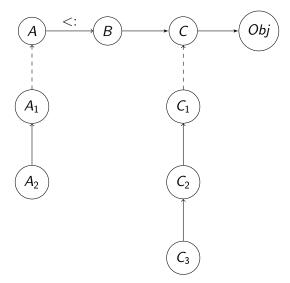
$$\begin{tabular}{c|c|c} \hline $C <: & D & D <: & E \\ \hline $C <: & C \\ \hline \end{tabular} \begin{tabular}{c|c|c} \hline $C <: & E \\ \hline \end{tabular} \begin{tabular}{c|c|c} \hline c & class C extends $D $\{ ... $\} \\ \hline \end{tabular}$$

Relação de Subtipo



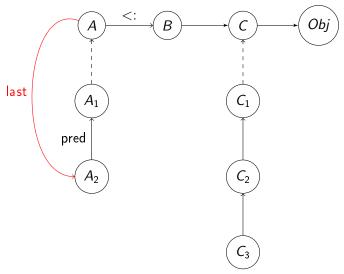
Feature Featherweight Java

Refinement Chain



Feature Featherweight Java

Refinement Chain



Sintaxe Abstrata

```
R : ::=
                              refinement names:
   C@feat
                               class refinements:
CR ::=
   refines class R \{ \bar{C} \ \bar{f} ; KD \ \bar{M} \ \bar{M}R \}
                          constructor refinements:
KD ::=
   refines C(\bar{E} \bar{h}, \bar{C} \bar{f}) \{ original(\bar{f}); this. \bar{f} = \bar{f}; \}
                            method refinements:
MR ::=
   refines C m (\overline{R} \ \overline{x}) {return e;}
```

$$\frac{\text{class C extends D }\{\bar{\text{C}}\ \bar{\text{f}};\ \text{K}\ \bar{\text{M}}\} \qquad \neg \textit{last C}}{\textit{fields C} = \textit{fields D},\ \bar{\text{C}}\ \bar{\text{f}}}$$

$$\frac{\text{class C extends D }\{\bar{\text{C}}\ \bar{\text{f}};\ \text{K}\ \bar{\text{M}}\}}{\textit{fields C} = \textit{fields D},\ \bar{\text{C}}\ \bar{\text{f}},\ \textit{fields_R}\ (\textit{last C})}$$

$$fields \; \texttt{Object} = \bullet$$

$$\frac{\texttt{class} \; \texttt{C} \; \texttt{extends} \; \texttt{D} \; \{\bar{\texttt{C}} \; \bar{\texttt{f}}; \; \texttt{K} \; \bar{\texttt{M}}\} \qquad \neg \textit{last} \; \texttt{C}}{\textit{fields} \; \texttt{C} = \textit{fields} \; \texttt{D}, \; \bar{\texttt{C}} \; \bar{\texttt{f}}}$$

$$\frac{\texttt{class} \; \texttt{C} \; \texttt{extends} \; \texttt{D} \; \{\bar{\texttt{C}} \; \bar{\texttt{f}}; \; \texttt{K} \; \bar{\texttt{M}}\}}{\textit{fields} \; \texttt{C} = \textit{fields} \; \texttt{D}, \; \bar{\texttt{C}} \; \bar{\texttt{f}}, \; \textit{fields}_{R} \; \textit{(last} \; \texttt{C})}$$

$$fields \; \texttt{Object} = \bullet$$

$$\frac{\texttt{class} \; \texttt{C} \; \texttt{extends} \; \texttt{D} \; \{ \overline{\texttt{C}} \; \overline{\texttt{f}} \; ; \; \texttt{K} \; \overline{\texttt{M}} \} \qquad \neg \textit{last} \; \texttt{C}}{\textit{fields} \; \texttt{C} = \textit{fields} \; \texttt{D}, \; \overline{\texttt{C}} \; \overline{\texttt{f}}}$$

$$\frac{\texttt{class} \; \texttt{C} \; \texttt{extends} \; \texttt{D} \; \{ \overline{\texttt{C}} \; \overline{\texttt{f}} \; ; \; \texttt{K} \; \overline{\texttt{M}} \}}{\textit{fields} \; \texttt{C} = \textit{fields} \; \texttt{D}, \; \overline{\texttt{C}} \; \overline{\texttt{f}} \; , \; \textit{fields}_{R} \; (\textit{last} \; \texttt{C})}$$

$$\frac{\text{class C extends D } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{K}\ \bar{\text{M}}\} \qquad \neg \textit{last C}}{\textit{fields C} = \textit{fields D},\ \bar{\text{C}}\ \bar{\text{f}}}$$

$$\frac{\text{class C extends D } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{K}\ \bar{\text{M}}\}}{\textit{fields C} = \textit{fields D},\ \bar{\text{C}}\ \bar{\text{f}},\ \textit{fields_R}\ (\textit{last C})}$$

Field Lookup (Refinement)

$$\frac{\text{refines R } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{KR}\ \bar{\text{M}}\ \bar{\text{MR}}\} \quad \neg \textit{pred}\ R}{\textit{fields}_R\ R\ =\ \bar{\text{C}}\ \bar{\text{f}}}$$

$$\frac{\text{refines R } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{KR}\ \bar{\text{M}}\ \bar{\text{MR}}\}}{\textit{fields}_R\ R\ =\ \textit{fields}_R\ (\textit{pred}\ R), \bar{\text{C}}\ \bar{\text{f}}}$$

Field Lookup (Refinement)

$$\frac{\text{refines R } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{KR $\bar{\text{M}}$ $\bar{\text{MR}}$}\} \quad \neg \textit{pred R}}{\textit{fields}_{\textit{R}}\ R\ =\ \bar{\text{C}}\ \bar{\text{f}}}$$

$$\frac{\text{refines R } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{KR $\bar{\text{M}}$ $\bar{\text{MR}}$}\}}{\textit{fields}_{\textit{R}}\ R\ =\ \textit{fields}_{\textit{R}}\ (\textit{pred R}), \bar{\text{C}}\ \bar{\text{f}}}$$

Field Lookup (Refinement)

$$\frac{\text{refines R } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{KR $\bar{\text{M}}$ $\bar{\text{MR}}$}\} \quad \neg \textit{pred R}}{\textit{fields}_{\textit{R}}\ R = \bar{\text{C}}\ \bar{\text{f}}}$$

$$\frac{\text{refines R } \{\bar{\text{C}}\ \bar{\text{f}};\ \text{KR $\bar{\text{M}}$ $\bar{\text{MR}}$}\}}{\textit{fields}_{\textit{R}}\ R = \textit{fields}_{\textit{R}}\ (\textit{pred R}), \bar{\text{C}}\ \bar{\text{f}}}$$

```
class C extends D \{\bar{C} \ \bar{f}; \ K \ \bar{M}\} B m (\bar{B} \ \bar{x}) \ \{\text{return e}; \} \in \bar{M}
                                   \neg mtype_R (m, last C)
                                mtvpe (m, C) = \overline{B} \rightarrow B
                 class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\}
                                                                         \mathtt{m} \notin \bar{\mathtt{M}}
                                   \neg mtype_R (m, last C)
                            mtype (m, C) = mtype (m, D)
                         class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\}
                        mtype (m, C) = mtype_R (m, last C)
```

```
class C extends D \{\bar{C} \ \bar{f}; \ K \ \bar{M}\} B m (\bar{B} \ \bar{x}) \ \{\text{return e}; \} \in \bar{M}
                                   \neg mtype_R (m, last C)
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                                                                         \mathtt{m} \notin \bar{\mathtt{M}}
                                   \neg mtype_R (m, last C)
                            mtype (m, C) = mtype (m, D)
                         class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\}
                        mtype (m, C) = mtype_R (m, last C)
```

```
class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\} B m (\bar{B} \ \bar{x}) \{ return \ e; \} \in \bar{M} \}
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                 class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\}
                                                                        \mathtt{m} \notin \bar{\mathtt{M}}
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```

```
class C extends D \{\bar{C} \ \bar{f}; \ K \ \bar{M}\} B m (\bar{B} \ \bar{x}) \ \{\text{return e}; \} \in \bar{M}
                                    \neg mtype_R (m, last C)
                                mtype (m, C) = \overline{B} \rightarrow B
                 class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\}
                                                                         \mathtt{m} \notin \bar{\mathtt{M}}
                                    \neg mtype_R (m, last C)
                             mtype (m, C) = mtype (m, D)
                         class C extends D \{\bar{C} \ \bar{f} : K \ \bar{M}\}
                        mtype (m, C) = mtype_R (m, last C)
```

```
class C extends D \{\bar{C} \ \bar{f}; \ K \ \bar{M}\} B m (\bar{B} \ \bar{x}) \ \{\text{return e}; \} \in \bar{M}
                                   \neg mtype_R (m, last C)
                                mtvpe (m, C) = \overline{B} \rightarrow B
                 class C extends D \{\bar{C} \ \bar{f} : K \ \bar{M}\}
                                                                     \mathtt{m}\notin \bar{\mathtt{M}}
                                   \neg mtype_R (m, last C)
                            mtype (m, C) = mtype (m, D)
                         class C extends D \{\bar{C} \ \bar{f}; K \bar{M}\}
                        mtype (m, C) = mtype_R (m, last C)
```

Method Type Lookup (Refinement)

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$$\frac{\text{refines class R }\{\bar{\textbf{C}}\ \bar{\textbf{f}};\ \text{KR}\ \bar{\textbf{M}}\ \bar{\textbf{MR}}\} \qquad \textbf{B}\ \textbf{m}\ (\bar{\textbf{B}}\ \bar{\textbf{x}})\ \{\text{return e};\}\in\bar{\textbf{M}}}{mtype_R\ (\textbf{m},\textbf{R})\ =\ \bar{\textbf{B}}\ \to\ \textbf{B}}$$

$$\frac{\text{refines class R }\{\bar{\textbf{C}}\ \bar{\textbf{f}};\ \text{KR}\ \bar{\textbf{M}}\ \bar{\textbf{MR}}\} \qquad \textbf{m}\notin\bar{\textbf{M}}}{refines\ \textbf{B}\ \textbf{m}\ (\bar{\textbf{B}}\ \bar{\textbf{x}})\ \{\text{return e};\}\in\bar{\textbf{MR}}}$$

$$\frac{mtype_R\ (\textbf{m},\textbf{R})\ =\ \bar{\textbf{B}}\ \to\ \textbf{B}}{refines\ \text{class R }\{\bar{\textbf{C}}\ \bar{\textbf{f}};\ \text{KR}\ \bar{\textbf{M}}\ \bar{\textbf{MR}}\}}$$

$$\frac{m\notin\bar{\textbf{M}}\ \bar{\textbf{M}}}{mtype_R\ (\textbf{m},\textbf{R})\ =\ mtype_R\ (\textbf{m},pred\ \textbf{P})}$$

Method Type Lookup (Refinement)

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$$\frac{\text{refines class R }\{\bar{\textbf{C}}\ \bar{\textbf{f}};\ \text{KR}\ \overline{\textbf{M}}\ \overline{\textbf{MR}}\} \qquad \textbf{B m }(\bar{\textbf{B}}\ \bar{\textbf{x}})\ \{\text{return e};\}\in \bar{\textbf{M}}}{mtype_R\ (\textbf{m},\textbf{R})\ =\ \bar{\textbf{B}}\ \to\ \textbf{B}}$$

$$\frac{\text{refines class R }\{\bar{\textbf{C}}\ \bar{\textbf{f}};\ \text{KR}\ \bar{\textbf{M}}\ \overline{\textbf{MR}}\} \qquad \textbf{m}\notin \bar{\textbf{M}}}{refines\ \textbf{B}\ \textbf{m}\ (\bar{\textbf{B}}\ \bar{\textbf{x}})\ \{\text{return e};\}\in \bar{\textbf{MR}}}$$

$$\frac{mtype_R\ (\textbf{m},\textbf{R})\ =\ \bar{\textbf{B}}\ \to\ \textbf{B}}{refines\ \text{class R }\{\bar{\textbf{C}}\ \bar{\textbf{f}};\ \text{KR}\ \bar{\textbf{M}}\ \bar{\textbf{MR}}\}}$$

$$\frac{m\notin \bar{\textbf{M}}\ \textbf{m}}{mtype_R\ (\textbf{m},\textbf{R})\ =\ mtype_R\ (\textbf{m},pred\ \textbf{P})}$$

Method Type Lookup (Refinement)

Method Body Lookup

class C extends D {
$$\bar{C}$$
 \bar{f} ; K \bar{M} } B m (\bar{B} \bar{x}) {return e;} $\in \bar{M}$ $\neg mbody_R$ (m, $last$ C) $mbody$ (m, C) = \bar{x} .e class C extends D { \bar{C} \bar{f} ; K \bar{M} } m $\notin \bar{M}$ $\neg mbody_R$ (m, $last$ C) $mbody$ (m, C) = $mbody$ (m, D) $mbody$ (m, C) = $mbody$ (m, D) $mbody$ (m, C) = $mbody$ (m, $last$ C)

Method Body Lookup (Refinement)

```
refines class R \{\bar{C}\ \bar{f};\ KR\ \bar{M}\ \bar{MR}\} B m (\bar{B}\ \bar{x})\ \{return\ e;\}\in \bar{M}
                                        mbody_R (m, R) = \bar{x}.e
                    refines class R \{\bar{C} \ \bar{f}; \ KR \ \overline{M} \ \overline{MR}\} m \notin \bar{M}
                         refines B m (\overline{B} \overline{x}) {return e;} \in \overline{MR}
                                        mbody_R (m, R) = \bar{x}.e
                            refines class R \{\bar{C}\ \bar{f};\ KR\ \overline{M}\ \overline{MR}\}
                                              m \notin \overline{M} \quad m \notin \overline{MR}
                           mbody_R (m, R) = mbody_R (m, last P)
```

Tipagem

 $\Gamma \vdash e : C$

Tipagem

$$\begin{array}{c} \Gamma \vdash e : C \\ \Downarrow \\ a : A, b : B, ..., x : X \vdash e : C \end{array}$$

Regras de Tipagem

$$\Gamma \vdash x : \Gamma(x) \tag{T-Var}$$

$$\frac{\Gamma \vdash e_0 : C_0 \quad \textit{fields} \ (C_0) = \overline{C} \ \overline{f}}{\Gamma \vdash e_0 \cdot f_i : C_i} \tag{T-Field}$$

$$\frac{\Gamma \vdash e_0 : C_0 \quad \textit{mtypes} \ (m, \ C_0) = \overline{D} \to C \quad \Gamma \vdash \overline{e} : \overline{C} \quad \overline{C} <: \ \overline{D}}{\Gamma \vdash e_0 \cdot m(\overline{e}) : C} \tag{T-Invk}$$

$$\frac{\textit{fields}(C) = \overline{D} \ \overline{f} \quad \Gamma \vdash \overline{e} : \overline{C} \quad \overline{C} <: \ \overline{D}}{\Gamma \vdash \textit{new} \ C(\overline{e}) : C} \tag{T-New}$$

$$\frac{\Gamma \vdash e_0 : D \quad D <: \ C}{\Gamma \vdash (C) \ e_0 : C} \tag{T-UCast}$$

$$\frac{\Gamma \vdash e_0 : D \qquad C <: D \qquad C \neq D}{\Gamma \vdash (C) e_0 : C}$$
(T-DCast)

$$\frac{\Gamma \vdash e_0 : D \qquad C \not<: \ D \qquad D \not<: \ C \qquad \textit{stupid warning}}{\Gamma \vdash \vdash (C) \ e_0 : \ C} \tag{T-SCast}$$



Computação



Regras de Computação

$$\frac{\mathit{fields}\;(C) = \bar{C}\bar{f}}{(\mathit{new}\;C(\bar{e})).f_i \to e_i} \tag{R-Field}$$

$$\frac{\mathit{mbody}\;(\mathit{m},C) = \bar{x}.e_0}{(\mathit{new}\;C\;(\bar{e})).\mathit{m}\;(\bar{d}) \to [\bar{d}/\bar{x},\mathit{new}\;C\;(\bar{e})/\mathit{this}]e_0} \tag{R-Invk}$$

$$\frac{\mathit{C}\;<:\mathit{D}}{(\mathit{D})(\mathit{new}\;C\;(\bar{e})) \to \mathit{new}\;C\;(\bar{e})} \tag{R-Cast}$$

Regras de Congruência

$$\frac{e_0 \rightarrow e_0'}{e_0.f \rightarrow e_0'.f} \qquad \qquad \text{(RC-Field)}$$

$$\frac{e_0 \rightarrow e_0'}{e_0.m \ (\overline{e}) \rightarrow e_0'.m \ (\overline{e})} \qquad \qquad \text{(RC-Invk-Recv)}$$

$$\frac{e_i \rightarrow e_i'}{e_0.m \ (\dots, e_i, \dots) \rightarrow e_0'.m \ (\dots, e_i, \dots)} \qquad \qquad \text{(RC-Invk-Arg)}$$

$$\frac{e_i \rightarrow e_i'}{new \ C \ (\dots, e_i, \dots) \rightarrow new \ C \ (\dots, e_i', \dots)} \qquad \qquad \text{(RC-New-Arg)}$$

$$\frac{e_0 \rightarrow e_0'}{(C)e_0 \rightarrow (C)e_0'} \qquad \qquad \text{(RC-Cast)}$$

Progress

• Um programa bem tipado não está preso

Progress

- Um programa bem tipado não está preso
 - Ou é um valor;

Progress

- Um programa bem tipado não está preso
 - Ou é um valor;
 - Ou dá um passo.

Preservation

 Se um termo bem tipado dá um passo, então o resultado também é bem tipado.

Type Safety (ou Soundness)

• Um termo bem tipado nunca fica preso durante a computação.

Safety = Progress + Preservation

• "Well-typed program cannot go wrong" - Robin Milner

Safety = Progress + Preservation

- "Well-typed program cannot go wrong" Robin Milner
- Formulado por Wright e Felleisen em 1994 como definição padrão de type safety para linguagens formuladas por *operational semantics*

Theoremas

Theorem (Preservation)

Se $\Gamma \vdash e : C \ e \ e \rightarrow e'$, então $\Gamma \vdash e' : C'$ para algum C' <: C.

Theorem (Progress)

 $Se \vdash e : C \ e \ n\~ao \ existe \ e' \ tal \ que \ e \rightarrow e'$

Então ou e é um valor, ou contém um downcast como subtermo.

Conclusão

Tudo isto foi devidamente implementado em Coq.^{2 3}

40 40 40 40 40 10 10 10 10

²https://github.com/hephaestus-pl/coqffj ³https://github.com/hephaestus-pl/coqfj

References 1

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 Practical foundations for programming languages.
 Cambridge University Press, 2012.
 - Igarashi, Atsushi, Benjamin C. Pierce, and Philip Wadler.
 Featherweight Java: a minimal core calculus for Java and GJ.
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 - Apel, Sven, Christian Kästner, and Christian Lengauer. Feature Featherweight Java: A calculus for feature-oriented programming and stepwise refinement Proceedings of the 7th international conference on Generative programming and component engineering. ACM, 2008.

Perguntas?

Appendix

Helper Functions

Class Name

$$\frac{{\tt R} = {\tt C@feat}}{{\it class_name} \; {\tt R} = {\tt C}}$$

Refinements of a class

$$\frac{\textit{filter} \ (\lambda R \cdot \textit{class}_\textit{name} \ R == C) \ RT = \bar{R}}{\textit{refinements} \ \textit{of} \ C = \bar{R}}$$

Predecessor

$$\frac{\textit{refinements}_\textit{of (class}_\textit{name } R) = \bar{R}}{\textit{index } R \; \bar{R} \; = \; n \quad \textit{get } (n-1) \; \bar{R} \; = \; P}}{\textit{pred } R \; = \; P}$$

Last

$$\frac{\textit{refinements}_\textit{of}\ \mathtt{C} = \overline{\mathtt{R}} \qquad \textit{tail}\ \overline{\mathtt{R}} \ = \ \mathtt{R}}{\textit{last}\ \mathtt{C} \ = \ \mathtt{R}}$$

Appendix

Override

$$\frac{\textit{mtype} \; (\texttt{m}, \texttt{D}) \; = \; \overline{\texttt{D}} \; \rightarrow \; \texttt{D} \; \textit{implies} \; \overline{\texttt{C}} \; = \; \overline{\texttt{D}} \; \textit{and} \; \texttt{C}_0 \; = \; \texttt{D}}{\textit{override} \; \texttt{m} \; \; \texttt{D} \; \overline{\texttt{C}} \; \texttt{C}_0}$$

 $override_R$ m R \bar{C} C₀

refines class P {
$$\overline{C}$$
 \overline{f} ; KR \overline{M} \overline{MR} } C₀ m (\overline{C} \overline{x}) {return e;} $\in \overline{M}$ pred R = P

 $override_R$ m R \bar{C} C_0

$$\frac{\textit{pred } R = P \quad \textit{override}_R \text{ m } P \ \bar{C} \ C_0}{\textit{override}_R \text{ m } R \ \bar{C} \ C_0}$$

Appendix Introduction

$$\frac{pred \ R \ = \ S \ \neg \ mtype_R \ (m,S)}{introduce \ m \ R}$$

introduce m R

Appendix

Method Typing

```
\overline{x}:\overline{C}, this: C \vdash t_0:E_0 \qquad E_0 <: C_0
               \operatorname{CT}(\operatorname{C}) = \operatorname{class} \operatorname{C} \operatorname{extends} \operatorname{D} \{\dots\} \qquad override(\operatorname{m}, \operatorname{D}, \overline{\operatorname{C}} \to \operatorname{C})
                                       C_0 \text{ m } (\overline{C} \overline{x}) \{ \text{return } t_0; \} \text{ OK in } C
                 \overline{x}:\overline{C}, this: C \vdash t_0:E_0 \qquad E_0 <: C_0 \qquad R = C@feat
CT(C) = class C extends D \{...\} RT(R) = refines R \{... \overline{M} ...\}
                          override(m, D, \overline{C} \to C) introduce m R m \in \overline{M}
                                       C_0 \text{ m } (\overline{C} \overline{x}) \{ \text{return } t_0; \} \text{ OK in } R
                 \overline{x}:\overline{C}, this: C \vdash t_0:E_0 \qquad E_0 <: C_0 \qquad R = C@feat
                  RT(R) = refines R \{ \dots \overline{M}, \overline{MR} \dots \} \qquad m \notin \overline{M} \qquad m \in \overline{MR}
                                   override_R(m, R, \overline{C} \to C) introduce m R
                              refines C_0 \text{ m} (\overline{C} \overline{x}) \{ \text{return } t_0; \} \text{ OK in R}
```

Appendix

Class and Refinement Typing

 $\frac{\overline{\text{M}} \text{ OK in R} \qquad \overline{\text{MR}} \text{ OK in R}}{\text{refines class R } \{\overline{\text{C}} \ \overline{\text{f}}; \ \text{KR} \ \overline{\text{M}} \ \overline{\text{MR}}\} \text{ OK}}$

Lemma (Typed method has body)

If $mtype(m, C) = B \rightarrow \overline{B}$

then $\exists \overline{x} \exists e \text{ such that mbody}(m, C) = \overline{x}.e$



Lemma (Typed method has body - Refinement)

If $mtype_R(m,R) = B \rightarrow Bs$ then $\exists \overline{x} \exists e \ such \ that \ mbody_R(m,R) = \overline{x}.e$

Lemma (Body method has type - Refinement)

If $mbody_R(m,R) = \overline{x}.e$ then $\exists \overline{B} \ \exists B \ such \ that \ mtype_R(m,R) = B \rightarrow Bs$



Lemma (Subtype respects method types)

```
If class C extends D \{\bar{C} \ \bar{f}; \ K \ \bar{M}\}\ then mtype(m,C)=mtype(m,D)
```



Lemma (Refinement respects method types)

```
If class C extends D \{\bar{C}\ \bar{f};\ K\ \bar{M}\}
then \forall feat,\ mtype_R(m,C@feat)=mtype(m,D)
```



Lemma (A1.4 - Method Body is Typable)

If mtype $(m, C) = \overline{D} \to D$ and mbody $(m, C) = \overline{x}.e$

then $\exists C <: D, \exists C_0 <: D_0, \text{ this }: D, \overline{x} : \overline{D} \vdash e : C_0$



Lemma (Method Body is Typable - Refinement)

If $mtype_R$ (m, C@feat) = $\overline{D} \rightarrow D$ and $mbody_R$ (m, C@feat) = $\overline{x}.e$

then $\exists C <: D, \exists C_0 <: D_0$, this $: D, \overline{x} : \overline{D} \vdash e : C_0$



Evaluation Context

$$E ::= \Box \mid E.f_i \mid E.m(\overline{e}) \mid e.m(\overline{e_I}, E, \overline{e_r}) \mid (C) \ E \mid new \ C(\overline{e_I}, E, \overline{e_r})$$

Table: Evaluation Context

Appendix

Theorems

Theorem (Progress – Using Evaluation Context)

Suppose e is closed, well-typed normal form.

Then either (1) e is a value, or (2) for some evaluation context E, we can expression e as $e = E[(C)(\text{new }D(\overline{e}))]$, with $D \nleq C$.