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Quadratura de Gauss - Legendre

I) Desenvolver polinômio

1) $P4(\alpha) = \frac{1}{2^4 . 4!} \frac{d^4}{d\alpha^4} \cdot \left[(\alpha^2 - 1)^4 \right]$

2) $\frac{1}{2^4 . 4!} \cdot \left[\alpha^8 - 2\alpha^6 + \alpha^4 - 2\alpha^6 + 4\alpha^4 - 2\alpha^2 + \alpha^4 - 2\alpha^2 + 1 \right] \frac{d^4}{d\alpha^4}$

3) $\frac{1}{4.4.4!} \cdot \left[\alpha^8 - 4\alpha^6 + 6\alpha^4 - 4\alpha^2 + 1 \right] \frac{d^4}{d\alpha^4}$

4) $\frac{1}{4.4.4!} \cdot \left[8.7.6.5\alpha^4 - 6.5.4.3.4\alpha^2 + 4.3.2.1.6 \right]$

5) $\frac{\cancel{8}.7.\cancel{6}.5\alpha^4}{\cancel{4}.4.\cancel{4}^{2}.\cancel{3}.\cancel{2}} - \frac{\cancel{6}.5.\cancel{4}.3.\cancel{4}\alpha^2}{\cancel{4}.\cancel{4}.4.\cancel{3}.\cancel{2}} + \frac{\cancel{4}.3.\cancel{2}.1.\cancel{6}}{\cancel{4}.\cancel{4}_{2}.4.\cancel{3}.\cancel{2}}$

6) $\frac{35}{8}\alpha^4 - \frac{15}{4}\alpha^2 + \frac{1,5}{4}$

7) $\frac{1}{4}\left(\frac{35}{2}\alpha^4 - 15\alpha^2 + 1,5\right)$

8) $\frac{1}{8}\left(35\alpha^4 - 30\alpha^2 + 3\right)$ //

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II Encontrando as raízes

1) $35\alpha^4 - 30\alpha^2 + 3 = 0$

2) Seja $x = \alpha^2$

3) $35x^2 - 30x + 3 = 0$

3.1) $\Delta = 900 - 420$

$\Delta = 480$

$\sqrt{\Delta} = 4\sqrt{30}$

3.2) $x = \dfrac{30 \pm 4\sqrt{30}}{70}$

$x = \dfrac{15 \pm 2\sqrt{30}}{35}$

$x_1 = \dfrac{15 + 2\sqrt{30}}{35}$

$x_2 = \dfrac{15 - 2\sqrt{30}}{35}$

4) Substituindo x temos, então:

$\alpha_1 = \sqrt{\dfrac{15 + 2\sqrt{30}}{35}} = 0,8611363116$

$\alpha_2 = -\sqrt{\dfrac{15 + 2\sqrt{30}}{35}} = -0,8611363116$

$\alpha_3 = \sqrt{\dfrac{15 - 2\sqrt{30}}{35}} = 0,3399810436$

$\alpha_4 = -\sqrt{\dfrac{15 - 2\sqrt{30}}{35}} = -0,3399810436$

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III) Calcular L_K; $K = 1, \dots, 4$

OBS: Importante notar que $L_1 = L_4$ e $L_2 = L_3$

III) 1) Fórmulas:

$$L_1(\alpha) = \frac{(\alpha - \alpha_2)}{(\alpha_1 - \alpha_2)} \cdot \frac{(\alpha - \alpha_3)}{(\alpha_1 - \alpha_3)} \cdot \frac{(\alpha - \alpha_4)}{(\alpha_1 - \alpha_4)}$$

$$L_2(\alpha) = \frac{(\alpha - \alpha_1)}{(\alpha_2 - \alpha_1)} \cdot \frac{(\alpha - \alpha_3)}{(\alpha_2 - \alpha_3)} \cdot \frac{(\alpha - \alpha_4)}{(\alpha_2 - \alpha_4)}$$

$$L_3(\alpha) = \frac{(\alpha - \alpha_1)}{(\alpha_3 - \alpha_1)} \cdot \frac{(\alpha - \alpha_2)}{(\alpha_3 - \alpha_2)} \cdot \frac{(\alpha - \alpha_4)}{(\alpha_3 - \alpha_4)}$$

$$L_4(\alpha) = \frac{(\alpha - \alpha_1)}{(\alpha_4 - \alpha_1)} \cdot \frac{(\alpha - \alpha_2)}{(\alpha_4 - \alpha_2)} \cdot \frac{(\alpha - \alpha_3)}{(\alpha_4 - \alpha_3)}$$

2) Calcular $L_1(\alpha)$

$$L_1(\alpha) = \frac{(\alpha - \alpha_2)}{(\alpha_1 - \alpha_2)} \cdot \frac{(\alpha^2 - \alpha \cdot \alpha_4 - \alpha \cdot \alpha_3 + \alpha_3 \cdot \alpha_4)}{(\alpha_1^2 - \alpha_1 \alpha_4 - \alpha_1 \alpha_3 + \alpha_3 \alpha_4)}$$

$$L_1(\alpha) = \frac{\alpha^3 + 0{,}11558771\,\alpha + 0{,}86117363116\,\alpha^2 - 0{,}09953626}{1{,}07808865}$$

OBS: Note que, no desenvolvimento, todos os multiplicações foram feitos e, por fim, os valores foram substituídos, por essa razão, os cálculos foram omitidos

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3) Calcular $L_2(\alpha)$

$$L_2(\alpha) = \frac{(\alpha - \alpha_1)}{(\alpha_2 - \alpha_1)} \cdot \frac{(\alpha^2 - \alpha \cdot \alpha_4 - \alpha \cdot \alpha_3 + \alpha_3 \alpha_4)}{(\alpha_2^2 - \alpha_2 \alpha_4 - \alpha_2 \alpha_3 + \alpha_3 \alpha_4)}$$

$$L_2(\alpha) = \frac{\alpha^3 - 0{,}339981\,\alpha^2 - 0{,}741557\,\alpha + 0{,}252114}{0{,}425634}$$

(IV) Calcularemos agora W_1 e W_4

$$W_4 = W_1 = \int_{-1}^{1} L_1(\alpha)$$

$$= \frac{\alpha^4}{4} + \frac{8611\,\alpha^3}{30000} + \frac{231\,\alpha^2}{4000} - \frac{199\,x}{2000} \Bigg|_{-1}^{1}$$

$$= \frac{\alpha \cdot (15000\,\alpha^3 + 17222\,x^2 + 3465\,x - 5970)}{60000} \Bigg|_{-1}^{1}$$

$$= 0{,}459404 - 0{,}111508$$

$$W_4 = W_1 = 0{,}347896$$

OBS: Note que o fator $\frac{1}{1{,}07808865}$

foi omitido para dar destaque

para a integral, mas foi

considerado no cálculo final

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(V) Calcularemos agora W_2 e W_3

$$W_3 = W_2 = \int_{-1}^{1} L_2(\alpha)$$

$$= \left. \frac{\alpha^4}{4} - \frac{113327\alpha^3}{1000000} - \frac{741557x^2}{2000000} + \frac{126057}{500000} \right|_{-1}^{1}$$

$$= \left. \frac{\alpha.(500000\alpha^3 - 226654x^2 - 741557x + 504228)}{2000000} \right|_{-1}^{1}$$

$$= 0,652145$$

OBS$_1$: Note que, novamente, o fator $\frac{1}{0,425634}$ foi omitido, entretanto, considerado no cálculo final.

OBS$_2$: Para evitar erros que estavam acontecendo durante a implementação, cálculos como: $\frac{0,3399810434\alpha^3}{3}$; foram transformados, e simplificados, para: $\frac{113327\alpha^3}{1000000}$.

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(VI) Com tudo que temos, podemos agora calcular I

$$I = \frac{x_f - x_i}{2} \cdot \left[f(x(\alpha_1)) \cdot W_1 + f(x(\alpha_2)) \cdot W_2 + f(x(\alpha_3)) \cdot W_3 + f(x(\alpha_4)) \cdot W_4 \right]$$

sendo $x(\alpha_k) = \frac{(x_i + x_f)}{2} + \frac{(x_f - x_i)}{2} \cdot \alpha_k$

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