Scalable Distributed Topologies

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MEIC SDLE 2021

Graphs

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A graph $G(V, E)$ can be defined by a set of vertices, and a set of edges that connect pairs of vertices. For instance, a path $G(V = \{a, b, c, d\}, E = \{(a, b), (b, c), (c, d)\})$ can be made into a ring by adding one edge connecting its ends $E = E \cup \{(d, a)\}$.

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- Having an edge (x, y), we say that those vertices are adjacent (or neighbours).
- A Weighted Graph is obtained by assigning a weight to each edge.
- Path is a sequence of vertices, $\ldots, v_i, v_{i+1}, \ldots$, with edges connecting them $(v_i, v_{i+1}) \in E$.

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Walk	Edges and vertices can be repeated
Trail	Only vertices can be repeated
Path	No repeated vertices or edges

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More topologies

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In a network context, graphs with cycles allow multi-path routing. This can be more robust but data handling can become more complex. Thus, distributed algorithms often construct trees to avoid cycles, while others try to work under multi-path.

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More concepts

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- Distance $d(v_i, v_j)$. Length of the shortest path connecting those nodes.
- Eccentricity of v_i. $ecc(v_i) = max(\{d(v_i, v_j) | v_j \in V\})$.
- Diameter. $D = max(\{ecc(v_i) | v_i \in V\})$
- Radius. $R = min(\{ecc(v_i) | v_i \in V\})$

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- Radius. $R = min(\{ecc(v_i)|v_i \in V\})$
- Center. $\{v_i|ecc(v_i) == R\}$
- Periphery. $\{v_i|ecc(v_i) == D\}$

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Questions

- How big is the center of a tree?
- How big is the center of a path?
- How big is the periphery of a ring? and the center?
- How to compute eccentricities?