DEPARTAMENTO DE ELETRÓNICA, TELECOMUNICAÇÕES
E INFORMÁTICA

MESTRADO INTEGRADO EM ENG. DE COMPUTADORES E TELEMÁTICA

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# DESEMPENHO E DIMENSIONAMENTO DE REDES

## ASSIGNMENT GUIDE NO. 2

## IMPACT OF TRANSMISSION ERRORS IN THE PERFORMANCE OF A WIRELESS NETWORK LINK

# 1. Assignment Description

Implement the following 2 tasks using MATLAB to obtain the requested numerical solutions. At the end, submit a report with the answers to all questions and including not only the numerical solutions but also the MATLAB codes used to obtain them.

## Task 1

RECALL FROM THEORETICAL CLASSES:

1. Bayes' law: consider a set of mutually exclusive events $F_1, F_2, \ldots, F_n$ such that its union is the set of all possible outcomes of a random experiment. Knowing that event $E$ has occurred, the probability of event $F_j$, with $j = 1, 2, \ldots, n$, is given by:

$$P(F_j|E) = \frac{P(EF_j)}{P(E)} = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

2. The probability function of a binomial random variable with parameters $n$ and $p$ is:

$$f(i) = \binom{n}{i} p^i (1-p)^{n-i} \quad , i = 0,1, \ldots n$$

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$

Consider a wireless link used by two stations for data communications. The link can be either in a normal state with a probability of $p$ or in an interference state with a probability of $1 - p$. Consider the following probability values: $p$ = 99%, $p$ = 99.9%, $p$ = 99.99% and $p$ = 99.999%. The bit error rate is $10^{-7}$ when the link is in the normal state and $10^{-3}$ when the link is in the interference state.

The two stations exchange from time to time a set of $n$ consecutive control frames of size 128 Bytes each to decide if the link is in interference state. Both stations determine with a 100% probability if the control frames have been received with errors. The stations decide that the link is in interference state when the $n$ consecutive control frames are received with errors. Consider the following definitions:

- a false positive is when a station decides wrongly that the link is in interference state (i.e., it receives $n$ consecutive control frames with error and the link is in the normal state)
- a false negative is when a station decides wrongly that the link is not in the interference state (i.e., at least one of the $n$ consecutive control frames is received without errors and the link is in the interference state)

**1.a.** For each value of $p$, determine the probability of the link being in the interference state and in the normal state when one control frame is received with errors (fulfil the following table). What do you conclude?

1. binomial to obtain probabilities of errors
2. then bayes laws

| | $p$(normal) | $p$(interference) |
|---|---|---|
| $p$ = 99% | 0.0156 | 0.9844 |
| $p$ = 99.9% | 0.1376 | 0.8624 |
| $p$ = 99.99% | 0.6150 | 0.3850 |
| $p$ = 99.999% | 0.9411 | 0.0589 |

**1.b.** For each value of $p$ and for $n$ = 2, 3, 4 and 5, determine the probability of false positives. Fulfil the follow table:

| | *Probability of false positives* | | | |
|---|---|---|---|---|
| | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ |
| $p = 99\%$ | | | | |
| $p = 99.9\%$ | | | | |
| $p = 99.99\%$ | | | | |
| $p = 99.999\%$ | | | | |

**1.c.** For each value of $p$ and for $n$ = 2, 3, 4 and 5, determine the probability of false negatives. Fulfil the follow table:

| | *Probability of false negatives* | | | |
|---|---|---|---|---|
| | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ |
| $p = 99\%$ | | | | |
| $p = 99.9\%$ | | | | |
| $p = 99.99\%$ | | | | |
| $p = 99.999\%$ | | | | |

**1.d.** Describe and justify the influence of the values of $p$ and $n$ observed in the results obtained in **1.b** and **1.c**.

**1.e.** Assume that we aim a system where both false positive and false negative probabilities are not higher than 0.1%. From the results obtained in **1.b** and **1.c**, what is the best value of $n$ to be used if the probability of the normal state is $p$ = 99.999%.

## Task 2

RECALL FROM THEORETICAL CLASSES:

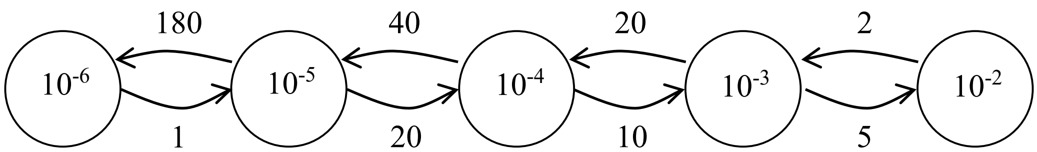
Birth-dead Markov chain: if $\lambda_i$ is the birth rate of state *i* and $\mu_i$ is the dead rate of state *i*, than, the steady-state probability of state 0 is:

$$\pi_0 = \frac{1}{1+\sum_{i=1}^{\infty}\frac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}}$$

and the steady-state probability of state $n > 0$ is:

$$\pi_n = \frac{\lambda_0\lambda_1\cdots\lambda_{n-1}}{\mu_1\mu_2\cdots\mu_n}\cdot\pi_0$$

Consider a wireless link between multiple stations for data communications. The bit error rate introduced by the wireless link due to the variation along with time of the propagation and interference factors is approximately given by the following Markov chain:




where the state transition rates are in number of transitions per hour. Consider that the link is in interference state when its bit error rate is $10^{-3}$ or higher.

**2.a.** What is the average percentage of time the link is on each of the five possible states?

**2.b.** What is the average bit error rate of the link?

**2.c.** What is the average time duration (in minutes) that the link is on each of the five possible states?

**2.d.** What is the probability of the link being in interference state?

**2.e.** What is the average bit error rate of the link when it is in the interference state?

**2.f.** What is the average time duration (in minutes) of the interference state?