

# How to Exploit Structure while Solving Weighted Model Integration Problems

Samuel Kolb, Pedro Zuidberg Dos Martires, Luc De Raedt

## Weighted Model Integration:

- Calculate the weight of an SMT-formula given a weight function.
- Generalizes weighted model counting (Boolean formula) to the discrete-continuous domain

$$2xy$$

$$[(x>0) \wedge (x<1)] \wedge [(y<1) \vee ((x>y) \wedge (y>1/2))]$$

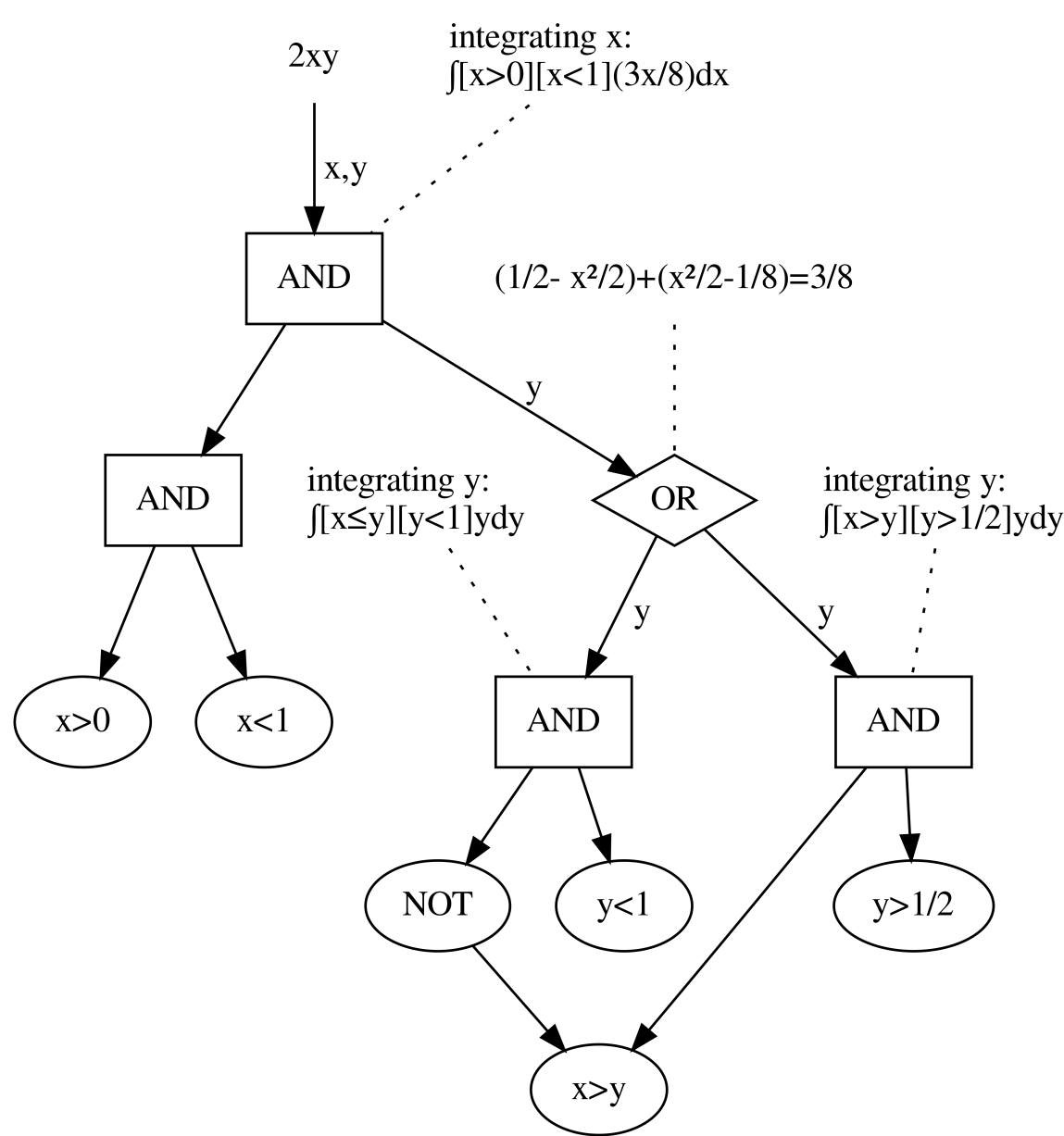
## $\lambda$ -SMT: search vs. compilation

- Find the set of all satisfying assignments

	PA	BR	Symbo	PRAiSE
$\lambda$ -SMT				
DPLL	✓			✓
Compilation		XADD	XSDD	
Integration				
Numeric	Latte			
Symbolic		XADD	PSI (Tree)	Exp. Tree

## Factorized Solving

1. Compile SMT formula to XSDD ( $\lambda$ -SMT).
2. Statically analyze circuit.
3. Push integration inside circuit.



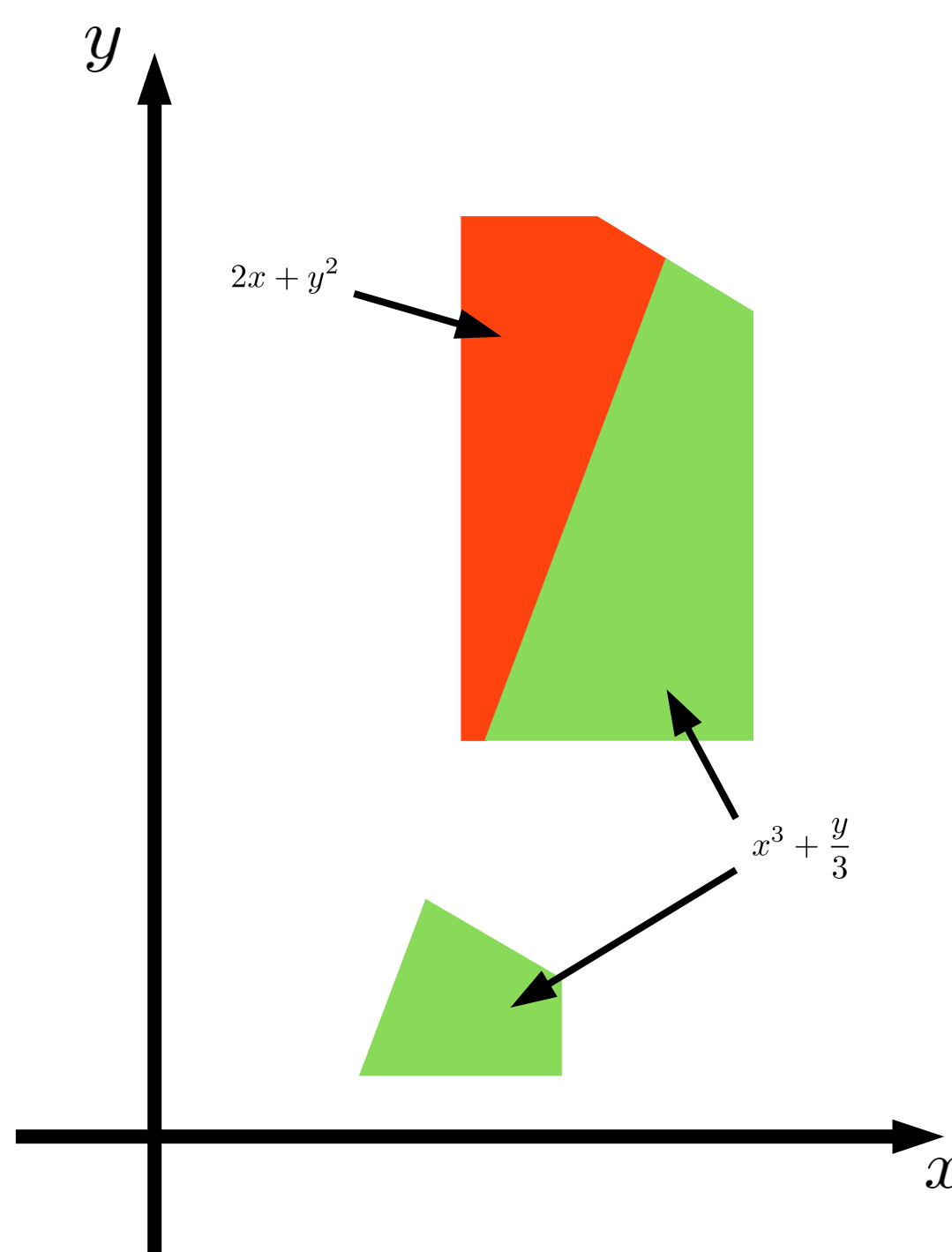
$$2 \int_{(x>0) \wedge (x<1)} \left( \int_{(y<1) \wedge (x \geq y)} y dy + \int_{(y>1/2) \wedge (x>y)} y dy \right) dx$$

Exploiting structure in discrete-continuous probabilistic inference can lead to exponential-to-linear speed-ups in inference time.

Check out our paper:



Check out our library:



### Algorithm 1 Factorized Integration

```
1: world-weight  $\omega$ 
2: procedure vol(XSDD  $D$ , vars  $\mathbf{x}$ )
3:   if  $\mathbf{x} = \emptyset$  then
4:     return  $\llbracket D \rrbracket$ 
5:   else if  $D$  is terminal then
6:     return  $\int \llbracket D \rrbracket \prod_{x \in \mathbf{x}} \omega_x(x) dx$ 
7:   else if  $D = \bigvee_c D_c$  then
8:     return  $\sum_c \text{vol}(D_c, \mathbf{x})$ 
9:   else if  $D = D_1 \wedge D_2$  then
10:     $\mathbf{x}_s = \mathbf{x} \cap \text{vars}(D_1) \cap \text{vars}(D_2)$ 
11:     $\mathbf{x}_1^*, \mathbf{x}_2^* = \text{vars}(D_1) \setminus \mathbf{x}_s, \text{vars}(D_2) \setminus \mathbf{x}_s$ 
12:     $r_1 = \text{vol}(D_1, \mathbf{x}_1^* \cap \mathbf{x})$ 
13:     $r_2 = \text{vol}(D_2, \mathbf{x}_2^* \cap \mathbf{x})$ 
14:    return  $\int r_1 \cdot r_2 \cdot \prod_{x \in \mathbf{x}_s} \omega_x(x) d\mathbf{x}_s$ 
```

