Differentiation and Weighted Model Integration

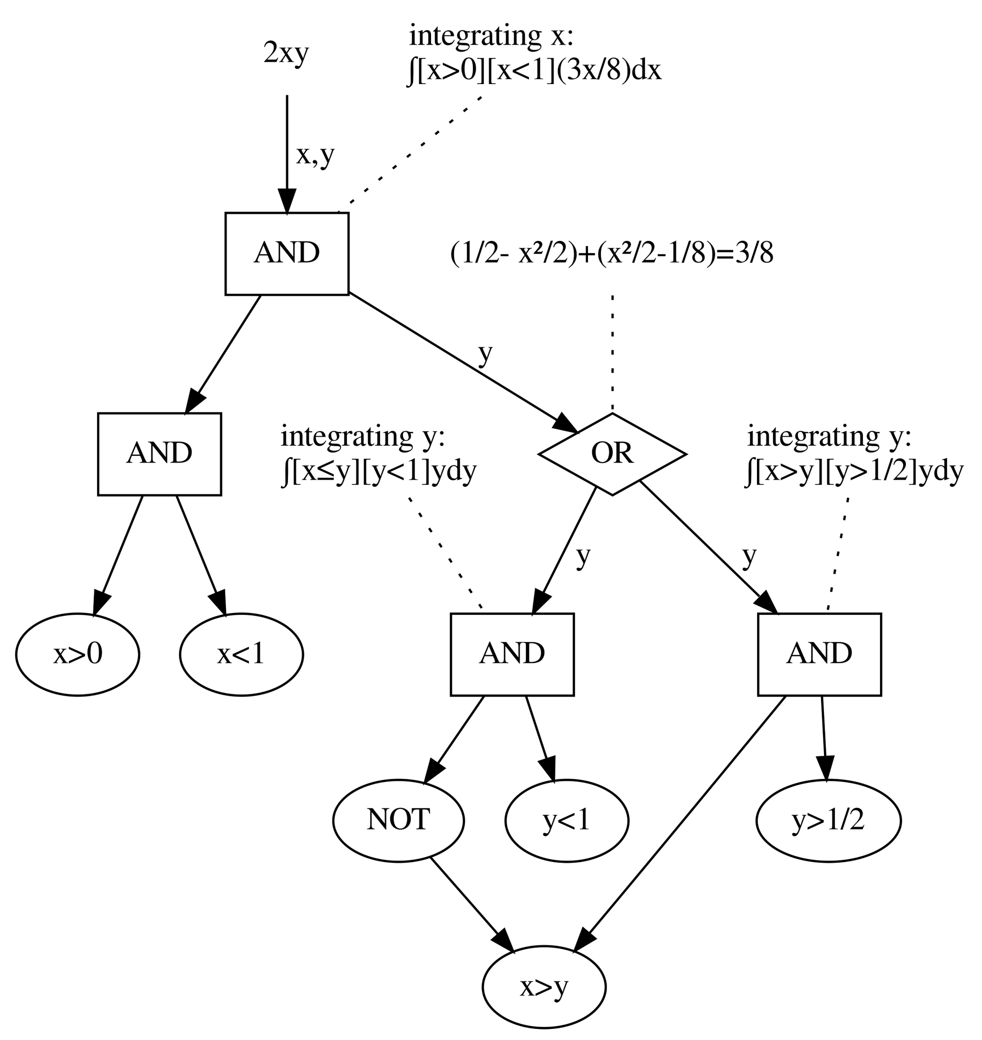
Pedro Zuidberg Dos Martires

Weighted Model Integration:

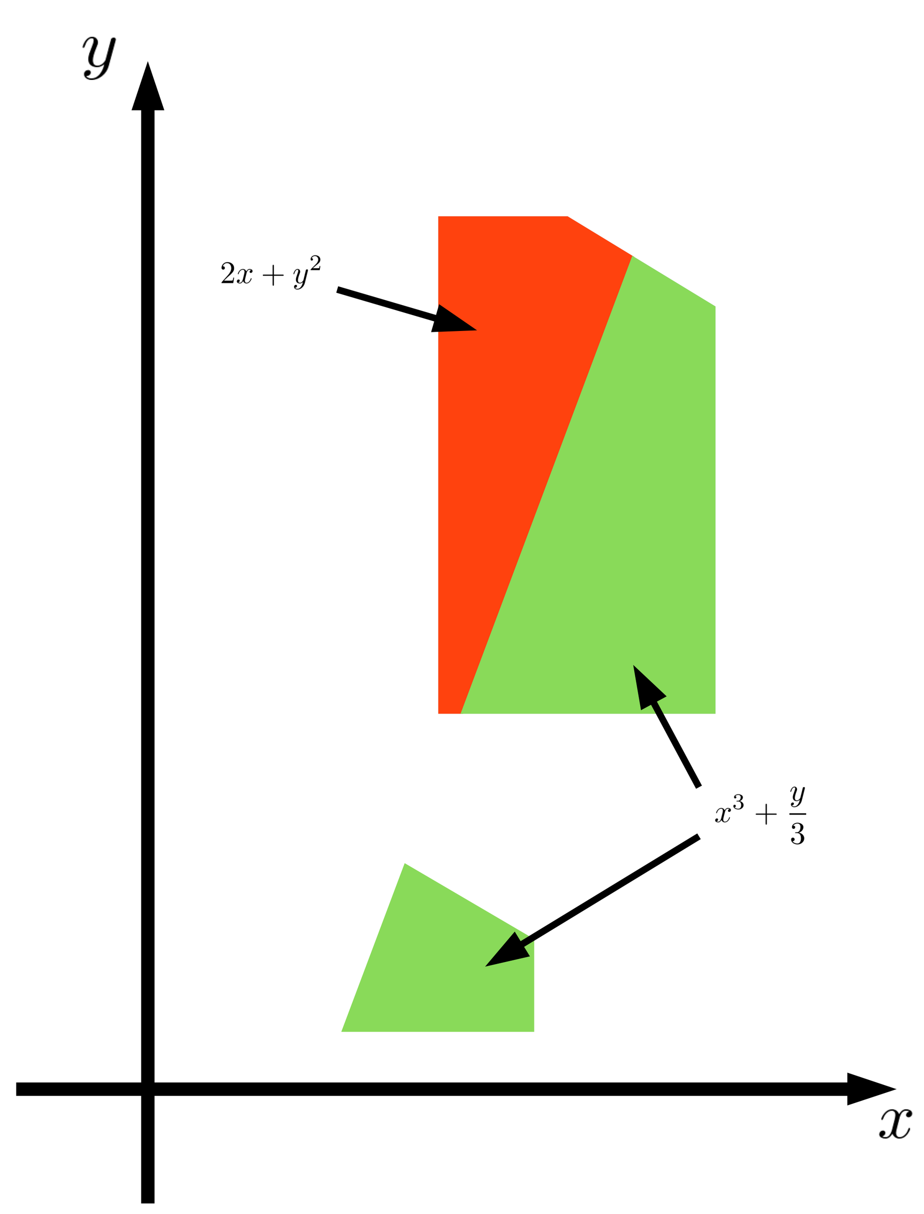
- Calculate the weight of an SMT-formula given a weight function.
- Generalizes weighted model counting (Boolean formula) to the discrete-continuous domain

$$2xy$$

$$[(x>0) \land (x<1)] \land [(y<1) \lor ((x>y) \land (y>1/2))]$$



$$2\int_{(x>0)\land(x<1)}\left(\int_{(y<1)\land(x\geq y)} ydy + \int_{(y>1/2)\land(x>y)} ydy\right)xdx$$



$$\mathrm{WMI}(\phi, w|\mathbf{x}, \mathbf{b})$$
$$= \int \sum_{\mathbf{b}_\mathcal{I}\in\mathcal{I}_{\mathbf{b},\mathbf{b}_a}(\phi_a)} \prod_{b_i\in\mathbf{b}_\mathcal{I}} \alpha_{b_i}(\mathbf{x}) w_x(\mathbf{x}) d\mathbf{x}$$
$$= \int \Psi(\mathbf{x}) w_x(\mathbf{x}) d\mathbf{x}$$
$$= \mathbb{E}_{w_x(\mathbf{x})}[\Psi(\mathbf{x})]$$

Transform a hybrid logic representation into an algebraic one to enable differentiation

The paper:

As Example: Gradient of Cross-Entropy

$$\nabla_{\boldsymbol{\theta}}\mathbb{E}_p[-\log q(\boldsymbol{\theta})] = \mathbb{E}_p[-\frac{1}{q(\boldsymbol{\theta})}\nabla_{\boldsymbol{\theta}} \sum_{\mathbf{b}_\mathcal{I}\in\mathcal{I}_{\mathbf{b},\mathbf{b}_a}(\phi_a)} \prod_{b_i\in\mathbf{b}_\mathcal{I}} \alpha_{b_i}(\boldsymbol{\theta})]$$
$$= \mathbb{E}_p[-\frac{1}{q(\boldsymbol{\theta})} \sum_{\mathbf{b}_\mathcal{I}\in\mathcal{I}_{\mathbf{b},\mathbf{b}_a}(\phi_a)} \sum_{b_i\in\mathbf{b}_\mathcal{I}} \nabla_{\boldsymbol{\theta}}(\alpha_{b_i}(\boldsymbol{\theta})) \prod_{b_j\in\mathbf{b}_\mathcal{I}\setminus\{b_i\}} \alpha_{b_j}(\boldsymbol{\theta})]$$
$$= \mathbb{E}_p[-\frac{1}{\Psi(\boldsymbol{\theta})}\nabla_{\boldsymbol{\theta}}\Psi(\theta)]$$

Challenges ahead

- Differentiation over SMT constraints leads to integration over boundaries.
- Integration over boundary conditions is hard.
- Find approximations for boundary integral:
  approximations? continuous relaxation?
  tackle simplified problem e.g. linear constraints.
- Implement within auto-differentiation system (JAX? PyTorch?).

KU LEUVEN

DTAI

Declaratieve Talen en Artificiële Intelligentie

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