

Solutions

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Problem (4-3).

For notation, take $\langle \cdot, \cdot \rangle$ to be $T(\cdot, \cdot)$. Let e_1, \dots, e_n be an orthonormal basis for V . By definition, $|\omega(e_1, \dots, e_n)| = 1$, and applying Theorem 4-6 gives

$$|\omega(e_1, \dots, e_n)| = \left| \det \begin{bmatrix} \ddots & & \\ & \langle w_i, e_j \rangle & \\ & & \ddots \end{bmatrix} \right|.$$

Using the common identity (see any quantum mechanics textbook)

$$\begin{bmatrix} \ddots & & \\ & \langle w_i, e_j \rangle & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \ddots & & \\ & \langle e_i, w_j \rangle & \\ & & \ddots \end{bmatrix} = \begin{bmatrix} \ddots & & \\ & \langle w_i, w_j \rangle & \\ & & \ddots \end{bmatrix},$$

we arrive at the desired result.

Problem.