Morphological Operations

FRA 626 Machine Vision in Smart Factory

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Mathematical Morphology

- A tool for extracting image components for representation and description of region shape
    - Boundaries
    - Skeletons
    - Convex hull
- Morphological techniques for pre- or post-processing such as morphological filtering, thinning, and pruning
- A set theory

Image & Structuring elements

Basic Set Theory

- If B is the set of pixels (2-D points) representing an object in an image.

- Translation:

$$(B)_z = \{c | c = b + z, \text{for } b \in B\}$$

- Reflection:

$$\hat{B} = \{w | w = -b, \text{for } b \in B\}$$

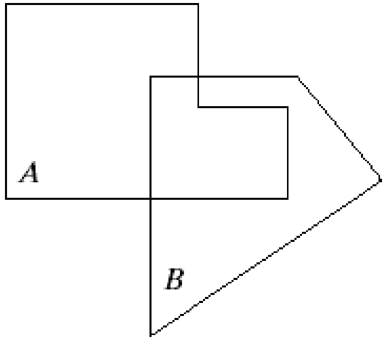
- Complement:

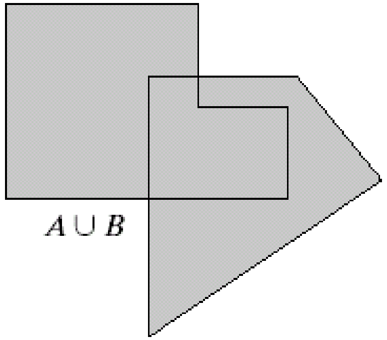
$$B^c = \{w | w \notin B\}$$

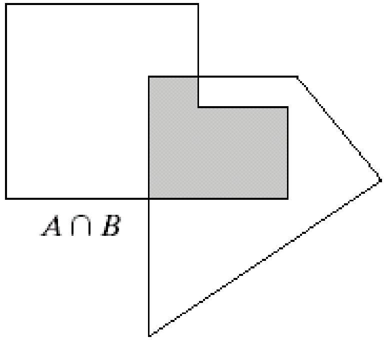
- Difference:

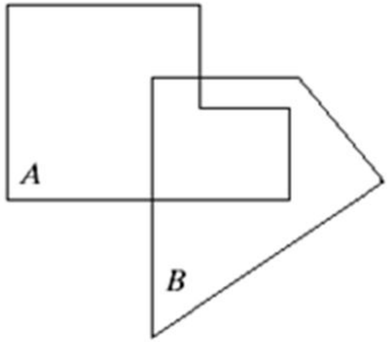
$$A - B = \{c | c \in A, c \notin B\} = A \cap B^c$$

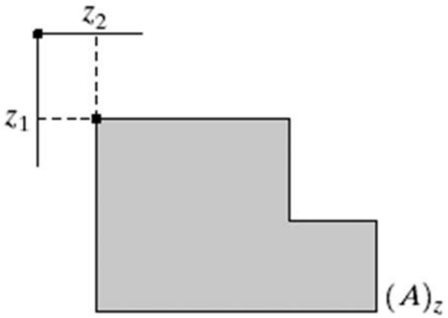
Basic Set Theory



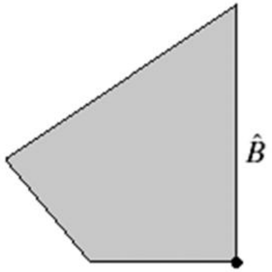




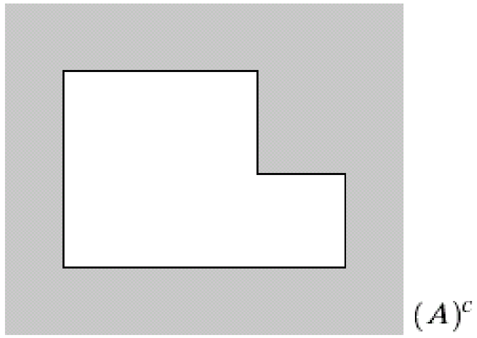


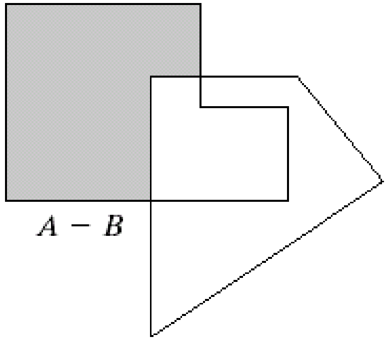


Translation of A by z



Reflection of B





p	q	p **AND** q (also $p \cdot q$)	p **OR** q (also $p + q$)	**NOT** (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

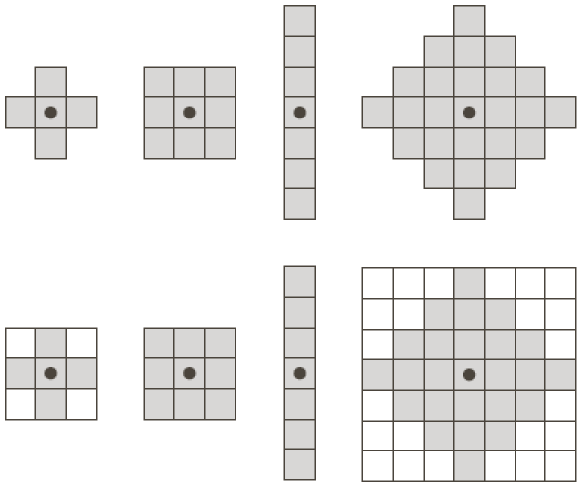
The three basic logical operations

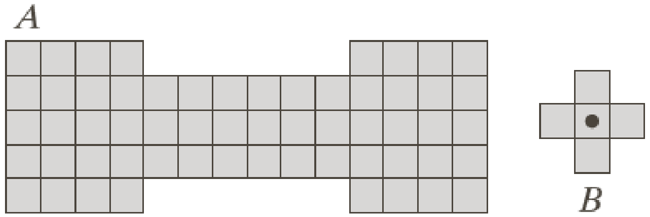
Source: Gonzalez+Woods

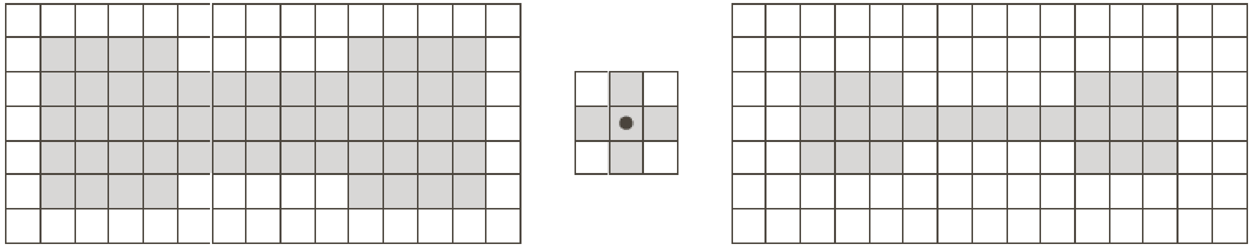
Structuring Elements (SEs)

Structuring elements

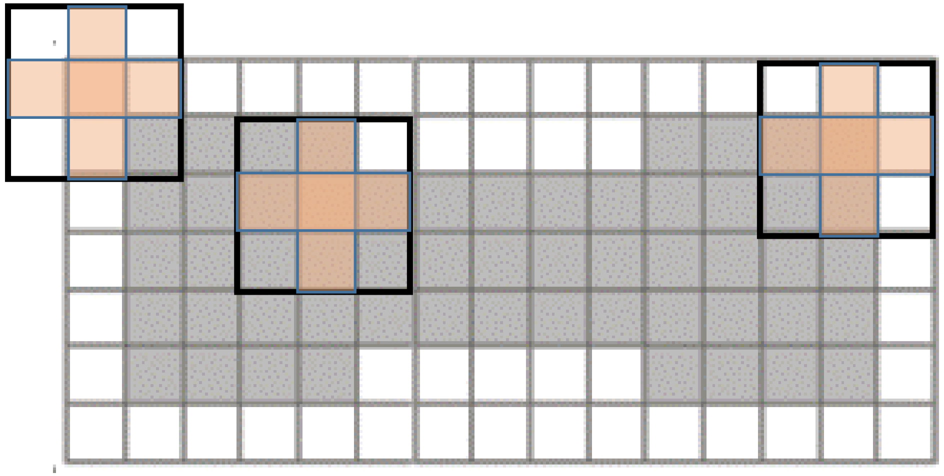
Rectangular arrays







B is completely contained in A



Source: Gonzalez+Woods

Erosion

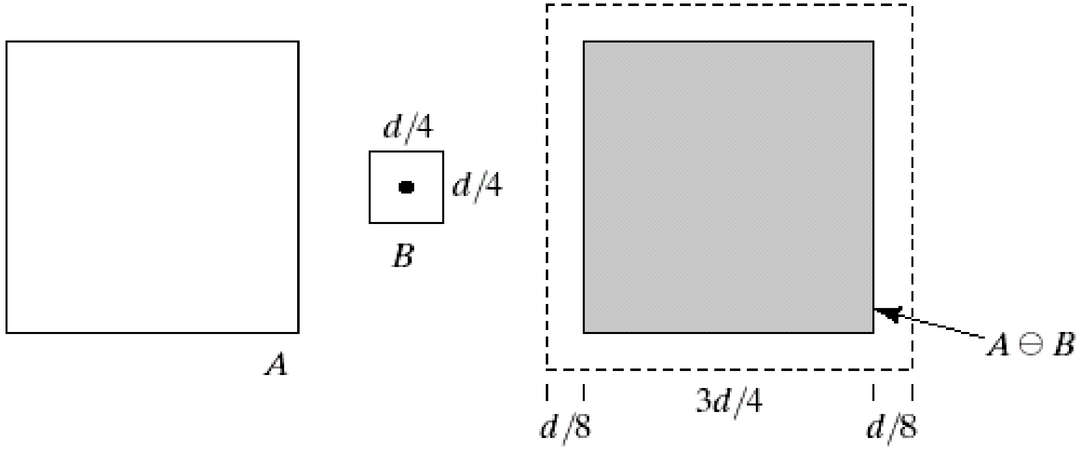
- The erosion of A by B is the set of all point z such that B, translated by z, is contained in A.

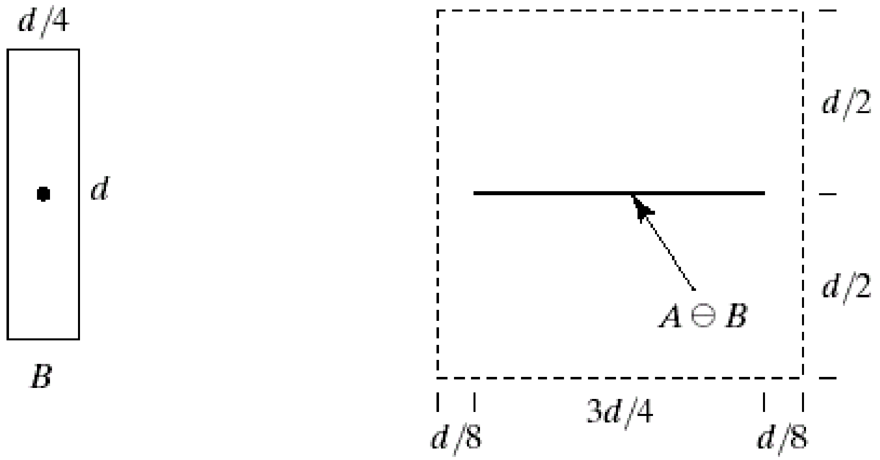
$$A \ominus B = \{z | (B)_z \subseteq A\}$$

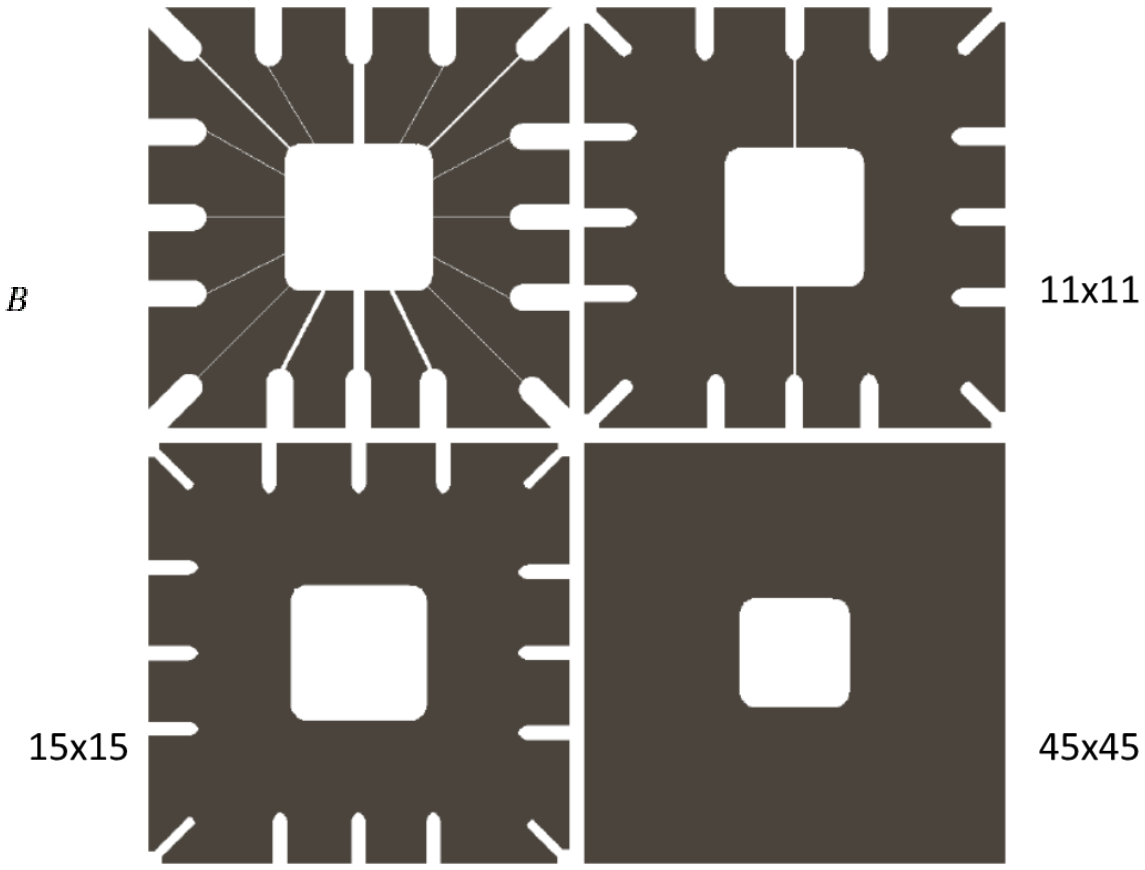
- "B has to be contained in A" is equivalent to "B not sharing any common elements with the background"

$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$

Erosion Example







A 486x486 binary image of a wire bond mask.

a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

Source: Gonzalez+Woods

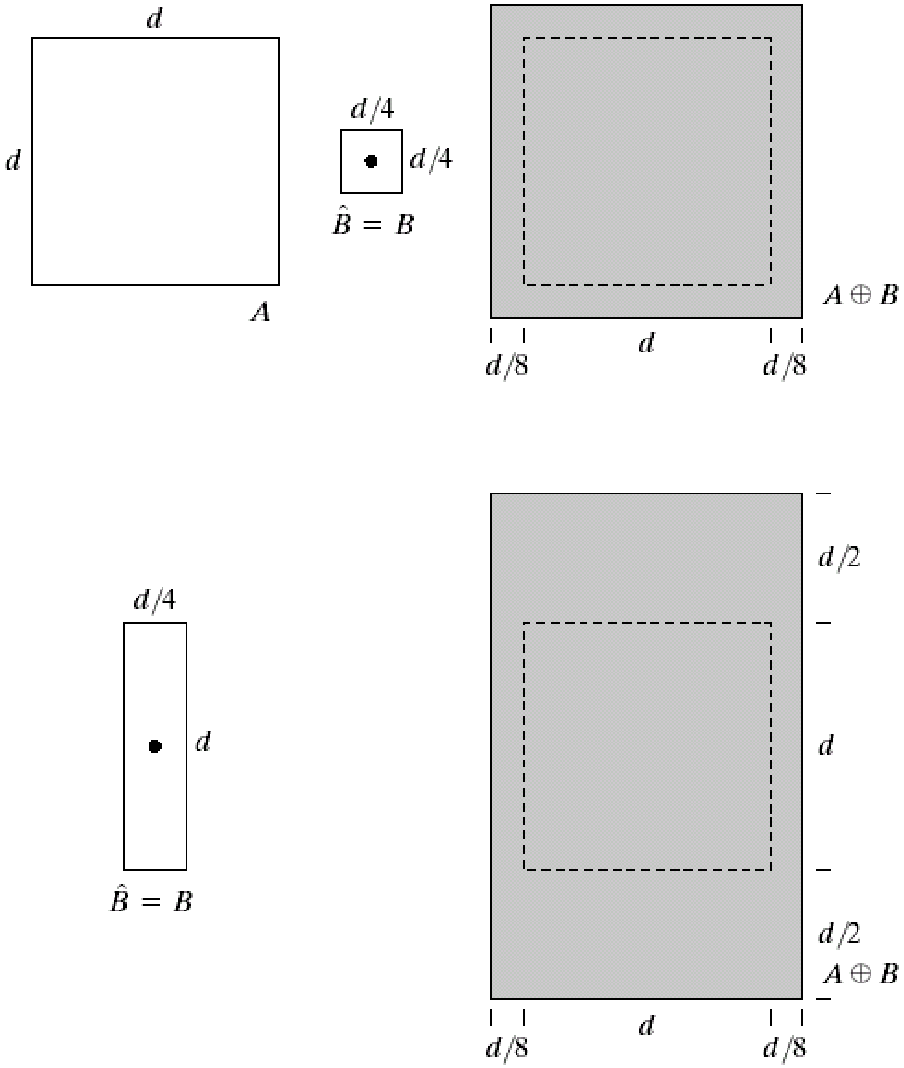
Dilation

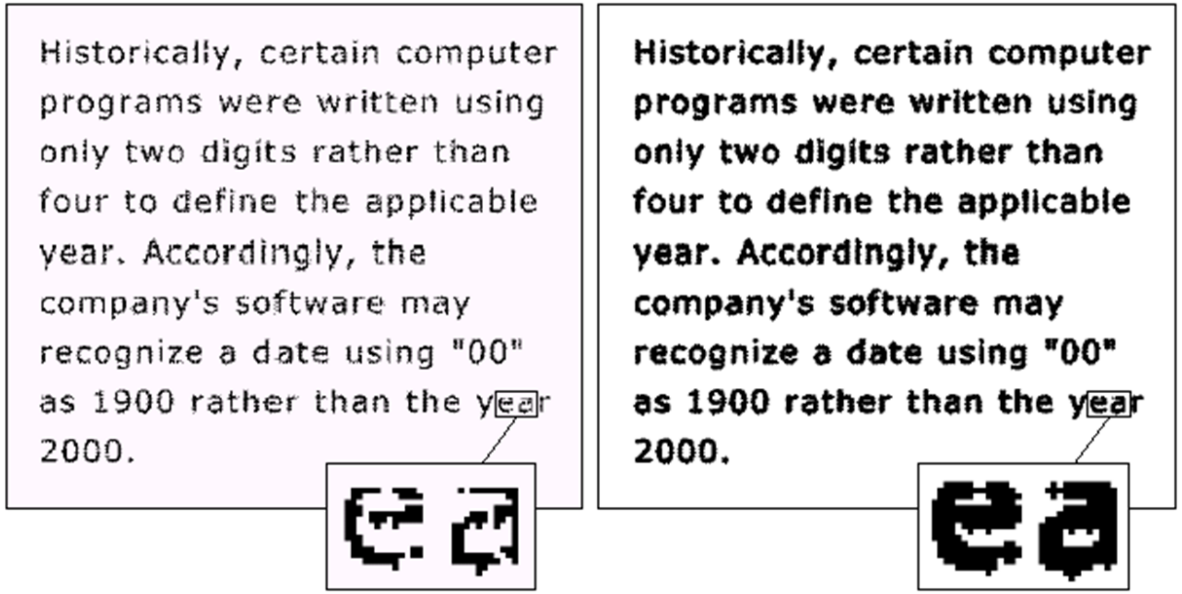
- The set of all z such that the reflection of B about its origin and then translated by z, overlaps with A by at least one nonzero element.

$$A \oplus B = \left\{ z | \left(\hat{B} \right)_z \cap A \neq \emptyset \right\}$$

$$A \oplus B = \left\{ z | \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \right\}$$

Dilation Example





0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Source: Gonzalez+Woods

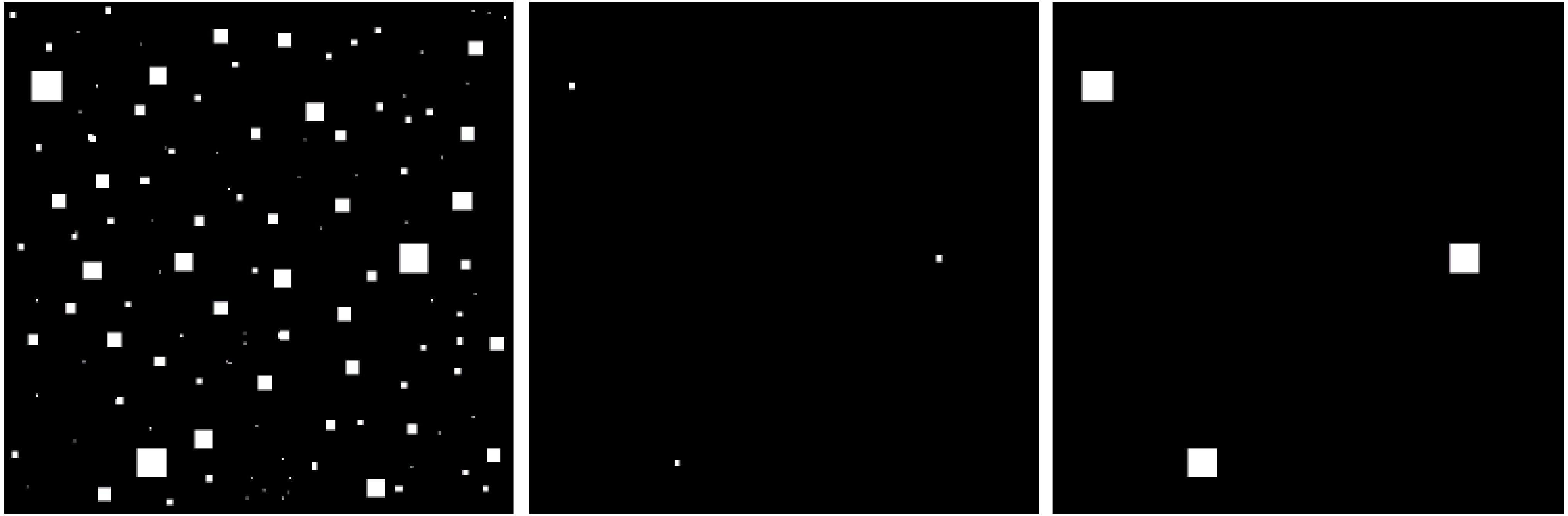
Duality

- Erosion : shrinking or thinning operation
- Dilation : growing or thickening operation
- Erosion and dilation are duals of each other

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Erosion and Dilation

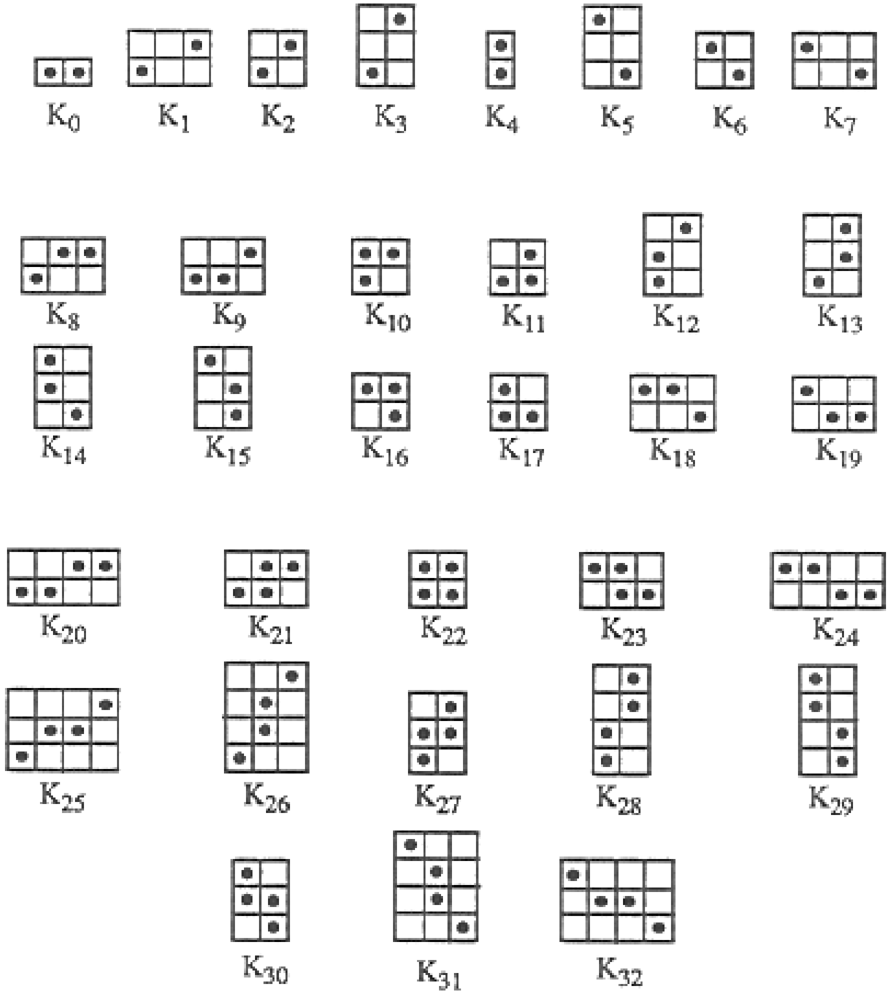


a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Source: Gonzalez+Woods

Typical Structuring Elements



Source: Gonzalez+Woods

Opening & Closing

- **Opening**

$$A \circ B = (A \ominus B) \oplus B$$
$$A \circ B = \bigcup\{(B)_z | (B)_z \subseteq A\}$$

Erosion follow by dilation

Smooth contour, break narrow isthmuses, eliminate thin protrusions

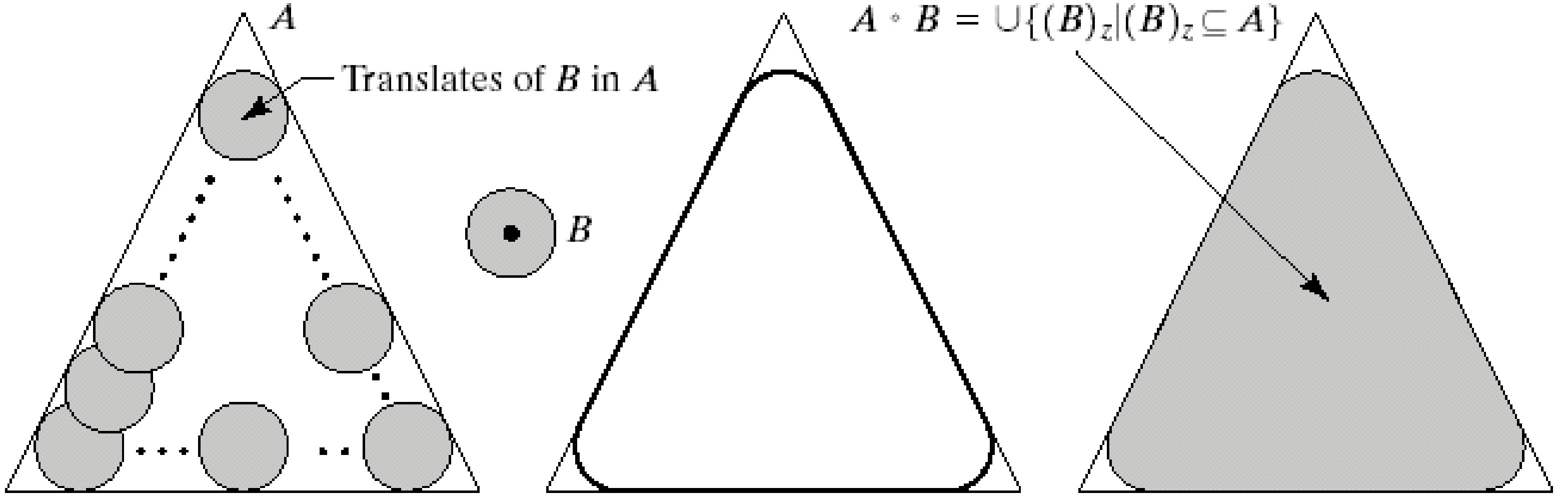
- **Closing**

$$A \bullet B = (A \oplus B) \ominus B$$

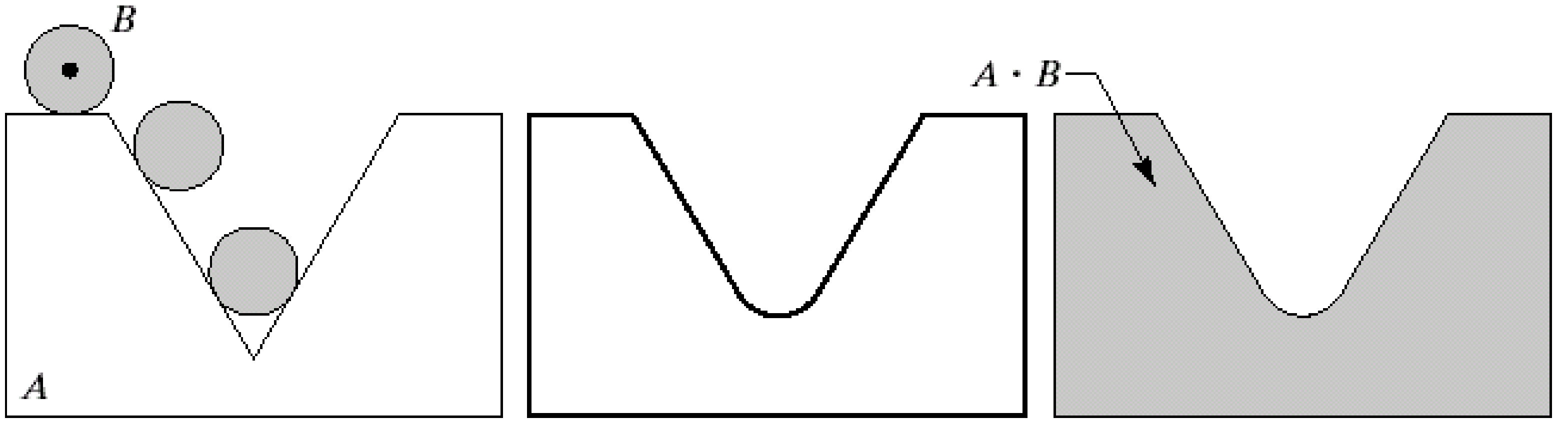
Dilation follow by erosion

Smooth contour, fuse narrow breaks, long thin gulfs, eliminate small holes, fill gaps in the contour

Opening and Closing Examples



Opening



Closing

Source: Gonzalez+Woods

Opening and Closing Examples

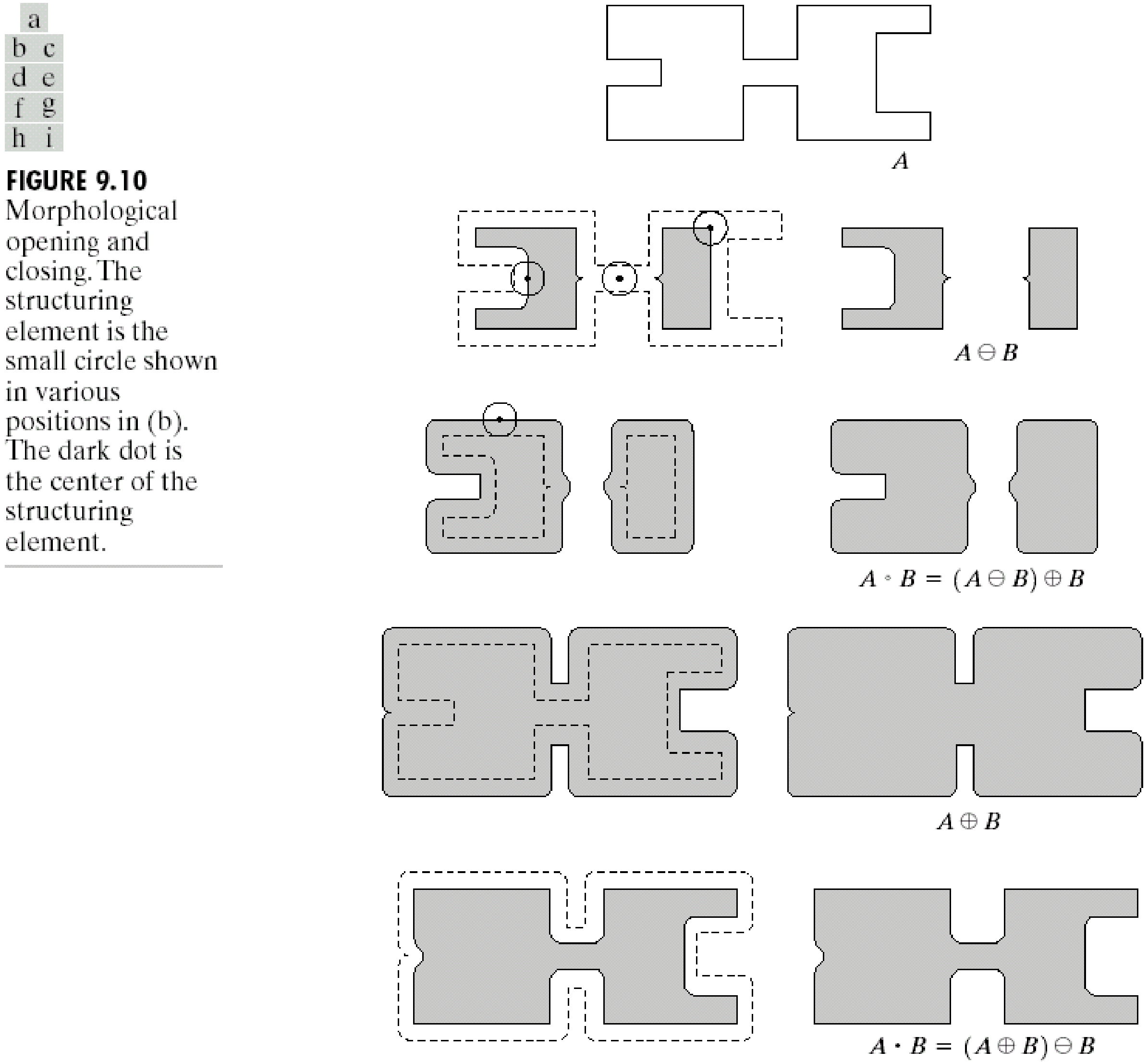


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

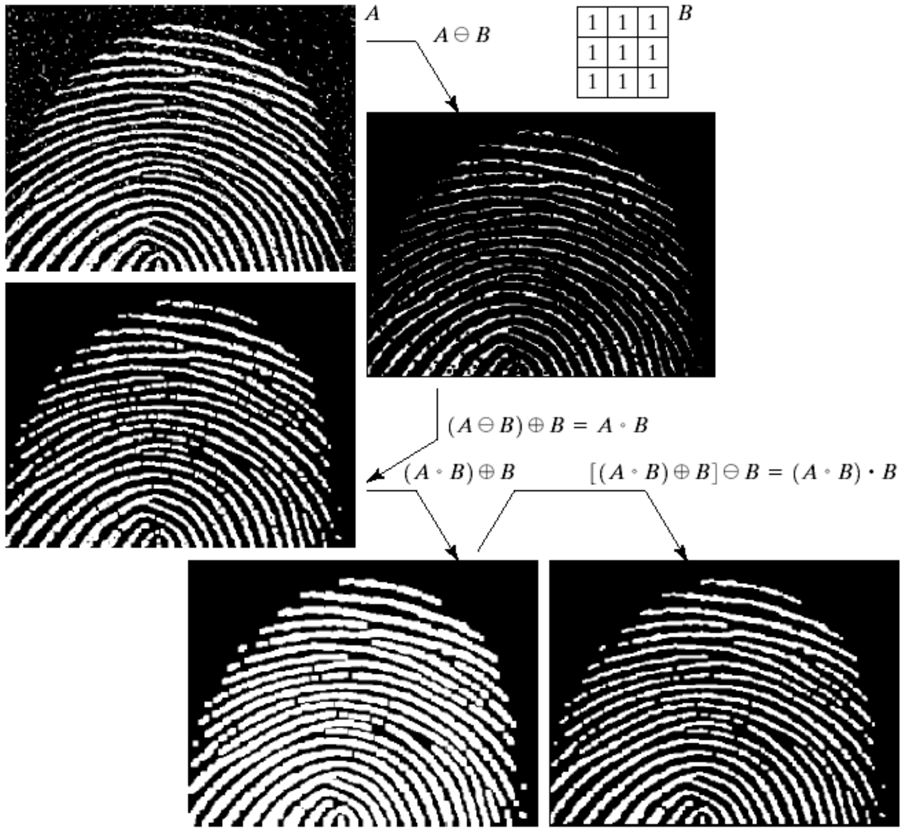


FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

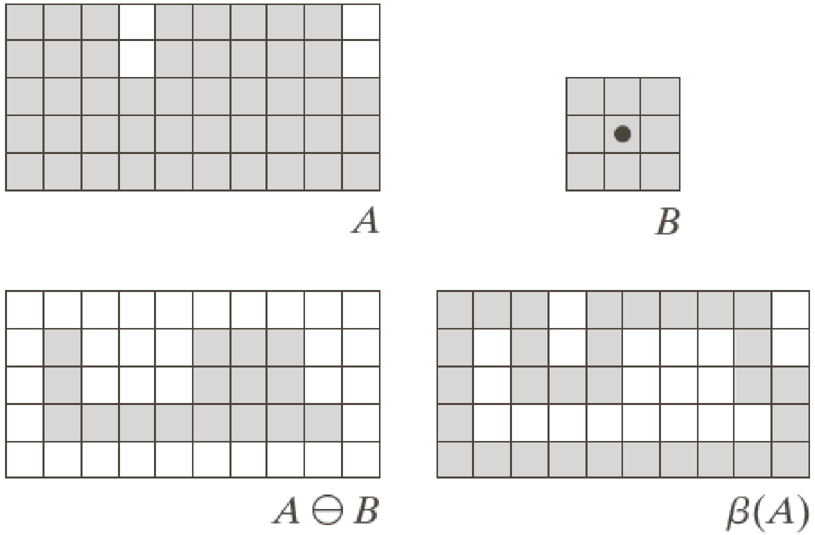
Source: Gonzalez+Woods

Basic Morphological Algorithm

- Boundary extraction
- Hole filling
- Extraction of connected components
- Convex hull
- Thinning
- Thickening
- Skeletons
- Pruning
- Morphological reconstruction

Boundary Extraction

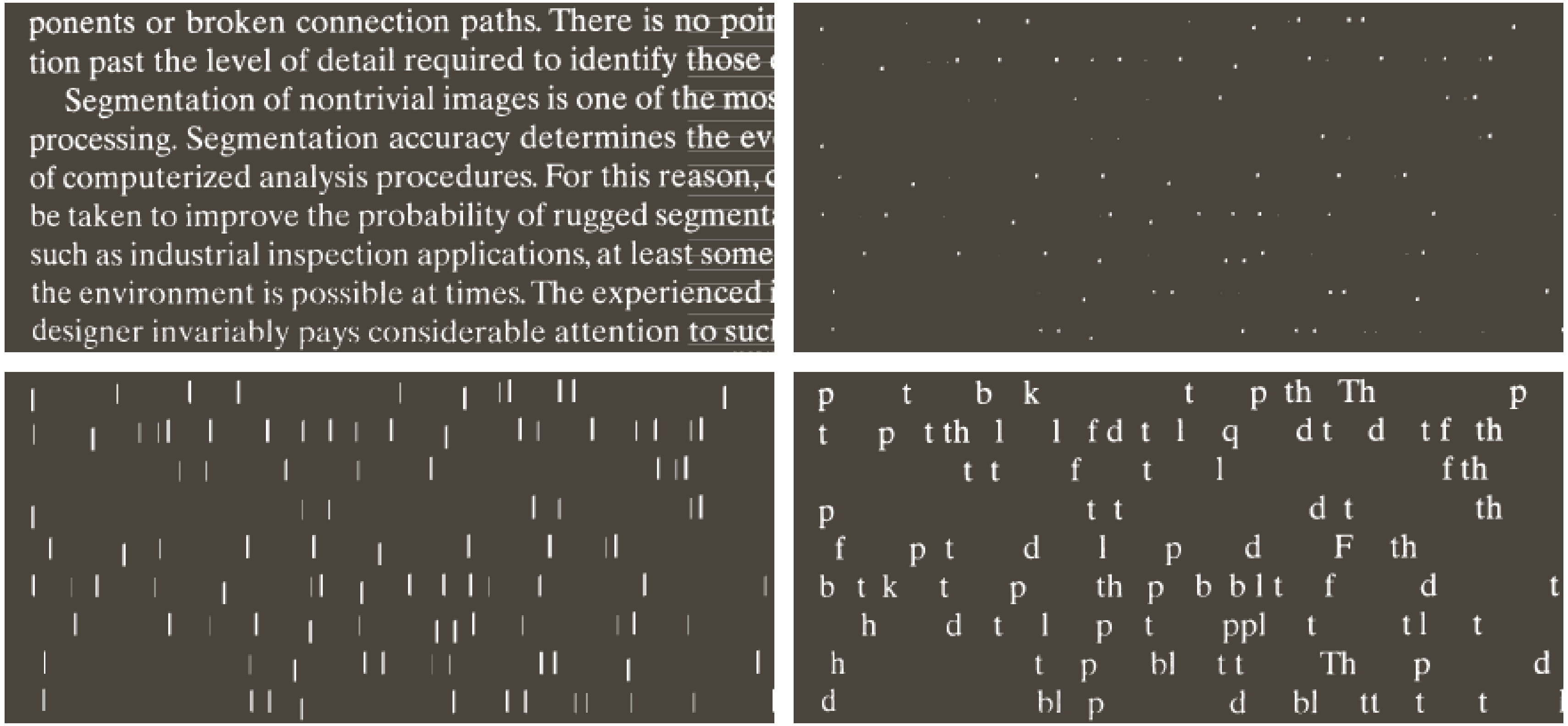
$$\beta(A) = A - (A \ominus B)$$



Source: Gonzalez+Woods

Opening by Reconstruction

$$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$$

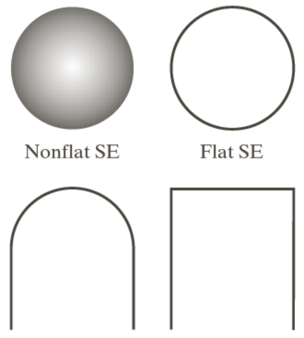


a	b
c	d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Source: Gonzalez+Woods

Gray-Scale Morphology

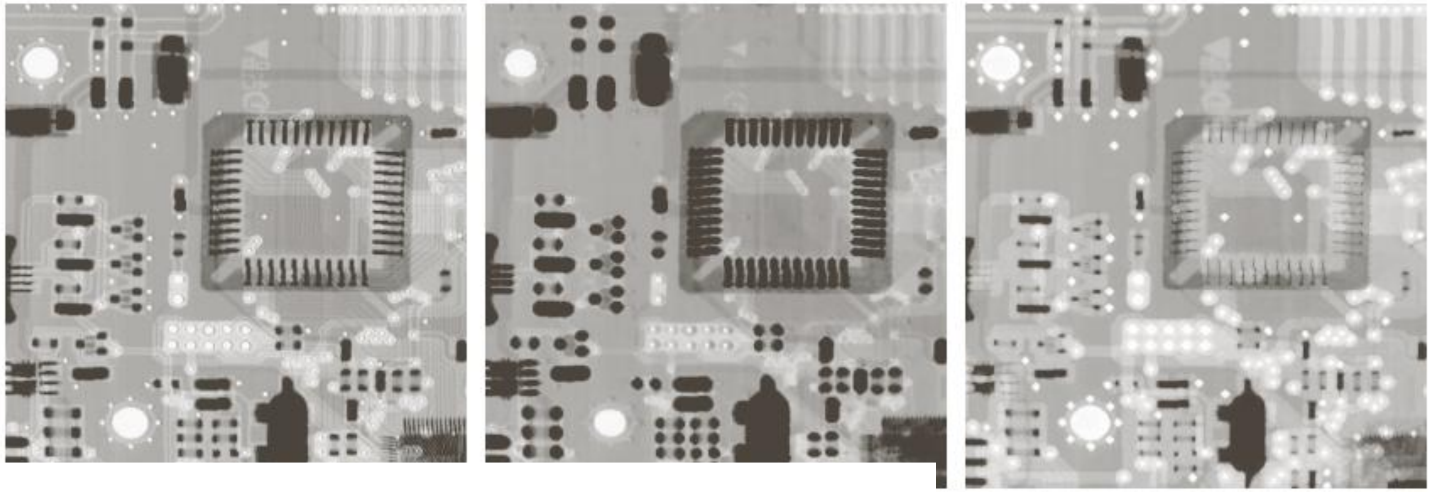


- Erosion

$$[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s,y+t)\}$$

- Dilation

$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s,y-t)\}$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Source: Gonzalez+Woods

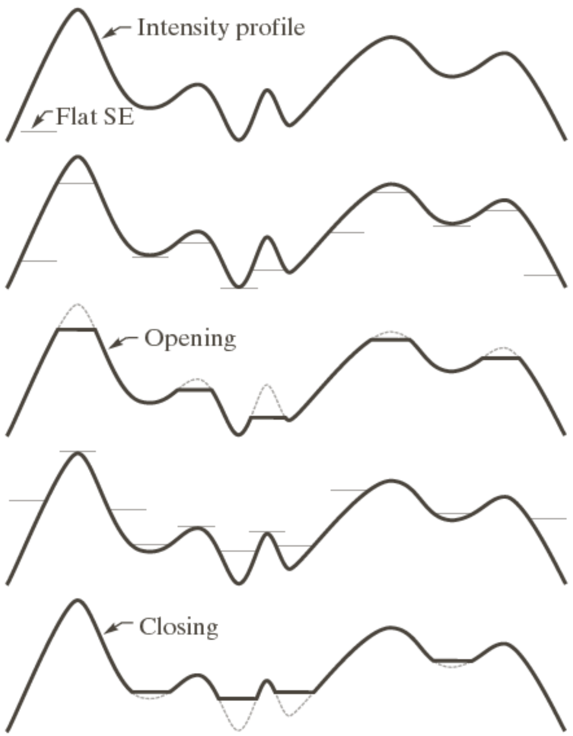
Gray-Scale Morphology

- Opening

$$f \circ b = (f \ominus b) \oplus b$$

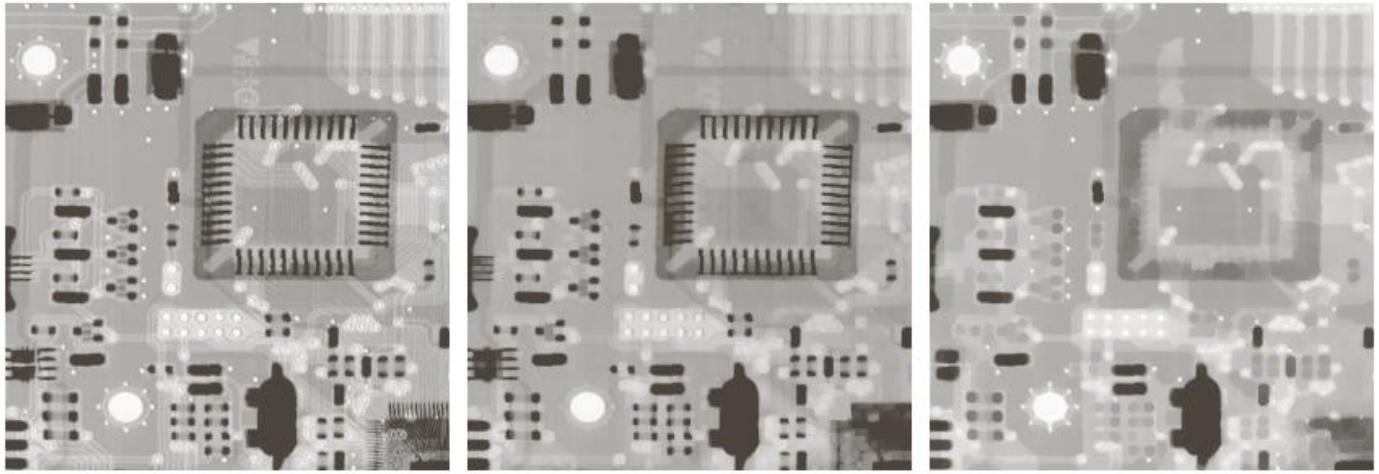
- Closing

$$f \bullet b = (f \oplus b) \ominus b$$



a
b
c
d
e

FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

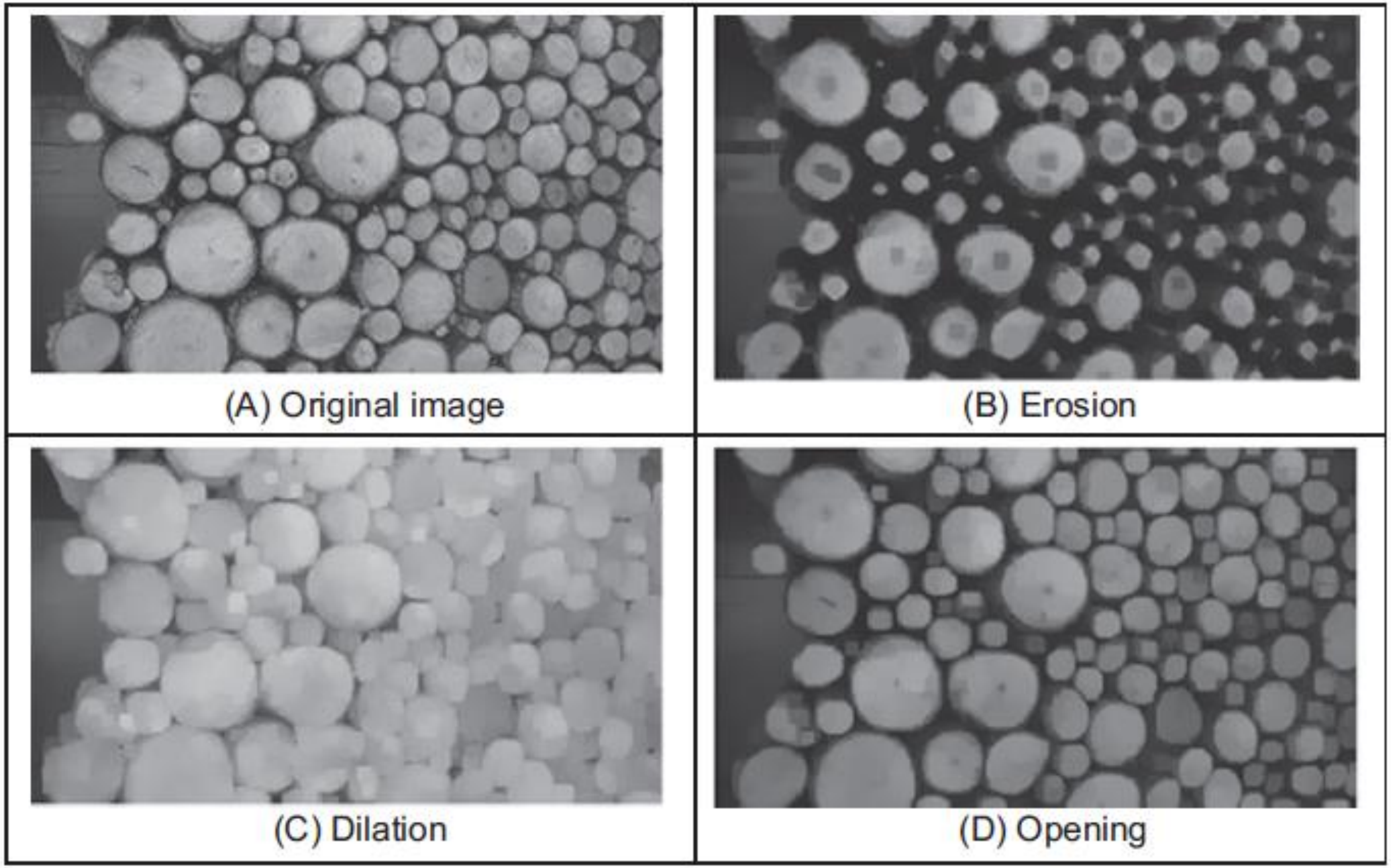


a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 $\times$ 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

Source: Gonzalez+Woods

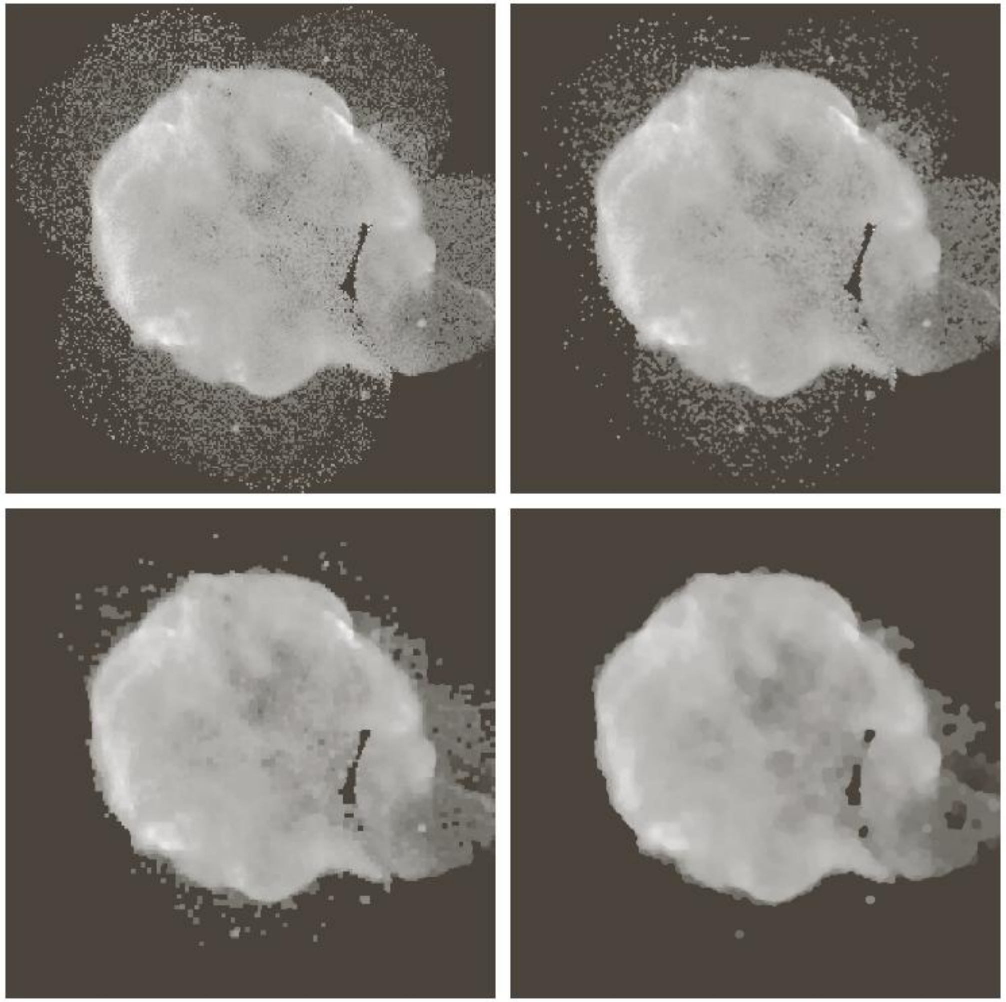
Examples of Gray-Scale Morphology Operators



Ref: Mark Nixon, Alberto Aguado, Feature Extraction and Image

Morphological Smoothing

Use opening and then closing



a b
c d

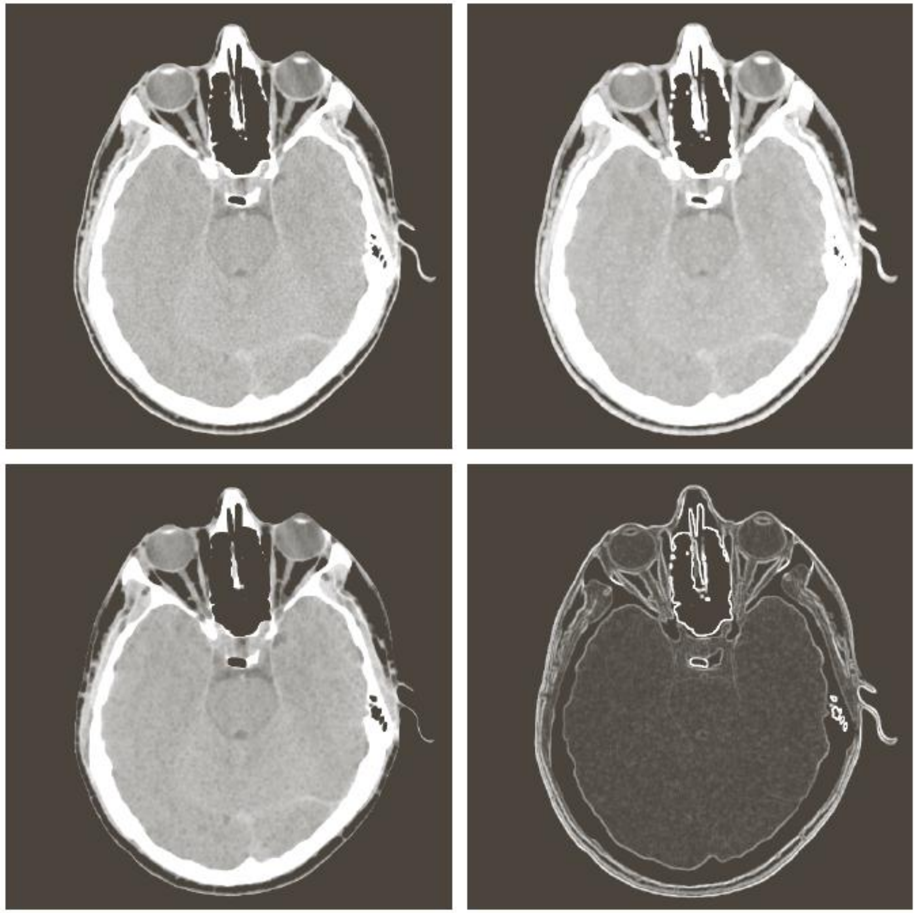
FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Source: Gonzalez+Woods

Morphological Gradient

"derivative-like" effect

$$g = (f \oplus b) - (f \ominus b)$$



a	b
c	d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Source: Gonzalez+Woods

Top-hat and Bottom-hat Transformation

$$T_{hat}(f) = f - (f \circ b)$$
$$B_{hat}(f) = (f \bullet b) - f$$

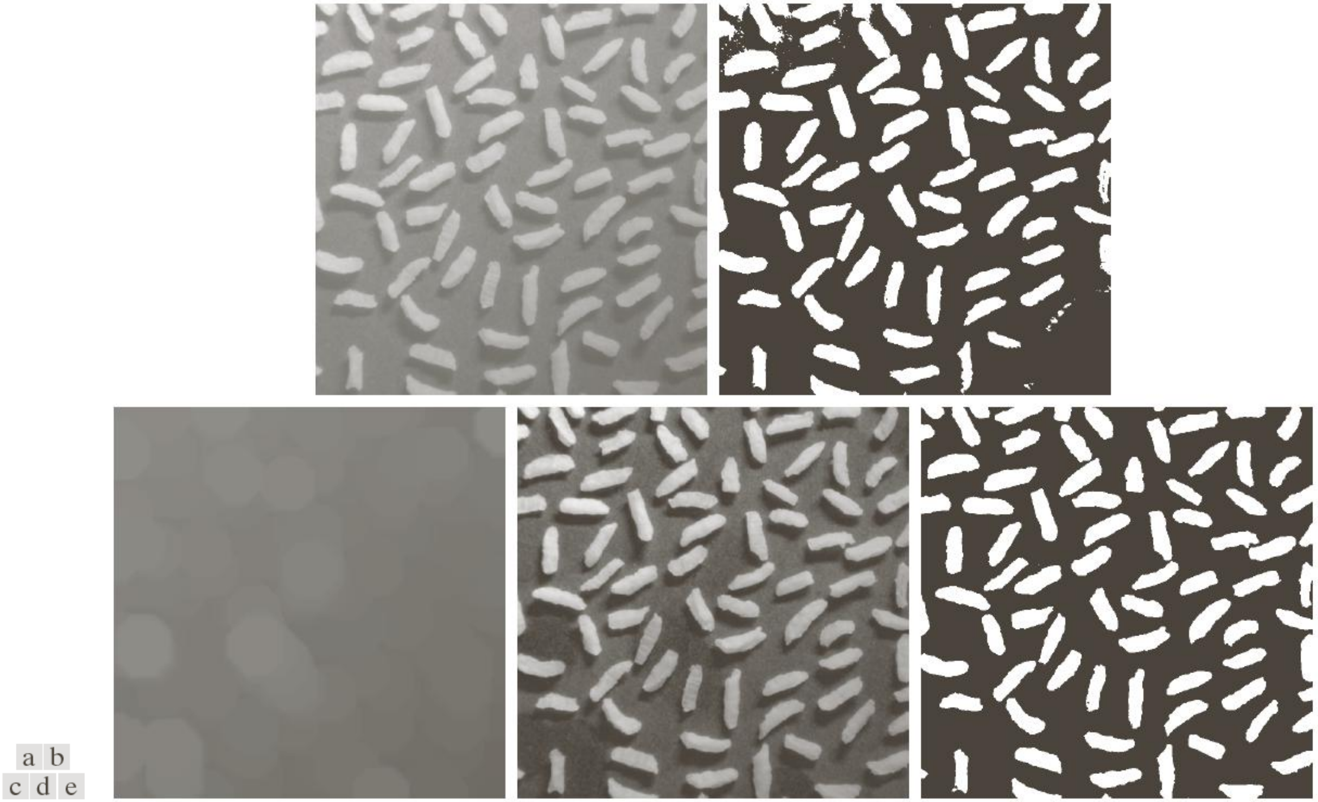
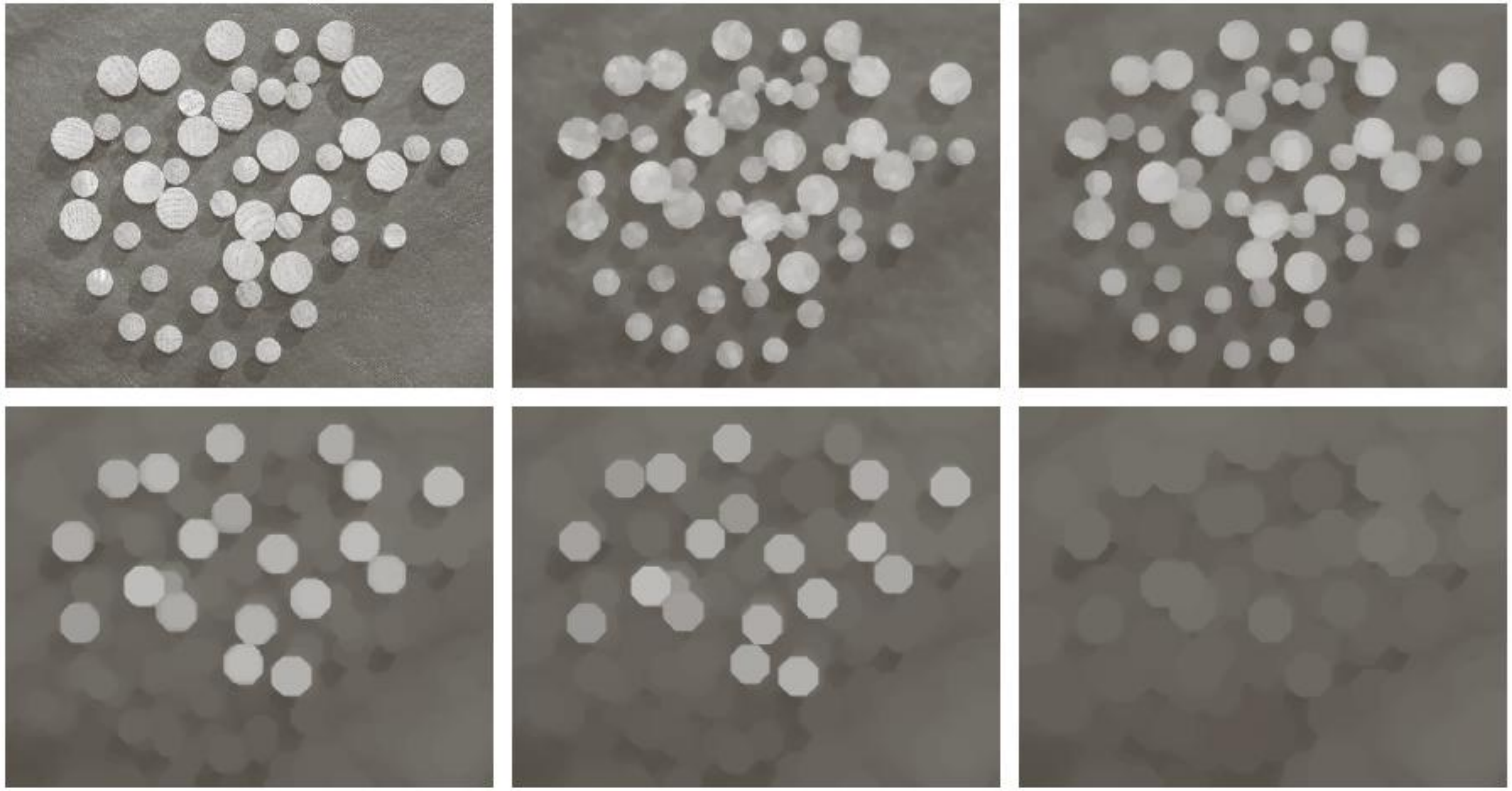


FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Source: Gonzalez+Woods

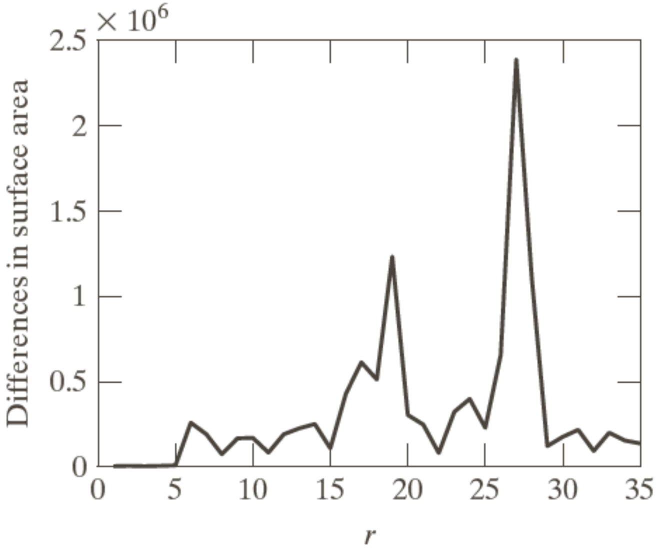
Granulometry

Determining the size distribution of particles in an image



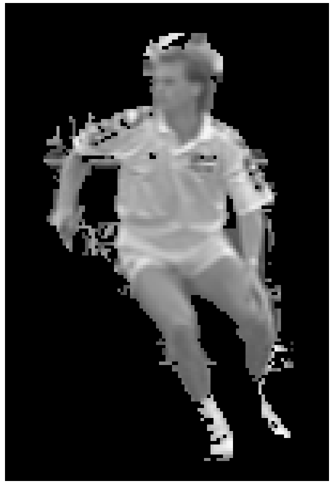
a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)



Source: Gonzalez+Woods

Example (using a 3x3 structure element)

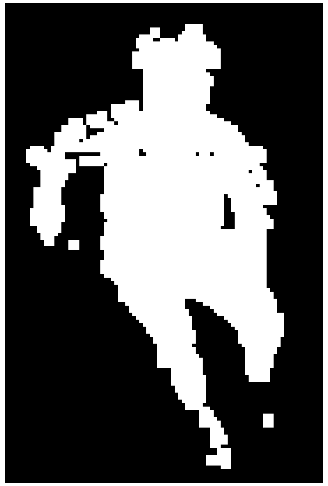


motion detection

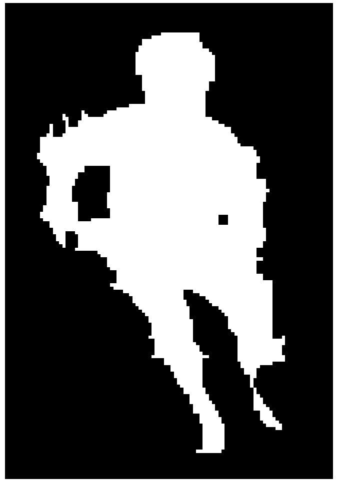
binary

erosion

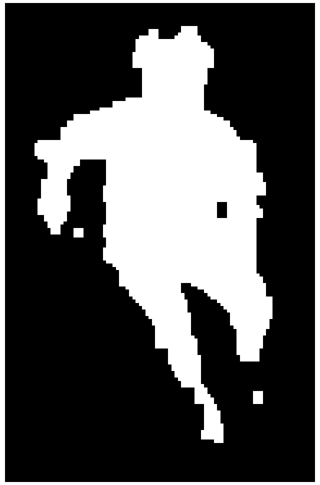
dilation



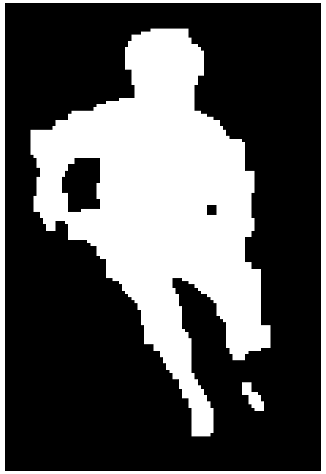
opening



closing



open-close



close-open

Summary of Morphological Operations and their Properties

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

Source: Gonzalez+Woods

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(*Continued*)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

Source: Gonzalez+Woods

Morphology in OpenCV

CreateStructuringElementEx(*cols, rows, anchorX, anchorY, shape, values=None*)→ kernel

- *cols* and *rows* is the number of columns and rows in the structuring element
- *anchor_x* and *anchor_y* point to the anchor pixel. The pixel that is checked for when the transformation should be made or not.
- *shape* lets you choose from three standard structuring elements. Or, if you want, you can set it to use a custom structuring element:
    - CV_SHAPE_RECT
    - CV_SHAPE_CROSS
    - CV_SHAPE_ELLIPSE
    - CV_SHAPE_CUSTOM

Dilate(*src, dst, element=None, iterations=1*) → None

Erode(*src, dst, element=None, iterations=1*) → None

- *src*: The image you want to dilate
- *dst*: This is where the dilated image is stored
- *element*: (optional) The structuring element (use CreateStructuringElementEx to create one). If not specified, a 3×3 square is used.
- *iterations*: (optional) Number of times you want to dilate *src*. If not specified, this is set to 1

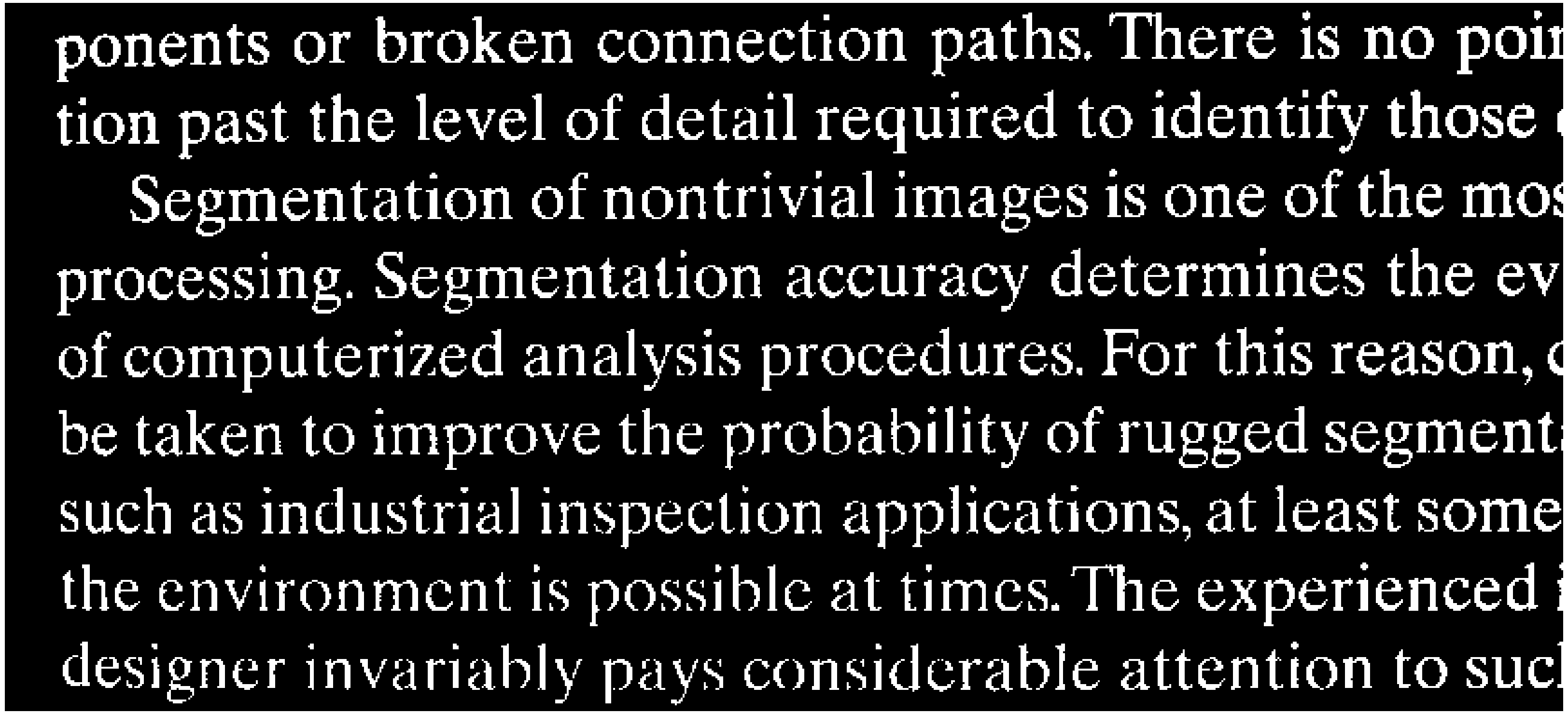
Morphology in OpenCV

MorphologyEx(*src*, *dst*, *temp*, *element*, *operation*, *iterations=1*) → None

- *src*: The image you want to work on
- *dst*: This is where the final result is stored
- *temp*: The temporary image
- *element*: (optional) The structuring element (use CreateStructuringElementEx to create one). If not specified, a 3×3 square is used.
- *operation*: This is the only parameter that differs from the previous functions. Its possible values are:
  - CV_MOP_OPEN
  - CV_MOP_CLOSE
  - CV_MOP_GRADIENT
  - CV_MOP_TOPHAT
  - CV_MOP_BLACKHAT
- *iterations*: (optional) Number of iterations. If not specified, this is set to 1.

Case Study #2 : Morphological Image Processing

- Identify ‘a’ in this picture


ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc