Context Free Languages 1 of 3

Context-Free Language (CFL)
Context-Free Grammar (CFG)

Ex 1: $S \to 0S1$, $S \to \varepsilon$ i.e. $S \to 0S1, \varepsilon$

$$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S111 \Rightarrow 000\varepsilon 111 = 000111$$

$$S \Rightarrow^* 000111$$

Definition: CFG

A CFG is a 4-tuple $G = (V, \Sigma, P, S)$ where
- ↳ V - set of variables (finite)
- ↳ Σ - alphabet (Set of terminals - Non-variable, non-ε that appear on RHS of production)
- ↳ P - set of productions (each has form $A \to \alpha$, where $A \in V$, $\alpha \in (V \cup \Sigma)^*$)
- ↳ S - start variable ($S \in V$)

Definition:

$\underbrace{\alpha \Rightarrow \beta}$ $(\alpha, \beta \in (V \cup \Sigma)^*)$

means that β can be derived (generated) by one application of production.

$\alpha \Rightarrow \beta$ means that β can be derived (generated) by 0 or more applications of production.

Definition:

Let $G = (V, \Sigma, P, S)$ be a CFG. The language of G (generated by G) is

$$\mathcal{L}(G) = \{ x \in \Sigma^* : S \Rightarrow^* x \}$$

Ex 2:

G: $S \to 0S1, \varepsilon$

$\mathcal{L}(G) = \{ 0^n 1^n : n \in \mathbb{N} \}$

Definition:

A language L is context-free iff $L = \mathcal{L}(G)$ for some CFG G.

Ex 3:

$$G \begin{cases} S \to \varepsilon,\ 0B,\ 1A \\ A \to 0S,\ 1AA \\ B \to 1S,\ 0BB \end{cases}$$

What's $\mathcal{L}(G)$?

$$\underbrace{\mathcal{L}(G) = \{ x \in \Sigma^* : \#_0(x) = \#_1(x) \}}_{L_e}$$

Left to right Method

Design:

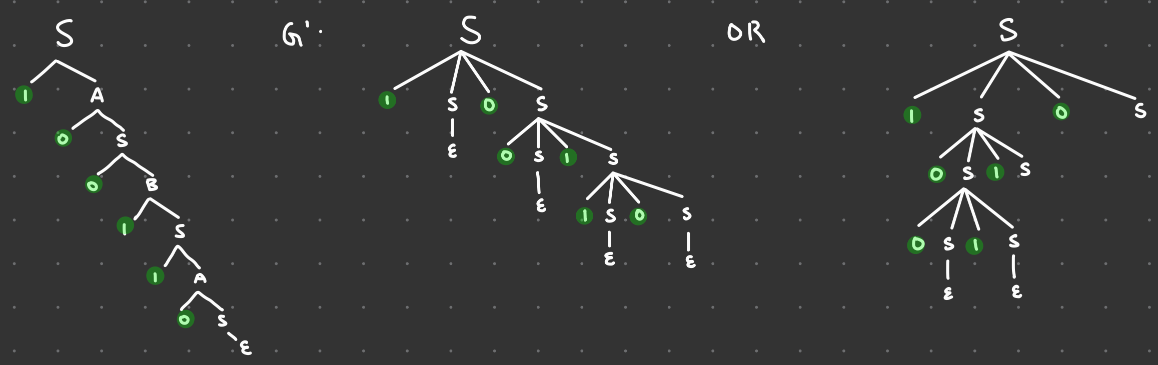
S generates L_e
B generates $\{ x \in \Sigma^*,\ \#_1(x) = \#_0(x) + 1 \}$
A generates $\{ x \in \Sigma^*,\ \#_0(x) = \#_1(x) + 1 \}$

Another CFG that generates L_e

$S \to \varepsilon,\ 0S1S,\ 1S0S$ — G'

ParseTree ($100110 \in L_e$)

G: G': OR



Definition:

A CFG G is ambiguous iff there's a string x s.t. G has more than one parse trees for generating x.

Definition:

A CFL is inherently ambiguous iff every CFG that generates it is ambiguous.