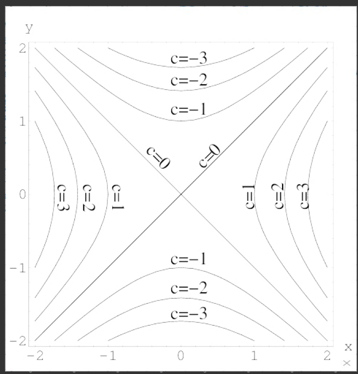
B41 Sept 24 Lec 2 Notes

Ex 1:



Find level set of $z = x^2 - y^2$.

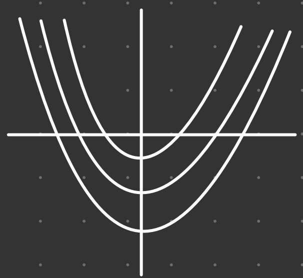
Set $x^2 - y^2 = k$

When $k=0$, $x^2 - y^2 = 0 \Rightarrow y = \pm x$

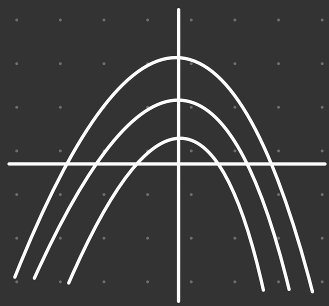
When $k<0$, $y^2 - x^2 = -k > 0 \Rightarrow y = \pm\sqrt{x^2 - k}$

When $k>0$, $x^2 - y^2 = k > 0 \Rightarrow x = \pm\sqrt{y^2 + k}$

Now set $y=c$, $z = x^2 - c^2$.



Set $x=c$, $z = c^2 - y^2$



Ex 2:

Find level set of $f(x,y) = 2 + \sin(x-y)$

Note that $-1 \le \sin(x-y) \le 1$. Then $1 \le f(x,y) \le 3$

Let $f(x,y) = k$, $1 \le k \le 3$

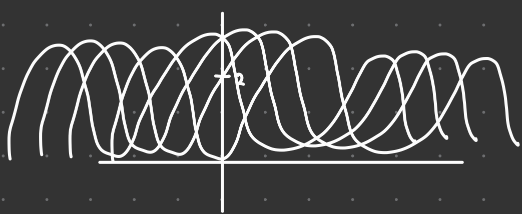
$2 + \sin(x-y) = k \Rightarrow \sin(x-y) = k-2$

When $k=1$, $\sin(x-y) = -1 \Rightarrow x - y = 2n\pi - \frac{\pi}{2}$, $n \in \mathbb{Z}$

When $k=2$, $\sin(x-y) = 0 \Rightarrow x - y = n\pi$

When $k=3$, $\sin(x-y) = 3 \Rightarrow x - y = 2n\pi + \frac{\pi}{2}$

Set $y=c$, $z = 2 + \sin(x-c)$

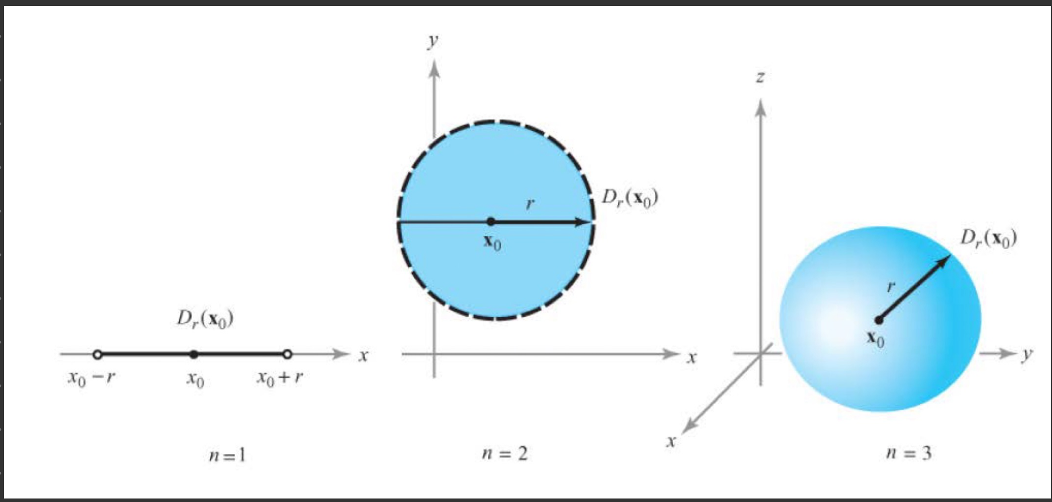


Set $x=c$, $z = 2 + \sin(c-y)$

(Similar to above)

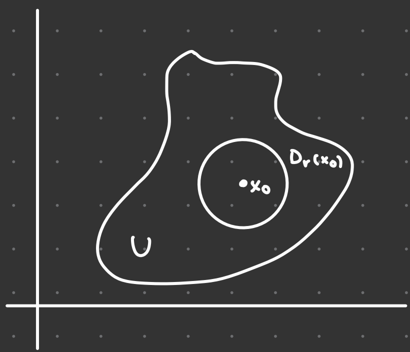
Limits

Let x_0 be a point in $\mathbb{R}^n$. Open disk or open ball $D_r(x_0) = \{ x \in \mathbb{R}^n \mid \|x - x_0\| < r \} \subset \mathbb{R}^n$.



Definition:

Let $U \subset \mathbb{R}^n$. For each point x_0 in U, there exists some $r > 0$ s.t. $D_r(x_0) \subset U$. Then U is called an open set of $\mathbb{R}^n$.



An open set U is one that completely encloses some disk or ball $D_r(x_0)$ about each of its point x_0.