# Webwork 11

1. Find det(M). 

$$M = \begin{array}{cccc} + & - & + & - \end{array}$$
$$\begin{bmatrix} -3 & 0 & 0 & -1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 2 & -1 & 0 \end{bmatrix}$$

$$\det M = -3 \begin{vmatrix} 0 & -2 & 0 \\ -1 & 0 & 2 \\ 2 & -1 & 0 \end{vmatrix} - (-1) \begin{vmatrix} -1 & 0 & -2 \\ 0 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix}$$

$$= -3 \left[ -(-2) \begin{vmatrix} -1 & 2 \\ 2 & 0 \end{vmatrix} \right] + (-1) \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix}$$

$$= -6(-4) - 1 = 23$$

2. If $A = \begin{bmatrix} 4+2i & -1-4i \\ -2-3i & 2+4i \end{bmatrix}$, find $|A|$.

$$\det A = (2+4i)(4+2i) - (-1-4i)(-2-3i)$$

$$= 8 + 16i + 4i + 8i^2 + (1+4i)(-2-3i)$$

$$= \quad '' \quad + (-2 - 3i - 8i - 12i^2)$$

$$= (8 - 2 - 8 + 12) + (16 + 4 - 3 - 8)i$$

$$= 10 + 9i$$

3. Find all values of $a$ that makes $\det A = \begin{vmatrix} a & 6 & 2 \\ a & -5 & 9 \\ 5 & 1 & a \end{vmatrix} = 0$

$$a \begin{vmatrix} -5 & 9 \\ 1 & a \end{vmatrix} - a \begin{vmatrix} 6 & 2 \\ 1 & a \end{vmatrix} + 5 \begin{vmatrix} 6 & 2 \\ -5 & 9 \end{vmatrix} = 0$$

$$a(-5a - 9) - a(6a - 2) + 5(54 + 10) = 0$$

$$-5a^2 - 9a - 6a^2 + 2a + 320 = 0$$
$$-11a^2 - 7a + 320 = 0$$

4. $\begin{vmatrix} 15 & 23 & 11 \\ -43 & -31 & -17 \\ -2 & -1 & 5 \end{vmatrix} = a \cdot \begin{vmatrix} -15 & -23 & -11 \\ 43 & 31 & 17 \\ 2 & 1 & -5 \end{vmatrix}$

$$= (-1)(-1)(-1) \begin{vmatrix} 15 & 23 & 11 \\ -43 & -31 & -17 \\ -2 & -1 & 5 \end{vmatrix}$$

$$= a$$

6a. Determine if set of vectors are L.I

$$\begin{bmatrix} -5 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 21 \\ 15 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 2 & 21 \\ -4 & 1 & 15 \\ 2 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} -5 & 2 & 21 \\ 0 & 5 & 15 \\ 2 & 2 & 0 \end{bmatrix}$$

$$\det A = -5 \begin{vmatrix} 5 & 15 \\ 2 & 0 \end{vmatrix} + 2 \begin{vmatrix} 2 & 21 \\ 5 & 15 \end{vmatrix}$$

$$= -5(-30) + 2(30 - 5 \cdot 21)$$

$$= 0$$

7. Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -2$. Find the following

$$\det \begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} = -2(-1)(-1)$$

8. If $\begin{vmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{vmatrix} = -2$ and $\begin{vmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{vmatrix} = 5$

$$\det \begin{vmatrix} a & 8 & d \\ b & 8 & e \\ c & 8 & f \end{vmatrix} = 8 \det \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ d & e & f \end{vmatrix} = 8(-2)$$

$$\det \begin{vmatrix} a & -1 & d \\ b & 0 & e \\ c & 1 & f \end{vmatrix} =$$

9. Find det A

$$A \to \underbrace{\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1} A \to \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2} E_1 A \to \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}}_{E_3} E_2 E_1 A \to \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{E_4} E_3 E_2 E_1 A$$

$$\underbrace{\begin{bmatrix} 7 & 4 & -9 \\ 0 & -4 & 6 \\ 0 & 0 & 6 \end{bmatrix}}_{B}$$

$$\det B = \det E_4 E_3 E_2 E_1 A$$

$$\det B = \det E_4 \det E_3 \det E_2 \det E_1 \det A \Rightarrow \det A = \frac{8}{3}$$

10. Consider the matrix

$$A = \begin{bmatrix} -2-x & -616 & -159 \\ 0 & 6-x & 8 \\ 0 & -18 & -18-x \end{bmatrix}$$

and let $B$ be a matrix similar to $A$, i.e, $B$ is of the form $S^{-1}AS$ for some non singular matrix $S$. Find all possible values of $x$ so that $\det B = 0$.

Since $S^{-1}$ and $S$ are invertible, $\det S$ and $\det S^{-1} \neq 0$. Thus $\det A = 0 \Rightarrow \det B = 0$.

$$\det A = (-2-x) \begin{vmatrix} 6-x & 8 \\ -18 & -18-x \end{vmatrix} = 0$$

$$= (-2-x) \left[ (6-x)(-18-x) + 18 \cdot 8 \right] = 0$$

$$= (-2-x) \left[ -108 + 18x - 6x + x^2 + 144 \right] = 0$$

$$= (-2-x) \left[ x^2 + 12x + 36 \right] = 0$$

$$= (x+2)(x+6)^2 = 0$$

$$x = -2, -6$$