B52 Nov 19 Lec 2 Notes

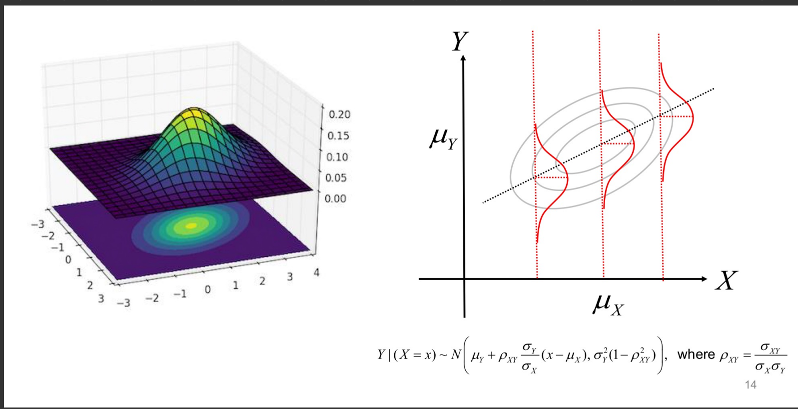
Conditional distributions

$$\begin{bmatrix}X\\Y\end{bmatrix} \sim N\left(\mu = \begin{bmatrix}\mu_X\\\mu_Y\end{bmatrix}, \Sigma = \begin{bmatrix}\sigma_X^2 & \sigma_{XY}\\ \sigma_{XY} & \sigma_Y^2\end{bmatrix}\right)$$

$$\Rightarrow Y|(X=x) \sim N\left(\mu_Y + \frac{\sigma_{XY}}{\sigma_X^2}(x-\mu_X),\ \sigma_Y^2 - \frac{\sigma_{XY}^2}{\sigma_X^2}\right)$$

Conditional mean of Y is linear function of given value of $X=x$.

Conditional variance of Y is constand & $\le$ unconditional variance.



Ex 1: Linear regression

Assume X = height^2 (m^2) & Y = weight (kg) follows bivariate Normal with $\mu_X = 3.1$, $\sigma_X = 0.15$, $\mu_Y = 70$, $\sigma_Y = 5$, $\rho_{XY} = 0.60$

(i) What's best guess of person's weight, knowing nothing else?
(ii) What's best guess of person's weight, if they are 1.6 m tall?
(iii) What's SD of previous guesses?

(i) 70 kg (marginal expected value of Y)
(ii) $X = 1.6^2 = 2.56$

Conditional mean of $Y = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x-\mu_X)$
$= 70 + .6\frac{5}{.15}(2.56 - 3.1)$
$= 57\ 2$ kg

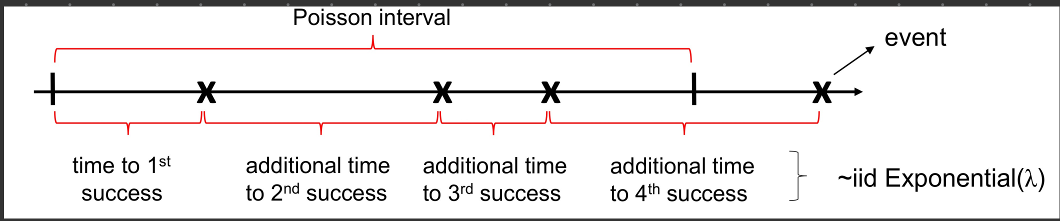
(iii) For marginal of Y, SD: 5
For conditional of Y, SD: $\sqrt{\sigma_Y^2(1-\rho^2)} = \sqrt{5^2(1-0.6^2)} = 4$

Poisson Distribution

Model # of events over continuum, e.g. # goals in Hockey game.
↳ No fixed # of trials
↳ Thus we model space between events

Poisson RV counts # of successes in some continuous interval, e.g. time or space (length, area, volume)

If area to successes follows (independent) Exp(λ), then a Poisson RV counts # of successes fitting in unit interval.



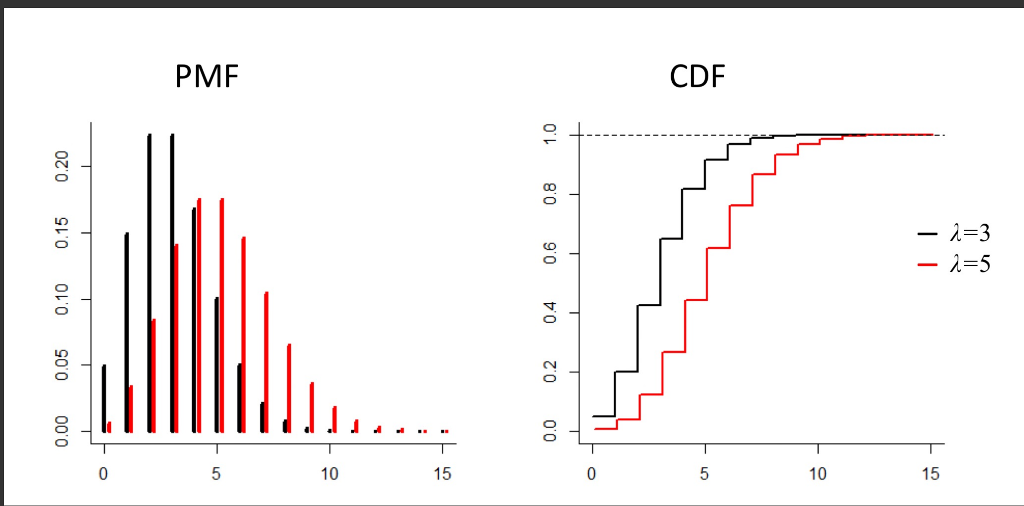
PMF: $P(X=x) = \frac{\lambda^x}{x!} e^{-\lambda}$, $x = 0, 1, 2, \dots$

Parameter $\lambda > 0$ represents average # of successes over interval.

Denoted $X \sim \text{Poisson}(\lambda)$

Verify this is a valid PMF:

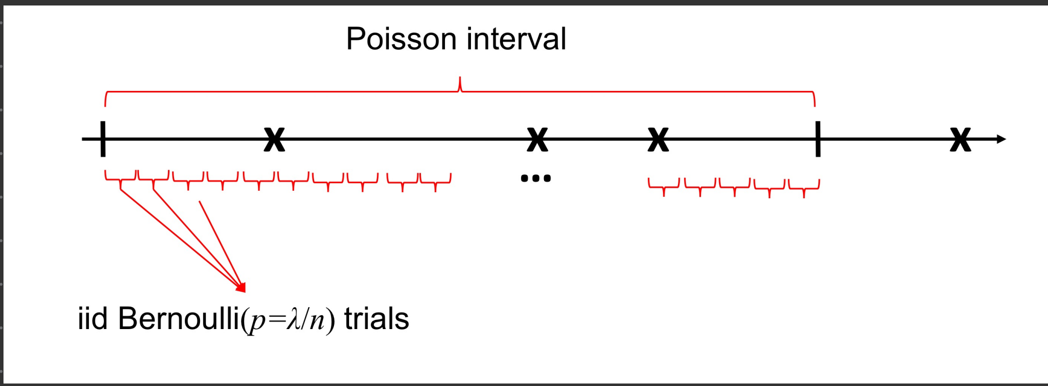
$$\sum_{x=0}^{\infty} P(X=x) = \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} e^{-\lambda} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda} e^{\lambda} = e^0 = 1$$



Poisson Approximation to Binomial

Poisson(λ) is approximated by Binomial$(n, \lambda/n)$ as $n \to \infty$

Think of dividing Poisson interval to #n pieces, each with $p = \lambda/n$ probability of containing a single success.



Binomial, Poisson & Normal

- ↳ Binomial $(n, \lambda/n) \to$ Poisson(λ), $n \to \infty$
- ↳ Binomial $(n, p) \to$ Normal (np, npq), $n \to \infty$
- ↳ Poisson $(\lambda) \to$ Normal (λ, λ), $\lambda \to \infty$