W9 Tutorial

1. Define the following predicates:

$R(x)$: "x is a right angle triangle"
$O(x)$: "x has an obtuse angle"

Now consider the following statements:

$S = \neg\exists x\,(R(x) \wedge O(x))$
$T = \forall x\,(R(x) \rightarrow \neg O(x))$

1a) Write S in ordinary English.

There isn't a triangle that is a right angle triangle and has an obtuse angle.

1b) Write T in ordinary English.

Every right angle triangle does not have an obtuse angle.

1c) Prove that $S \Leftrightarrow T$

$$\begin{aligned} \neg\exists x\,(R(x) \wedge O(x)) &\Leftrightarrow \forall x\,\neg(R(x) \wedge O(x)) && \text{Negate Quantifier} \\ &\Leftrightarrow \forall x\,\neg R(x) \vee \neg O(x) && \text{De Morgans} \\ &\Leftrightarrow \forall x\; R(x) \rightarrow \neg O(x) && \text{Conditional} \end{aligned}$$

2. For each set of sentences, define the domain X, the value of $a \in X$ (for part b), and the predicates $A(x)$ and $B(x)$ such that the last sentence is false and the other sentences are true.

Requirement : $|X| \leq 2$ (this means the size of the domain X must be less than or equal to 2.

2a)
(T) $\forall x \in X, A(x) \rightarrow B(x)$
(F) $\exists x \in X, A(x) \wedge B(x)$

X = set of all types of meat
$A(x)$ = x is cooked meat
$B(x)$ = x is safe to eat

$X = \{1\}, A(1) = F, B(1) = T$

2b) (T) $\forall x \in X, A(x) \rightarrow B(x)$

(T) $\neg A(a)$

(F) $\neg B(a)$

X = set of all food

$A(x)$ = x is tasty

$B(x)$ = consumers will want to eat more of x

a = Ginseng (food that is not tasty, but people want more of it)

$X = \{a\}$, $A(a) = F$, $B(a) = T$