B52 Sept 22 Lec 1 Notes

For events A, B the probability of A given that B has occurred is called conditional probability and given by

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \text{ , provided } P(B) > 0$$

Conditioning on event B restricts the sample space to B. Thus we divide by P(B).

Conditional Probabilities behave just like regular probabilities. They follow the probabity axioms.

Regular probabilities can be thought of as being conditional on S.

Ex 2:

(i) Find $P(A|B)$ when $A \subseteq B$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

(ii) Find $P(A|B)$ when A, B are disjoint

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(\emptyset)}{P(B)} = 0$$

Independence

Two events A, B are called independent with respect to probabity measure P when

$$P(A\cap B) = P(A)\cdot P(B)$$

For $P(A), P(B) \neq 0$, equivalent to:

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

Independence implies that info on occurrence of one event does not affect probability of other.

Independence is property of probability functions, not just events.

Ex 3:

If A, B are independent (with respect to P), show that A, B^c are also independent.

We know that: $P(A\cap B) = P(A)\cdot P(B)$

From law of total probability: $P(A) = P(A\cap B) + P(A\cap B^c)$

$\Rightarrow P(A\cap B^c) = P(A) - P(A)P(B) = P(A)(1-P(B))$

$= P(A)P(B^c) \Rightarrow A, B^c$ are independent.

Baye's Rule

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Ex 4:

Let SR be the event that the side is red.

$$\begin{aligned} P(SR) &= P(SR \cap RR) + P(SR \cap RB) + P(SR \cap BB) \\ &= P(SR|RR)\cdot P(RR) + P(SR|RB)\cdot P(RB) + P(SR|BB)\cdot P(BB) \\ &= 1 \quad \tfrac{1}{3} + \tfrac{1}{2} \cdot \tfrac{1}{3} + 0 \cdot \tfrac{1}{3} \\ &= \tfrac{3}{6} = \tfrac{1}{2} \end{aligned}$$

For partition $A_1, A_2, \dots$, Baye's rule is equivalently expressed as

$$P(A_j|B) = \frac{P(B|A_j)\,P(A_j)}{\sum_{i=1}^{n} P(B|A_i)\,P(A_i)}$$

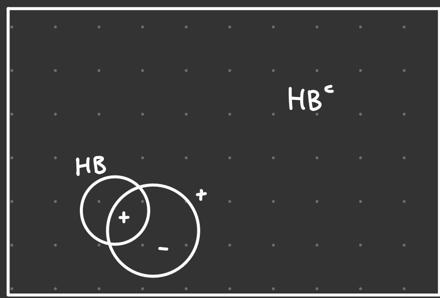
Ex 5:

(i) $P(\text{Having HB}) = 2\%$

(ii) $$P(HB|+) = \frac{P(+|HB)\,P(HB)}{P(+|HB)\cdot P(HB) + P(+|HB^c)\cdot P(HB^c)}$$

$$= \frac{.96 \cdot .02}{.96 \cdot .02 + .02 \cdot .98}$$

$$= 0.4948$$



(iii) $$P(HB|-) = \frac{P(-|HB)\,P(HB)}{P(-|HB)\cdot P(HB) + P(-|HB^c)\cdot P(HB^c)}$$

$$= \frac{.04 \cdot .02}{.04 \cdot .02 + .98 \cdot .98}$$

$$= .0008322$$

Ex 6:

$$P(RR|SR) = \frac{P(SR|RR)\cdot P(RR)}{P(SR)}$$

$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}}$$

$$= \frac{2}{3}$$