O S O
G T G T
5 E 5
N O N O
O S O
G T G T
5 E 5
N O N O
O S O
G T G T
5 E 5

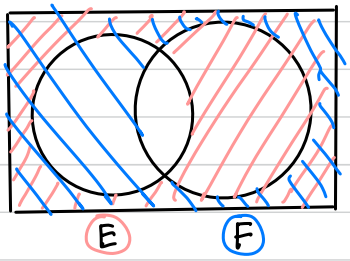
W6 Tutorial Exercises

1. Suppose that two events E and F are independent. Are $\bar{E}$ and $\bar{F}$ independent?

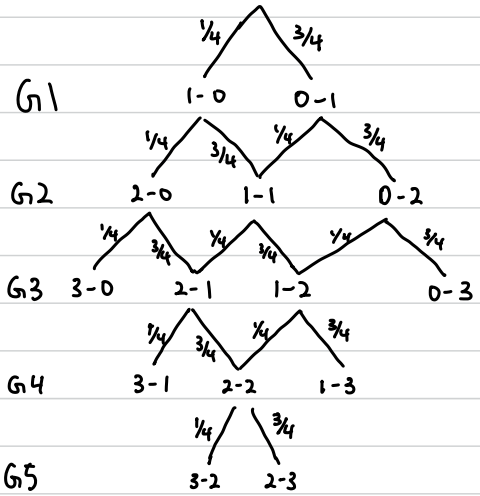
We need to show that $P(\bar{E} \cap \bar{F}) = P(\bar{E}) \cdot P(\bar{F})$

$$\begin{aligned} P(\bar{E}) \cdot P(\bar{F}) &= (1 - P(E)) \cdot (1 - P(F)) \\ &= 1 - P(E) - P(F) + P(E) \cdot P(F) \\ &= 1 - (P(E) + P(F) - P(E \cap F)) \\ &= 1 - P(E \cup F) \\ &= P(\bar{E} \cap \bar{F}) \end{aligned}$$

$\bar{E} \cap \bar{F} = \overline{E \cup F}$



2. If two teams A and B play a best of five series where team A has a $\frac{1}{4}$ probability of winning any given game, what is the expected number of games in the series?



X = # of games played in the series.

$$P(3) = \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^0$$
$$P(4) = C(3,1) \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^1 + C(3,1) \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^1$$
$$P(5) = C(4,2) \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 + C(4,2) \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$$

$$E(x) = 3 \cdot P(3) + 4 \cdot P(4) + 5P(5)$$
$$= 3.77$$

3. Two marbles are chosen from a bag containing 3 red, 5 white, 8 green marbles.

a) What is the probability that both are red?

$$\frac{3}{16} \cdot \frac{2}{15} = \frac{1}{40}$$

b) What is the probability that one is white and one is green?

W_1 = 1st one is white G_1 = 1st one is green
W_2 = 2nd one is white G_2 = 2nd one is green

$$\begin{aligned} P(W_1 \cap G_2) + P(G_1 \cap W_2) &= P(W_1) \cdot P(G_2 | W_1) + P(G_1) \, P(W_2 | G_1) \\ &= \frac{5}{16} \cdot \frac{8}{15} + \frac{8}{16} \cdot \frac{5}{15} \\ &= \frac{1}{3} \end{aligned}$$

4. When athlete has used steroids. the test result is correct with probability 0.995 and incorrect with probability 0.005.

When athlete has not used steroids. the test result is correct with probability 0.98 and incorrect with probability 0.02

An athlete at a certain event is randomly chosen for drug testing. Let C be the event that the steroid test is positive and let S be the event that the athlete has used steroids. Assuming that 3% of all athletes at that event are using illegal steroids (ie., $P(S) = 0.03$), what is $P(C \cap S)$?

$$P(C \cap S) = P(C) \cdot P(S|C) = P(S) \cdot P(C|S)$$
$$= 0.03 \cdot 0.995$$
$$= 0.02985$$

	Test correct	Test incorrect
Athlete uses	0.995	0.005
Athlete Doesn't	0.98	0.02