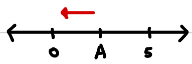
Ex 1

Does $\int_0^5 \frac{\ln x}{x}\,dx$ converge or diverge? type II

$$\int_0^5 \frac{\ln x}{x}\,dx = \lim_{A\to 0^+} \int_A^5 \frac{\ln x}{x}\,dx$$

if A is lower limit, we want right hand limit and vice versa.



$$= \lim_{A\to 0^+} \frac{1}{2}(\ln x)^2 \Big|_A^5$$

$$= \lim_{A\to 0^+} \left[\frac{1}{2}(\ln 5)^2 - \frac{1}{2}(\ln A)^2 \right]$$

$$= \lim_{A\to 0^+} \frac{1}{2}(\ln 5)^2 - \lim_{A\to 0^+} \frac{1}{2}(\ln A)^2$$

$$= \frac{1}{2}(\ln 5)^2 - \frac{1}{2}(-\infty)^2$$

$= -\infty$ $\therefore$ limit DNE

$\therefore \int_0^5 \frac{\ln x}{x}\,dx$ diverges

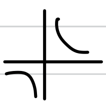
Ex 2

Does $\int_{-1}^1 x^{-2}\,dx$ converge or diverge? $x=0\in[-1,1]$ type II

$$\int_{-1}^1 x^{-2}\,dx = \int_{-1}^0 x^{-2}\,dx + \int_0^1 x^{-2}\,dx$$

$$= \lim_{A\to 0^-} \int_{-1}^A x^{-2}\,dx + \lim_{A\to 0^+} \int_0^1 x^{-2}\,dx$$

$$= \lim_{A\to 0^-} -x^{-1}\Big|_{-1}^A + \lim_{A\to 0^+} -x^{-1}\Big|_A^1$$



$$= \lim_{A\to 0^-} \left(-A^{-1} + (-1)^{-1}\right) + \lim_{A\to 0^+} \left(-1^{-1} + A^{-1}\right)$$

$$= -(-\infty) - 1 - 1 + \infty$$

$$= \infty$$

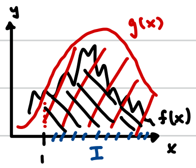
Theorem 5.23: Comparison Test for Improper Integrals

Let f, g, h be cont. functions on interval I. Consider the improper integral $\int_I f(x)\,dx$.

Case 1: Convergent

If $0 \le f(x) \le g(x)$ $\forall x \in I$, and $\int_I g(x)\,dx$ conv., then $\int_I f(x)$ also conv.

eg.



$\int_I g(x)\,dx$ converges. $\Rightarrow$ g(x) has finite area

Case 2: Divergent

If $0 \le h(x) \le f(x)$ $\forall x \in I$, and $\int_I h(x)\,dx$ div., then $\int_I f(x)\,dx$ also div.

Ex 3

Does $\int_1^\infty \frac{\cos^2(x+1)}{3+2x^2}\,dx$ converge or diverge?

$f(x) = \frac{\cos^2(x+1)}{3+2x^2}$ is positive on $[-1, \infty)$