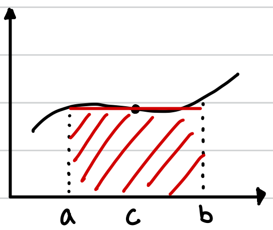
MVT for Integrals

Let $a, b \in \mathbb{R}$, $a < b$

If f is continuous on $[a,b]$, then $\exists c \in [a,b]$ s.t.

$$\int_a^b f(x)\,dx = f(c)(b-a)$$



Theorem 4.33: FTOC - Part II

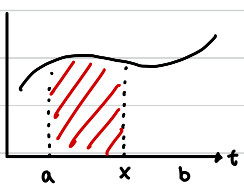
Let $a, b \in \mathbb{R}$, $a<b$. If f is continuous on $[a,b]$ and we define $F(x) = \int_a^x f(t)\,dt$ for all $x \in [a,b]$, then

(a) F is continuous on $[a,b]$
(b) F is diff. on (a,b)
(c) F is an antiderivative of f, $\boxed{F'(x) = f(x) \quad \forall x \in [a,b].}$

$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$, $\forall x \in [a,b]$

we evaluate deriv at the endpoints with one sided derivs.

$F(x) = \int_a^x f(t)\,dt$ is an example of an area accumulation function of f on $[a,b]$.



Ex 1

Find $\frac{d}{dx}\left(\int_7^x \frac{1}{1+t^4}\,dt\right)$

$f(t) = \frac{1}{1+t^4}$ is a rational function so $f(t)$ is continuous on $\text{Dom}(f) = \mathbb{R}$.

In particular, f is continuous on $[7, x] \subset \mathbb{R}$

Thus $\frac{d}{dx}\left(\int_7^x \frac{1}{1+t^4}\,dt\right) = f(x) = \frac{1}{1+x^4}$

Ex 2

Let $H(x) = \int_{\pi}^{x} e^{t^2+1}\, dt$

Find $H'(x)$

$f(t) = e^{t^2+1}$

e^t is an exp. function so e^t is continuous on $\mathrm{dom}(e^t) = \mathbb{R}$

t^2+1 is a polynomial so t^2+1 is continuous on $\mathrm{dom}(t^2+1) = \mathbb{R}$

$\therefore$ Composition of continuous everywhere functions are continuous everywhere.

In particular, f is continuous $[x, \pi] \subset \mathbb{R}$

$H'(x) = e^{x^2+1}$