B41 Sept 20 Lec 1 Notes

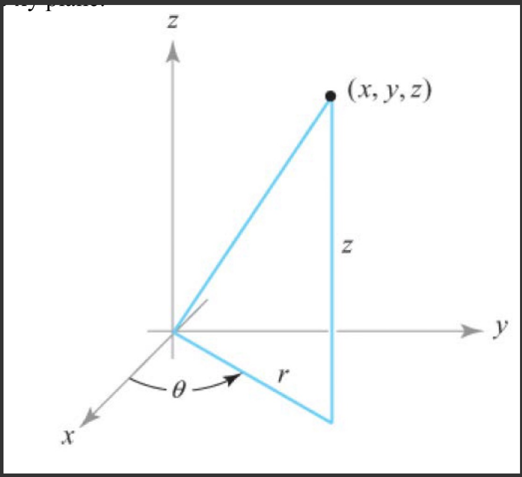
Cylindrical Coordinates

In cylindrical coordinates, a point $P(x,y,z)$ has coordinates (r,θ,z).

(i) r is the distance between P and Z axis.

(ii) θ is the usual polar angle measured counterclockwise from the positive x-axis.

(iii) As in cartesian coordinates, the z coordinate is the signed vector distance between P and the XY plane.



$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$0 \le r < \infty, \quad 0 \le \theta \le 2\pi$

$-\infty < z < \infty$

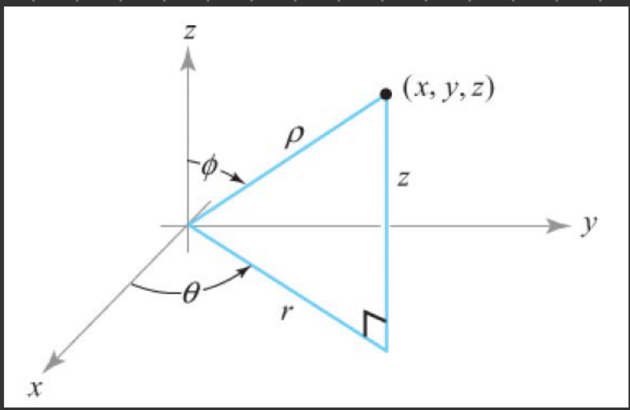
Spherical Coordinates

In spherical coordinates, a point $P(x,y,z)$ has coordinates (ρ,θ,ϕ)

(i) ρ is the distance from the origin to P.

(ii) θ is the same angle as in cylindrical coordinates

(iii) ϕ is the angle between the positive z axis and the line OP.



$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$0 \le \rho < \infty$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi$

$\sqrt{x^2+y^2+z^2} = \rho$

$\theta = \arctan\frac{y}{x}$

$\phi = \pm\arctan\frac{\sqrt{x^2+y^2}}{z}$

Graphs of functions

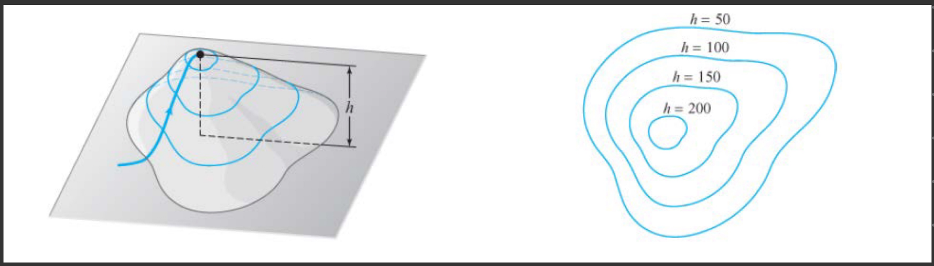
Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function $z = f(x,y)$ with $(x,y) \in U \subset \mathbb{R}^2$.

$$\text{Graph of } f = \{(x, y, f(x,y)) \mid (x,y) \in U\}$$

Definition: Let $f: U \to \mathbb{R}$ with $U \subset \mathbb{R}^n$.

$$\text{Graph of } f = \{(x_1, x_2, \cdots, x_n, f(x_1, x_2, \cdots, x_n)) \mid (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n\}$$

Level Sets, curves, and surfaces



Definition:

Let $f: U \subset \mathbb{R}^n \to \mathbb{R}$ and let $k \subset \mathbb{R}$. The level set of f at value k is defined to be the set of those point $x \in U$ at which $f(x) = k$.

$$\text{The level set of } f = \{(x, k) \mid x \in U\}$$

If $n = 2$, it is said to be level curve
If $n = 3$, it is said to be level surface.

The level curves of a function f of two variables are the curves $f(x,y) = k$, where $k \in \text{range}(f)$.