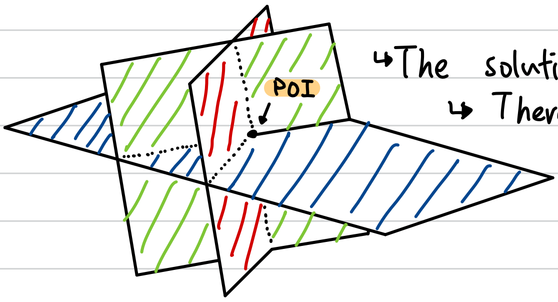
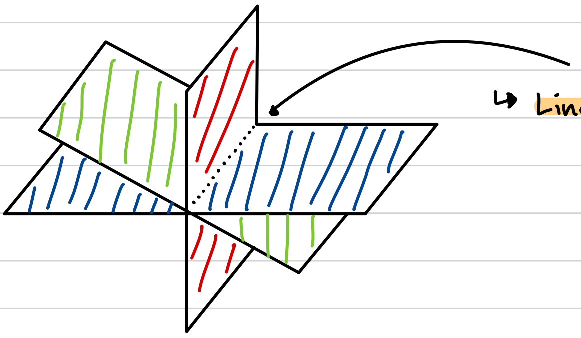
Sec 1.1 Introduction to Linear Systems

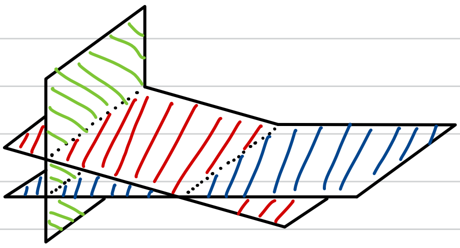
Geometric interpretation of linear systems:



↳ The solution is the point of intersection (POI).
↳ There is a unique solution.



↳ Line of intersection has infinitely many solutions.



↳ No intersections = No solutions

Exercises 1.1

3. Find solutions of $\left|\begin{array}{l} 2x+4y=3 \\ 3x+6y=2 \end{array}\right|$

$$\left|\begin{array}{l} 2x+4y=3 \\ 3x+6y=2 \end{array}\right| \div 2 \rightarrow \left|\begin{array}{l} x+2y=1.5 \\ 3x+6y=2 \end{array}\right| -3(I) \rightarrow \left|\begin{array}{r} x+2y=1.5 \\ 0=-2.5 \end{array}\right|$$

No solutions

9. Similar to 3. $\left|\begin{array}{l} x+2y+3z=1 \\ 3x+2y+z=1 \\ 7x+2y-3z=1 \end{array}\right| \begin{array}{l} \\ -(I) \\ -(I) \end{array} \rightarrow \left|\begin{array}{l} x+2y+3z=1 \\ 2x-2z=0 \\ 6x-6z=0 \end{array}\right| \rightarrow \left|\begin{array}{c} x+2y+3z=1 \\ x=z \\ z=x \end{array}\right|$

Let $z=t$:

$$\begin{array}{l} t+2y+3t=1 \\ x=t \\ z=t \end{array} \rightarrow \begin{array}{l} y=\frac{1-4t}{2} \\ x=t \\ z=t \end{array} \rightarrow \text{Solution} = \langle x,y,z\rangle = \langle t, \tfrac{1-4t}{2}, t\rangle$$
$$= \langle 0, \tfrac{1}{2}, 0\rangle + t\langle 1,-2,1\rangle$$

18. Find all solutions of
$$\left|\begin{array}{l} x+2y+3z=a \\ x+3y+8z=b \\ x+2y+2z=c \end{array}\right| \begin{array}{l} \\ -(\mathrm{I}) \\ -(\mathrm{I}) \end{array} \rightarrow \left|\begin{array}{l} x+2y+3z=a \\ y+5z=b-a \\ -z=c-a \end{array}\right| \rightarrow \left|\begin{array}{l} x+2y+3z=a \\ y+5z=b-a \\ z=a-c \end{array}\right| \begin{array}{l} -3(\mathrm{III}) \\ -5(\mathrm{III}) \\ \end{array}$$

$$\rightarrow \left|\begin{array}{l} x+2y=-2a+3c \\ y=b-6a+5c \\ z=a-c \end{array}\right| \begin{array}{l} -2(\mathrm{II}) \\ \\ \end{array} \rightarrow \left|\begin{array}{l} x=-2b+10a-7c \\ y=b-6a+5c \\ z=a-c \end{array}\right|$$

Solution $= \langle x,y,z\rangle = \langle -2b+10a-7c,\ -b-6a+5c,\ a-c\rangle$

19. Consider the linear system
$$\left|\begin{array}{l} x+y-z=-2 \\ 3x-5y+13z=18 \\ x-2y+5z=K \end{array}\right|$$

where K is an arbitrary number.

a. For which value(s) of K does this system have one or infinite solutions?

$$\left|\begin{array}{l} \\ -3(\mathrm{I}) \\ -(\mathrm{I}) \end{array}\right| \rightarrow \left|\begin{array}{l} x+y-z=-2 \\ -8y+16z=24 \\ -3y+6z=K+2 \end{array}\right| \begin{array}{l} \\ \div(-8) \\ \end{array} \rightarrow \left|\begin{array}{l} x+y-z=-2 \\ y-2z=-3 \\ -3y+6z=K+2 \end{array}\right| \begin{array}{l} -(\mathrm{II}) \\ \\ +3(\mathrm{II}) \end{array} \rightarrow$$

$$\rightarrow \left|\begin{array}{l} x+z=1 \\ y-2z=-3 \\ 0=K-7 \end{array}\right| \Rightarrow z=1-x \Rightarrow y-2(1-x)=-3 \Rightarrow y-2+2x=-3$$
$$\Rightarrow y+2x=-1,\quad K=7$$

when $K=7$

b. For each value of K in part (a), how many solutions does the system have?

$K=7 \Rightarrow$ Infinite Solutions.

c. Find all solutions for each value of K.

$\langle x,y,z\rangle = \langle 1-t,\ -3+2t,\ t\rangle$

21. The sums of any two of three real numbers are 24, 28, and 30. Find these three numbers.

$$\left|\begin{array}{l} x + y = 24 \\ y + z = 28 \\ x + z = 30 \end{array}\right| \begin{array}{l} \\ \\ -(\mathrm{II}) \end{array} \rightarrow \left|\begin{array}{l} x + y = 24 \\ y + z = 28 \\ x - y = 2 \end{array}\right| \begin{array}{l} -(\mathrm{III}) \\ \\ \\ \end{array} \rightarrow$$

$$\rightarrow \left|\begin{array}{l} 2y = 22 \\ y + z = 28 \\ x - y = 2 \end{array}\right| \begin{array}{l} \div 2 \\ \\ \\ \end{array} \rightarrow \left|\begin{array}{l} y = 11 \\ y + z = 28 \\ x - y = 2 \end{array}\right| \begin{array}{l} \\ -(\mathrm{I}) \\ +(\mathrm{I}) \end{array} \rightarrow$$

$$\rightarrow \left|\begin{array}{l} y = 11 \\ z = 17 \\ x = 13 \end{array}\right|$$

Solution : $\langle x, y, z \rangle = \langle 13, 11, 17 \rangle$