Tut 2

Warm-Up I:

1. Rank of a matrix - The rank of a matrix A is the number of leading 1's in rref(A), denoted rank(A).

2. Leading variable - The variables whose column in rref contain leading 1's are called leading variables.

3. Free variable - A variable whose column in the rref does not contain a leading 1 is called a free variable.

4. Pivot Entry - If a matrix is in ref, then the first nonzero entry of each row is called a pivot.

5. Pivot Column - The columns in which pivots appear in.

A2:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 1 & 3 & 2 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & -0.583 \\ 0 & 0 & 1 & 1.25 \end{bmatrix}$$

A3:

$$A = \begin{bmatrix} 0 & a & 0 & 0 & b \\ c & 0 & d & 0 & e \\ 0 & 0 & 0 & 1 & f \end{bmatrix}$$

$c = 0$, $a = 1$, $d = 1$, $b, e, f \in \mathbb{R}$

A4:

$$A = \begin{bmatrix} 1 & a & b & 2 & 0 & c \\ 0 & 0 & d & 1 & e & 3 \\ 0 & f & 0 & 0 & g & h \end{bmatrix}$$

A is in rref.

$f = 0$ $\quad d = 1 \Rightarrow b = 0$ $\quad c \in \mathbb{R}$

$g = 1 \Rightarrow e = 0$ $\quad a \in \mathbb{R}$

$h \in \mathbb{R}$

B1. $x_1 + 3x_2 - x_3 + 2x_4 = 5$
$2x_1 + 6x_2 - x_3 - x_4 = 6$

(a)(b) $\left[\begin{array}{cccc|c} 1 & 3 & -1 & 2 & 5 \\ 2 & 6 & -1 & -1 & 6 \end{array}\right] \underset{R_2 \leftrightarrow R_2 - 2R_1}{\sim}$

$\left[\begin{array}{cccc|c} 1 & 3 & -1 & 2 & 5 \\ 0 & 0 & 1 & -5 & -4 \end{array}\right] \underset{R_1 \leftrightarrow R_1 + R_2}{\sim}$

$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -3 & 1 \\ 0 & 0 & 1 & -5 & -4 \end{array}\right]$

(c) $1x_1 + 3x_2 - 3x_4 = 1$
$x_3 - 5x_4 = -4$

(d) $x_1 = 1 - 3x_2 + 3x_4$
$x_3 = -4 + 5x_4$

(e) x_1, x_3 = dependent
x_2, x_4 = free

Let $x_2, x_4 = t, s$

$x_1 = 1 - 3t + 3s$
$x_3 = -4 + 5s$

C1 Let A be an $n \times m$ matrix. Rank $(A) = K$.

(a)

	$K<m$	$K=m$
$K<n$	∞ or 0	0 or 1
$K=n$	∞	1

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$x_1 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

(bi) If the system is inconsistent, then rank $(A) < n$

(bii) If rank$(A) < m$, then the system has no solution or infinitely many.

(biii) A linear system with $n = m$ has a unique solution iff rank $(A) = n$.

Cool - off

(a) Point, line, plane

(b) where $a = md = ng \quad , m, n \in \mathbb{R}$

$b = me = nh \quad , m, n \in \mathbb{R}$

$c = mf = nk \quad , m, n \in \mathbb{R}$

$p = mq = mr \quad , \quad m, n, \in \mathbb{R}$