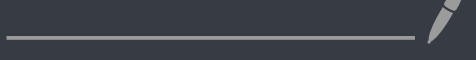

Math Hygeine

Sets

The number of elements in a set S is called the **cardinality** of the set.

There are two ways to specify a set:

(i) **Enumeration**: List the contents of a set. i.e. $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

(ii) **Comprehension**: Describe contents of the set by a property. If $P(a)$ is a property of the object a, then the set of all objects a such $P(a)$ is true is denoted by $\{a \mid P(a)\}$.

Set Operations

(i) Union

(ii) Intersection

(iii) Given sets X and Y, the difference of X and Y, denoted $X \setminus Y$ or $X - Y$ is the set

$$\{x \in X \mid x \notin Y\}$$

(iv) Given a set Y inside some larger set X, the complement of Y with respect to X, denoted Y^c is $X \setminus Y$.

Example:

1. For each rational number q, let $q\mathbb{Z} = \{qm \mid m \in \mathbb{Z}\}$, so that we have $q\mathbb{Z} \in \mathbb{Q}$

(a) Use enumeration to describe the sets $\frac{1}{2}\mathbb{Z}$, $\frac{1}{3}\mathbb{Z}$, $\frac{1}{2}\mathbb{Z} \cap \frac{1}{3}\mathbb{Z}$, $\frac{1}{2}\mathbb{Z} \cup \frac{1}{3}\mathbb{Z}$, $\frac{1}{2}\mathbb{Z} \setminus \frac{1}{3}\mathbb{Z}$, and $(3\mathbb{Z})^c$.

$$\tfrac{1}{2}\mathbb{Z} = \{\dots, \tfrac{-2}{2}, \tfrac{-1}{2}, 0, \tfrac{1}{2}, \tfrac{2}{2}, \dots\}$$
$$\tfrac{1}{3}\mathbb{Z} = \{\dots, \tfrac{-2}{3}, \tfrac{-1}{3}, 0, \tfrac{1}{3}, \tfrac{2}{3}, \dots\}$$
$$\tfrac{1}{2}\mathbb{Z} \cap \tfrac{1}{3}\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$
$$\tfrac{1}{2}\mathbb{Z} \cup \tfrac{1}{3}\mathbb{Z} = \{\dots, -1, -\tfrac{1}{2}, -\tfrac{1}{3}, 0, \tfrac{1}{3}, \tfrac{1}{2}, 1, \dots\}$$
$$\tfrac{1}{2}\mathbb{Z} \setminus \tfrac{1}{3}\mathbb{Z} = \{\dots, -\tfrac{5}{2}, -\tfrac{3}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \dots\}$$
$$(3\mathbb{Z})^c = \{\dots, -5, -4, -2, -1, 1, 2, 4, 5, \dots\}$$

(b) What is the smallest natural number n s.t. every set from (a) is contained in $\frac{1}{n}\mathbb{Z}$?

$n = 6$.