B52 Sept 10 Lec 2 Notes

Theorem: Law of Total Probability

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \ldots$$

$$\Rightarrow P(A_1) + P(A_2) + \ldots = 1$$

Proposition: Complement rule

$$P(A^c) = 1 - P(A)$$

Theorem: Inclusion-exclusion principle

For any events $A_1, \ldots, A_n$,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \ldots + (-1)^{n+1} P(A_1 \cap \cdots \cap A_n)$$

Sample spaces with distinct outcomes called **discrete**; can be

- **Finite**: finite # of elements
- **Countably infinite**: elements can be put in a 1-1 correspondence with natural numbers ($\mathbb{N} = \{1, 2, 3, \ldots\}$)

For discrete S, probability function P is uniquely determined by outcome probabilities $P(s_i)$, $\forall s_i \in S$

$$A = \{s_1, s_2, \ldots\} \Rightarrow P(A) = P(s_1) + P(s_2) + \ldots$$

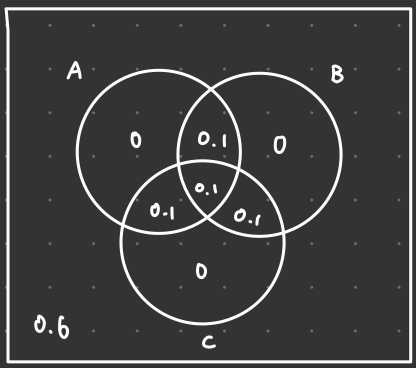
Finite spaces whose outcomes have equal probability are called (discrete) uniform probabity spaces

↳ $S = \{s_1, \ldots, s_n\}$, where $n = |S|$ is size of S
↳ $P(s_i) = 1/|S| > 0$, $\forall i = 1, \ldots, n$

For finite uniform probability spaces and any event A,

$$P(A) = \frac{|A|}{|S|} = \frac{\text{\# of outcomes in } A}{\text{\# of outcomes in } S}$$

Ex 1:



$P(A) = P(B) = P(C) = 0.3$

$P(A \cap B) = P(A \cap C) = P(B \cap C) = 0.2$

$P(A \cap B \cap C) = 0.1$

$0.4 = 0.9 - 0.6 + 0.1$

Probability of exactly two clubs = $0.1 + 0.1 + 0.1 = 0.3$