# B41 Oct 1 Lec 2 Notes

## Ex 1:

Find $\lim_{(x,y)\to(1,0)} \frac{y\ln y}{x}$.

$\lim_{(x,y)\to(1,0)} y\ln y = \lim_{(x,y)\to(1,0)} \frac{\ln y}{\frac{1}{y}}$ $\qquad \lim_{(x,y)\to(1,0)} x = 1$

$\overset{LH}{=} \lim_{(x,y)\to(1,0)} \frac{\frac{1}{y}}{-\frac{1}{y^2}}$

$= \lim_{(x,y)\to(1,0)} (-y) = 0$

So $\lim_{(x,y)\to(1,0)} \frac{y\ln y}{x} = \frac{\lim_{(x,y)\to(1,0)} y\ln y}{\lim_{(x,y)\to(1,0)} x}$

$= 0$

## Ex 2:

$\lim_{(x,y,z)\to(0,0,0)} z\sin(\frac{x}{y})$

Note $-1 \le \sin(\frac{x}{y}) \le 1$

If $z \ge 0$, $-z \le z\sin(\frac{x}{y}) \le z$

$0 = \lim_{(x,y,z)\to(0,0,0)} (-z) \le \lim_{(x,y,z)\to(0,0,0)} z\sin(\frac{x}{y}) \le \lim_{(x,y,z)\to(0,0,0)} (z) = 0$

Therefore, $\lim_{(x,y,z)\to(0,0,0)} z\sin(\frac{x}{y}) = 0$

## Definition:

Let $f: U \subset \mathbb{R}^n \to \mathbb{R}^m$ be a given function with domain $U$. Let $x_0 \in U$. We say $f$ is continuous at $x_0$ iff $\lim_{x\to x_0} f(x) = f(x_0)$

$\lim_{x\to x_0} f(x) = f(x_0)$ means

(i) $x_0 \in U$

(ii) $\lim_{x\to x_0} f(x) = L$

(iii) $f(x_0) = L$

If $f$ does not satisfy one of the three, we say $f$ is not continuous at $x_0$.