A37 Apr 1 Lec 2 Notes

Def (pg 641):

A series of the form

$$b_1 - b_2 + b_3 - b_4 + \dots = \sum_{n=1}^{\infty} (-1)^{n+1} b_n \quad , b_n > 0$$

OR

$$-b_1 + b_2 - b_3 + b_4 - \dots$$

is called an alternating series or Leibniz series

e.g. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$, $b_n = \frac{1}{\ln(n)}$ ✓

e.g. $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n!}$, $b_n = \frac{1}{n!}$ ✓

Theorem (pg 642): Alternating Series Test (AST)

Suppose $\sum (-1)^{n+1} b_n$, $b_n > 0$

If (i) $b_n \geq b_{n+1}$, $\forall n \in \mathbb{N}$

(ii) $\lim_{n \to \infty} b_n = 0$

Then $\sum (-1)^{n+1} b_n$ converges

AST cannot prove div.

Ex 1

$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$ conv or div?

Proof:

$b_n = \frac{1}{\ln(n)} > 0$

$\lim_{n \to \infty} b_n = 0$

WTS $\forall n \in \mathbb{N}$, $n \geq 2$ $b_n \geq b_{n+1}$

Let $n \in \mathbb{N}$ be arbitrary s.t. $n \geq 2$

$0 < \ln(n) < \ln(n+1) \Rightarrow \frac{1}{\ln(n)} > \frac{1}{\ln(n+1)} \Rightarrow b_n > b_{n+1}$

$\therefore \forall n \in \mathbb{N}$, $n \geq 2$, $b_n > b_{n+1}$

$\therefore$ By AST, $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$ converges.

Proof: AST

Suppose (i) $\sum (-1)^{n+1} b_n$, $b_n > 0$

(ii) $\forall n \in \mathbb{N}$, $b_n \geq b_{n+1}$

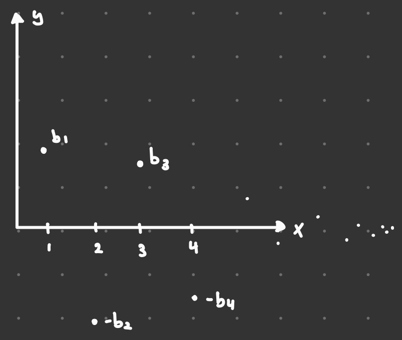
(iii) $\lim_{n\to\infty} b_n = 0$

WTS $\sum (-1)^{n+1} b_n$ conv

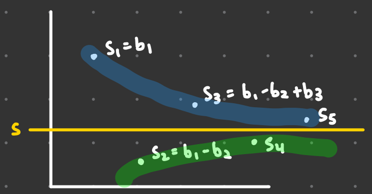
WTS $\lim_{n\to\infty} S_n$ exists

WTS $\{S_n\}$ conv by BMCT

e.g.



$\{S_n\} = \{ \underset{b_1}{S_1}, \underset{b_1-b_2}{S_2}, \underset{b_1-b_2+b_3}{S_3}, \dots \}$



(i) bounded below and decreasing $\{S_{2n}\}$ conv by BMCT

(ii) bounded above and increasing $\{S_{2n+1}\}$ conv by BMCT

(iii) $\lim_{n\to\infty} S_{2n} = \lim_{n\to\infty} S_{2n+1} = S$