Ex 1

$$\int \frac{4x^3+2x}{x^4+x^2\ln(x^4+x^2)}\,dx = \int \frac{1}{u}\,du$$

Let $u = \ln(x^4+x^2)$

$du = \dfrac{4x^3+2x}{x^4+x^2}\,dx$

$$= \ln(|\ln(x^4+x^2)|) + C$$

Ex 2

$$\int \sqrt{1+x^2}\,x^5\,dx = \int (1+x^2)^{1/2}\,x^4 x\,dx$$

$$= \int u^{\frac{1}{2}}(u-1)^2\,\tfrac{1}{2}\,du$$

Let $u = 1+x^2$

$du = 2x\,dx$

$\dfrac{du}{2} = x\,dx$

$$= \frac{1}{2}\int (u^2-2u+1)\,u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\int u^{5/2} - 2u^{3/2} + u^{1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{7}u^{7/2} - 2\left(\frac{2}{5}\right)u^{5/2} + \frac{2}{3}u^{3/2}\right) + c$$

$$= \frac{1}{7}u^{7/2} - \frac{2}{5}u^{5/2} + \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C$$

Ex 3

What possible u-subs can be applied to.

$$\int_0^9 \sqrt{4-\sqrt{x}}\,dx$$

$u = 4-\sqrt{x}$

$u = \sqrt{x}$

$u = \sqrt{4-\sqrt{x}} \Rightarrow 4-\sqrt{x} = u^2$

$\sqrt{x} = 4-u^2$

$(4-u^2)^2 = x$

$2(4-u^2)(-2u)\,du = dx$

$$= \int_0^9 u\,(2)(4-u^2)(-2u)\,du$$

$$=$$

Theorem 5.10: Integration by Parts for Definite Integrals

If $u = f(x)$ and $v = g(x)$ are diff. on $[a, b]$, then

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Theorem 5.7: Integration By Parts

If $u = f(x)$ and $v = g(x)$ are diff. on $[a, b]$, then

$$\int u\,dv = uv - \int v\,du$$

Proof: thm 5.10

Suppose $u = f(x)$ and $v = g(x)$ are diff. on $[a, b]$.

WTS $\int u\,dv = uv - \int v\,du$

Consider product rule:

$$(uv)' = uv' + vu'$$

$$\int (uv)' = \int uv' + vu'$$

$$uv = \int uv' + \int vu'$$

$$\int u\,dv = uv - \int v\,du \quad //$$

Ex 4

$$\int_1^2 x \ln x \,dx$$

$u = \ln x$ $\quad du = \frac{1}{x}\,dx$

$v = \frac{1}{2}x^2$ $\quad dv = x\,dx$

$$= \ln x \left(\tfrac{1}{2}x^2\right)\Big|_1^2 - \int_1^2 \tfrac{1}{2}x^2 \cdot \tfrac{1}{x}\,dx$$

$$= \ln x \left(\tfrac{1}{2}x^2\right) - \tfrac{1}{4}x^2 \Big|_1^2$$

$$= 2\ln 2 - \tfrac{3}{4}$$

Ex 5

$$\int t^2 e^t \, dt$$

$u_1 = t^2$ $\quad du_1 = 2t \, dt$

$v_1 = e^t$ $\quad dv_1 = e^t \, dt$

$$\begin{aligned}\int t^2 e^t \, dt &= t^2 e^t - \int e^t \, 2t \, dt \\ &= t^2 e^t - \left[2te^t - \int 2e^t \, dt \right] \\ &= t^2 e^t - \left[2te^t - 2e^t \right] + C \\ &= t^2 e^t - 2te^t + 2e^t\end{aligned}$$

$u_2 = 2t$ $\quad du_2 = 2 \, dt$

$v_2 = e^t$ $\quad dv_2 = e^t \, dt$

Ex 6

$$\int_0^1 \arctan x \, dx$$

$u = \arctan x$ $\quad du = \frac{1}{1+x^2}$

$V = x$ $\quad dv = 1$

$$\begin{aligned}\int_0^1 \arctan x \, dx &= x \arctan x - \int_0^1 \frac{x}{1+x^2} \, dx \\ &= x \arctan x - \frac{1}{2} \ln(1+x^2) \Big|_0^1 \\ &= \frac{\pi}{4} - \frac{1}{2} \ln 2 - (0 - \ln 1) \\ &= \frac{\pi}{4} - \frac{\ln 2}{2}\end{aligned}$$