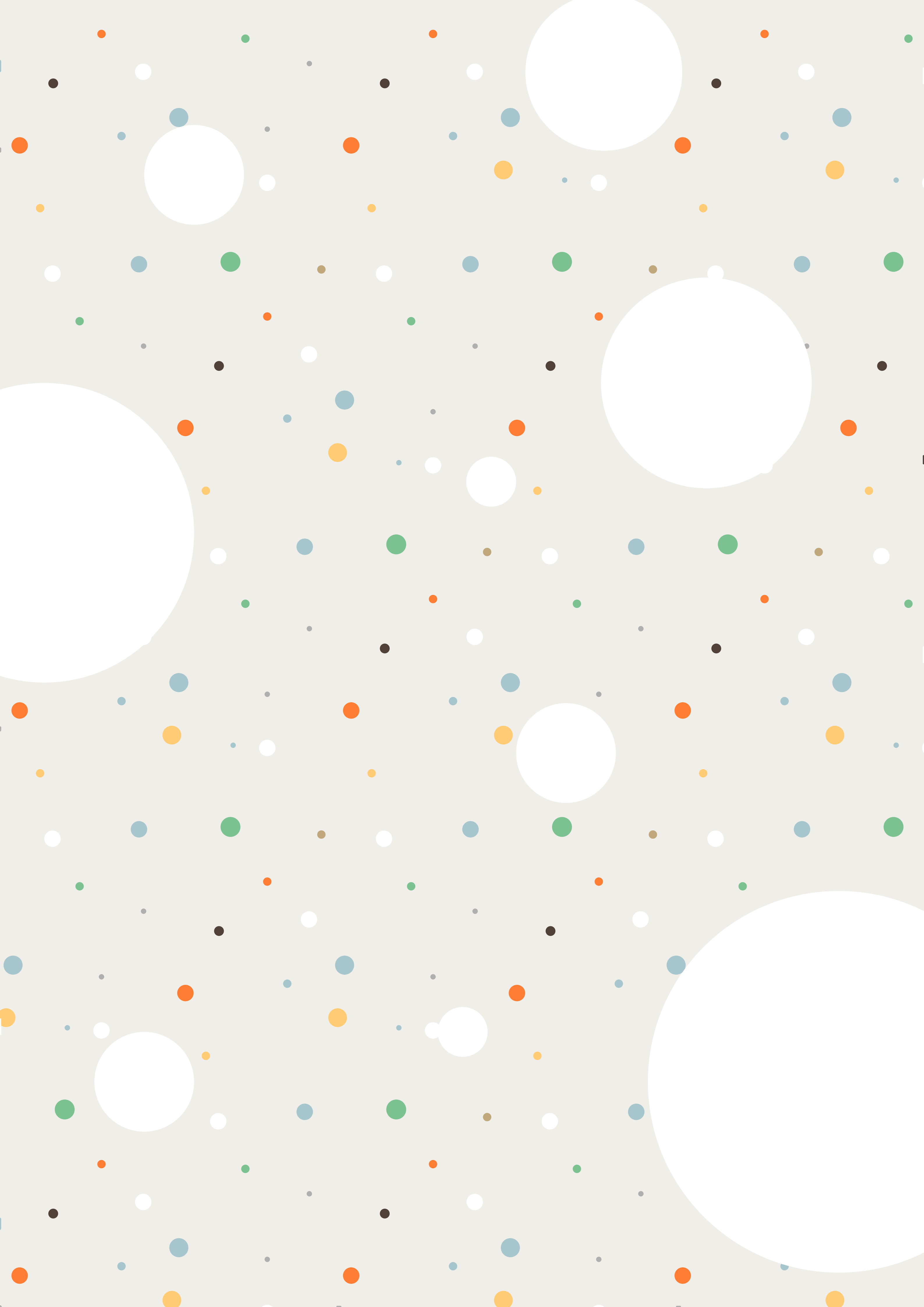

Propositional logic 2 of 2

Theorem: DNF Theorem

Every formula is LEQV to a DNF formula

Ex 1: Let $F = (x \leftrightarrow y) \vee \neg(x \rightarrow z)$

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

DNF formula that is LEQV to F:

$(\neg x \wedge \neg y \wedge \neg z) \vee$
$(\neg x \wedge \neg y \wedge z) \vee$
$(x \wedge \neg y \wedge \neg z) \vee$
$(x \wedge y \wedge \neg z) \vee$
$(x \wedge y \wedge z)$

Theorem: CNF Theorem

Every formula is LEQV to a CNF formula

Ex 2 Let $F = (x \leftrightarrow y) \vee \neg(x \rightarrow z)$

x	y	z	F	¬F
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

DNF for $\neg F$:

$\Leftrightarrow (\neg x \wedge y \wedge \neg z) \vee (\neg x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z)$

Then negate above
Then apply De Morgan's

$\Leftrightarrow \neg(\neg x \wedge y \wedge \neg z) \wedge \neg(\neg x \wedge y \wedge z) \wedge \neg(x \wedge \neg y \wedge z)$
$\Leftrightarrow (x \vee \neg y \vee z) \wedge (x \vee \neg y \wedge \neg z) \wedge (\neg x \vee y \vee \neg z)$

CNF formula that is LEQV to F

Completeness of a set of Connectives

By DNF theorem, $\{\neg, \wedge, \vee\}$ is complete.

$\{\neg, \wedge\}$, $\{\neg, \vee\}$ are complete.

Ex 3:

Define unary connective 0 (zero)

P	0P
0	0
1	0

Prove that $\{0, \to\}$ is complete

Step 1: Define the set of formulas that uoc $\{\neg, \vee\}$

Let G be the smallest set s.t.

Basis: If x is a variable, then $x \in G$.

I.S.: If $F_1, F_2 \in G$, then $\neg F_1$, $(F_1 \vee F_2) \in G$

Step 2:

Prove that for every $F \in G$, there's a F' s.t. F' uses $\{0, \to\}$ and F' LEQV F

Use structural Induction:

Basis: Let $F = x$, where x is a variable

We let $F' = x$

Then F' uoc $\{0, \to\}$ [F' uses no connectives]

and F' LEQV F [$F' = F$]

I.S.: Let $F_1, F_2 \in G$

Suppose there are formulas F_1', F_2' s.t. F_1', F_2' uoc $\{0, \to\}$ and F_1' LEQV F_1, F_2' LEQV F_2

Case 1: Let $F = \neg F_1$

Let $F' = F_1' \to 0F_1'$

by I.H.

Then F' uoc $\{0, \to\}$ and F' LEQV F

Case 2: Let $F = F_1 \vee F_2$

Let $F' = (F_1' \to 0F_1') \to F_2'$

Then F' uoc $\{0, \to\}$ and F' LEQV F

Ex 4: Define unary connective 1 (one)

P	$1P$
0	1
1	1

Prove that $\{1, \to\}$ is not complete.

Step 1: Define the set of formulas that uoc $\{1, \to\}$

Let H be the smallest set s.t.

Basis: If x is a variable, then $x \in H$.

I.S: If $F_1, F_2 \in H$, then $1F_1$, $F_1 \to F_2 \in H$

Step 2: Find a predicate $P(F)$ s.t.

$P(F)$ holds for all $F \in H$, but $P(F)$ does not hold for some formula F.

Step 2: Define predicate

$P(F)$: $\tau_1^*(F) = 1$, where τ_1 is the t.a. that assigns 1 to every variable i.e. $\tau_1(x) = 1$ $\forall x \in PV$

Step 3: Prove that $P(F)$ holds for every $F \in H$.

(Use structural induction)

Step 4: Find formula F s.t. $P(F)$ doesn't hold (find counterexample)

Let $F = \neg x$
Then $\tau_1^*(F) = \tau_1^*(\neg x) = 1 - \tau_1(x) = 0$

$\therefore$ $P(F)$ doesnt hold.
Therefore $\{1, \to\}$ is not complete.