W6 Reading Notes

Reading

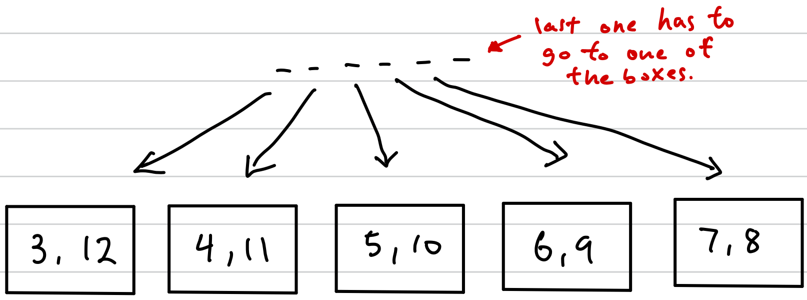
Pigeonhole Principle

If $(n+1)$ pigeons occupy n holes, then some hole must have at least 2 pigeons.

If you have n pigeons in k holes, and $\left(\frac{n}{k}\right)$ is not an integer, then some hole must have strictly more than $\left(\frac{n}{k}\right)$ pigeons.

Video

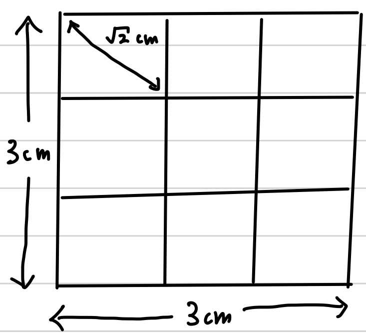
1. Prove that if 6 integers are selected from $\{3,4,5,6,7,8,9,10,11,12\}$ there must be 2 integers whose sum is 15.



Label 5 boxes with pairs of numbers that sum to 15 as shown. Every selected integer is placed into the boxes.

We need to place 6 selected integers into 5 boxes. By PHP, one box must have at least 2 integers.

2. Prove that if 10 points are placed in a 3cm by 3cm square then 2 points must be less than or equal to $\sqrt{2}$ cm apart.



We have 10 points to place in 9 squares. By PHP, there must be a square that has more than 1 point on or within its boundaries.

3. Prove that there are 2 people who have shaken hands the same number of times.

Suppose that there are n people. The min # of handshakes is 0. Max is $n-1$. If someone has shaken hands with $n-1$ ppl then there can't be someone who has shaken hands with 0 ppl. Vice versa.

So we have n people and at most $n-1$ possible # of hand shakes. By PHP, there must be 2 people who have shaken hands the same # of times.