# B41 Nov 15 Lec 1 Notes

**Definition:** Double integral

$$\iint_R f(x,y)\,dA = \lim_{n\to\infty}\lim_{m\to\infty}\sum_{j=1}^{n}\sum_{i=1}^{m} f(x_i^*, y_i^*)\Delta A$$

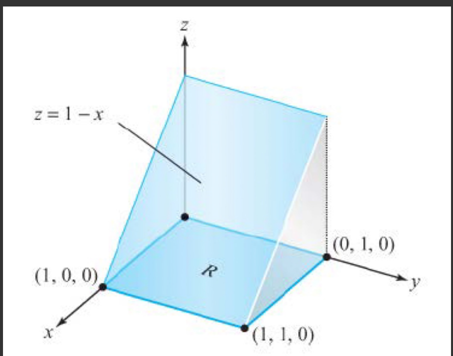
**Definition:** Midpoint rule

If we choose the sample points to be the center of the subrectangle $R_{ij}$, that is, $\bar{x}_i^*$ is the midpoint of $[x_{i-1}, x_i]$ and $\bar{y}_j^*$ is the midpoint of $[y_{j-1}, y_j]$, then we have the midpoint rule.

$$\iint_R f(x,y)\,dA \approx \sum_{j=1}^{n}\sum_{i=1}^{m} f(\bar{x}_i^*, \bar{y}_i^*)\Delta A$$

**Ex 1:**

Let $f(x,y) = 1-x$ over $R = [0,1]\times[0,1]$.




Volume of triangular solid = ½

$$\text{Average} = \frac{\iint_R f(x,y)\,dA}{A} = \frac{1/2}{1} = 1/2$$

Area of R = 1

**Theorem:** MVT

If $f(x,y)$ is continuous on a compact set $R$ (Area = $A$) in xy-plane, then there exists a point $(x_0, y_0)$ in $R$ s.t.

$$\iint_R f(x,y)\,dA = f(x_0, y_0)A$$

**Theorem:** Fubini's theorem

Let $f$ be continuous on the rectangle region $R = [a,b]\times[c,d]$. The double integral of $f$ over $R$ may be evaluated by either of two iterated integrals.

$$\iint_R f(x,y)\,dA = \int_c^d\int_a^b f(x,y)\,dx\,dy = \int_a^b\int_c^d f(x,y)\,dy\,dx$$