CH 1.3 Naive def. of Probability

Definition: (1.3.1) Naive def. of Probability

Let A be an event for an experiment with a finite sample space S. The naive probability of A is

$$P_{naive}(A) = \frac{|A|}{|S|} = \frac{\#\text{ of outcomes favorable to } A}{\text{total }\#\text{ of outcomes in } S}$$

The naive definition is restrictive because S is required to be finite, with equal mass for each pebble.

We can assume the naive definition when:

(i) There is symmetry in the problem that makes outcomes equally likely.

(ii) The outcomes are equally likely by design.

(iii) The naive definition serves as a null model. We assume the naive definition to apply just to see what predictions it will yield.

CH 1.4 How to Count

Theorem: (1.4.1) Multiplication Rule

Consider a compound experiment consisting of two sub-experiments, Experiment A and Experiment B. Suppose that Experiment A has a possible outcomes and B has b. Then the compound Experiment has ab possible outcomes.

Theorem: (1.4.7) Sampling with Replacement

Consider n objects and making k choices from them, one at a time with replacement (i.e., choosing a certain object does not preclude it from being chosen again.) Then there are n^k possible outcomes.

Theorem: (1.4.8) Sampling without Replacement

Consider n objects and making k choices from them, one at a time without replacement (i.e. choosing a certain object precludes it from being chosen again). Then there are $n(n-1)\cdots(n-k+1)$ possible outcomes for $1 \le k \le n$, and 0 possibilities for $k > n$ (where order matters). By convention, $n(n-1)\cdots(n-k+1) = n$ for $k=1$.

Definition: (1.4.14) Binomial Coefficient

For any nonnegative integers k and n, the binomial coefficient $\binom{n}{k}$, read as "n choose k", is the number of subsets of size k for a set of size n.

Theorem: (1.4.15) Binomial Coefficient Formula

For $k \le n$, we have

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!} = \frac{n!}{(n-k)!\,k!}$$

For $k > n$, we have $\binom{n}{k} = 0$

Proof:

Let A be a set with $|A| = n$. Any subset of A has size at most n, so $\binom{n}{k} = 0$ for $k > n$. Now let $k \leq n$. By Theorem 1.4.8, there are $n(n-1)\cdots(n-k+1)$ ways to make an ordered choice of k elements without replacement. This overcounts each subset of interest by a factor of $k!$ (since we don't care how these elements are ordered). So we can get the correct count by dividing by $k!$.

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