W8 Tutorial Notes

1. Construct a truth table for the following statement : $(p \leftrightarrow q) \rightarrow r$

p	q	r	$p \leftrightarrow q$	$(p \leftrightarrow q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	F

2. Derive from this truth table an equivalent formula to $(p \leftrightarrow q) \rightarrow r$ that only uses connectives from $\{\wedge, \vee, \neg\}$ and has exactly 6 propositional variables in it (for example, $(p \wedge q) \vee (\neg q \wedge p) \vee r$ has exactly 4 propositional variables)

$\neg$, since we found that $(p \leftrightarrow q) \rightarrow r$ is false in table above. $\rightarrow$

$$\neg(p \leftrightarrow q) \rightarrow r = ((p \wedge q) \wedge \neg r) \wedge (\neg p \wedge \neg q \wedge \neg r)$$

$$(p \leftrightarrow q) \rightarrow r = \text{not}((p \wedge q) \wedge \neg r) \wedge (\neg p \wedge \neg q \wedge \neg r)$$

double negation from line above

3. Explain how you derived your formula

First we look at the results that evaluate to False for $(p \leftrightarrow q) \rightarrow r$. We can write the formulas for p, q, r and then negate the whole formula. This is when it is false. Double negation on both sides would give not $((p \wedge q) \wedge \neg r) \wedge (\neg p \wedge \neg q \wedge \neg r)$

4. Prove that $(p \leftrightarrow q) \rightarrow r$ is logically equivalent to $(\neg p \wedge q) \vee (\neg q \wedge p) \vee r$ using the logical equivalence laws.

$\Leftrightarrow (p \leftrightarrow q) \rightarrow r$	
$\Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \rightarrow r$	Biconditional
$\Leftrightarrow \neg((\neg p \vee q) \wedge (\neg q \vee p)) \vee r$	$\Rightarrow$ Law (3 times)
$\Leftrightarrow [(\neg(\neg p \vee q)) \vee (\neg(\neg q \vee p))] \vee r$	De morgan
$\Leftrightarrow ((\neg\neg p) \wedge \neg q) \vee ((\neg\neg q) \wedge (\neg p))) \vee r$	De morgan
$\Leftrightarrow ((p \wedge \neg q) \vee (q \wedge \neg p)) \vee r$	Double negation (2 times)
$\Leftrightarrow (p \wedge \neg q) \vee (q \wedge \neg p) \vee r$	Associative
$\Leftrightarrow (\neg p \wedge q) \vee (\neg q \wedge p) \vee r$	Commutative (3 times)