B41 Nov 26 Lec 2 Notes

Ex 1.

Evaluate $\iiint_W z \, dV$ where W is the region bounded by the coordinate planes, $y+z=1$ and $x+z=1$.

$D = \{(x,z) \mid 0 \le x \le 1, \ 0 \le z \le 1-x\}$

$W = \{(x,y,z) \mid (x,z) \in D, \ 0 \le y \le 1-z\}$

$$\iiint_W z \, dV = \int_0^1 \int_0^{1-x} \int_0^{1-z} z \, dy \, dz \, dx$$

$$= \frac{1}{12}$$

Ex 2:

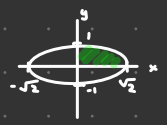
Integrate $f(x,y,z) = x$ over the region W bounded below by $z = 4-x^2-y^2$ and above $z = x^2+3y^2$ in the octant $x,y,z \ge 0$.

$4-x^2-y^2 = x^2+3y^2$

$2x^2+4y^2 = 4$

$x^2+2y^2 = 2$

$\frac{1}{2}x^2+y^2 = 1$



$$\int_0^{\sqrt{2}} \int_0^{\sqrt{1-\frac{x^2}{2}}} \int_{x^2+3y}^{4-x^2-y^2} f(x,y,z) \, dz \, dy \, dx = \frac{16}{15}$$