B41 Sept 10 Lec 2 Notes

In $\mathbb{R}^n$, the equation of a line may be decided by a: two points on the line; or
b: a point on the line and direction of the line

Vector equation of the line: $x = x^0 + tv$, $t \in \mathbb{R}$

Parametric equation of the line: $x_i = x_i^0 + tv_i$, $i = 1, 2, \cdots, n$

Symmetric equation of the line: $\frac{x_1 - x_1^0}{v_1} = \frac{x_2 - x_2^0}{v_2} = \cdots = \frac{x_n - x_n^0}{v_n}$

Plane: A plane in $\mathbb{R}^n$ may be decided by a point on the plane and a vector n that is orthogonal to the plane.

A plane may be described by

$$ax + by + cz = d$$

which is a rectangular description of the plane; (i.e., in terms of its rectangular coordinates.

A parametric description would be:

$$p + sv + tw, \quad s, t \in \mathbb{R}$$

where p is a point on the plane and v and w are vectors in the plane.

$ax + by + cz = d$ is equivalent to $x = p + sv + tw$, $s, t \in \mathbb{R}$.

For instance, let $x = s$ and $y = t$, then

$$z = d/c - (a/c)s - (b/c)t \text{ if } c \neq 0$$

Rearranging, we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ d/c \end{bmatrix}}_{p} + s\underbrace{\begin{bmatrix} 1 \\ 0 \\ -a/c \end{bmatrix}}_{v} + t\underbrace{\begin{bmatrix} 0 \\ 1 \\ -b/c \end{bmatrix}}_{w}$$

Ex 1: Find intersection of $\begin{cases} 2x + 2y - z = 4 \\ x - 2y + z = -1 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & 2 & -1 & 4 \\ 1 & -2 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 6 & -3 & 6 \end{array}\right]$$

Solution $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 + 2t \\ 1 + \frac{3}{2}t \\ t \end{bmatrix}$