B52 Oct 1 Lec 2 Notes

Bernoulli Distribution

Experiments often have binary results

We can encode such results with binary RV X, called Bernoulli RV

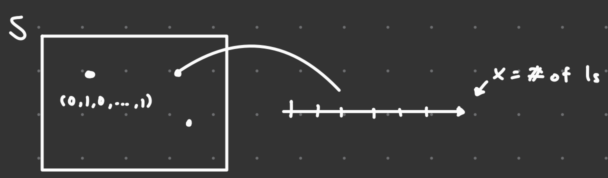
$$X = \begin{cases} 1, & \text{Success} \\ 0, & \text{Failure} \end{cases}$$

Bernoulli RVs are equivalent to indicator RVs

$$X \sim \text{Bernoulli}(p) \Rightarrow I_{\{X=1\}} \sim \text{Bernoulli}(p)$$

Binomial Distribution

For arbitrary $n \in \mathbb{N}$, what are the possible values of Binomial RV X ?



Possible values : $\{0, 1, 2, \cdots, n\}$

What is the probability of $x = 3$ successes in $n = 5$ trials?

i.e. what is $P(X=3) = P(\{s \in S : X(s) = 3\})$

$$= \binom{5}{3} p^3 (1-p)^2$$

More generally, the PMF of a Binomial RV is

$$p_X(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}, \quad \forall x \in \{0, 1, 2, \cdots, n\}$$

Denoted by $X \sim \text{Binomial}(n, p)$

Verify this is a valid PMF.

(i) $p_X(x) \geq 0$

(ii) $\sum_{x=0}^{n} p_X(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = (p + (1-p))^n$ Binomial theorem

$= 1^n = 1$

Ex 1:

(i) perfect score (6/6)

$X \sim \text{Binomial}(n=6, p=1/4)$

$$P(X=6) = \binom{6}{6}(1/4)^6(1-1/4)^0$$
$$= 1 \cdot (\tfrac{1}{4})^6 \cdot 1$$
$$= \frac{1}{4^6}$$

(ii) at least (3/6)

$$P(X \in \{3,4,5,6\}) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$$
$$= \binom{6}{3}(\tfrac{1}{4})^3(3/4)^3 + \dots + (\tfrac{1}{4})^6$$
$$= 17\%$$

Geometric Distribution

Consider infinite sequence of independent Bernoulli trials with the same probability of "success" p.

Let RV X/Y count # of trials / failures until 1st success.

More generally, the PMF of a geometric RV is

$$P(X=x) = pq^{x-1}, \quad \forall x \in \mathbb{N}_+$$

Denoted $X \sim \text{Geometric}(p)$

Verify

(i) $p_X(x) \geq 0$

(ii) $\sum_{x=1}^{\infty} p \cdot q^{x-1} = p \cdot \sum_{j=0}^{\infty} q^j = p \cdot \frac{1}{1-q} = p \cdot \frac{1}{p} = 1$

$$F_X(x) = P(X \leq x) = \sum_{j=1}^{x} p \cdot q^{j-1} = p \sum_{j=0}^{x-1} q^j = p \cdot \frac{1-q^x}{1-q} = 1-q^x$$