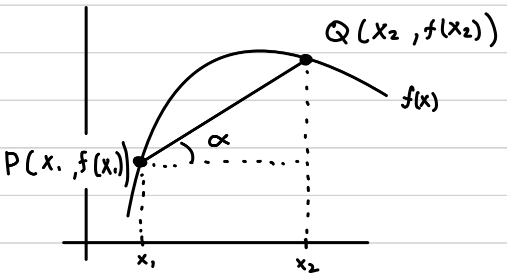
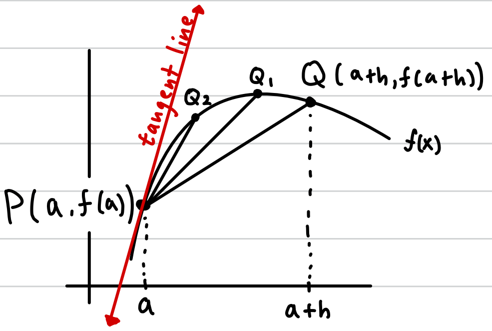
W7 Lecture 14 Notes



$$m_{PQ} = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{\Delta f(x)}{\Delta x} = \frac{\Delta y}{\Delta x}$$

Equation of secant line PQ:

$$y = f(x_1) + m_{PQ}(x-x_1)$$



Slope of tangent line:

$$m = \lim_{Q \to P} m_{PQ} = \lim_{h \to 0} m_{PQ}$$

Equation of tangent line to f(x) at a

$$y = f(a) + m(x-a)$$
$$y = f(a) + f'(a)(x-a)$$

$$f'(a) = m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

Derivative of f(x) at point a:

Examples:

1. Find $f'(a)$ if $f(x) = \frac{x}{x+3}$, $a=1$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h \to 0} \frac{\frac{a+h}{a+h+3} - \frac{a}{a+3}}{h}$$

$$= \lim_{h \to 0} \frac{(a+h)(a+3) - a(a+h+3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h \to 0} \frac{\cancel{a^2} + \cancel{3a} + 3\cancel{h} + \cancel{ah} - \cancel{a^2} - \cancel{ah} - \cancel{3a}}{\cancel{h}(a+h+3)(a+3)}$$

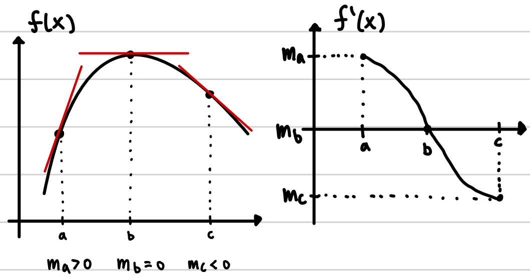
$$= \lim_{h \to 0} \frac{3}{(a+h+3)(a+3)} = \frac{3}{(a+3)^2}$$

$$f'(1) = \frac{3}{16}$$

Equation of tangent line:

$$y = f(1) + f'(1)(x-1)$$
$$y = \frac{1}{4} + \frac{3}{16}(x-1)$$

Derivative of a Function



$$f'(a) = f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Examples:

2. Find $f'(x)$ if $f(x) = \frac{1}{\sqrt{x}}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\cdot\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\cdot\sqrt{x}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{x - x - h}{h\sqrt{x+h}\cdot\sqrt{x}\left(\sqrt{x} + \sqrt{x+h}\right)} = \frac{-1}{2x\sqrt{x}}$$

Notation

$$f'(x) = \frac{df(x)}{dx} = \frac{d}{dx}(f(x))$$

$$f''(x) = \frac{d^2}{dx^2}(f(x))$$

Definition

$f(x)$ is differentiable at $x = a$ if $\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ does exist.

Example:

3. Where is $f(x) = |x-1|$ differentiable?

$$f(x) = \begin{cases} 1 - x, & x < 1 \\ 0, & x = 1 \\ x - 1, & x > 1 \end{cases}$$

$(-\infty, 1)$: $f'(x) = \lim_{h\to 0} \frac{1 - (x+h) - (1-x)}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$

$(1, \infty)$: $f'(x) = \lim_{h\to 0} \frac{(x+h) - 1 - (x-1)}{h} = \lim_{h\to 0} \frac{h}{h} = 1$

$f'(x<1) \neq f'(x>1) \Rightarrow f'(x)$ is not differentiable at $x=1$