W10 Pre-Lecture

1. Prove the statement for a collection of natural numbers $x_1, x_2, \dots, x_n$ and the set $\mathcal{I} = \{1, 2, \dots, n\}$.

$$\forall x_i \in \mathbb{N},\ (x_1 + x_2 + \dots + x_n) > \frac{n(n+1)}{2} \rightarrow \exists i \in \mathcal{I},\ x_i > i$$

Contrapositive: $\exists i \in \mathcal{I},\ x_i \leq i \rightarrow \forall x_i \in \mathbb{N},\ (x_1 + x_2 + \dots + x_n) \leq \frac{n(n+1)}{2}$

There exists some i that belongs to $\mathcal{I}$

Let X_i be an abritrary natural number

Suppose that $X_i \leq i$:

Since we know that if $a \leq b$ and $c \leq d$

Then $a + c \leq b + c$, $c + b \leq d + b \Rightarrow a + c \leq b + d$

Then $x_i + x_{i+1} + \dots + x_n \leq i + (i+1) + \dots + n$

Let $i = 1$.

Then $x_1 + x_2 + \dots + x_n \leq 1 + 2 + \dots + n = \frac{n(n+1)}{2}$ by the sum of natural numbers formula.

Since we assumed $x_i \leq i$, $x_i \leq i \rightarrow (x_1 + x_2 + \dots x_n) \leq \frac{n(n+1)}{2}$

Since X_i is an arbritrary natural number, $x_i \leq i \rightarrow \forall x_i \in \mathbb{N},\ (x_1 + x_2 + \dots + x_n) \leq \frac{n(n+1)}{2}$

Since there exists some i that belongs to $\mathcal{I}$, $x_i \leq i \rightarrow \forall x_i \in \mathbb{N},\ (x_1 + x_2 + \dots + x_n) \leq \frac{n(n+1)}{2}$

and by contraposition,

$$\forall x_i \in \mathbb{N},\ (x_1 + x_2 + \dots + x_n) > \frac{n(n+1)}{2} \rightarrow \exists i \in \mathcal{I},\ x_i > i$$