B41 Sept 13 Lec 1 Notes

Ex 1: Find equation of the plane that passes through the point $(1,1,-1)$ and is perpendicular to the line:

$$\begin{aligned} x &= 1+t \\ y &= 1-3t \qquad t \in \mathbb{R} \\ z &= -7t \end{aligned}$$

The normal vector would be $n = (1,-3,-7)$.

The vector

$$(x,y,z)-(1,1,-1) = (x-1, y-1, z+1) \quad , \; x,y,z \text{ are arbitrary vectors}$$

is on the plane which is perpendicular to n

Then

$$n \cdot (x-1, y-1, z+1) = 0$$

$$\Rightarrow \quad x-3y-7z=5$$

Remark: Two planes are parallel if they have parallel normal vectors.

If two planes are not parallel, they must intercept in a line and the angle between the two planes is the angle between their normal vectors.

Ex 2: Find angle between $\begin{cases} -2x+6y+14z=5 \\ x-3y-7z=5 \end{cases}$

$n_1 = (-2,6,14)$
$n_2 = (1,-3,-7)$

$$\cos\theta = \frac{n_1 \cdot n_2}{\|n_1\|\|n_2\|} = \frac{7}{14} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

A function of one single variable in $\mathbb{R}^n$ may be expressed in the form of $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))$ where $x_i(t)$ are component functions, $i = 1,2,\cdots,n$ are real functions of $t \in \mathbb{R}$.

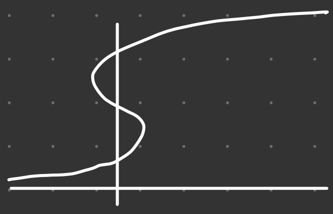
A path in $\mathbb{R}^n$ is a map $c: [a,b] \to \mathbb{R}^n$,

$$x(t) = (x_1(t), x_2(t), \cdots, x_n(t)), \quad t \in [a,b]$$

The collection C of points $x(t)$ as t varies in $[a,b]$ is called a curve.

Ex 3: Sketch the curve given by $x = t^3 - 4t^2 + 2$, $y = t+3$, $-2 \le t \le 5$

t	-2	-1	0	1	2	3	4	5
x	-22	-3	2	-1	-6	-7	2	27
y	1	2	3	4	5	6	7	8



This makes us think that x is a cubic function of y.

$y = t+3 \Rightarrow t = y-3$

$$x = (y-3)^3 - 4(y-3)^2 + 2$$

Ex 4: Find parametric equations that represent the ellipse curve $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$

Set $\frac{x-x_0}{a} = \sin t$, $\frac{y-y_0}{b} = \cos t$

Then $x = a\sin t + x_0$, $y = b\cos t + y_0$, $0 \le t \le 2\pi$

Let a path in $\mathbb{R}^n$ is a map $c:[a,b] \to \mathbb{R}^n$

$$c(t) = (c_1(t), c_2(t), \cdots, c_n(t)), \quad t \in [a,b]$$

The velocity of c at time t is defined by $c'(t) = \lim_{h\to 0} \frac{c(t+h)-c(t)}{h} = (c_1'(t), c_2'(t), \cdots, c_n'(t))$ which is the vector tangent to the path $c(t)$.

$\|c'(t)\|$ is the speed of the path.

Definition: The tangent line to c at point $a = (x(t_0), y(t_0), z(t_0))$ is defined to be the line through a with direction $c'(t_0)$, which is the tangent vector of c at a.

$$x - a = c'(t_0)(t - t_0)$$

Polar Coordinates