O S O

G T G T

5 E 5

N O N O

O S O

G T G T

5 E 5

N O N O

O S O

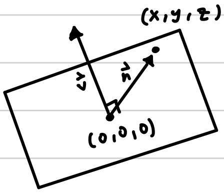
G T G T

5 E 5

Jan 20 Lec 1 Notes

Equation of a plane

(Plane goes through origin)



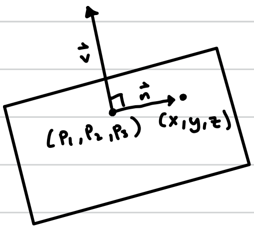
$\vec{v} = \langle x, y, z \rangle$

$\vec{n} = \langle n_1, n_2, n_3 \rangle$

$\vec{n} \cdot \vec{v} = 0$

Equation of plane P perpendicular to $\vec{v}$ $\quad n_1 x + n_2 y + n_3 z = 0$

(Plane does not go through origin)



$\vec{v} \cdot \vec{n} = 0$

$\vec{v} = \langle x - P_1, y - P_2, z - P_3 \rangle$

$\vec{v} \cdot \vec{n} = n_1(x - P_1) + n_2(y - P_2) + n_3(z - P_3)$

$= n_1 x + n_2 y + n_3 z - n_1 P_1 - n_2 P_2 - n_3 P_3 = 0$

Equation of plane P perpendicular to $\vec{v}$ $\quad n_1 x + n_2 y + n_3 z = n_1 P_1 + n_2 P_2 + n_3 P_3$

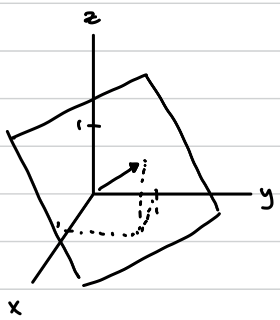
Example:

1. What is the equation of the plane through origin and is perpendicular to z-axis.

$\vec{n} = \langle 0, 0, 1 \rangle$ $\qquad$ Plane: $0 + 0 + z = 0 \Rightarrow z = 0$ (x and y plane)

$P = \langle 0, 0, 0 \rangle$

2. Visualize $x + y + z = 0$



Systems of linear equations

Example:

3. $$\left|\begin{array}{l} 2x + 3y - z = 1 \\ \quad\;\; y - 3z = 2 \\ 4x + 5y - 2z = 1 \end{array}\right| \rightarrow \left|\begin{array}{l} x + \frac{3}{2}y - \frac{1}{2}z = \frac{1}{2} \\ \quad\;\; y - 3z = 2 \\ \qquad\quad z = -\frac{1}{3} \end{array}\right| \rightarrow \left|\begin{array}{l} x = -\frac{7}{6} \\ y = 1 \\ z = -\frac{1}{3} \end{array}\right|$$

Each equation is a plane, thus the solution is the intersection of the planes.

A Solution set, or general solution to C (a augmented matrix)

$$\left\{ (c_1, c_2, c_3) \;\middle|\; (c_1, c_2, c_3) \text{ is a solution to } C \right\}$$

Row Reduction Steps

(i) Switch the order of rows, or we can exchange rows $R_i \Leftrightarrow R_j$

(ii) mutiply a row by a non-zero scalar $R_i \Leftrightarrow kR_j$, $k \in \mathbb{R}$, $k \neq 0$

(iii) Replace a row with itself plus a multiple of another row $R_i \Leftrightarrow R_i + kR_j$