**Ex 1**

Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

in $\mathbb{R}^3$, and define the plane $V = \text{span}(\vec{v}_1, \vec{v}_2)$ in $\mathbb{R}^3$. Is the vector

$$\vec{x} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

on the plane $V$?

We have to solve $M = \left[\begin{array}{cc:c} 1 & 1 & 5 \\ 1 & 2 & 7 \\ 1 & 3 & 9 \end{array}\right] \sim \left[\begin{array}{cc:c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

Thus $\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2$
$= 3\vec{v}_1 + 2\vec{v}_2$

**Def 3.4.1:** Coordinates in a Subspace of $\mathbb{R}^n$

Consider a basis $\mathfrak{B} = (\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m)$ of a subspace $V$ of $\mathbb{R}^n$. By theorem 3.2.10, any vector $\vec{x}$ in $V$ can be written uniquely as

$$\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_m\vec{v}_m$$

The scalars $c_1, c_2, \dots, c_m$ are called $\mathfrak{B}$ coordinates of $\vec{x}$, and the vector

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}$$

is the $\mathfrak{B}$ coordinate vector of $\vec{x}$, denoted by $[\vec{x}]_{\mathfrak{B}}$. Thus,

$$[\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}$$

**Theorem 3.4.2** : Linearity of Coordinates

If $B$ is a basis of a subspace $V$ of $\mathbb{R}^n$, then

(a) $[\vec{x}+\vec{y}]_B = [\vec{x}]_B + [\vec{y}]_B$, for all vectors $\vec{x}$ and $\vec{y}$ in $V$, and

(b) $[k\vec{x}]_B = k[\vec{x}]_B$, for all $\vec{x}$ in $V$ and for all scalars $k$.