


Ex 1

$$\int e^x \sin x \, dx$$

$$\begin{aligned} \text{Choose } u &= e^x & dv &= \sin x \, dx \\ du &= e^x dx & v &= -\cos x \end{aligned}$$

$$\begin{aligned} &= -e^x \cos x - \int -\cos x e^x \, dx \\ &= -e^x \cos x + \int \cos x e^x \, dx \end{aligned}$$

$$\begin{aligned} \text{Choose } u_2 &= e^x & dv_2 &= \cos x \, dx \\ du_2 &= e^x dx & v_2 &= \sin x \, dx \end{aligned}$$

$$\int \sin x e^x dx = -e^x \cos x + (e^x \sin x - \int \sin x e^x dx)$$

$$\begin{aligned} 2 \int \sin x e^x dx &= -e^x \cos x + e^x \sin x \\ &= \frac{1}{2}(-e^x \cos x + e^x \sin x) \end{aligned}$$

Partial Fraction Decomposition

Ex 2

$$\begin{aligned} \int \frac{5x+11}{(x+3)(x+2)} dx &= \int \frac{4}{x+3} + \frac{1}{x+2} dx \\ &= 4 \ln|x+3| + \ln|x+2| + C \end{aligned}$$

Ex 3

Determine the PFD for the following:

$$(i) \quad f(x) = \frac{8x-42}{(x+6)(x-3)} = \frac{A}{x+6} + \frac{B}{x-3}$$

$$(ii) \quad f(x) = \frac{25}{x^3(x^2+4)} = \frac{Ax+B}{(x^2+4)} + \frac{C}{x} + \frac{D}{x^2} + \frac{E}{x^3}$$

$$\begin{aligned} (iii) \quad f(x) = \frac{\sqrt{2}x - e + 10^{100}x^4}{(x^4+2)(x-1)^6 x} &= \frac{Ax^3+Bx^2+Cx+D}{(x^4+2)} + \frac{E}{x} + \frac{F}{(x-1)} + \frac{G}{(x-1)^2} + \frac{H}{(x-1)^3} \\ &\quad + \frac{I}{(x-1)^4} + \frac{J}{(x-1)^5} + \frac{K}{(x-1)^6} \end{aligned}$$