Def:

Let $m, n, l, p \in \mathbb{Z}^{\geq 0}$. Let $a, b \in \mathbb{R}$, $a < b$.

Integrals of the form:

$$\int \sin^{l}(x) \cos^{p}(x)\, dx \quad \text{or} \quad \int_a^b \sin^{l}(x) \cos^{p}(x)\, dx$$

$$\int \sec^{n}(x) \tan^{m}(x)\, dx \quad \text{or} \quad \int_a^b \sec^{n}(x) \tan^{m}(x)$$

are called trig integrals.

Ex 1

$= \int \cos^2 x \sin^5(x)\, dx$ Even / ODD sin - cos

$= \int \sin x \,(\cos^2 x \sin^4 x)\, dx$

$= \int \sin x \,(\cos^2 x)(1 - \cos^2 x)^2\, dx$

Let $u = \cos x$

$du = -\sin x\, dx$

$= \int -u^2 (1-u^2)^2\, du$

$= \int -u^2(1) - u^2(-2u^2) - u^2(u^4)\, du$

$= \int -u^2 + 2u^4 - u^6\, du$

$= -\frac{1}{3}u^3 + \frac{2}{5}u^5 - \frac{1}{7}u^7 + C$

$= -\frac{1}{3}(\cos x)^3 + \frac{2}{5}(\cos x)^5 - \frac{1}{7}(\cos x)^7 + C$

Ex 2

$$= \int \cos^5 x \sin^7 x \, dx$$ ODD / ODD

sin - cos

$$= \int \cos x \,(1-\sin^2 x)^2 \sin^7 x \, dx$$

Let $u = \sin x$

$du = \cos x$

$$= \int (1-u^2)^2 u^7 \, du$$

$$= \int u^7 - 2u^7u^2 + u^7u^4 \, du$$

$$= \int u^7 - 2u^9 + u^{11} \, du$$

$$= \frac{1}{8}u^8 - \frac{2}{10}u^{10} + \frac{1}{12}u^{12} + C$$

$$= \frac{1}{8}(\sin x)^8 - \frac{1}{5}(\sin x)^{10} + \frac{1}{12}(\sin x)^{12} + C$$

Ex 3

$$= \int \sin^2(3x) \cos^2(3x) \, dx$$ Even / Even sin - cos

$$= \int \left(\frac{1-\cos(2(3x))}{2}\right)\left(\frac{1+\cos(2(3x))}{2}\right) dx$$ $\sin^2(A) = \frac{1-\cos(2A)}{2}$

$$= \int \frac{1-\cos^2(6x)}{4} \, dx$$ $\cos^2(A) = \frac{1+\cos(2A)}{2}$

$$= \int \frac{\sin^2(6x)}{4} \, dx$$

$$= \frac{1}{4} \int \frac{1-\cos(12x)}{2} \, dx$$

$$= \frac{1}{8} \int 1 \, dx - \frac{1}{8} \int \cos(12x) \, dx$$

$$= \frac{1}{8} x - \frac{1}{8 \cdot 12} \sin(12x) + C$$