B52 Sept 15 Lec 1 Notes

Ex 1:

(i) 2^n
(ii) 6^n
(iii) 2^n

Ex 2:

(i) 10^4
(ii) $\frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7$

Permutation: ordered arrangement or k objects, chosen without replacement from n possible objects.

Stirling's approximation: $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

Ex 3:

$$P(2 \text{ or more}) = 1 - P(\text{no one shares same birthday})$$
$$= 1 - \frac{\frac{365!}{335!}}{365^{30}}$$
$$\approx 70.63\%$$

Ex 4:

$|S| = 9!$

$P(A) = \frac{3!\,2!\,3!\,4!}{9!}$

$|A| = \underbrace{3!}_{\text{ways to arrange subjects as groups}} \cdot \underbrace{2! \cdot 3! \cdot 4!}_{\text{ways to arrange the books within the subject}}$

Ex 5:

$= 6!/6 = 5!$

Ex 6:

$= C^{49}_{6}$

Binomial Theorem : $(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$

Every term in expression will be of form $x^i \cdot y^{n-i}$ # times $x^i y^{n-i}$ appears (for fixed i) = # ways to choose the i positions of the x's from the n-tuple $(x, y, \ldots)$

In particular $\sum_{r=0}^{n} \binom{n}{r} = 2^n$

i.e. (# subsets of S) = $\sum_{i=0}^{n}$ (# of subsets of size i)

Binomial Coefficient Properties

(i) $\binom{n}{0} = \binom{n}{n} = 1$ and $\binom{n}{1} = \binom{n}{n-1} = n$
(ii) $\binom{n}{k} = \binom{n}{n-k}$
(iii) $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$