Propositional logic 1 of 2

Definition:

Let PV be a set of propositional variables. The set of propositional formulas, denoted F (or F_{PV}), is the smallest set s.t.

Basis: if $x \in PV$, then $x \in F$

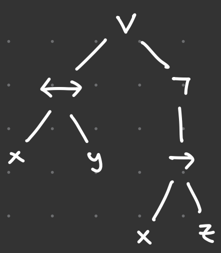
I.S.: If $P_1, P_2 \in F$, then

$$\neg P_1, (P_1 \wedge P_2), (P_1 \vee P_2), (P_1 \to P_2), (P_1 \leftrightarrow P_2) \in F$$

Ex 1:

$$((x \leftrightarrow y) \vee \neg(x \to z))$$

Parse Tree:



Definition:

A truth assignment (t.a.) is a function $\overset{\text{\tau}}{\tau} : PV \to \{\overset{\text{False}}{0}, \overset{\text{True}}{1}\}$

Definition: Extended Truth Assignment

$$\tau^* : F \to \{0, 1\}$$

$$\tau^*(P) = \begin{cases} \tau(x) & \text{if } P = x \in PV \\ \tau^*(P_1) \cdot \tau^*(P_2) & \text{if } P = (P_1 \wedge P_2) \\ \max(\tau^*(P_1), \tau^*(P_2)) & \text{if } P = (P_1 \vee P_2) \end{cases}$$

$\min(\tau^*(P_1), \tau^*(P_2))$ also works

$$\tau^*((P_1 \to P_2)) = \begin{cases} 1 & \text{, if } \tau^*(P_1) = 0 \text{ or } \tau^*(P_2) = 1 \\ 0 & \text{, o/w} \end{cases}$$

$$\tau^*((P_1 \leftrightarrow P_2)) = \begin{cases} 1 & \text{, if } \tau^*(P_1) = \tau^*(P_2) \\ 0 & \text{, o/w} \end{cases}$$

Conventions:

Precedence (high to low)

(i) $\neg$

(ii) $\wedge$

(iii) $\vee$

(iv) $\rightarrow$, $\leftrightarrow$ (right associate)

Omit outermost parantheses

Definition:

2 formulas P_1, P_2 are logically equivalent iff for any t.a. τ, $\tau^*(P_1) = \tau^*(P_2)$.

Notation: P_1 LEQV P_2 , or $(P_1 \Leftrightarrow P_2)$

Definition:

A formula P_1 logically implies a formula P_2 iff for any t.a. τ, if $\tau^*(P_1) = 1$, then $\tau^*(P_2) = 1$.

Definition:

τ satisfies P means $\tau^*(P) = 1$
τ falsifies P means $\tau^*(P) = 0$

Definition:

P is a tautology (valid formula) means every t.a. satisfies P.

P is satisfiable means some t.a. satisfies P.

P is unsatisfiable (contradiction) means every t.a. falsifies P.

P is falsifiable means some t.a. falsifies P.

Theorem: (5.7, 5.9)

For any formula P_1, P_2,

(i) P_1 LEQV P_2 iff $P_1 \leftrightarrow P_2$ is a tautology
(ii) P_1 logically implies P_2 iff $P_1 \rightarrow P_2$ is a tautology

Truth Tables

e.g.

x	y	z	$(x \leftrightarrow y)$	$\vee$	$\neg$	$(x \to z)$
0	0	0	1	1	0	1
0	0	1	1	1	0	1
0	1	0	0	0	0	1
0	1	1	0	0	0	1
1	0	0	0	1	1	0
1	0	1	0	0	0	1
1	1	0	1	1	1	0
1	1	1	1	1	0	1

Definition: Normal Forms (5.9)

(i) A literal is a variable or the negation of a variable.
(ii) A term is a literal or the conjunction (AND) of 2 or more literals.
(iii) A clause is a literal or the disjunction (OR) of 2 or more literals.
(iv) A disjunctive normal form (DNF) formula is a term or the disjunction of 2 or more terms.
(v) A conjunctive normal form (CNF) formula is a clause or the conjunction of 2 or more clauses.