Week 4 Reading Notes

An **experiment** is a clearly defined procedure that results in one of a possible set of **outcomes** or **elementary events**.

A **Sample (probability) space** of a random experiment is a set S that includes all possible outcomes of the experiment.

A **size** of a set S is the number of items in S and is denoted by $|S|$.

A **compound event** is a subset of S consisting of several elementary events.

Let S be the sample space of an experiment and E be an event in S. Then Laplace's definition of probability says that the probability of E is :

$$P(E) = \frac{|E|}{|S|}$$

Examples:

1a. What is the probability of rolling a 3 on a standard die?

$$\boxed{\frac{|\{3\}|}{|S|} = \frac{1}{6}}$$

1b. Probability of rolling a power of two on a standard die?

$$\boxed{\frac{|\{1,2,4\}|}{|S|} = \frac{3}{6}}$$

1c. Consider the experiment of rolling two dice and the probability of their sum being 7. What is the sample space?

$$\boxed{\{(i,j) \mid 1 \leq i \leq 6,\ 1 \leq j \leq 6\}}$$

2. If the experiment is to throw a standard die and record the outcome then:

$$\text{Sample space } S = \{1,2,3,4,5,6\}$$

and the probability $P(x)$ of rolling an x is $1/6$, then what is the following sum?

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$\boxed{\sum_{i=1}^{6} P(i) = 6\left(\tfrac{1}{6}\right) = 1}$$

A **probability function** P assigns to each outcome x in a sample space S a number $P(x)$ so that:

$$0 \leq P(x) \leq 1 \quad , \forall x \in S$$

and

$$\sum_{x \in S} P(x) = 1$$

This then tells us that if we know $P(E)$, the probability of an event E occuring, then we can easily determine the probability of the event not occuring. We indicate that the event that E does not occur as $\bar{E}$.

Theorem

Let E be an event. The probability of $\bar{E}$, the complement of E satisfies:

$$P(E) + P(\bar{E}) = 1$$

Examples:

3. Considering tossing a coin five times. What is the probability of getting the same result on the first two tosses or the last two tosses?

Let E be the event that the first two tosses are the same and F be the event that the last two tosses are the same.

$|E| = 2 \cdot 1 \cdot 2 \cdot 2 \cdot 2 = 16$ $\qquad |E \wedge F| = 2 \cdot 2 \cdot 2 = 8$

$|F| = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 16$

$|S| = 2^5 = 32$

$$P(E) = \frac{|E|}{|S|} = \frac{16}{32} = \frac{1}{2}$$

$$P(F) = \frac{|F|}{|S|} = \frac{16}{32} = \frac{1}{2}$$

$$P(E \cap F) = \frac{8}{32} = \frac{1}{4}$$

$$P(E \cup F) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$$

$$\boxed{= 3/4}$$

Theorem (The Sum Rule)

If E and F are events in an experiment then the probability that E or F occurs is given by:

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$