W6 TUT

Discrete RV has point mass probability

For joint dist., two RVs happen at the same time.

$\Omega_X = S$
$\Omega_Y = T$
$\Omega_{X,Y} = S \times T$

$P(X=x, Y=y) = f_{X,Y}(x,y)$

$P(X=x, Y=y)$	1	2	3	$P_Y(y)$
1	0	$1/6$	$1/6$	$1/3$
2	$1/6$	0	$1/6$	$1/3$
3	$1/6$	$1/6$	0	$1/3$
$P_X(x)$	$1/3$	$1/3$	$1/3$	

Marginal distribution $\begin{cases} P_X(x) = \sum_{y \in Y} f_{X,Y}(x,y) \\ P_Y(y) = \sum_{x \in X} f_{X,Y}(x,y) \end{cases}$

Conditional Distributions

$$P(X=x \mid Y=y) = f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{P_Y(y)}$$

$$P(X \in A \mid Y \in B) = \sum_{a \in A} \sum_{b \in B} \frac{P(X=a, Y=b)}{P(Y \in B)}$$

$$= \sum_{a \in A} \sum_{b \in B} \frac{f_{X,Y}(a,b)}{\sum_{b \in B} P_Y(b)}$$

Independence of RV.

$$X, Y \text{ independent} \iff P(X=x, Y=y) = P(X=x) \cdot P(Y=y) \iff f_{X,Y} = P_X P_Y$$

PSS Q6

X_1 = # of obj in 1
X_2 = " in 2

$P(X_1 = x, X_2 = y, [X_3 = n-x-y])$

	1	2	3
n	x	y	$n-x-y$
	p_1	p_2	p_3

$$P(X_1=x, X_2=y) = \binom{n}{x;y} p_1^x p_2^y p_3^{n-x-y}$$

$$P_{X_1}(x) = \sum_{y \in X_2} \binom{n}{x;y} p_1^x p_2^y p_3^{n-x-y}$$

PS5 Q2

$H \begin{cases} 0 \\ 1 \end{cases}$

$P(H) = p \Rightarrow P(H=1) = p$

$P(H=y, X=x)$

If $H=1$, $Bin(n, a)$
If $H=0$, $Bin(n, b)$

$p\ Bin(n, a) + (1-p)\ Bin(n, b)$