B52 Nov 12 Lec 2 Notes

Weak Law of Large Numbers

Statistics: estimate mean $\mu = E(x)$ of unknown distribution by averaging random values $X_1, X_2, \ldots$ (aka samples)

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \to \mu, \text{ as } n \to \infty$$

Simulation: approximate probability of event A by repeating experiment & counting average # times event occurs.

$$\bar{P}_n = \frac{1}{n}\sum_{i=1}^{n} I_i(A) \to P(A), \text{ as } n \to \infty \quad \left(\begin{array}{l} \text{where } I_i(A) = \begin{cases} 1, & A \text{ occurs} \\ 0, & \text{o/w} \end{cases} \\ \Rightarrow E(I_i) = P(A) \end{array} \right)$$

Ex 1:

Consider two dependent RVs X_1, X_2 with mean 0 & variance 1, and apply Chebyshev's inequality to their average when:

(i) RVs are perfectly positively correlated (i.e. $X_1 = X_2$)

Since they are perfectly correlated, $Corr(X_1, X_2) = 1 \Rightarrow Cov(X_1, X_2) = \frac{1}{\sqrt{1 \cdot 1}} = 1$

$$E(\bar{X}_2) = E\left[\frac{X_1 + X_2}{2}\right] = \frac{1}{2}\left[E(X_1) + E(X_2)\right]$$
$$= 0$$

$$V(\bar{X}_2) = V\left(\frac{1}{2}(X_1 + X_2)\right) = \left(\frac{1}{2}\right)^2\left[V(X_1) + V(X_2) + 2Cov(X_1, X_2)\right]$$
$$= \frac{1}{4}\left[1 + 1 + 2(1)\right]$$
$$= 1$$

(ii) RVs are perfectly negatively correlated (i.e. $X_1 = -X_2$)

$$E(\bar{X}_2) = 0 \quad \& \quad V(\bar{X}_2) = \frac{1}{4}\left[V(X_1) + V(X_2) + 2Cov(X_1, X_2)\right]$$
$$= \frac{1}{4}\left[1 + 1 + 2(-1)\right]$$
$$= 0$$