B52 Sept 8 Lec 1 Notes

Random experiment: an experiment whose result is not known in advance.
Outcome: result of experiment which cannot be decomposed to simpler results.
Sample space: collection of all possible outcomes.
Event: arbitrary collection of outcomes, i.e. a subset of sample space

Probability are assigned to events

Sets are unordered collections of elements.
e.g. set $A = \{1, 2, 3\}$ has 3 elements.
Sets can be defined by listing or describing their elements

Universal Set (S) contains all possible elements

Null or **empty set** (Φ) contains no elements.

In probability,
Sample space → Universal set
Outcomes → Elements
Events → sets

Ex 1:

$E_1 = \{M, F, MF, MM, FF, FMM, FFM, FFF, MMM\}$

Ex 2:

$$E_2 = \begin{cases} (M), (F), \\ (MM), (MF), (FM), (FF), \\ (MMM), (MMF), (MFM), (FMM), (MFF), (FMF), (FFM), (FFF) \end{cases}$$

De Morgan's Laws:

$$(A \cap B)^c = A^c \cup B^c$$
$$(A \cup B)^c = A^c \cap B^c$$

Generally,

$$\left(\bigcap_{i=1}^{n} A_i\right)^c = A_1^c \cup A_2^c \cup \cdots \cup A_n^c$$
$$\left(\bigcap_{i=1}^{n} A_i\right)^c = A_1^c \cap A_2^c \cap \cdots \cap A_n^c$$

Ex 3:

$A \cap B^c$

A is a **subset** of B if every element in A is also in B.

Denoted by $A \subseteq B$

Φ is a subset of any set

If $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$

Two or more events are **disjoint** or **mutually exclusive** if they have no elements in common.

Two or more events form a **partition** if they are disjoint and their union is the sample space.

Definition: Probability Axioms

(i) $0 \leq P(A) \leq 1, \ \forall A \subseteq S$

(ii) $P(S) = 1$

(iii) If $A_1, A_2, A_3, \cdots$ mutually exclusive $\Rightarrow P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Probabity model consists of triple $(S, \{A, B, \ldots\}, P)$. If S is discrete, then the set of events are all possible subsets.

S: sample space; $\{A, B, \ldots\}$: set of events; P: probability functions

Ex 4

$A \cap (B \cup C)^c$