B52 Sept 17 Lec 2 Notes

$(x+y)^3 = (x+y)(x+y)(x+y)$

$= \sum_{i=0}^{3} \binom{3}{i} x^i y^{3-i}$

Let $i=2$, x^2y^1. 3 ways to get x^2y.

Recall $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$

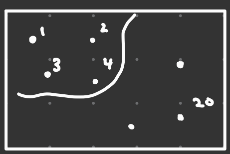
$\binom{n}{k+1}$: # ways to select k+1 out of n+1, where n+1^st element is not included

$\binom{n}{k}$: # ways to select k+1 out of n+1, where n+1^st element is included

Proof: (non-formal)

$|S| = n+1$

Ex 1:



Committee is in favour:
- 3/5 in favour : A_1
- 4/5 in favour : A_2
- 5/5 ~~in favour~~ (not possible)

$P(\text{committee is in favour}) = P(A_1 \cup A_2)$

$= P(A_1) + P(A_2)$ Since A_1 and A_2 are disjoint

$= \frac{|A_1|}{|S|} + \frac{|A_2|}{|S|}$

$|S| = C_5^{20}$

$|A_1| = C_3^4 C_2^{16}$ $\qquad |A_2| = C_4^4 C_1^{16}$

Composed of two sets

$\{\{\_,\_,\_\},\{\_,\_\}\}$

$C_3^4 \quad C_2^{16} \longrightarrow$ Choose 2 ppl not in favour out of 16.

$C_3^4 \downarrow$ Choose 3 ppl in favour out of 4.

Ex 2:

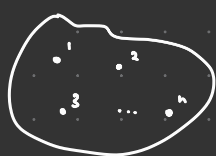
	Order matters	order doesn't matter
w/o replacement	P_k^n	C_k^n
w/ replacement	n^k	$\binom{n+k-1}{k}$

(Stars & bars)

Consider a bin for each distinct object, and place a star for every time object is selected:

| * * * | * | * | . . | |

bin for obj 1 — bin for obj 2 — bin for obj 3 — bin for obj n



$\{1, 1, 2, 3, 1\}$

There are n-1 bars, k stars

ways to arrange stars in bins =

= # ways to select the positions of stars/bars $= \binom{n+k-1}{k} = \binom{n+k-1}{n-1}$

Multinomial Rule

Number of ways to partition n objects into ℓ sets, each with $k_1, \dots, k_\ell$ objects respectively.

$$C^n_{k_1 k_2 \cdots k_\ell} = \binom{n}{k_1 k_2 \cdots k_\ell} = \frac{n!}{k_1! k_2! \cdots k_\ell!} \text{, where } \sum_{i=1}^{\ell} k_i = n$$