O S O

G T G T

5 E 5

N O N O

O S O

G T G T

5 E 5

N O N O

O S O

G T G T

5 E 5

# W11 Pre-Lecture

Reading

## Mathematical Induction

2 Steps:
1) Show it is true for the base case, usually $n=1$.
2) Show that if $n=K$ is true then $n=K+1$ is also true.

### Examples:

1. Is $3^n - 1$ a multiple of 2?

Step 1: Show it is true for $n=1$

$3^1 - 1 = 3 - 1 = 2 \Rightarrow 3^1 - 1$ is true for $n=1$

Step 2: Assume it is true for $n=K$

Assume $3^K - 1$ is true

Now, prove that $3^{K+1} - 1$ is a multiple of 2

$$3^{K+1} - 1 \Rightarrow 3 \times 3^K - 1$$
$$\Rightarrow \underbrace{2 \times 3^K}_{\text{multiple of 2}} + \underbrace{3^K - 1}_{\text{we assumed } n=K \text{ is true (multiple of 2)}}$$

So:

$3^{K+1} - 1$ is true

2. $1 + 3 + 5 + \ldots + (2n-1) = n^2$

Step 1: Show it is true for $n=1$

$1 = 1^2$ is true

Step 2: Assume it is true for $n=K$

Assume $1 + 3 + 5 + \ldots + (2K-1) = K^2$ is true

Now, prove it is true for $K+1$

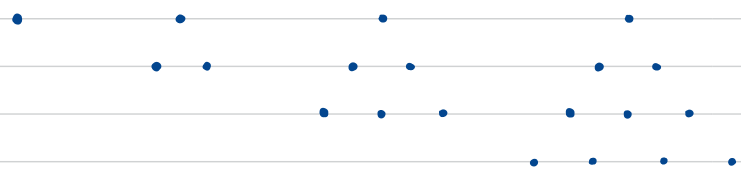
$$1+3+5+\ldots+(2K-1)+(2(K+1)-1)=(K+1)^2 \Rightarrow$$
$$\Rightarrow \quad K^2+(2(K+1)-1)=(K+1)^2$$
$$\Rightarrow \quad K^2+2K+2-1=K^2+2K+1$$
$$\Rightarrow \quad K^2+2K+1=K^2+2K+1$$

So:

$$1+3+5+\ldots+(2(K+1)-1)=(K+1)^2 \text{ is true}$$

3. Triangular numbers are numbers that can make a triangular dot pattern.

1 dot     3 dots     6 dots     10 dots



Prove that the $n^{th}$ triangular number is:

$$T_n=\frac{n(n+1)}{2}$$

Step 1: Show that it is true for $n=1$

$$T_1=1=\frac{1(1+1)}{2}$$

Step 2: Assume it is true for $n=K$.

$$T_K=\frac{K(K+1)}{2}$$

Prove it is true for $K+1$

$$T_{K+1}=\frac{K+1(K+2)}{2} \Rightarrow$$
$$\Rightarrow T_K+(K+1)=\frac{(K+1)(K+2)}{2}$$
$$\Rightarrow \frac{K(K+1)}{2}+(K+1)=\frac{(K+1)(K+2)}{2}$$
$$\Rightarrow \frac{K(K+1)+2(K+1)}{2}=\frac{K^2+K+2K+2}{2}$$
$$\Rightarrow \frac{K^2+3K+2}{2}=\frac{K^2+3K+2}{2}$$

So:

$$T_{K+1}=\frac{(K+1)(K+2)}{2} \text{ is true}$$

4. Prove that:

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

Step 1: Show that $n=1$ is true

$$1^3 = \frac{1^2(1+1)^2}{4}$$

$$1 = 1$$

Step 2: Assume $n=k$ is true

$$1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$$

Prove that $k+1$ is true:

$$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4} \Rightarrow$$

$$\Rightarrow \frac{k^2(k+1)^2}{4} + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$$

$$\Rightarrow \frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2(k+2)^2}{4}$$

$$\Rightarrow k^2 + 4(k+1) = (k+2)^2$$

$$\Rightarrow k^2 + 4k + 4 = k^2 + 4k + 4$$

So:

$$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4} \text{ is true}$$

Gradescope

1. Let $S(n)$ be:

$$\sum_{i=0}^{n-1} 2^i = 2^n - 1$$

Prove $S(n)$ is true for all $n \geq 1$ by answering the follow subquestions.

a) Base Case. Prove $S(1)$.

$$\sum_{i=0}^{1-1} 2^i = 2^1 - 1$$

$$2^0 = 2^1 - 1$$

$$1 = 1$$

b) Induction Hypothesis. State your induction hypothesis $S(k)$.

$\sum_{i=0}^{k-1} 2^i = 2^k - 1$ is true

c) Prove $S(k) \rightarrow S(k+1)$

$$\sum_{i=0}^{(k+1)-1} 2^i = 2^{k+1} - 1 \Rightarrow$$

$$\Rightarrow \sum_{i=0}^{k-1} 2^i + 2^k = 2^{k+1} - 1$$

$$\Rightarrow 2^k - 1 + 2^k = 2^{k+1} - 1$$

$$\Rightarrow 2 \cdot 2^k - 1 = 2^{k+1} - 1$$

$$\Rightarrow 2^{k+1} - 1 = 2^{k+1} - 1$$