Sec 1.2 Reading

Two matrices A and B are equal if they are the same size and if corresponding entries are equal: $a_{ij} = b_{ij}$.

If # rows equal # columns in a matrix, then the matrix is called a square matrix and the entries $a_{11}, a_{22}, \dots, a_{nn}$ form the main diagonal of the matrix.

A matrix is called diagonal if all its entries above and below the main diagonal are zero; that is, $a_{ij} = 0$ whenever $i \neq j$

A square matrix A is called upper triangular if all its entries below the main diagonal are zero; that is, $a_{ij} = 0$ whenever $i > j$

Example:

1. $$\left|\begin{array}{rrrrrcl} x_1 & - x_2 & & & + 4x_5 & = & 2 \\ & & x_3 & & - x_5 & = & 2 \\ & & & x_4 & - x_5 & = & 3 \end{array}\right|$$

We solve each equation for the leading variable:

$$\left|\begin{array}{lcl} x_1 & = & 2 + x_2 - 4x_5 \\ x_3 & = & 2 \quad\quad + x_5 \\ x_4 & = & 3 \quad\quad + x_5 \end{array}\right|$$

Let $x_2 = t$, $x_5 = r$ $\Rightarrow$ $x_1 = 2 + t - 4r$, $x_3 = 2 + r$, $x_4 = 3 + r$

The system has infinitely many solutions.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 2 + t - 4r \\ t \\ 2 + r \\ 3 + r \\ r \end{pmatrix}$$

Def: Reduced Row-Echelon Form

A matrix is said to be in reduced row-echelon form (rref) if it satisfies all of the following conditions

(a) If a row has nonzero entries, then the first nonzero entry is a 1, called the leading 1 (or pivot) in this row.
(b) If a column contains a leading 1, then all the other entries in that column are 0.
(c) If a row contains a leading 1, then each row above it contains a leading 1 further to the left.

Condition C implies that rows of 0's, if any, appear at the bottom of the matrix.