2. If A, B, C are matrices s.t. $AB = C$ and 2 of the matrices are singular (non-invertible) then so is the third.

Let $A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 10 & 7 \\ 3 & 4 \end{bmatrix}$

Which is invertible. $\Rightarrow$ False

3. If the set $S = \{ v_1, v_2, \ldots, v_n \}$ is L.I. then the set $S' = \{ v_1, v_1+v_2, v_1+v_2+v_3, \ldots, v_1+v_2+v_3+v_n \}$ is also L.I.

$$c_1(v_1) + c_2(v_1+v_2) + \overset{c_3(v_1+v_2+v_3)}{\ldots} + c_n(v_1+v_2+\ldots+v_n) = 0$$

$$c_1 v_1 + c_2 v_1 + c_2 v_2 + \ldots + c_n v_1 + c_n v_2 + \ldots + c_n v_n = 0$$

$$(c_1 v_1 + c_2 v_2 + \ldots + c_n v_n) + (c_2 v_1 + c_3 v_2$$

4. Is $W = \{ v \in \mathbb{R}^4 \mid v \cdot [3,1,0,-1] \geq 0 \} \subseteq \mathbb{R}^4$ a subspace of $\mathbb{R}^4$

Let $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \in W$

$= 1 \cdot 3 + 1 \cdot 1 + 0 + 1 \cdot (-1)$
$= 3 + 1 - 1$
$= 3 \geq 0$ ✓

If W is a subspace $r\vec{v} \in W$

Let $r = -1$

$$r\vec{v} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$r\vec{v} \cdot [3,1,0,-1] = (-1)3 + (-1)(1) + 0 + (-1)(-1)$
$= -4 \geq 0$
✗

$\Rightarrow$ False

5. Is $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \;\middle|\; x_1 + x_2 + x_3 + x_4 = 0 \right\}$ a subspace of $\mathbb{R}^4$?

True

12. If $A^2 = I$ then $A = \pm I$ where I is the $n \times n$ Id.

$AA = I$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $A^2 = I$. $\Rightarrow$ False

15. True

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

$\text{Col } A = \left\{ c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \cdots \right.$

16. $\text{Nul } A = \left\{ \vec{x} \in V \mid T(\vec{x}) = \vec{0} \right.$

$T_A : \mathbb{R}^n \to \mathbb{R}^m$

$m \vdots \cdots$ n

$\text{Nul} \in \mathbb{R}^n$