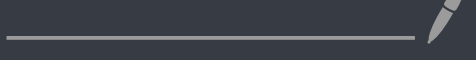

PHP Practice

1. Suppose you randomly select K of the 2016 positive integers. What is the smallest K that guarantees that at least one pair of the selected teenagers will sum to 2017?

$\frac{2016}{2} + 1 = 1009$

There are $\frac{2016}{2}$ boxes with the pairs of integers in 2016 that add up to 2017. By PHP, 1009 integers guarantees that at least two of the in integers add up to 2017

2. Missy and Mussy are very messy. Their drawer consists of 43 white socks, 2 black socks, 23 blue socks, and 8 red socks. What is the minimum number of socks they must remove from the drawer, in order to be certain that they have removed 4 socks of the same color?

The # of socks we can get without getting 4 socks of the same color is: $3 + 2 + 3 + 3 = 11$, where there are 2 for the black socks. Picking 12 socks will then guarantee that we get 4 socks with the same color.

3. 7 pairs of red socks, 6 pairs of blue socks, and 5 pairs of green socks in my drawer. If I randomly pick out 4 socks, what is the probability that I will get (at least) a matching pair?

100%. By PHP, you pick 4 socks and there are 3 colors Thus there must be 2 socks with the same color.

4. If we pick 77 numbers randomly from the set $\{1, 2, 3, 4, \ldots, 150\}$, we are guaranteed to have at least K pairs of numbers where the difference between each pair is 19. What is the max possible value of K?

$x_1, x_2, \ldots, x_{77}$

$x_1 + 19, x_2 + 19, \ldots x_{77} + 19$

$|x_1, x_2, \ldots, x_{77}, x_1 + 19, x_2 + 19, \ldots x_{77} + 19| = 154$

5. Given any 10-element subset M of $\{1, 2, 3, \ldots, 117\}$, does there exists 2 non-empty disjoint subsets of M that have the same sum?

There are $2^{10} - 2 = 1022$ subsets. The greatest possible sum is