W7 Reading Notes

Propositional Logic

A statement is a declarative sentence that can be True (1) or False (0)

Examples
↳ Milk is white T
↳ $|\phi| = 0$ T
↳ Humans are just fish with legs F

Questions and imperatives cannot be statements

Syntax

Propositions are denoted with capital letters $P, Q, R,$

P = I cheated
Q = I wrote an exam

Lowercase letters $p, q, r,$ are used for general propositions that have no meaning
These are used for general proofs

Connectives

p is a well-formed formula (wff)

$\neg p \Rightarrow$ not p
$p \wedge q \Rightarrow$ p and q
$p \vee q \Rightarrow$ p or q
$p \rightarrow q \Rightarrow$ if p, then q

Truth Tables

Negation ($\neg, \sim$)

P	$\neg P$
1	0
0	1

neg $P = 1 - P$

Conjunction ($\wedge, \&, \cdot$)

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

$p \wedge q = \min\{p, q\}$

# of rows = $2^{\text{\# of statements}}$

Disjunction ($\vee$, $+$)

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

$p \vee q = \max\{p, q\}$

Conditionals ($\rightarrow$, $\supset$)

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

$p \rightarrow q = 1$ iff $p \leq q$

Biconditional ($\leftrightarrow$, $\equiv$)

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

$p = q$ then $p \Leftrightarrow q = 1$

Exclusive Or ($\oplus$, $\veebar$)

p	q	$p \oplus q$
1	1	0
1	0	1
0	1	1
0	0	0

$p \neq q$, then $p \oplus q = 1$

Proofs Using Truth Tables

Formulas p and q are logically equivalent iff the truth conditions of p are the same as the truth conditions of q

$p \Leftrightarrow q$ iff

p	q
x	x
y	y

Examples

1 Is $(p \wedge q) \Leftrightarrow \neg(p \vee q)$?

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$
1	1	1	1	0
1	0	0	1	0
0	1	0	1	0
0	0	0	0	1

Truth table for $p \wedge q$ and $\neg(p \vee q)$ does not equal Thus $(p \wedge q)$ is not $\Leftrightarrow \neg(p \vee q)$

Logically equivalent

2 Show that $(p \vee \neg p)$ is $\underbrace{\text{always true}}_{\text{a tautology}}$

p	$\neg p$	$p \vee \neg p$
1	0	1
0	1	1

The truth table shows that $(p \vee \neg p)$ is always true

3 Show that $(p \wedge \neg p)$ is $\underbrace{\text{always false}}_{\text{a contradiction}}$

p	$\neg p$	$p \wedge \neg p$
1	0	0
0	1	0

The truth table shows that $(p \wedge \neg p)$ is always false

Working with Connectives

The following are all equivalent

$$(P \wedge Q) \vee R$$

Commutative Law

$$R \vee (P \wedge Q)$$

Distributive Law

$$(R \vee P) \wedge (R \vee Q)$$

Order of precedence for connectives $\neg, \wedge, \vee, \rightarrow$

So

$$\neg P \vee Q \wedge T \rightarrow S \wedge R \vee \neg Q$$

is

$$(\neg P \vee (Q \wedge T)) \rightarrow ((S \wedge R) \vee \neg Q)$$

Gradescope Pre-Exercise 7

1 Determine the value of $p \wedge \neg(q \wedge r)$

p	q	r	$q \wedge r$	$\neg(q \wedge r)$	$p \wedge \neg(q \wedge r)$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	T	F

True

2 Determine the value of $(p \wedge \neg q) \vee (p \wedge \neg r)$

p	q	r	$\neg q$	$\neg r$	$p \wedge \neg q$	$p \wedge \neg r$	$(p \wedge \neg q) \vee (p \wedge \neg r)$
T	T	T	F	F	F	F	F
T	T	F	F	T	F	T	T
T	F	T	T	F	T	F	T
T	F	F	T	T	T	T	T
F	T	T	F	F	F	F	F
F	T	F	F	T	F	F	F
F	F	T	T	F	F	F	F
F	F	F	T	T	F	F	F

True

3 What do you notice about Q1 and Q2

They are the same, so equivalent