```
In [197]: import pandas as pd
          import numpy as np
          import statistics
          # Visualizations
          import seaborn as sns
          import matplotlib.pyplot as plt
          # Pre-Processing
          from sklearn.model_selection import train_test_split
          from sklearn.preprocessing import StandardScaler
          # Models
          from sklearn.linear_model import LogisticRegression
          from sklearn.naive_bayes import GaussianNB
          from sklearn.linear_model import RidgeClassifierCV
          from sklearn.neural_network import MLPClassifier
          from sklearn import ensemble
          # Linear Regression
          from sklearn import linear_model
          from sklearn.linear_model import LinearRegression
          import statsmodels.api as sm
          from scipy import stats
          # Evaluation
          from sklearn.metrics import confusion_matrix
```

1. Data Pre-Processing/Cleaning

Data Reading and Overview

```
In [112]: df=pd.read_csv("census_income_data.csv")
    df.head(10)
```

Out[112]:

age		workclass	fnlwgt	education	education_num	marital_status	occupation	relationship	race	sex	capital_gain	capital_loss	hours_per_week	native_country	income
0	39	State-gov	77516	Bachelors	13	Never-married	Adm-clerical	Not-in-family	White	Male	2174	0	40	United-States	<=50K
1	50	Self-emp-not-inc	83311	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	13	United-States	<=50K
2	38	Private	215646	HS-grad	9	Divorced	Handlers-cleaners	Not-in-family	White	Male	0	0	40	United-States	<=50K
3	53	Private	234721	11th	7	Married-civ-spouse	Handlers-cleaners	Husband	Black	Male	0	0	40	United-States	<=50K
4	28	Private	338409	Bachelors	13	Married-civ-spouse	Prof-specialty	Wife	Black	Female	0	0	40	Cuba	<=50K
5	37	Private	284582	Masters	14	Married-civ-spouse	Exec-managerial	Wife	White	Female	0	0	40	United-States	<=50K
6	49	Private	160187	9th	5	Married-spouse-absent	Other-service	Not-in-family	Black	Female	0	0	16	Jamaica	<=50K
7	52	Self-emp-not-inc	209642	HS-grad	9	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	45	United-States	>50K
8	31	Private	45781	Masters	14	Never-married	Prof-specialty	Not-in-family	White	Female	14084	0	50	United-States	>50K
9	42	Private	159449	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	5178	0	40	United-States	>50K

```
In [113]: df.shape
Out[113]: (48842, 15)
```

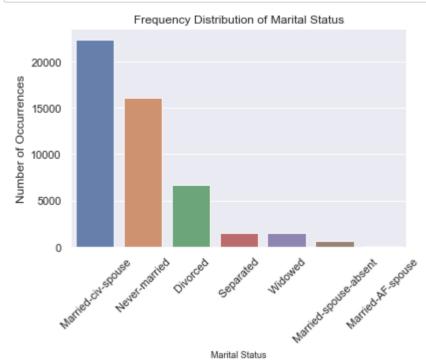
Data Type Overview

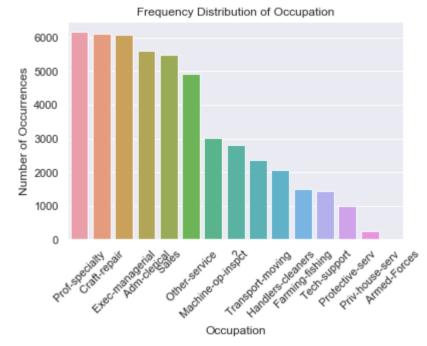
```
In [114]: df.info()
          <class 'pandas.core.frame.DataFrame'>
          RangeIndex: 48842 entries, 0 to 48841
          Data columns (total 15 columns):
          age
                            48842 non-null int64
                            48842 non-null object
          workclass
                            48842 non-null int64
          fnlwgt
          education
                            48842 non-null object
                            48842 non-null int64
          education_num
                            48842 non-null object
          marital_status
          occupation
                            48842 non-null object
                            48842 non-null object
          relationship
                            48842 non-null object
          race
                            48842 non-null object
          sex
                            48842 non-null int64
          capital_gain
          capital_loss
                            48842 non-null int64
          hours_per_week
                            48842 non-null int64
          native_country
                            48842 non-null object
          income
                            48842 non-null object
          dtypes: int64(6), object(9)
          memory usage: 5.6+ MB
```

Check for Missing Values

```
In [115]: df.isnull().values.sum()
Out[115]: 0
```

Visualizations on some features





Data Cleaning

Feature Selection

```
In [120]: df=df.drop(["fnlwgt","education"], axis=1)
```

Target Variable Overview

Problem: Same class is coded differently

```
In [121]: df.groupby("income").count()
```

Out[121]:

	age	workclass	education_num	marital_status	occupation	relationship	race	sex	capital_gain	capital_loss	hours_per_week	native_country
income												
<=50K	24720	24720	24720	24720	24720	24720	24720	24720	24720	24720	24720	24720
<=50K.	12435	12435	12435	12435	12435	12435	12435	12435	12435	12435	12435	12435
>50K	7841	7841	7841	7841	7841	7841	7841	7841	7841	7841	7841	7841
>50K.	3846	3846	3846	3846	3846	3846	3846	3846	3846	3846	3846	3846

Cleaning Target Variable

The Problem

```
In [122]: # The Problem
df.groupby("income").mean()
```

Out[122]:

	age	education_num	capital_gain	capital_loss	hours_per_week
income					
<=50K	36.783738	9.595065	148.752468	53.142921	38.840210
<=50K.	37.048010	9.605308	143.547004	56.157780	38.839727
>50K	44.249841	11.611657	4006.142456	195.001530	45.473026
>50K.	44.326833	11.584763	4115.832033	190.526781	45.411856

The Solution

```
In [123]: #The solution
            df["income"] = df["income"].astype('category')
            df["income"] = df["income"].cat.codes
            df.loc[df.income==1, 'income'] = 0
           df.loc[df.income==2, 'income'] = 1
df.loc[df.income==3, 'income'] = 1
            df["income"] = df["income"].astype('category')
            df.groupby("income").mean()
Out[123]:
                          age education_num capital_gain capital_loss hours_per_week
             income
                  0 36.872184
                                    9.598493
                                             147.010308
                                                           54.151931
                                                                           38.840048
                  1 44.275178
                                    11.602807 4042.239497 193.528964
                                                                           45.452896
```

Transforming categorical features

```
In [124]: df.info()
          <class 'pandas.core.frame.DataFrame'>
          RangeIndex: 48842 entries, 0 to 48841
          Data columns (total 13 columns):
                            48842 non-null int64
          age
          workclass
                            48842 non-null object
                            48842 non-null int64
          education_num
                            48842 non-null object
          marital_status
                            48842 non-null object
          occupation
          relationship
                            48842 non-null object
                            48842 non-null object
          race
                            48842 non-null object
          sex
          capital_gain
                            48842 non-null int64
          capital_loss
                            48842 non-null int64
                            48842 non-null int64
          hours_per_week
          native country
                            48842 non-null object
                            48842 non-null category
          income
          dtypes: category(1), int64(5), object(7)
          memory usage: 4.5+ MB
In [125]: categoricals=df.select_dtypes(include="object").columns
          df=pd.get_dummies(df,columns=categoricals,drop_first=True)
In [126]: columns = df.columns.tolist()
          columns.remove("income")
          columns.append("income")
In [127]: df=df[columns]
          df.shape
Out[127]: (48842, 85)
```

Standardize Numerical Variable

_	age	education_num	capital_gain	capital_loss	hours_per_week	workclass_ Federal- gov	workclass_ Local-gov	workclass_ Never- worked	workclass_ Private	workclass_ Self-emp- inc	 native_country_ Puerto-Rico	native_country_ Scotland	native_country_ South	native_country_ Taiwan	native_country Thailan
0	0.025996	1.136512	0.146932	-0.217127	-0.034087	0	0	0	0	0	 0	0	0	0	
1	0.828308	1.136512	-0.144804	-0.217127	-2.213032	0	0	0	0	0	 0	0	0	0	
2	-0.046942	-0.419335	-0.144804	-0.217127	-0.034087	0	0	0	1	0	 0	0	0	0	
3	rows × 85 c	columns													
4															>

Summary Data Processing

- 2 variables are dropped because they are (1) irrelevant: "fnlwgt" or (2) redundant: "education"
- the Target variable is unified and transformed to a dummy
- all numerical variables are standarized
- all categorical variables are transformed into dummy variables with m-1 columns for m manifestations

Linear Regression for Variable Effect Overview

OLS Regression Results

Dep. Variable: income R-squared: 0.362 Model: OLS Adj. R-squared: 0.361 Least Squares F-statistic: Method: 333.9 Date: Tue, 15 Oct 2019 Prob (F-statistic): 0.00 Time: 15:21:55 Log-Likelihood: -16708. No. Observations: 48842 AIC: 3.358e+04 3.432e+04 Df Residuals: 48758 BIC: Df Model: 83 nonrobust Covariance Type:

Covariance Type: nonrobust						
	coef	std err	t	P> t	[0.025	0.975]
const	-0.4532	0.031	-14.643	0.000	-0.514	-0.393
age	0.0026	0.000	17.935	0.000	0.002	0.003
education_num	0.0313	0.001	41.091	0.000	0.030	0.033
<pre>capital_gain capital loss</pre>	8.065e-06 9.285e-05	2.11e-07 3.87e-06	38.164 24.012	0.000 0.000	7.65e-06 8.53e-05	8.48e-06 0.000
hours_per_week	0.0028	0.000	20.261	0.000	0.003	0.003
workclass_ Federal-gov	0.1119	0.011	10.144	0.000	0.090	0.134
workclass_ Local-gov	0.0164	0.009	1.747	0.081	-0.002	0.035
workclass_ Never-worked	0.0739	0.108	0.684	0.494	-0.138	0.286
<pre>workclass_ Private workclass_ Self-emp-inc</pre>	0.0386 0.0958	0.007 0.011	5.167 8.730	0.000 0.000	0.024 0.074	0.053 0.117
workclass_ Self-emp-not-inc	-0.0313	0.009	-3.445	0.001	-0.049	-0.013
workclass_ State-gov	-0.0008	0.010	-0.075	0.940	-0.021	0.020
workclass_ Without-pay	-0.0601	0.071	-0.844	0.399	-0.200	0.080
<pre>marital_status_ Married-AF-spouse marital_status_ Married-civ-spouse</pre>	0.0833 0.1257	0.059 0.019	1.407 6.557	0.159 0.000	-0.033 0.088	0.199 0.163
marital_status_ Married-spouse-absent	0.0432	0.014	3.003	0.003	0.015	0.071
marital_status_ Never-married	-0.0027	0.006	-0.470	0.638	-0.014	0.009
marital_status_ Separated	0.0198	0.010	2.033	0.042	0.001	0.039
marital_status_ Widowed	0.0190	0.010	1.888	0.059	-0.001	0.039
occupation_ Adm-clerical occupation_ Armed-Forces	-0.0023 0.0336	0.008 0.084	-0.303 0.398	0.762 0.691	-0.017 -0.132	0.013 0.199
occupation_ Craft-repair	-0.0186	0.008	-2.441	0.015	-0.034	-0.004
occupation_ Exec-managerial	0.1368	0.008	17.926	0.000	0.122	0.152
occupation_ Farming-fishing	-0.0973	0.011	-9.133	0.000	-0.118	-0.076
occupation_ Handlers-cleaners occupation_ Machine-op-inspct	-0.0536 -0.0476	0.010 0.009	-5.584 -5.463	0.000 0.000	-0.072 -0.065	-0.035 -0.031
occupation_ Other-service	-0.0218	0.008	-2.786	0.005	-0.037	-0.006
occupation_ Priv-house-serv	0.0158	0.022	0.712	0.476	-0.028	0.059
occupation_ Prof-specialty	0.1057	0.008	13.461	0.000	0.090	0.121
occupation_ Protective-serv	0.0558	0.012	4.507	0.000	0.032	0.080
occupation_ Sales occupation_ Tech-support	0.0432 0.0620	0.008 0.011	5.621 5.846	0.000 0.000	0.028 0.041	0.058 0.083
occupation_ Transport-moving	-0.0412	0.009	-4.439	0.000	-0.059	-0.023
relationship_ Not-in-family	-0.1613	0.019	-8.447	0.000	-0.199	-0.124
relationship_ Other-relative	-0.1373	0.019	-7.315	0.000	-0.174	-0.100
relationship_ Own-child	-0.1326 -0.1514	0.019	-6.952 -7.645	0.000 0.000	-0.170 -0.190	-0.095
relationship_ Unmarried relationship_ Wife	0.0956	0.020 0.009	10.972	0.000	0.079	-0.113 0.113
race_ Asian-Pac-Islander	0.0478	0.021	2.320	0.020	0.007	0.088
race_ Black	0.0299	0.017	1.803	0.071	-0.003	0.062
race_ Other	0.0354	0.023	1.509	0.131	-0.011	0.081
race_ White sex_ Male	0.0476 0.0553	0.016 0.005	2.998 11.963	0.003 0.000	0.016 0.046	0.079 0.064
native_country_ Cambodia	0.1231	0.066	1.859	0.063	-0.007	0.253
native_country_ Canada	0.0685	0.028	2.453	0.014	0.014	0.123
native_country_ China	-0.0509	0.035	-1.463	0.144	-0.119	0.017
native_country_ Columbia	-0.0858 0.0172	0.039 0.031	-2.207 0.549	0.027 0.583	-0.162 -0.044	-0.010 0.079
<pre>native_country_ Cuba native_country_ Dominican-Republic</pre>	-0.0056	0.031	-0.157	0.875	-0.044 -0.076	0.065
native_country_ Ecuador	-0.0084	0.052	-0.161	0.872	-0.111	0.094
<pre>native_country_ El-Salvador</pre>	0.0597	0.030	1.987	0.047	0.001	0.118
native_country_ England	0.0900	0.033	2.770	0.006	0.026	0.154
<pre>native_country_ France native_country_ Germany</pre>	0.1148 0.0310	0.057 0.027	2.029 1.169	0.042 0.243	0.004 -0.021	0.226 0.083
native_country_ Greece	0.0109	0.050	0.217	0.829	-0.087	0.109
native_country_ Guatemala	0.0615	0.039	1.597	0.110	-0.014	0.137
native_country_ Haiti	0.0353	0.041	0.854	0.393	-0.046	0.116
<pre>native_country_ Holand-Netherlands native_country_ Honduras</pre>	-0.1589 0.0444	0.341 0.077	-0.465 0.575	0.642 0.566	-0.828 -0.107	0.510 0.196
native_country_ Hong	-0.0328	0.064	-0.512	0.609	-0.158	0.093
native_country_ Hungary	0.0446	0.079	0.564	0.573	-0.111	0.200
native_country_ India	0.0157	0.031	0.499	0.618	-0.046	0.077
native_country_ Iran	0.0207	0.046	0.451	0.652	-0.069	0.111
<pre>native_country_ Ireland native_country_ Italy</pre>	0.1228 0.0802	0.057 0.035	2.142 2.267	0.032 0.023	0.010 0.011	0.235 0.150
native_country_ Jamaica	0.0247	0.035	0.698	0.485	-0.045	0.094
native_country_ Japan	0.0413		1.092	0.275		0.116
native_country_ Laos	-0.0794	0.073				0.064
native_country_ Mexico native_country_ Nicaragua	0.0234 -0.0301	0.017 0.050	1.413 -0.599	0.158 0.549	-0.009 -0.128	0.056 0.068
native_country_ Outlying-US(Guam-USVI-etc)	-0.0792	0.072	-1.099	0.272	-0.220	0.062
native_country_ Peru	-0.0574	0.052	-1.110	0.267	-0.159	0.044
native_country_ Philippines	0.0408	0.025	1.607	0.108	-0.009	0.091
native_country_ Poland	-0.0167	0.038	-0.434 1 212	0.664	-0.092 -0.038	0.059
<pre>native_country_ Portugal native_country_ Puerto-Rico</pre>	0.0570 -0.0030	0.043 0.028	1.313 -0.107	0.189 0.915	-0.028 -0.058	0.142 0.052
native_country_ Scotland	-0.1063	0.028	-1.411	0.158	-0.254	0.032
native_country_ South	-0.0836	0.036	-2.351	0.019	-0.153	-0.014
native_country_ Taiwan	0.0064	0.045	0.142	0.887	-0.082	0.095
native_country_ Thailand	-0.0673	0.064	-1.052 -1.202	0.293	-0.193	0.058
<pre>native_country_ Trinadad&Tobago native_country_ United-States</pre>	-0.0870 0.0287	0.067 0.012	-1.303 2.404	0.193 0.016	-0.218 0.005	0.044 0.052
native_country_ Vietnam	-0.0592	0.040			-0.138	0.020
native_country_ Yugoslavia	0.0989	0.072	1.372	0.170	-0.042	0.240
2226 226 Du		=======				
Omnibus: 2236.226 Dur	bin-Watson:		1.999	1		

Omnibus: 2236.226 Durbin-Watson: 1.999 Prob(Omnibus): 0.000 Jarque-Bera (JB): 2489.501 Skew: 0.541 Prob(JB): 0.00 Kurtosis: 2.770 Cond. No. 1.49e+17 _____

^[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

^[2] The smallest eigenvalue is 1.25e-22. This might indicate that there are

strong multicollinearity problems or that the design matrix is singular.

2. Prediction

This is a supervised 2-class classification problem.

2.1 Comparison of Default Models

Five different models in their default setting are trained on a train set (80%) and evaluated on the test set (20%) with their prediction accuracy.

```
In [135]: def prediction(df, model="logit", seed=1):
               '''This function returns the accuracy of a
              prediction dependent on the model used.'''
              # Create Train and Test Sets
              X=df.loc[:,df.columns!="income"]
              y=df["income"].values.ravel()
              X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.2, random_state = seed)
              # Select Model
              if model=="Logit":
                  model= LogisticRegression(random_state=1, solver='liblinear')#regularization is applied by default
              elif model=="Naive Bayes":
                  model= GaussianNB()
              elif model=="Multi-layer Perceptron":
                  model= MLPClassifier()
              elif model=="Ridge Classifier":
                  model= RidgeClassifierCV()
              elif model=="Gradient Boosting":
                  model= ensemble.GradientBoostingClassifier()
              # Fit Model and Return Accuracy
              model.fit(X_train, y_train)
              accuracy=model.score(X_test,y_test)
              return(accuracy)
```

The results are illustrated in a table

```
In [131]: def create results(result table,n,model):
               '''This functions shows the mean accuracy (and its variance)
               over n different train-test-splits.'''
               acc=[]
               for seed in range(n):
                  acc_n=prediction(df,seed=seed,model=model)
                  acc.append(acc_n)
              mean = sum(acc)/len(acc)
               sd = statistics.stdev(acc)
               result_table.loc[model,"Mean"] = mean
               result_table.loc[model,"SD"] = sd
               return(result_table)
  In [ ]: | columns=["Mean", "SD"]
           models=["Naive Bayes",
                   "Logit",
                   "Ridge Classifier",
                   "Gradient Boosting",
                   "Multi-layer Perceptron"]
           result_df=pd.DataFrame(columns=columns, index=models)
           for model in models:
               results=create_results(result_df,10,model)
```

Results with default parameters

```
    In [133]:
    result_df

    Out[133]:
    Mean
    SD

    Naive Bayes
    0.58007
    0.026521

    Logit
    0.851377
    0.00390552

    Ridge Classifier
    0.84087
    0.00274896

    XGB
    0.867407
    0.00288284

    MLP
    0.850537
    0.00539972
```

2.2 Manual Optimization of Parameters for Gradient Boost Model

Since the Gradient Boosting Classifier has 2 percentage points higher accuracy than the other classifiers in its default settings, this classifier is further optimized. (It is possible that another classifier is even superior when optimized in its parameters.)

The following parameters are chosen for a simultaneous optimization

- subsample (default 1.0) "Choosing subsample < 1.0 leads to a reduction of variance and an increase in bias"
- max_depth (default 3) "The maximum depth limits the number of nodes in the tree. Tune this parameter for best performance; the best value depends on the interaction of the input variables."

 $from \ its \ \underline{scikit\text{-learn documentation } (\underline{https://scikitlearn.org/stable/modules/generated/sklearn.ensemble.GradientBoostingClassifier.html})}{}$

```
In [ ]: | def gradient_boost(dataframe,n,subsample=subsample, max_depth = max_depth):
               '''This function returns the accuracy of a
              gradient boost prediction dependent on the
              parameters subsample and max_depth used.'''
              X = df.loc[:,df.columns!="income"]
              y = df["income"].values.ravel()
              acc = []
              for seed in range(n):
                  # Create Train and Test Sets
                  X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.2, random_state = seed)
                  # Set Parameters
                  model = ensemble.GradientBoostingClassifier(subsample=subsample,max_depth=max_depth)
                  # Fit Model and Return Accuracy
                  model.fit(X_train, y_train)
                  accuracy = model.score(X_test,y_test)
                  acc.append(accuracy)
              mean_accuracy = sum(acc)/len(acc)
               return(mean_accuracy)
  In [ ]: | for subsample in subsamples:
               print(subsample)
                  for max_depth in max_depths:
                      subsample_num = float(subsample)
                      max_depth_num = float(max_depth)
                      result=gradient_boost(dataframe=df, n=3, subsample=subsample_num, max_depth=max_depth_num)
                      parameter_selection_1.loc[max_depth,subsample]=result
In [187]: | parameter_selection_1
```

Out[187]:

```
0.3
                0.6
                          1
1 0.85471 0.855222 0.855666
2 0.861876 0.861774 0.861228
3 0.866073 0.867677 0.867506
6 0.870782
              0.873 0.873921
9 0.868496 0.870202 0.871259
```

Zooming closer to the optimal area

```
In [ ]: | subsamples_2 = ["0.5","0.75","0.9","1"]
        max_depths_2 = ["5","6","7","8"]
        parameter_selection_2=pd.DataFrame(columns=subsamples_2, index=max_depths_2)
        for subsample in subsamples_2:
            print(subsample)
            for max_depth in max_depths_2:
                subsample_num = float(subsample)
                max_depth_num = int(max_depth)
                result=gradient_boost(dataframe=df, n=3, subsample=subsample_num, max_depth=max_depth_num)
                print(result)
                parameter_selection_2.loc[max_depth,subsample]=result
```

In [191]: parameter_selection_2

Out[191]:

```
0.75
                         0.9
                                    1
       0.5
5 0.872215 0.872863 0.873341 0.872761
6 0.872283 0.873307 0.873546 0.873443
7 0.873068 0.873136 0.873716 0.873068
8 0.870782 0.871737 0.872693 0.872658
```

Result

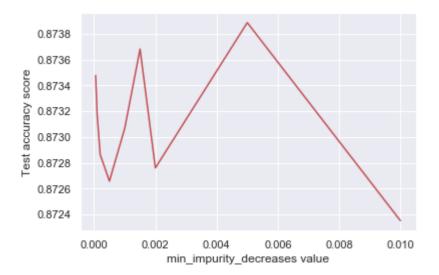
The results do not change much in these areas of the two considered parameters. We chose 0.9 for subsample and and 7 for the maximal depth as this results in the highest accuracy of 0.8737.

Iterative Optimization Approach

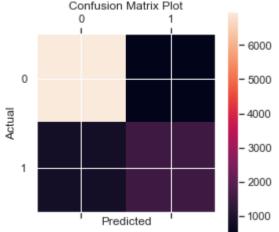
After the simultaneous optimization in a two-dimensional space, another, third, parameter (min_impurity_decrease) is optimized at the optimum of the first two parameters. (It is possible that the overall optimum of these three parameters is different from this result. The computational costs are significantly lower for the iterative optimization though.)

```
In [196]: min_impurity_decreases = [0.00005,0.0001,0.0002,0.0005,0.001,0.0015,0.002,0.005,0.01]
          test_acc = []
          for min_impurity_decrease in min_impurity_decreases:
              X = df.loc[:,df.columns!="income"]
              y = df["income"].values.ravel()
              X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.2, random_state = 1)
              model = ensemble.GradientBoostingClassifier(subsample=0.9, max_depth=7, min_impurity_decrease=min_impurity_decrease)
              model.fit(X train, y train)
               accuracy = model.score(X_test,y_test)
              test_acc.append(accuracy)
```

```
In [203]: plt.plot(min_impurity_decreases, test_acc, "r")
          plt.ylabel("Test accuracy score")
          plt.xlabel("min_impurity_decreases value")
          plt.show()
```



There does not seem to be a significant change corresponding to the min_impurity_decrease variable. The fluctuations indicate a lack of significance. Nevertheless, we chose 0.005 as it leads to the maximal accuracy.



In [207]: accuracy = model.score(X_test,y_test)
accuracy

Out[207]: 0.8725560446309756