Intermediate State in Type I superconducting sphere: pinning and size effect

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Irina Shapiro and Boris Ya. Shapiro

Bar Ilan University, Department of Physics and Institute of Superconductivity, Ramat Gan, Israel

E-mail: shapib@biu.ac.il

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Abstract

Simulations, based on the time dependent Ginzburg-Landau equations, show that the magnetization and spatial structure of the intermediate state strongly affected both by the radius of the sphere and by pinning centres concentration. The intermediate states undergoes transformation from one-domain state in small clean sphere to multi-domain structure in big spheres. In spheres where part of the superconducting material replaced by the 0.5% randomly distributed normal phase (dirty case) the intermediate state demonstrates a well pronounced turbulence behaviour.

Keywords: intermediate state, flux turbulence, type-I superconductors

1. Introduction

The phenomenon of a bulk superconductivity in a magnetic field was a topic of many years efforts and well descried in numerous text books [1]. Long before BCS [2] microscopic theory some very successful phenomenological theories of superconductors had been conceived. The most powerful tool is the Ginzburg-Landau (GL) theory, conceived in 1950 [3]. Considering superconductivity as a phase transition they constructed gauge invariant theory with Lagrangian similar to that considered in field theory. Microscopic superconducting theory reads $\kappa = \lambda/\xi$ (Abrikosov parameter, where λ and ξ are the penetration magnetic field length and the coherence length correspondingly) is a constant material Ginzburg-Landau (GL) parameter. In their original publication, Ginzburg and Landau showed that the solutions of their GL equations behave quite differently when $\kappa < 1/\sqrt{2}$ and $\kappa > 1/\sqrt{2}$. This property distinguishes what have come to be called type-I and type-II superconductors. The cylindrical, infinite in the field direction type-I superconductor expels the magnetic field from its interior at fields smaller than the thermodynamic critical field H_c while for applied fields larger than the critical field, the sample is in the normal state, fully penetrated by the magnetic field. Magnetization in type-I superconductor demonstrate hysteresis at small κ parameter (κ<0.42) [4]. In a real system geometry of the sample external magnetic field reaches its critical values in some parts of the system while it still smaller in the rest (diamagnetic factor). This factor leads to the appearance of an intermediate state (IS), in which regions of both normal and Meissner state coexist. Due to the proximity effect these regions are overlapped blurring the borders between domains. superconducting sphere is the oldest studied example of the system where the diamagnetic factor (n=1/3) strongly affects the intermediate state (IS) [5] appears at the magnetic fields $2H_c/3 < H < H_c$ where the normal and superconducting domains coexist. Generally, the type-I superconductors represent a perfect physical system where it is relatively easy to tune the parameters and try to understand the physics behind the observed topology of the intermediate state establishment [6]. Experimental and theoretical effort is growing to obtain a general understanding of the problem [6–12]. The fundamental problem, however, is that in a finite system, it is impossible to predict the topology of the intermediate state based solely on the energy minimization [7]. In fact in this approach we have to guess the topology of the IS and minimize the energy after then. A simple stripe model [7] allows analyze analytically the intermediate state in type – I superconductors [7-9] while later refinements (such as domain widening and/or branching) tried to address apparent inconsistencies between the model and the experiment [10–17]. Still, a comprehensive description has never been achieved, with the main problem being multiple observations of the closed-topology structures (flux tubes) [18] in best samples, laminar [19] and turbulent [13] structures even assumption on the final steady IS cannot be done. From this point of view exact numerical simulation of the time dependent GL equations in mesoscopic samples is the only way to study dynamics of the intermediate state from the "first principle" with no initial assumption on the spatial distribution of the order parameter and magnetic field inside the sample. There are several factors that affect and often determine the topology of the intermediate state. The issue crucial to understand the topology of the intermediate state in superconducting sphere is sample size (size effect) while the inclusions of the normal phase (pinning centers) are also the essentially important factor strongly modified the IS [20-25]. In spite of considering problem of the IS in superconducting sphere the problem is still far from understanding. First of all most of the numerical simulation were performed for large GL parameter (κ=0.4 for Pb in the Ref. [26]) so the dependence of the IS on κ was ignored along with the hysteresis behavior of the magnetization. Secondly, the size effect and pinning effect affecting the IS structure is the problem which has not been studied systematically. (In fact in the overwhelming majority of published papers flux pinning was omitted). In last, our calculations method in contrast to others allow to study dynamic of the IS inside the sphere. Restricting here by spherical geometry of the sample we study numerically the IS in two spheres "big" and "small" in a wide ranges of the magnetic fields at small GL parameter κ =0.18 (typical for Sn). We study the pinning effect on the IS and found the topologically caused change in magnetization in the big spheres where IS has the multi-domain form. Hysteresis in magnetization recorded in the clean spheres with practically absent in dirty samples.

2. Model and numerical simulation.

The theory considered magnetization as a function of the external magnetic field for type I superconducting spheres with radiuses in the range $\lambda \ll \xi_{eff} \ll R \ll \xi$, where λ is the magnetic penetration depth, ξ is the coherence length of the clear materials, while $\xi_{eff}^{-1} \quad = \xi^{-1} + l^{-1}$ (l is the mean free electron path) is the effective coherence length in dirty superconductors. The magnetic moment of a sphere with radius R subjected to an external magnetic field H was studied by solving numerically the time dependent Ginzburg-Landau (GL) scheme [27]. Starting from the dimensionless GL Hamiltonian [28]

$$G\{A,\psi\} = \iiint d^3r \left(-(1-T)|\psi|^2 + \frac{1}{2}|\psi|^4 + \left|\left(\frac{\partial}{\partial r_i} - -iA_i\right)\psi\right|^2 + \kappa^2(\partial\times A - H)^2\right); \qquad (1)$$

Here Abrikosov parameter κ(T) is the ratio κ=λ/ξ (where λ and ξ are the magnetic penetration and coherence lengths correspondingly), ψ is the order parameter and A is the vector potential in $\sqrt{2}\,\lambda H_{cm}$ units, H_{cm} is the bulk thermodynamic magnetic field, the applied magnetic field H and the magnetic moment M is in $\sqrt{2}\,H_{cm}$ units), while coordinate is scaled by the coherence length. In dirty superconductors $\kappa = 0.96\lambda_L/l$ [29]. It is assumed that parameter κ is reduced in big spheres.The relaxation equations [30] have the form:

$$\frac{\partial\psi}{\partial t} = -\frac{\delta G}{\delta\psi^*} + f_\psi; \ \frac{\partial A_\nu}{\partial t} = -\frac{1}{2}\frac{\delta G}{\delta A_\nu} + f_A \qquad (2)$$

where f_ψ, f_A are the magnitude of the order parameter and magnetic vector potential random noise. Using rectangular Cartesian greed (h is the step of the greed) and completed set of the equations by the boundary conditions with $H = H_z$ far from the sphere, one obtain taking the boundary condition at the sphere border (zero of the normal component of the order parameter gauge gradient). A link-variables approach method [31] was applied to rectangular Cartesian grid with steps h in all directions. The set of the equations (2) was solved numerically up to stationary state for any external magnetic field. The stationary state solution provides the local magnetic field inside the sphere $H(r)$ and the magnetic induction $B = \int H(r)d^3r / \int d^3r$, while the magnetic moment has usual form: $M(H) = (B-H)/4\pi$. The topology of the intermediate state and its characteristics strongly depends on radius of the sphere where the order parameter inside the sphere is presented for various magnetic field at the magnetization curve. We consider type-I superconductivity with Abrikosov parameter κ=0.18 (in clean case) in two spherical samples: small, with radius $R = 6\xi$ (here ξ is the coherence length) and big. With $R = 15\xi$. (All calculations performed for reduced temperature $\theta = \frac{T}{T_c} = 0.5$).

2.1 Small Sphere R=6 ξ, κ=0.18.

Initial state of the superconducting sphere in zero magnetic field is plotted in Fig. 1 where colors scale denotes the magnitude of the superconducting electrons density changing from 0.5 (red colour corresponds to the uniform superconducting state where $|\psi|^2 = 1 - \theta = 1/2$) to zero (black).

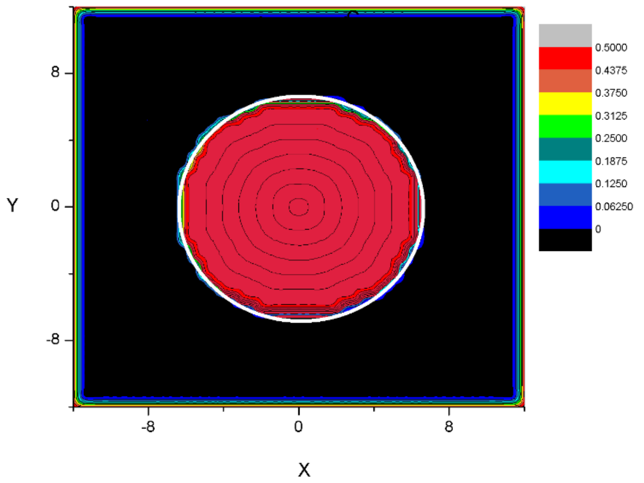

Fig.1. Spatial distribution of the superconducting electrons in zero magnetic field. Colours scale defines the magnitude of the superconducting electrons density $|\psi|^2$. Magnetic field is directed in z-direction.

Magnetization curve $M(H)$ is calculated both for increasing external magnetic field from zero to critical field H_{sh} denoted as forward direction (here $M(H_{sh}) = 0$) and in the "back" direction if the magnetic field decreases from H_s to zero. Lower, more deep, curve demonstrates magnetization of the clean superconducting sphere while upper, more shallow curve shows magnetization of the sphere with normal phase inclusions ("dirty case") containing 0.5% of the total volume of the sphere.

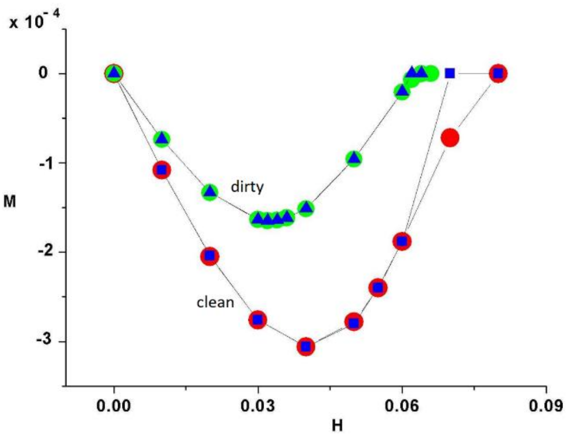

Fig.2 Forward and back magnetization of small superconducting sphere as a function of external magnetic field

This figure demonstrates weak hysteresis for clean sphere superconductor (red circles for "forward" magnetization and blue square for "back" magnetization) and for "dirty" sample. Green circles for "forward" magnetization coincide with "back" magnetization (blue triangles). Two curves represent magnetization both of the clean magnetization while in dirty sphere hysteresis is absent at all. The magnetization curves for a small sphere demonstrates reversibility at most values of the magnetic field values. Intermediate state inside the sphere is strongly different in clean and dirty samples as it is presented in Fig.3 where density of the superconducting electrons $n_s = |\psi|^2$ is plotted for the same magnitudes of the magnetic fields in both cases in x-y and y-z projections.

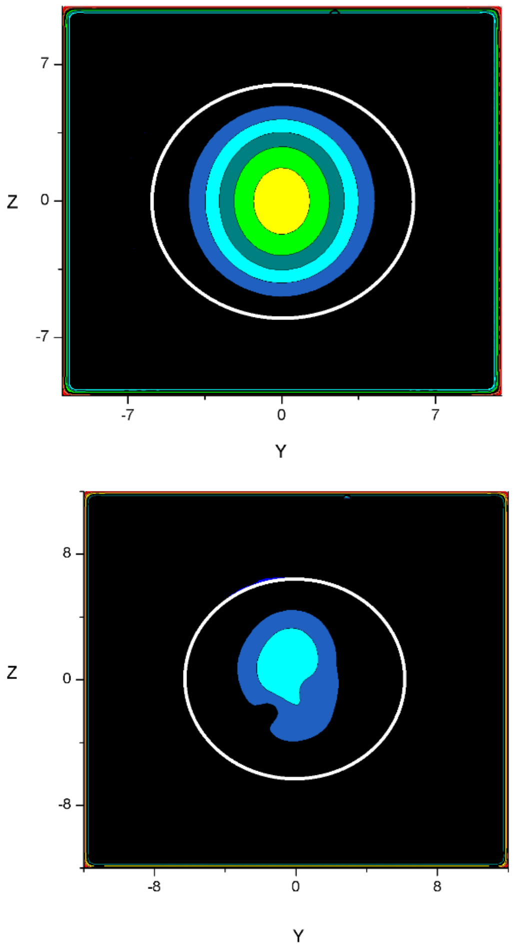

Fig.3. Density of the superconducting electrons inside small sphere in Y-Z projection for clean and "dirty" superconducting spheres. Magnetic field is 0.05.

Intermediate state in this case has an one domain structure with domain shape strongly dependent even on small concentration of the normal inclusions. This tendency is well pronounced at external magnetic field close to the critical field H_s (see Fig.4). In clean sphere the superconducting domain stretches along the magnetic field becoming narrower in perpendicular direction while in dirty superconducting sphere superconductivity disappears inside the sample.

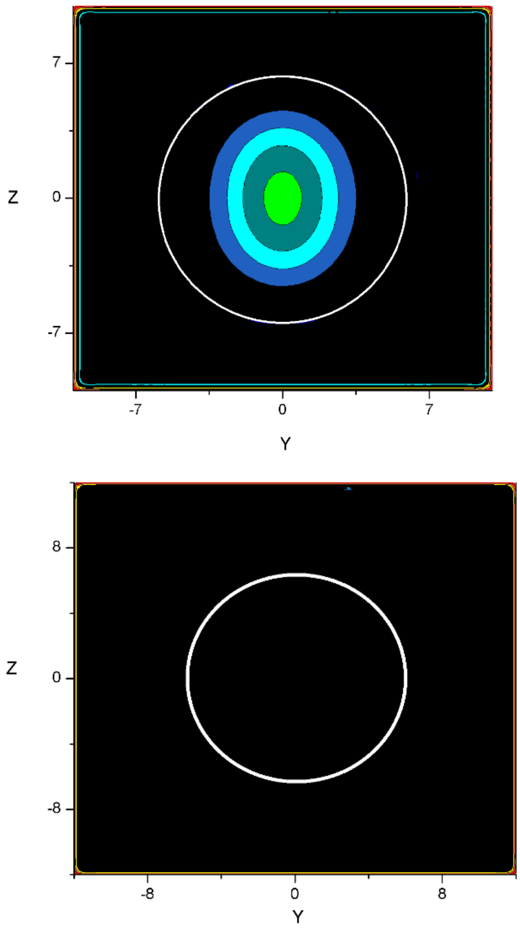

Fig. 4 Density of the superconducting electrons inside small sphere in Y-Z projection for clean (left) and "dirty" spheres. $R = 6\xi, \kappa = 0.18$. Magnetic field is 0.06.

It should be concluded that in a small clean sphere there is one-domain state while in dirty sphere the unformulated turbulent state appears.

2.2. Big sphere $R = 15\xi, \kappa = 0.18$.

Magnetic moment of superconducting sphere in this case is more complicated. It demonstrates hysteretic behavior typical for bulk type-I superconductors with small κ parameter [29]. Magnetic moment as a function on the external magnetic field was calculated in four different protocols: "forward" direction for a. clean and b. dirty spheres subjected to the external magnetic field increases from zero to the critical field H_{sh} and c. for magnetic field decreases from H_{sh} to zero for clean and d. for dirty samples ("back" direction). Results are presented in Fig.5.

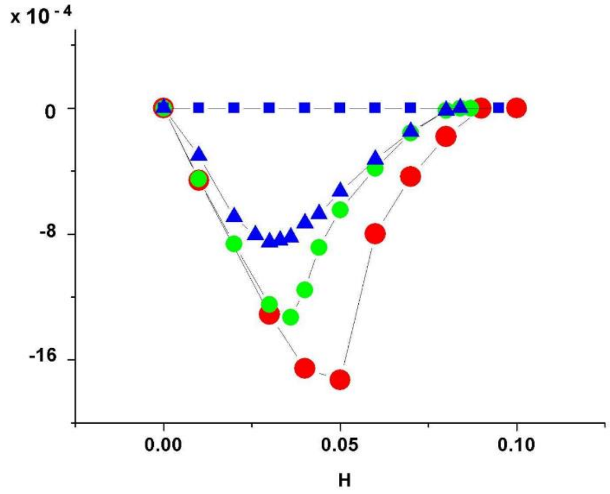

Fig.5. Magnetic moment of the superconducting sphere with radius $R = 15\xi$ and κ=0.18. The grid step was chosen h=0.5. Magnetizations for "forward" regimes are presented by red (clean) and green circles (dirty) while blue squares and triangles represent back direction in clean and dirty cases correspondingly.

Magnetization demonstrates both Meissner behaviour at small magnetic field and well pronounced intermediate state on the long tail extended to H_{sh} where superconducting state is suppressed by applied magnetic field and $M(H_{sh}) = 0$. Magnetization of the clean spheres demonstrates well pronounced hysteresis typical for bulk type I superconductors where metastable normal state appears in magnetic fields in the range $H_{sc} < H < H_{sh}$ where H_{sc} is the surface critical magnetic field (here $H_{sc} = 0.69H_{c2}$ and H_{c2} is the second critical magnetic field). In our case, however, the unstable, hysteretic region spreads up to zero magnetic field $(H_{sc} \to 0)$. In a dirty sphere magnetization demonstrates properties more typical for the type II superconductors. The intermediate state in a big spheres is complicated and topologically diverse. It contains the multi-domain superconducting to normal structure where proximity effect smash the boundaries between domains typical for macroscopic type-I superconductors structure while the density of superconducting electrons is spatially modulated and there is no sharp border between the domains. Typical spatial structures of the intermediate state domains are presented in Fig.6,7

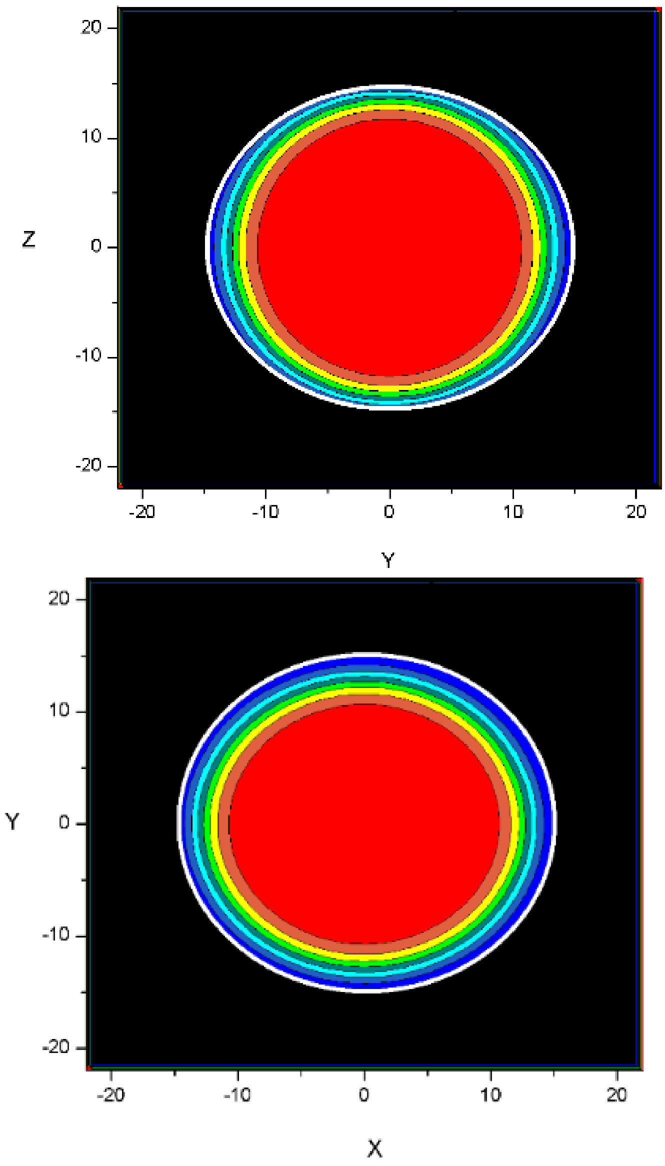

Fig.6 The intermediate state in big type I superconducting sphere ($R = 15\xi, \kappa = 0.18$) in external magnetic field H=0.04 in clean sphere where Meissner state is well pronounced.

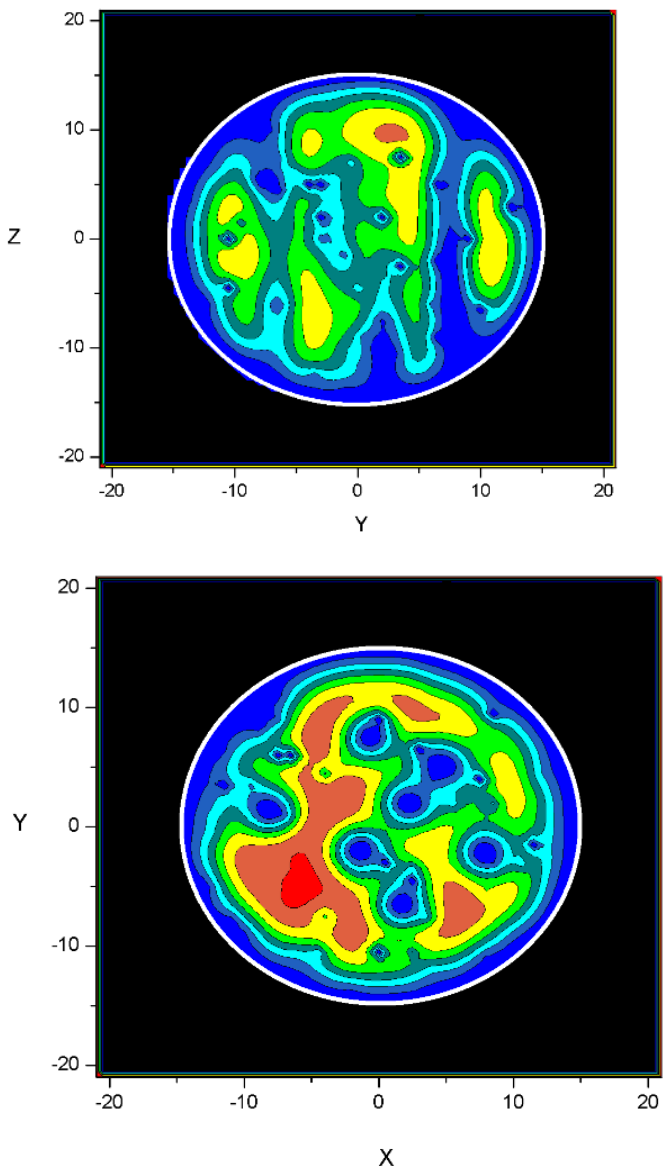

Fig.7 The intermediate state in big type I superconducting sphere ($R = 15\xi, \kappa = 0.18$) in external magnetic field H=0.04 in dirty spheres where flux turbulence is exhibited.

In the external magnetic field H=0.04 the intermediate state in big clean sphere has typical for the Meissner state shape while in dirty case the IS manifests turbulent behaviour.

In increasing magnetic field $H = 0.07$ the Meissner state in a clean sphere is broken and the intermediate state containing the set of domains (tubes) separated by normal regions (where a weak superconductivity is induced by the proximity effect). In dirty sphere with randomly inclusions of the normal phase the turbulent state becomes more pronounced and normal state percolation domains cross the sample (Fig.8,9) from one side to another.

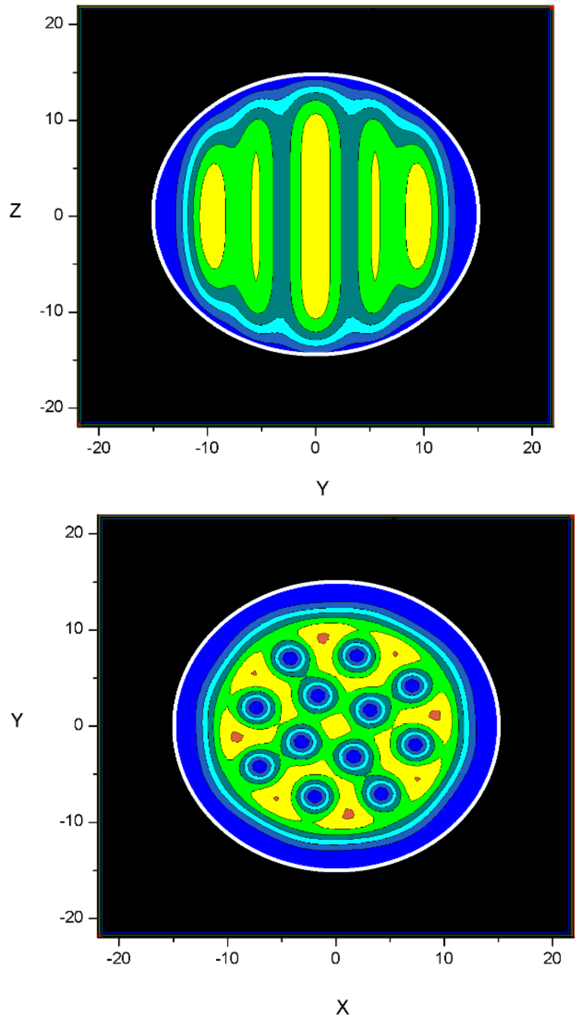

Fig.8 The intermediate state in big type I superconducting sphere ($R = 15\xi, \kappa = 0.18$) in external magnetic field H=0.06 in clean sphere.

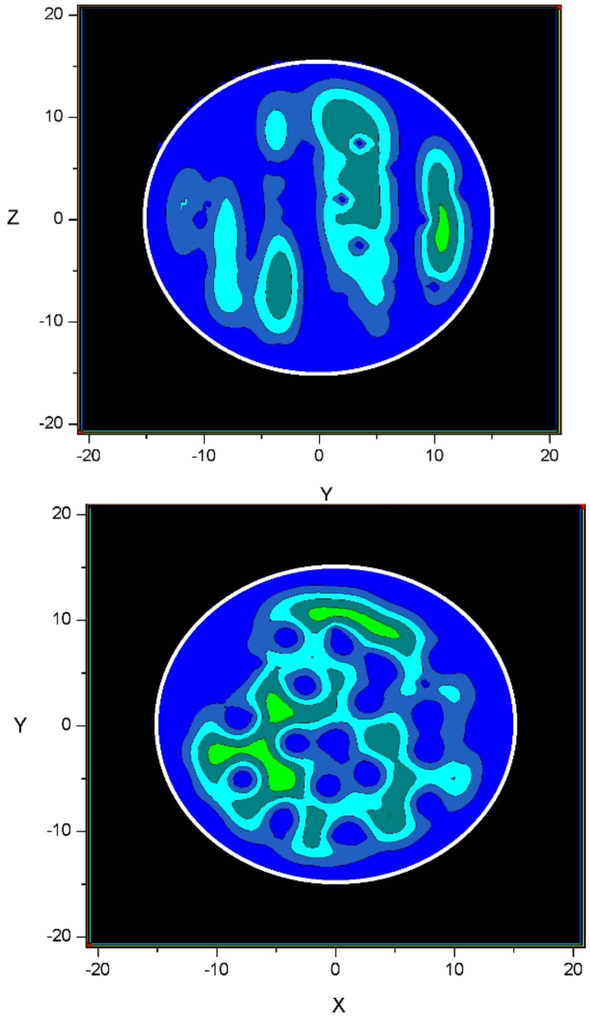

Fig. 9 The intermediate state in big type I superconducting sphere ($R = 15\xi, \kappa = 0.18$) in external magnetic field H=0.06 in dirty sphere.

If the external magnetic field approaches $H = 0.06$ the Meissner intermediate state in clean big spheres split by several domains (Fig.8) while at dirty samples the turbulence state becomes more pronounced (Fig.9). Domains in clean sphere forms Abrikosov lattice similar to those in type-II superconductors.

At magnetic field close to the critical magnetic field ($H_{sh} = 0.1$) $H = 0.07$, $H = 0.08$ the amplitude of the domains decreases dramatically while the shape of the intermediate state and domains number remain. The turbulence intermediate state in this case completely disappears while the sphere undergoes transition to normal state (Fig, 10,11.).

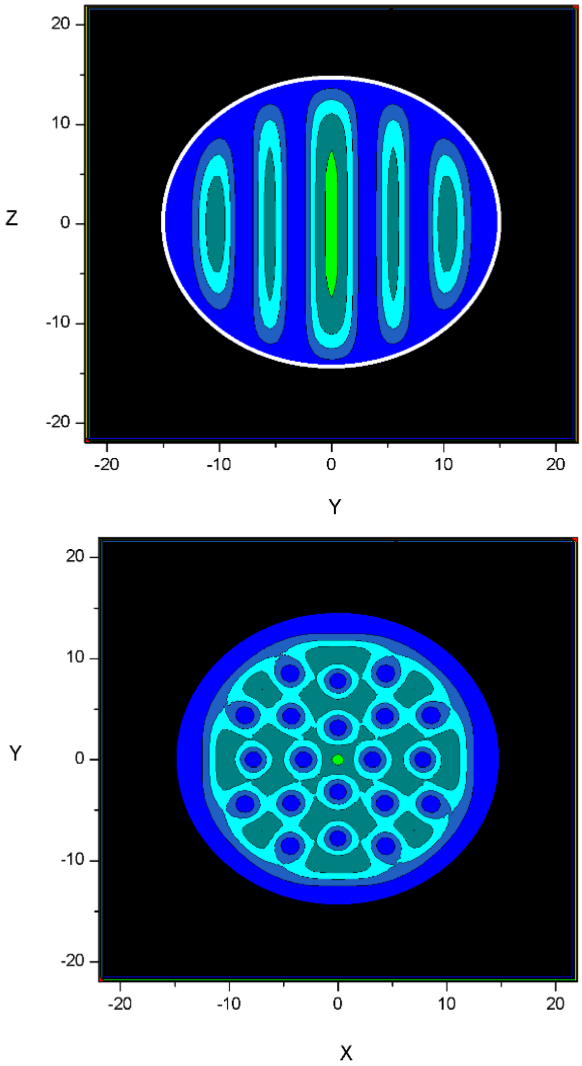

Fig.10. Density of superconducting electrons inside big clean sphere (R=15, k=0.18,$T = 0.5T_c$) in the intermediate state for applied magnetic fields H=0.07.

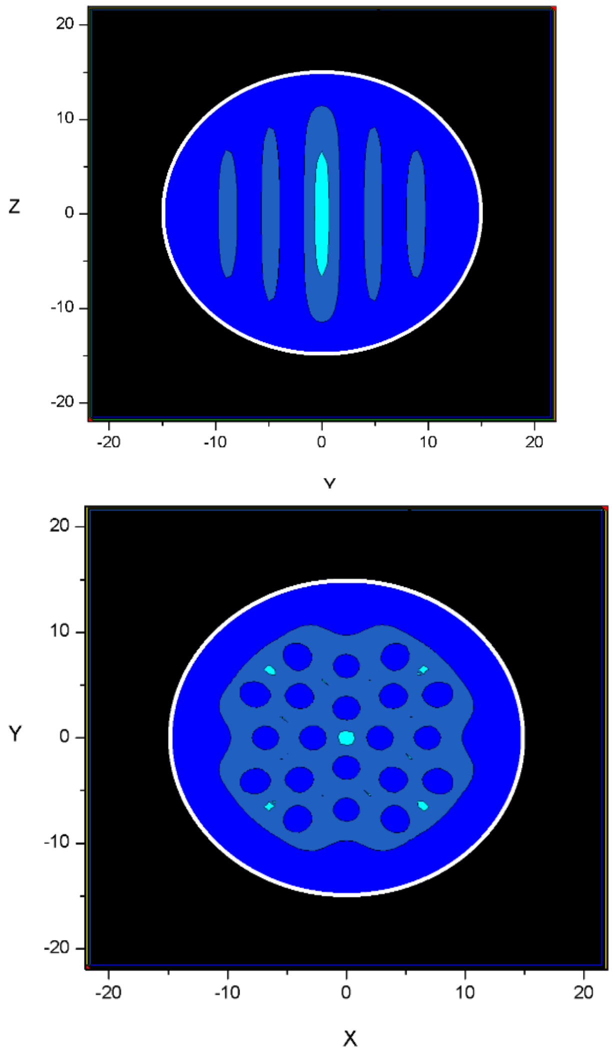

Fig.11. Density of superconducting electrons inside big clean sphere (R=15, k=0.18,$T = 0.5T_c$) in the intermediate state for applied magnetic fields. H=0.08.

3. Summary

The magnetic moment and the intermediate state as a function of the external magnetic field for small ($R = 6\xi$) and big ($R = 15\xi$) type I superconducting spheres have been calculated numerically in the framework of time dependent Ginzburg-Landau equations. Both clean and dirty spherical samples (with 0.5% of the superconducting material of the spheres was replaced by the normal phase (dirty case) playing the role of the pinning centres), were studied. It was shown that the magnetic moments of the small sphere (κ=0.18, $T = 0.5T_c$) does not show hysteresis and irreversibility (Fig.2). In a big sphere the magnetization of the clean sphere demonstrates behaviour similar for bulk system with a well pronounced hysteresis. The unstable normal hysteretic line (blue squares in Fig. 5) extends up to zero field due to small surface barrier. The intermediate state in spheres demonstrates both domains (tubes) in clean case and turbulence behaviour in dirty samples at the same magnitude of the external magnetic fields. The size of the sphere plays extremely important role strongly affecting structure of the intermediate state. In particular the intermediate state in small clean sphere consists of one superconducting domain. This domain stretches along the magnetic field while in dirty sphere at the same magnetic field the intermediate state demonstrates onset of the turbulence behaviour (see Figs 6-11). These results are in a good qualitative agreement with experiment presented in Ref. [6].

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