M.23 Transport properties of Copper

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Abstract

In this lab report we present the results from an experiment to determine the thermal conductivity and electrical resistivity of copper. There were two measurements taken, one at cryo temperatures from 10K up until room temperature. The second one was performed at room temperature, for ambient pressure and for a vacuum. Using these measurements we were able to calculate the thermal conductivity and electrical resistivity values at room temperature. Combining these with the cryo measurements we were able to calculate the RRR of the copper sample, demonstrate the Wiedemann Franz law and the Hall coefficient which allowed us to calculate the charge carrier density, Fermi velocity and the Fermi energy.

Contents

1 Introduction

In order to understand the properties of a metal it is important to know the dynamics of the electrons in the metal when any external field is applied. These properties are usually grouped under the topic "Transport properties". In this experiment we study the transport properties especially the electrical and thermal conductivity of copper. We use the Drude model to understand the basic overview of the electron dynamics of the copper, we measure the thermal conductivity, electrical conductivity and also study the magneto-resistance using the Hall configuration. In this report we discuss the theory of the experiment first, then we talk about the experimental techniques we used to measure the transport properties. in the end we give the analysis of our data and we finish off with a discussion of our results.

2 Theory

2.1 Transport properties

When a metal is placed in an external field the motion of the electrons inside the metal leads to a current proportional to the applied field. Two such important properties are the electrical and thermal conductivity in metals.

Electrical conductivity When a metal is placed in an external electric field an electric current($\vec{j_e}$) is produced in the material proportional to the applied electric field. The proportionality constant between the applied electric field and the current is the electrical conductivity (σ) of the material.

$$\vec{j_e} = \sigma \vec{E} \tag{2.1}$$

The electric field can be expressed as the negative gradient of the electrical potential, $\vec{E} = -\vec{\nabla} U$. hence,

$$\vec{j_e} = -\sigma \vec{\nabla} U \tag{2.2}$$

Thermal conductivity Similar to electrical conductivity, when there is a temperature gradient in the material there is a current developed inside the material called the thermal current ($\vec{j_t}$). this thermal current is dependent on the proportionality constant (κ) called the thermal conductivity.

$$\vec{j_t} = -\kappa \vec{\nabla} T \tag{2.3}$$

2.2 Drude model

We start our discussion of electronic transport in solids with the classical Drude model.This model was introduced to explain the conductivity of metals. It is based on the following assumptions:

- When metal atoms are brought together to form a solid, the weakly bound valence electrons become detached and are free to move in the solid. Each metal atom contributes Z electrons to these so-called conduction electrons. The nuclei and the tightly bound core electrons form the metallic ions.

- A conduction electron does not interact with other conduction electrons (independent electron approximation) or with the metallic ions (free electron approximation)between collisions.
- The mean time between successive collisions of a conduction electron is given by the relaxation time τ . Each collision results in an abrupt change of the velocity of the corresponding conduction electron and the collision rate is independent of the position and the momentum of the electron

These electrons have a random thermal motion in the absence of an external fields and hence the net current is averaged out. When an external perturbation is present, like an electrical potential gradient or a thermal gradient the free electrons are set into uniform motion leading to a net current (electrical or thermal) within the conductor. The electron dynamics leading to such a current is discussed in the next section.

2.3 Electron Dynamics of the free electron model

Electrical Conductivity

When a D.C electric field is applied to a metal, the electrons in the thermal motion accumulates a infinitesimal momentum between the collisions which occurs once every τ seconds. The total momentum accumulated by the electron in time t is,

$$\Delta\langle p\rangle = q\vec{E}\tau \tag{2.4}$$

we know, $\langle p\rangle = m\langle v\rangle$. also the electrical current flowing in a metal per unit cross-section area is given by,

$$\vec{j_e} = ne\langle v\rangle \tag{2.5}$$

hence we get,

$$\vec{j_e} = \frac{ne^2\tau}{m}\vec{E} \tag{2.6}$$

from this comparing (2.2) we get the conductivity as,

$$\sigma = \frac{ne^2\tau}{m} \tag{2.7}$$

Hall effect

In this we see the electron dynamics in a metal in the presence of both electric and magnetic fields. A schematic picture of the Hall configuration is as shown in the Fig 17. The external electric field $\vec{E_x}$ is applied along the x-axis, the Magnetic field is applied along $\vec{B_y}$.
Due to the Lorentz force, the electrons moving in the x-direction get deflected due to the presence of the external magnetic field.The corresponding equation of motion reads,

$$\vec{F} = e[\vec{E} + \vec{v} \times \vec{B}] \tag{2.8}$$

The individual components are,

$$\begin{aligned} \vec{F_x} =& e[\vec{E_x} + \vec{v_y} \times \vec{B_z}] \\ \vec{F_y} =& e[\vec{E_y} + \vec{v_x} \times \vec{B_z}] \\ \vec{F_z} =& e[\vec{E_x}] \end{aligned} \tag{2.9}$$

the moving electrons would be deflected under the influence of the magnetic field in negative y-direction. In steady state,this effect is however exactly compensated by an electric field $\vec{E_y}$ that results from the surplus charges at the sides of the Hall bar as shown in Fig 17 This is the so-called Hall effect and the field $\vec{E_y}$ is the Hall field.
Using 2.4 and 2.6 we can write,

$$\vec{E_y} = \frac{m}{e^2\tau n}[\frac{e\tau\vec{B_z}}{m}\vec{j_x} + \vec{j_y}] = R_h\vec{B_z}\vec{j_x} \tag{2.10}$$

here, $R_h = \frac{1}{ne}$ os the Hall coefficient. Using this the carrier concentration can be calculated.

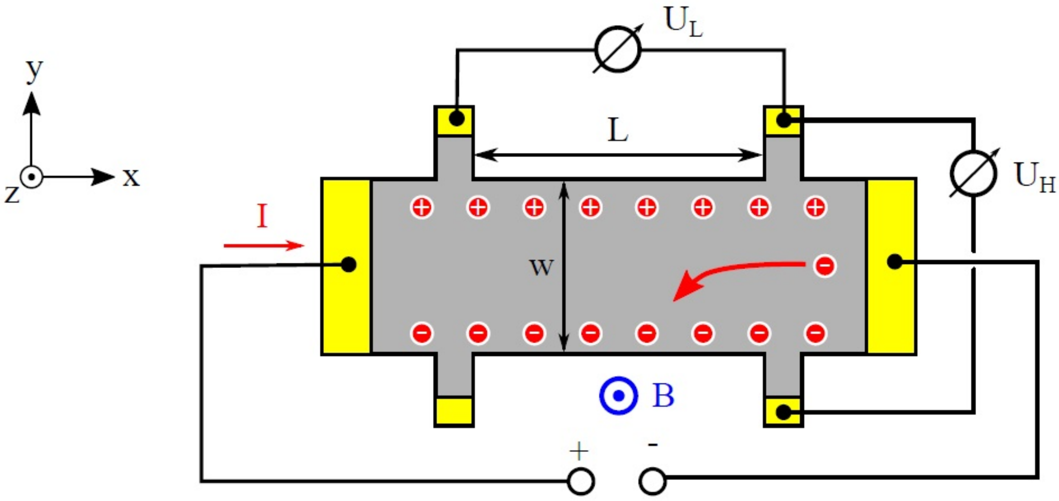


Figure 1: This figure shows the configuration of the Hall bar. An electric field is applied along the x-direction, Magnetic field along z-direction and a Hall voltage is developed along the y-axis due to the deflection of the electrons.

Thermal Conductivity

Similar to the electrical conductivity, we can use the Drude model to understand the thermal conductivity in the material, To apply Drude's model to study the thermal conductivity of a metal, we need to remember the last assumption which implies that after a collision, an electron carries the thermal energy of the local environment. We also need to assume that T varies a little over l (the mean free path of the electron) i.e. T is a function of x
To calculate the heat delivered to a point we need to simplify this to a one dimensional problem. We write the thermal energy of an electron at temperature T as $E(T)$. Then, at a point x the average electron coming from the left brings with it an energy $E(T[x + v\tau])$ the average electron from the right delivers $E(T[x + v\tau])$. Thus, for electrons of density n and average velocity v (remembering that half will travel towards the point x and half away) leaves us with:

$$\begin{aligned} j_q =& \frac{nv}{2}\left(E(T(x - v\tau)) - E(T[x + v\tau])\right) \\ j_q =& \frac{nv}{2}\left(\frac{dE}{dt}\right)\left(-\frac{dT}{dx}\right)2v\tau. \end{aligned} \tag{2.11}$$

We know that, $n\frac{dE}{dT} = c_v$ i.e the specific heat of the solid, extending the same to three dimentions we can see that

$$\kappa = \frac{\langle v \rangle^2 \tau}{3} c_v \tag{2.12}$$

2.4 Wiedmann Franz law

We have seen that the conduction electrons are responsible for the thermal and electrical conductivity of the materials. Wiedmann Franz law relates the thermal conductivity σ and the electrical conductivity κ of the material to the Temperature T. It states that the ratio of the ratio of the thermal and electrical conductivity is proportional to the temperature of the material.

$$\frac{\kappa}{\sigma} = LT. \tag{2.13}$$

The constant L is called the Lorenz number $L = 2.44 \times 10^{-8}\ W\Omega K^{-2}$ In general this law is not always true. It is observed that the law holds good for low temperatures and high temperatures. But in the intermediate temperatures due to the properties of the materials the law does not hold.

2.5 Magneto resistance

The electrical resistance of the material increases when an external magnetic field is applied to a metal. This resistance due to the applied magnetic field is called as the magneto-resistance. We have seen in the previous section that when a magnetic field is applied to a material in the z-direction a electric potential is developed in the y-direction of the metal called as the Hall voltage such that all the forces are in equilibrium and hence there should not be any resistance in the metal. also when an magnetic field is applied in the x-direction then there is no Lorentz force acting on the electron which also cannot explain the magneto resistance. So these simple models cannot explain the magneto-resistance. A few more assumptions and models have to be constructed in order to explain this phenomenon.
One can explain this effect by assuming two types of charge carriers in the metal, the electrons and the holes. Generally the electric field is given by,

$$\vec{E} = \rho_i \vec{j_i} + \frac{e\tau}{m} \vec{B} \times \rho_i \vec{j_i} \tag{2.14}$$

where ρ_i represents the zero field resistivity for the current $\vec{j_i}$. Here, $i = 1, 2$ corresponding to the electrons and holes respectively. Hence the total current is given by,

$$\vec{j_{tot}} = \vec{j_1} + \vec{j_2}. \tag{2.15}$$

Hence the current contributions can be written as,

$$\vec{j_i} = \sigma_i \frac{E - \beta_1 \vec{B} \times \vec{E}}{1 + \beta_i^2 B^2} \tag{2.16}$$

where $\beta_i = \frac{e\tau_i}{m_i}$.
Hence the total current is,

$$\vec{j_{tot}} = \left(\frac{\sigma_1}{1 + \beta_1^2 B^2} + \frac{\sigma_2}{1 + \beta_2^2 B^2} \right) \vec{E} - \left(\frac{\sigma_1 \beta_1}{1 + \beta_1^2 B^2} + \frac{\sigma_2 \beta_2}{1 + \beta_2^2 B^2} \right) \vec{B} \times \vec{E}. \tag{2.17}$$

Projecting the electric field in the direction of the current and comparing the results with the specific resistance without the magnetic field,$\rho_0 = \frac{1}{\sigma_1+\sigma_2}$ and rearranging

$$\frac{\Delta\rho}{\rho_0} = \frac{\sigma_1\sigma_2(\beta_1 - \beta_2)^2 B^2}{(\sigma_1\sigma_2)^2 + B^2(\beta_1\sigma_1 + \beta_2\sigma_2)^2} \tag{2.18}$$

Hence we obtain a resistance as a function of the applied magnetic field. This model cannot explain the longitudinal magnetic field. However this model is sufficient to understand the transverse magnetic field. Hence the presence of magneto-resistance in the presence of different charge carriers is proven. Assuming that the different types of charge carriers have the same relaxation time, the ratio $\frac{\Delta\rho}{\rho_0}$ is a function of the product of τ and B. τ is inversely proportional to ρ_0, so we obtain

$$\frac{\Delta\rho}{\rho_0} = F\left(\frac{B}{\rho_0}\right) \tag{2.19}$$

This is known as Kohler's Rule. The function F depends on the type of metal only. To prevent false conclusions, it should be emphasized that this formula is just an approximation.

2.6 Scattering effects

We have got a general simple picture of how the electric and the thermal properties are studied. To get into a little more details, we think about the scattering of the conducting electrons in the metal. We can assume the metal as a crystal lattice and the lattice points are occupied by the metal atoms. Various scattering effects can be caused in a crystal lattice, the most significant ones are the phonon scattering and the scattering due to the lattice imperfections and impurities. These effects get significant affect in different temperature regimes. The phonons are the quasi particles caused due to the vibration of the lattice. At very low temperatures, the lattice vibrations are minimal and the phonon scattering is low so the impurities get a major affect in the electron dynamics. At high temperatures, almost all the vibrational modes of the phonons take effect and hence the scattering is dominated by the phonons that affects the electron dynamics. These effects cause changes in the motion of the electrons leading to the change in its conducting properties.

2.7 Debye temperature and Fermi level

The properties such as the Debye temperature and Fermi level helps in the understanding the temperature dependence of the phonons and the coupling of the electrons.

Debye temperature Debye temperature Θ_D is the temperature above which all the phonon modes are excited. This was proposed by Debye to explain the Temperature dependence of the specific heat capacity of a lattice.

Fermi level Fermi level is the highest energy state occupied by electrons in a material at absolute zero temperature. As the temperature is increased, electrons start be excited to the higher energy states too. The temperature dependence of the Fermi level is shown in the graph Fig 2.

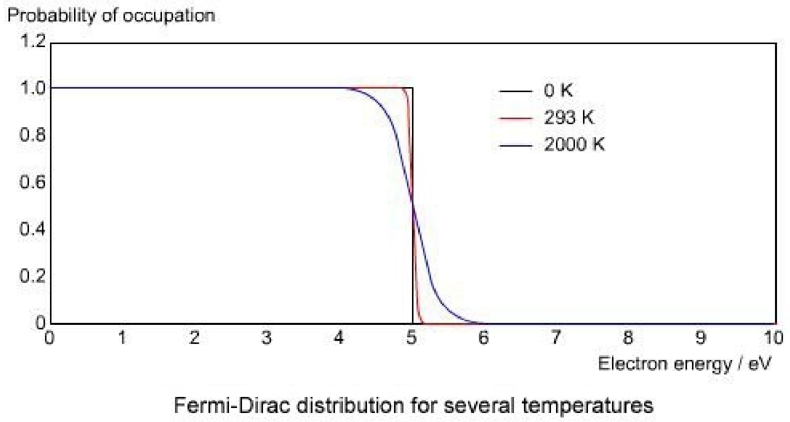


Figure 2: This graph shows the step function for absolute zero and the higher states are occupied at higher temperatures.

This helps in characterising the materials. Based on the position of the Fermi level, the materials are classified into conductors, insulators and semi-conductors. Based on the position of the Fermi level in the energy band diagram we can know if the material will have free electrons for conduction

3 Experiment

In this section we will see the various experimental techniques used in measuring the electrical conductivity and the thermal conductivity.

3.1 Measurement Techniques

Four point method

This method is used in measuring the resistance of the sample. There are four terminals, the first and the last terminals are connected across the current source whereas the second an the third are connected across the volt meter. The advantage of this method is the voltage drop across the wires are eliminated and only the resistance of the sample is measured. The resistance is calculated by using the well known Ohm's law

$$R = \frac{V}{I} \tag{3.1}$$

The resistivity of the metal is related to the resistance by,

$$R = \rho \frac{l}{A} \tag{3.2}$$

using this relation the resistivity can be calculated.

Steady state method

In this method the material is supplied with a constant heat from one end and the temperature change is studied using the Seebeck effect. The Seebeck voltage between the two ends of the material can be converted back to the temperature gradient. The outline of this method is shown in the Fig 4

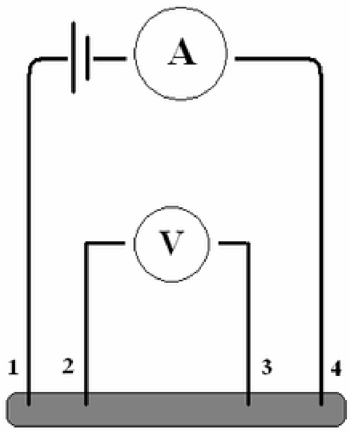


Figure 3: This figure shows how the resistance of the sample is measured using the four point method. The voltage measurements are taken in points 2 and 3 when the current is applied across points 1 and 4.

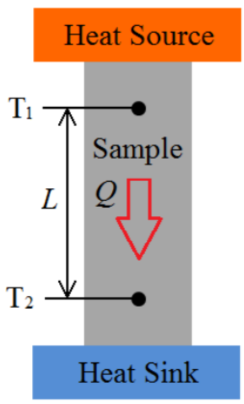


Figure 4: This image shows how the steady state measurements are done a heater supplies heat from one end and the other end acts as the sink,the direction of geat flow is given by Q, and the temperature difference between two points T_1 and T_2 is measured to get the response function[1]

Hall Measurement configuration

In order to measure the Hall effect the sample is first first prepared in Hall configuration Hall configuration as shown in the Fig 5. This configuration is very similar to the four-point method included the Hall voltage is taken along the vertical axis also. The thickness of the vertical hands have to be very small as possible to reduce the error.

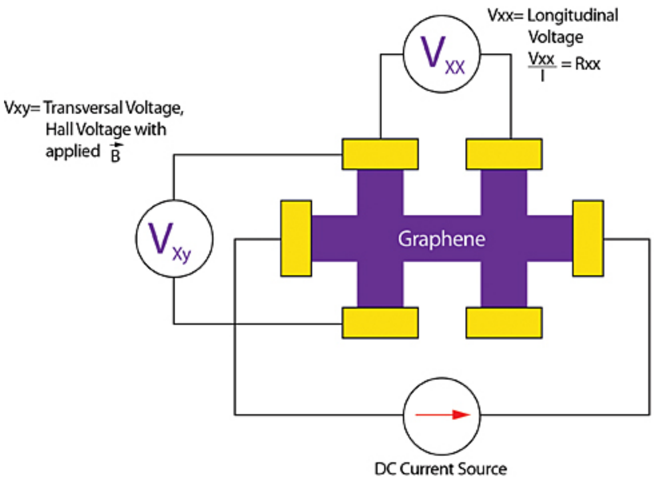


Figure 5: The blue print of the Hall configuration is as shown in this figure. The voltage is measured in both horizontal and vertical directions.[2]

3.2 Measuring Conditions

The thermal conductivity and the electrical conductivity of the copper was measured in various measuring conditions, the room temperature measurements were made in the regular environment (at normal atmospheric pressure) and in vacuum. The same was measured at very low temperatures and for different magnetic fields. The data of these low temperature measurements was given to us for analysis.

3.3 Measuring Methods

Resistivity

To determine the resistivity ρ a constant current I_R is applied to the sample and the voltage drop U_R between two points saperated by l_R caused by the sample is measured. This is done in the 4-point method. The resistivity is then determined by

$$\rho = \frac{AU_R}{I_R l_R} \tag{3.3}$$

where A is the cross-sectional area of the sample and l_R is the distance between the inner contacts.

Thermal Conductivity

One end of the sample is thermally coupled to the sample holder, which has temperature T_0. On the same end of the sample a heater is arranged to produce the stationary heat flow through the sample. The temperature gradient is measured by a thermocouple. Because copper is a good heat conductor, the thermal resistance of the couplings between heater and sample as well as between sample and sample holder cannot be neglected. Therefore a 4-point method is chosen similar to the measurement of the resistivity. The relation between heat flux j_x, heater power P and cross-sectional area A is given by $j_x = P/A$. The heater power is given by $P = U_{PH}I_{PH}$. With the temperature difference between the tips of the thermocouple ΔT and their distance l_{TE} the temperature gradient is given by

$$\vec{\nabla}T = \frac{\Delta T}{l_{TE}} \tag{3.4}$$

$$\frac{U_{PH}I_{PH}}{A} = \kappa\frac{\Delta T}{l_{TE}} \tag{3.5}$$

For a thermocouple it is $\Delta T = U_{TE}/S(T,B)$ with the thermo-power S(T,B) and the thermoelectric voltage U_{TE}. We obtain

$$\kappa = \frac{U_{PH}T_{PH}S(T,B)l_{TE}}{U_{TE}A} \tag{3.6}$$

Hall effect

If we apply a constant current I_H to the sample and a magnetic field B perpendicular to the current, we can measure the so called Hall voltage U_H perpendicular to the current and the field. The Hall constant is given by

$$A_H = \frac{U_H d}{BI_H} \tag{3.7}$$

where d is the thickness of the sample.

4 Analysis and Results

In this section we will present the results that we have obtained from the measurements and we will also discuss and analyse them.

4.1 Room Temperature Measurements

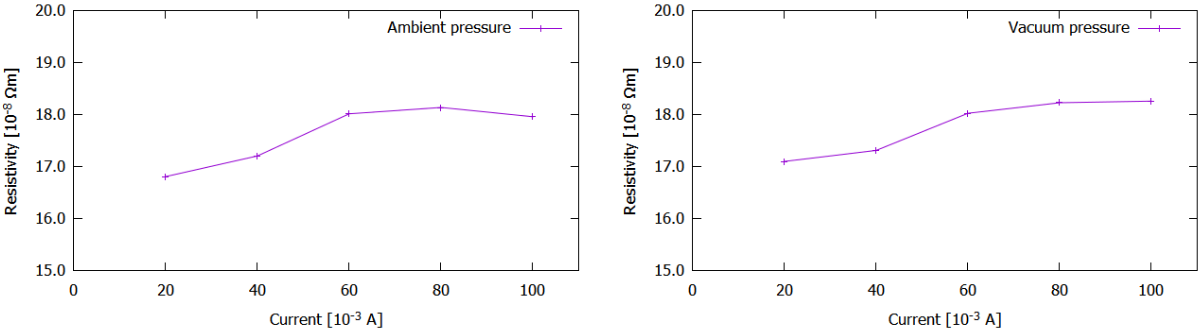
4.1.1 Electrical Resistivity

To measure the electrical resistivty we took 5 different readings from 20 mA up to 100 mA. To correct for the background we have also done measurements from -20 mA to -100 mA, and then corrected the resistivity according to

$$\rho(I) = \frac{\rho(I_+) - \rho(I_-)}{2} \tag{4.1}$$

where I_+ denotes the positive current, and I_- denotes the negative current.
As we can see in Fig. 6 the resistivity does not change much for different values of the current, and there is almost no change in different pressures. We can find the resistivity for both the ambient case and vacuum case to be

$$\begin{aligned}\rho_{amb} &= (1.76 \pm 0.02) \times 10^{-8}\Omega\ m \\ \rho_{vac} &= (1.78 \pm 0.02) \times 10^{-8}\Omega\ m\end{aligned} \tag{4.2}$$



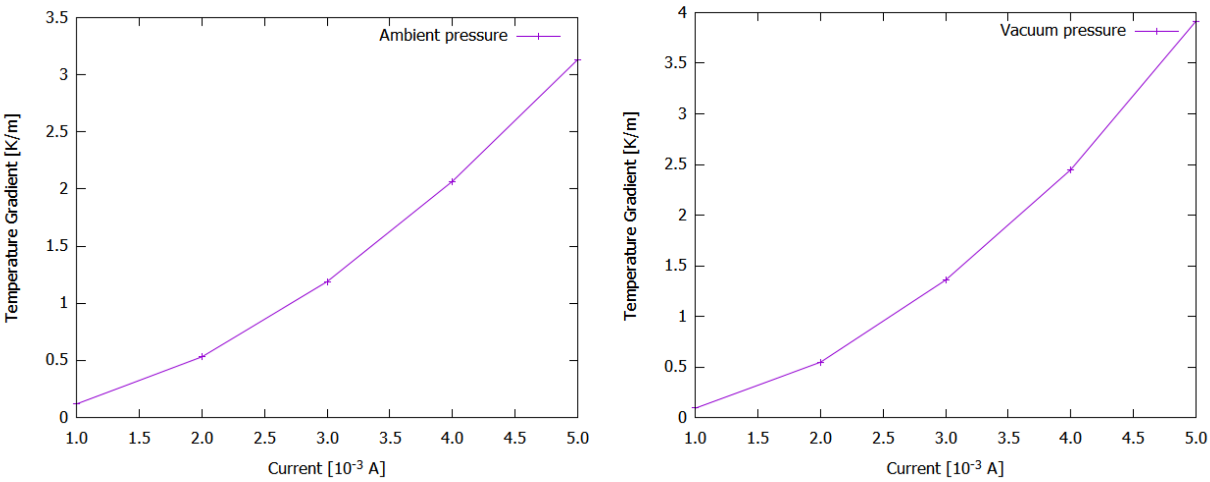
(a) (b)

Figure 6: a) Resistivity versus current at room temperature with ambient pressure b) Resistivity versus current at room temperature in a vacuum.

The accepted value for the resistivity of copper is $1.68 \times 10^{-8}\Omega m$ [4], so our result agrees quite well with the accepted value.

4.1.2 Thermal Conductivity

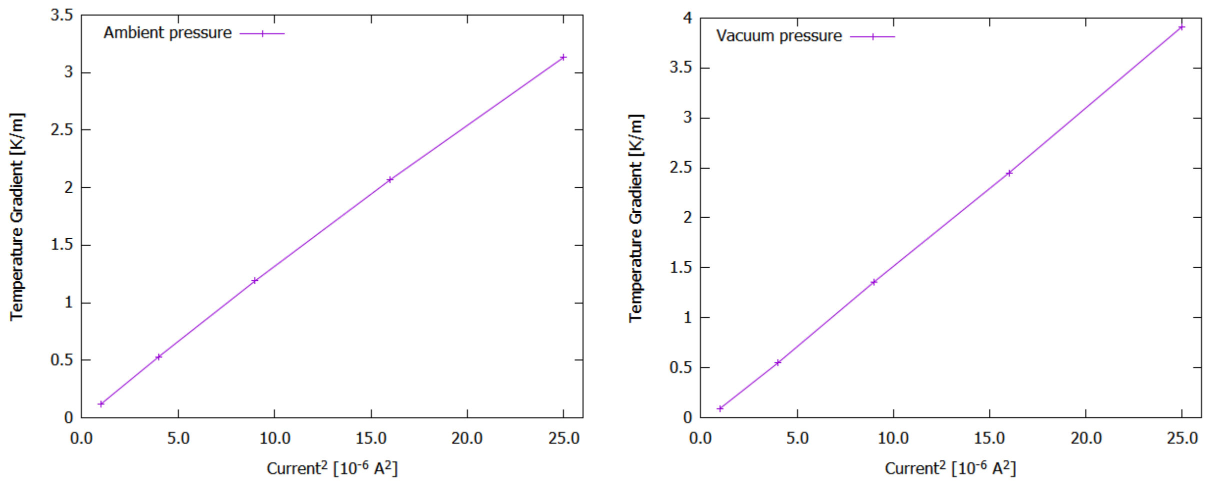
Here we have taken 5 measurements again but this time from 1 mA up to 5 mA, the values for the current are very low so to not damage the heater. This time to correct for background we took the background reading for every value of the current and subtracted the background from calculation to determine κ at each value for the current. First let us plot the temperature gradient versus the current see Fig. 7. From eq. (3.6) we can see that we are expecting an I^2 dependence, to visualise this a plot of the temperature gradient versus the current squared is given in Fig. 8. As we can see the plot is linear so the expected behaviour is realised.



(a) (b)

Figure 7: a) Temperature gradient versus current with ambient pressure b) Temperature gradient versus current in a vacuum.

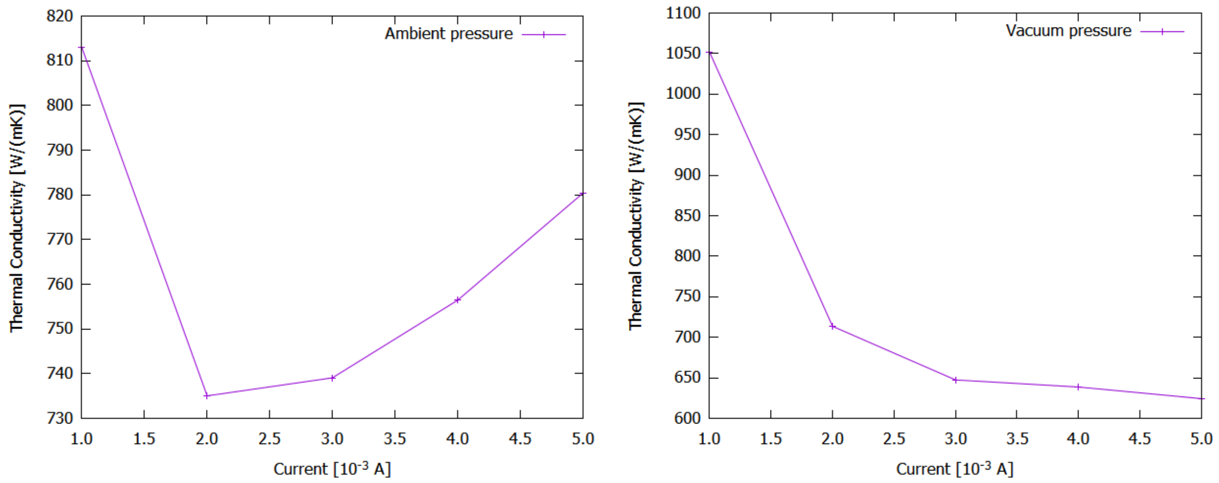
The thermal conductivity does change depending on the pressure as can be seen in Fig. 9. This is expected because when there is no vacuum the air can also conduct heat,



(a) (b)

Figure 8: a) Temperature gradient versus current squared with ambient pressure b) Temperature gradient versus current squared in a vacuum.

therefore we expect the thermal conductivity to be higher in the ambient case compared to the vacuum case. The value for the thermal conductivity at $I = 1$ mA shows a



(a) (b)

Figure 9: a) Conductivity versus current at room temperature with ambient pressure b) Conductivity versus current at room temperature in a vacuum.

huge increase in the thermal conductivity than expected, this is due to the heating being small compared to the natural temperature drift, therefore this value was omitted when calculating κ. We find the thermal conductivity for both the ambient case and vacuum case to be

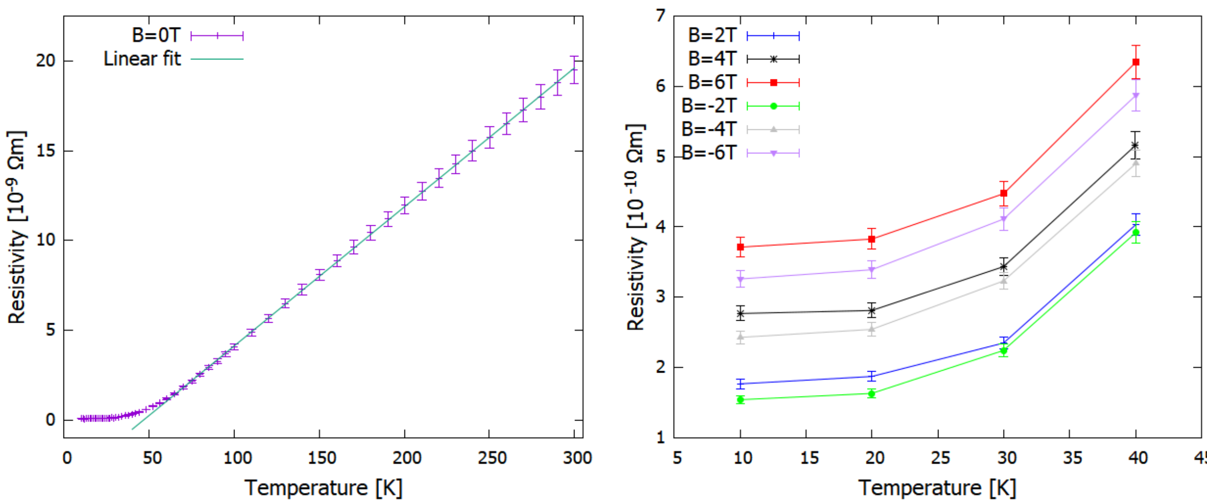
$$\begin{aligned} \kappa_{amb} &= (752 \pm 13) W/mK \\ \kappa_{vac} &= (655 \pm 6) W/mK \end{aligned} \tag{4.3}$$

The accepted value for the thermal conductivity of copper is $401 W/mK$ [5] which is quite far from our result. This could be from the fact that a steady state was not reached because there were a lot of fluctuations in the vacuum measurements.

4.2 Cryo Measurements

4.2.1 Electrical Resistivity

The temperature dependent measurements were taken for a temperature range of 10K - 300K, during this measurement the magnetic field was taken to be 0T. The magnetic field dependence measurement was performed for $B = \pm 2$T, $B = \pm 4$T and $B = \pm 6$T, at temperatures $T = 10$K, $T = 20$K, $T = 30$K and $T = 40$K. First let us discuss the temperature dependency of the resistivity, see Fig. 10. When $B = 0T$ we can see the



(a) (b)

Figure 10: a) Resistivity versus temperature for B=0T, with a linear fit for high temperatures. b) Resistivity versus temperature for $B = \pm 2$T , $B = \pm 4$T and $B = \pm 6$T.

resistivity plateaus for low temperatures, as the temperature rises the resistivity increases. At high temperatures the resistivity behaves linearly with the temperature. In Fig. 10a the linear fit is given by

$$\rho = (7.74 \pm 0.02) \times 10^{-11} T + (-3.62 \pm 0.03) \times 10^{-9} \tag{4.4}$$

In Fig. 10b we see that the resistivity rises with the magnetic field. We have expected that the resistivity does not depend on the sign of the magnetic field, but it can be seen that there is a small difference. This could be due to the misalignment of the terminals used to measure the voltage. If the alignment is not perfect there will be a small component of the electric field which will contribute to the voltage, that component will depend on the sign of the magnetic field which would therefore create a deviation in the curves. Using eq. (4.4) we can now calculate the residual resistivity ratio (RRR). As the resistivity plateaus at low temperatures we can approximate $\rho_{(0K)}$ to be approximately $\rho_{(10K)}$, the numerical value is $\rho_{(10K)} = (6.6 \pm 0.2) \times 10^{-11} \Omega$ m.

$$RRR = \frac{\rho_{(300K)}}{\rho_{(10K)}} = 296 \pm 8 \tag{4.5}$$

This suggests that our copper sample is quite a pure sample, as a comparison the RRR of copper wire is around 40. Now let us determine the power law of the resistivity in the range of $T = 15K$ to $T = 55K$. To do that we plot the data on a double log scale and make a linear fit to the data. We also have to subtract the residual resistivity for low temperatures, as a power law has to start from 0. The fit can be seen in Fig. 11, the values obtained are

$$\ln(\rho - \rho_0) = (4.3 \pm 0.1)\ln(T) + (-38.2 \pm 0.4) \tag{4.6}$$

The expected value is $\rho \sim T^5$ [3], so our result $\rho \sim T^{(4.3\pm0.01)}$ is around 15% away of the expected value.

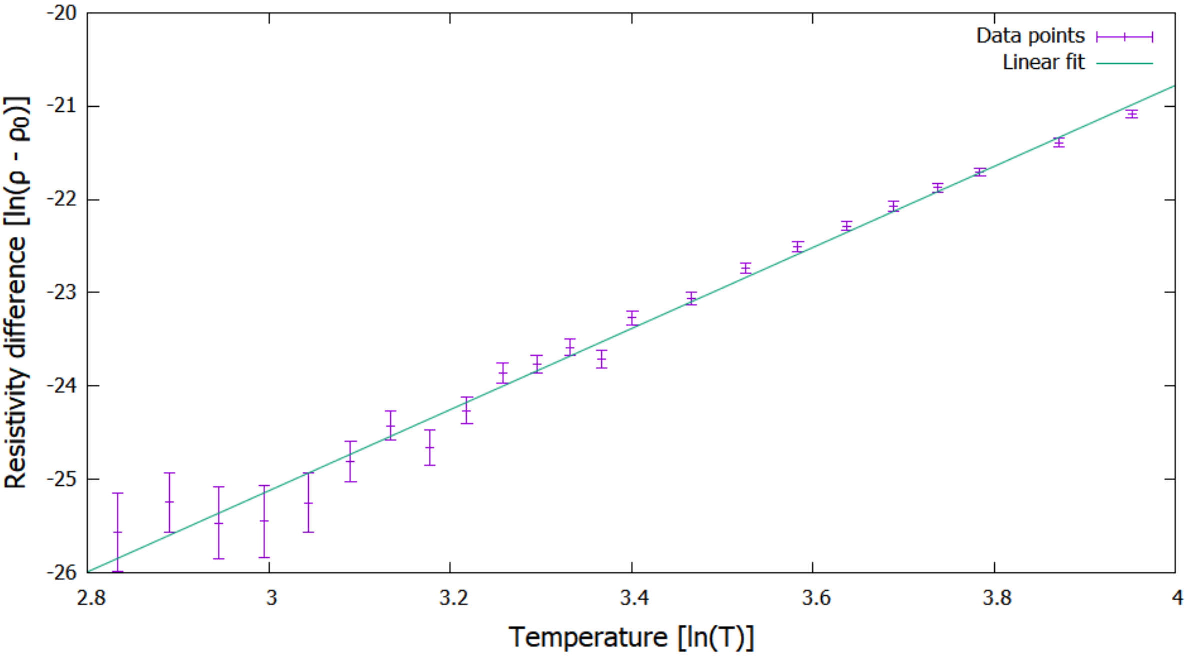


Figure 11: Resistivity vs temperature on a double log graph for temperatures in the range of $T = 15K$ to $T = 55K$ with a linear fit of the data.

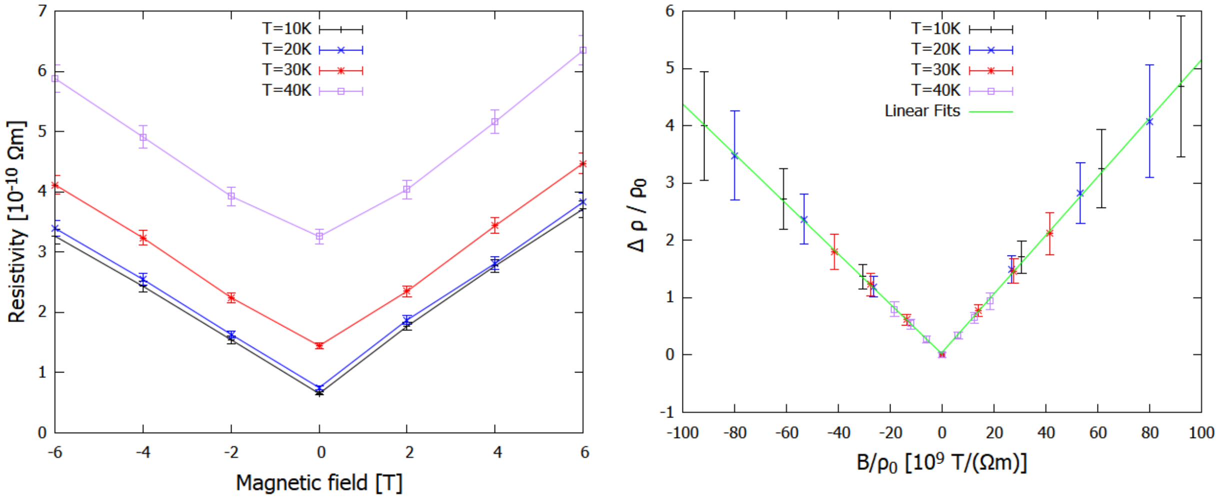
Now let us discuss the dependence of the magnetic field on the resistivity. Fig. 12a shows the resistivity as a function of the magnetic field. We can see that at higher temperatures the resistivity value is higher, but the slope increase is similar at each temperature.
Fig. 12b shows the Kohler plot, as we can see the data does indeed follow a linear relation, so a linear fit is reasonable. The positive and the negative values were fitted seperately, the equation for the linear fits are

$$\begin{aligned} y_{pos}(x) &= (5.10 \pm 0.06) \times 10^{-11}x + (0.05 \pm 0.03) \text{ for } x = \frac{B}{\rho_0} > 0 \\ y_{neg}(x) &= (-4.36 \pm 0.02) \times 10^{-11}x + (0.01 \pm 0.01) \text{ for } x = \frac{B}{\rho_0} < 0 \end{aligned} \tag{4.7}$$

We can see that the slopes vary slightly for the positive and negative fit.

4.2.2 Thermal Conductivity

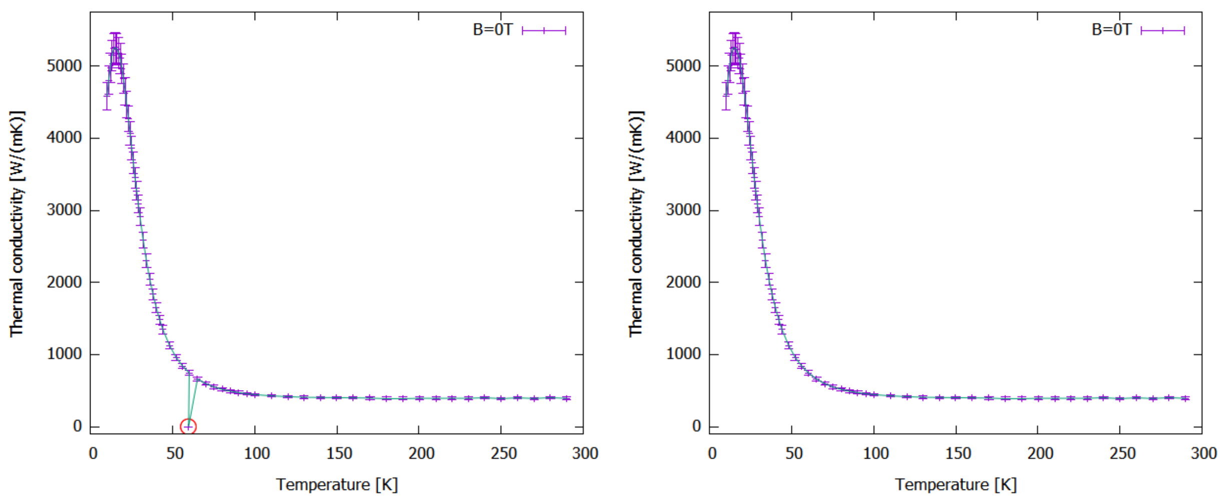
Similarly for the conductivity the temperature measurement was also done for $T = 10$K - $T = 300$K at $B = 0T$. Whereas the magnetic measurements were performed at $B = \pm 2$T,



(a) (b)

Figure 12: a) Resistivity versus magnetic field for $T = 10K, 20K, 30K$ and $T = 40K$ b) Kohler plot for $T = 10K, 20K, 30K$ and $T = 40K$, with a linear fit.

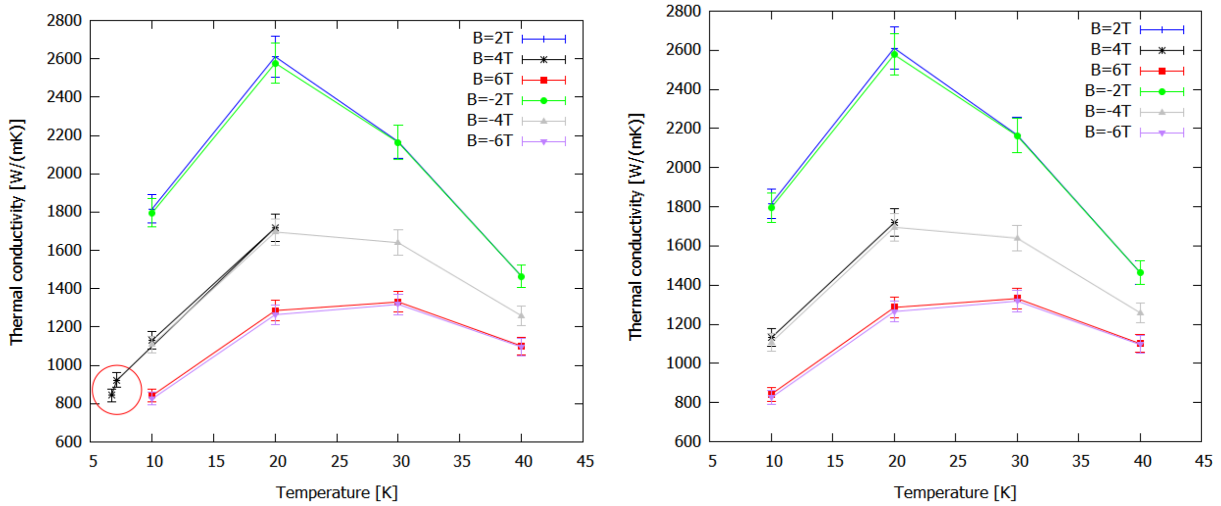
$B = \pm 4$T and $B = \pm 6$T, at temperatures $T = 10$K, $T = 20$K, $T = 30$K and $T = 40$K. Let us begin with the temperature dependence. In our data set, while measuring the temperature dependence of the thermal conductivity at B=0, at around $T = 50$K the current for the heater turned off for one reading. This gave a thermal conductivity of 0 at that point which can be seen in Fig. 13a as the circled point. This anomaly was removed and we did not consider it anymore.



(a) (b)

Figure 13: Thermal conductivity versus temperature for B=0T a) shows the anomaly present near $T = 50$K with a red circle b) Anomaly is removed.

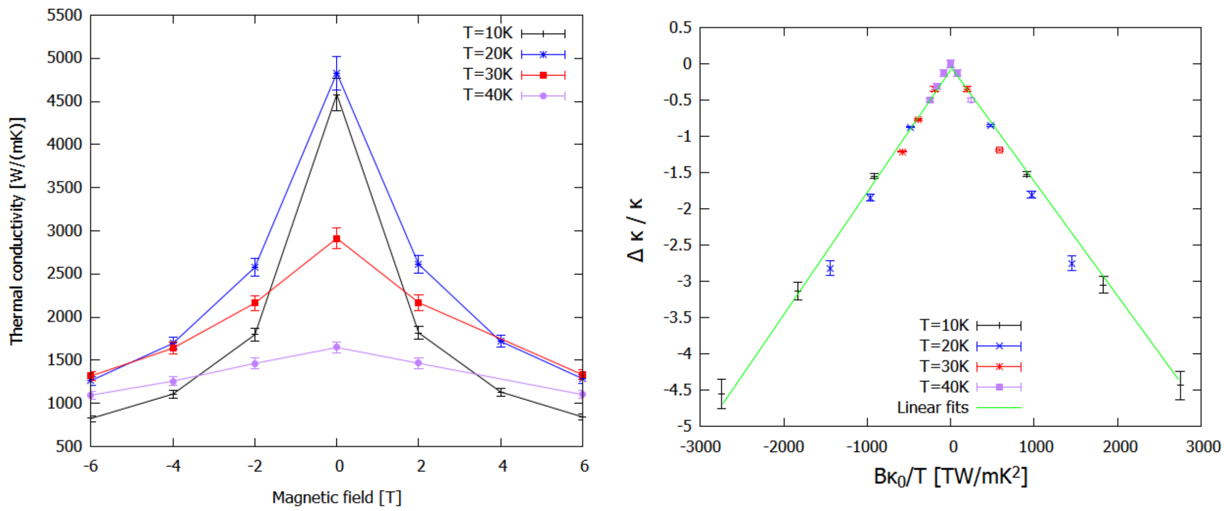
Now let us discuss the temperature dependency for $B \neq 0$. Once again we have some anomalies, this time the measurement for $B = 4T$ was not performed at the required temperatures $T = 30K$ and $T = 40K$, these can be seen circled in Fig. 14a. Removing these only leaves us with 2 data points for $B = 4T$, so we cannot draw concise conclusions for such an applied magnetic field, but we can continue with the analysis for the other magnetic field strengths. We can see in Fig. 14b that the thermal conductivity rises with a stronger magnetic field, and if considering error bars there is almost no discrepancy between the curves for positive and negative applied magnetic field, which is as expected.



(a) (b)

Figure 14: Thermal conductivity versus temperature for $B \neq 0T$ a) shows the anomalies present for $B = 4T$ with a red circle b) Anomaly is removed.

Now let us discuss the magnetic field dependence, this is shown in Fig. 15a. We can see that the shape of the peak depends on the temperature. For lower temperatures we have a sharper peak around $B = 0T$ and when we go to higher temperatures the peaks spread out more. As we compare these for a constant value of temperature the anomaly was not included in the plot.



(a) (b)

Figure 15: a) Thermal conductivity versus applied magnetic field B for different temperatures. b)Kohler plot for T=10K,20K,30K and 40K, with a linear fit.

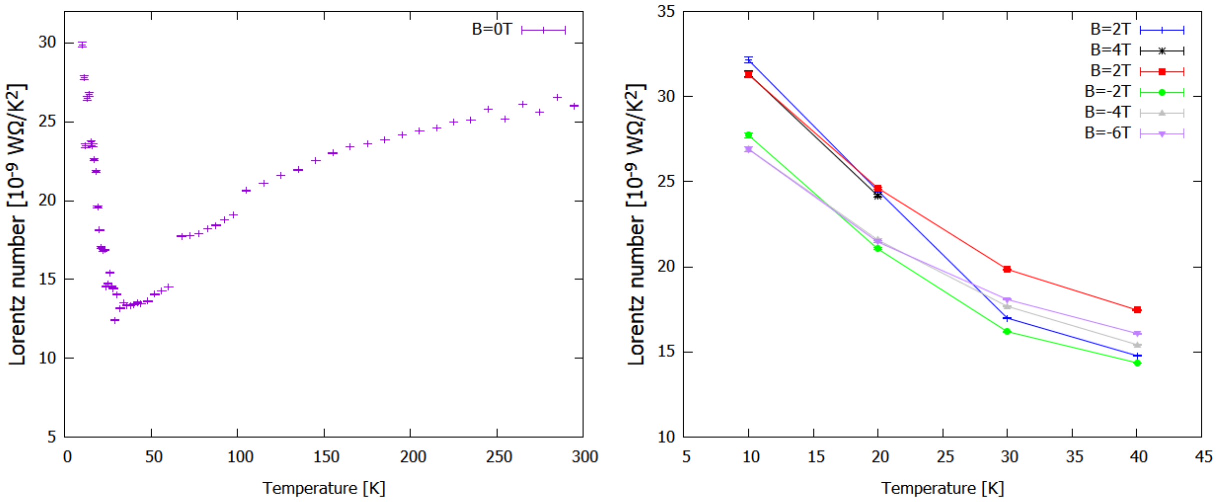
Fig. 15b shows the Kohler plot for the thermal conductivity, once again we see that the data follows a linear relation. The positive and negative values were fitted seperately, the equation for each of the linear fits is given as

$$\begin{aligned} y_{pos}(x) &= (-1.6 \pm 0.1) \times 10^{-3} x + (-0.04 \pm 0.1) \text{ for } x = \frac{B\kappa_0}{T} > 0 \\ y_{neg}(x) &= (1.7 \pm 0.05) \times 10^{-3} x + (-0.08 \pm 0.05) \text{ for } x = \frac{B\kappa_0}{T} < 0 \end{aligned} \tag{4.8}$$

We can see that the slopes agree within error bars, which suggests that the Kohler approximation is realised well in this measurement.

4.2.3 Wiedemann-Franz law

We now have enough information to plot the Lorenz number as a function of temperature, this is done in Fig. 16. The Sommerfeld value of L is 2.4×10^{-8} WΩ/K^2 [3]. We can see that for $B = 0$T we reach that value for high temperatures, but for low temperatures there is a dip. For $B \neq 0$T the Lorentz number is close to the Sommerfeld value, but the temperature range is very small. Note that $B = 4T$ only has 2 data points, due to the anomalies.



(a) (b)

Figure 16: a) Lorentz number versus temperature for $B = 0$T b) Lorentz number versus temperature for $B \neq 0$T.

4.2.4 Hall effect

To calculate the Hall coefficient for each temperature we first plot the Hall voltage with respect to the magnetic field and fit the data linearly.
As we can see in Fig. 17 the linear dependence for the data points is almost indistinguishable. The slopes and the Hall coefficients for each temperature is summarised in Tab. 1.

4.2.5 Further analysis

Now that we know the Hall coefficient we can calculate some additional parameters, in particular the charge carrier density n, the Fermi velocity v_F, the Fermi energy E_F can

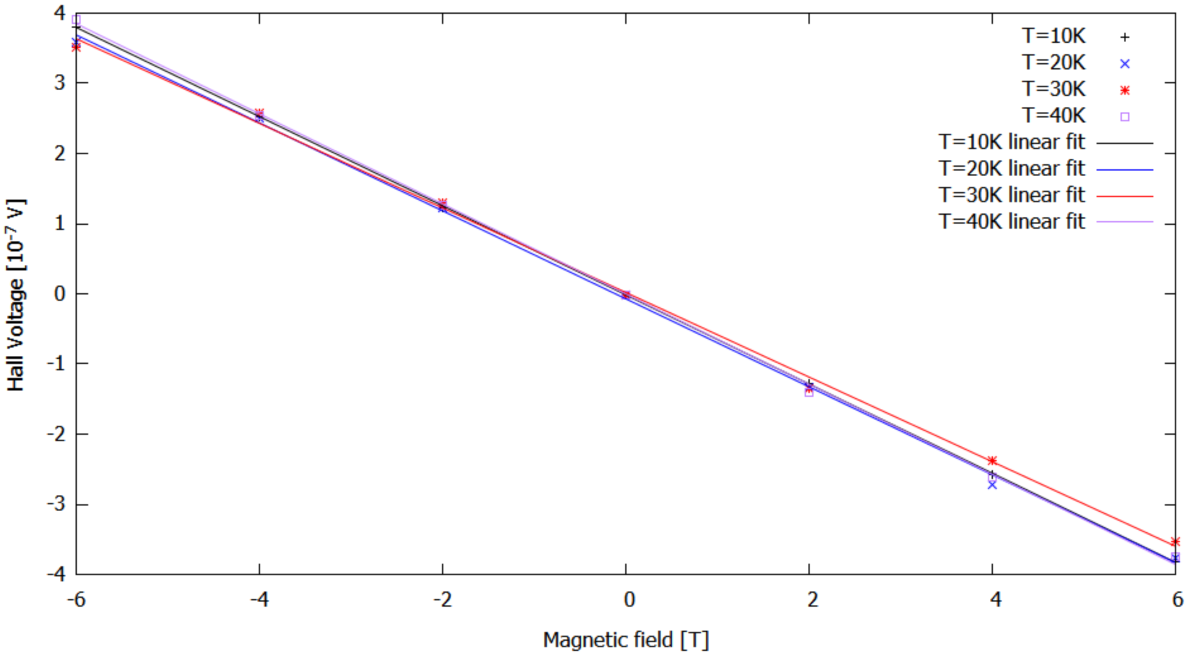


Figure 17: Hall voltage against Magnetic field for T = 10K,20K,30K and 40K.

Temperature [K]	Slope [10^{-8} V/T]	Hall coefficient [10^{-11}m^3/C]
10	-6.35 ± 0.01	-7.06 $\pm$ 0.10
20	-6.27 ± 0.09	-6.97 $\pm$ 0.14
30	-6.02 ± 0.11	-6.69 $\pm$ 0.16
40	-6.42 ± 0.07	-7.13 $\pm$ 0.13

Table 1: Table showing the slope of the linear fittings performed in Fig. 17 for each temperature. The third column gives the Hall coefficient for that corresponding temperature.

be calculated.

$$n = \frac{1}{R_h q} = (8.98 \pm 0.08) \times 10^{28} m^{-3} \tag{4.9}$$

$$v_F = \frac{\hbar}{m}(3\pi^2 n)^{1/3} = (1.60 \pm 0.01) \times 10^6 \ \frac{m}{s} \tag{4.10}$$

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = (1.17 \pm 0.01) \times 10^{-18} \ J \tag{4.11}$$

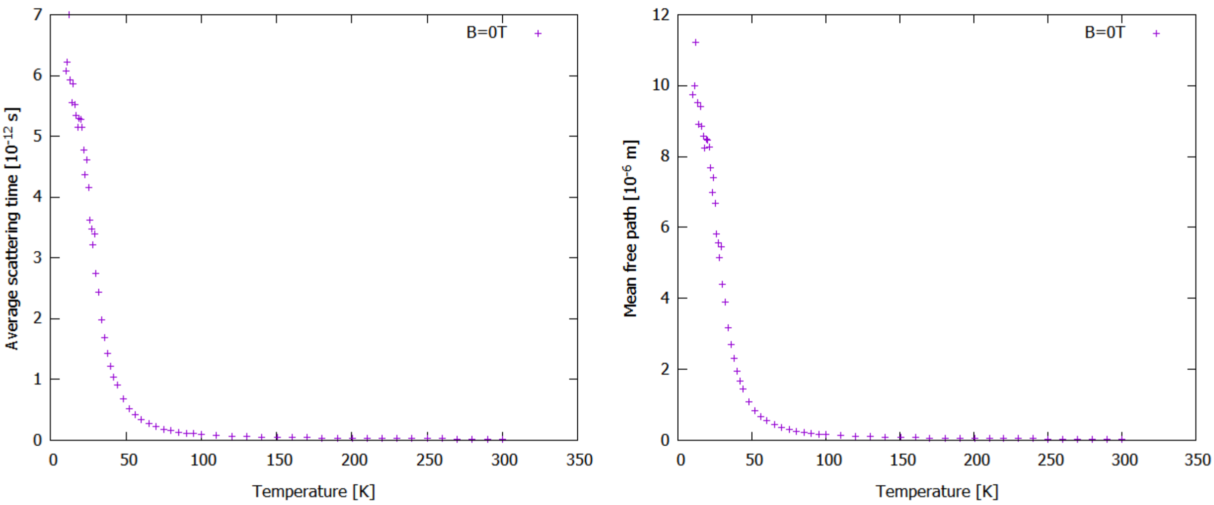
These values agree quite well with the expected values $v_F = 1.57 \times 10^6 \frac{m}{s}$ and $E_F = 1.12 \times 10^{-18} \ J$ [3]
Also a graph of the average scattering time $\tau(T)$ and the mean free path $l(T)$ of the electrons can be plotted against temperature. The average scattering time is given by

$$\tau = \frac{m}{nq^2} \frac{1}{\rho(T)} \tag{4.12}$$

while the mean free path is given by

$$l(\tau) = \tau(T) v_F. \tag{4.13}$$

Both these quantities are given in Fig. 18



(a) (b)

Figure 18: a) Average scattering time versus temperature at $B = 0$T b) Mean free path length versus temperature at $B = 0$T.

5 Discussion

In this experiment we have performed room temperature measurements of the electrical resistivity and thermal conductivity of copper at ambient and at vacuum pressure. The electrical resistivity was obtained to be $\rho_{amb} = (1.76 \pm 0.02) \times 10^{-8} \Omega m$ and $\rho_{vac} = (1.78 \pm 0.02) \times 10^{-8} \Omega m$, which agreed well with the literature results. The thermal conductivity was obtained to be $\kappa_{amb} = (752 \pm 13) W/mK$ and $\kappa_{vac} = (655 \pm 6) W/mK$, these values were quite far away from the expected results from literature. After that we have analysed data from a measurement at low temperatures. This allowed us to calculate the RRR for our sample of copper to be 296 ± 9, which suggests that the copper sample was quite pure. We also showed that the Kohler plot shows a linear behaviour which agrees with Kohler's rule. We have plotted the Widemann-Franz law and calculated the Lorenz number as a function of temperature, we showed that at temperatures above $150K$ the Lorenz number converges to the Sommerfeld value. We calculated the Hall coefficient for copper at certain temperature values given in Tab. 1. At the end from the Hall coefficient we were able to deduce the sign of the charger carrier, as well as calculate the charger carrier density, Fermi velocity and the Fermi energy. We obtained $n = (8.98 \pm 0.08) \times 10^{28} m^{-3}$, $v_F = (1.60 \pm 0.01) \times 10^6 \frac{m}{s}$, $E_F = (1.17 \pm 0.01) \times 10^{-18}\ J$ these values also agreed well with literature values.

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