M 2.3 - Transport Properties of Copper

Appendix

1 Theory

1.1 Magnetoresistance

The electrical resistivity of a metal usually changes when applying an external magnetic field. This change in resistivity is called magnetoresistance. Commonly, the resistivity gets larger in a magnetic field. Let us first try to explain this phenomenon with the electron-gas model.

The conduction electrons move under the influence of an external electric field with an average drift velocity v_D through the metal. The electric field lines are supposed to align in x-direction.

When a magnetic field is applied perpendicular to the current in z-direction, the charge carriers will be deflected to the y-direction due to the Lorentz force. This creates the Hall voltage and thus an electric field in y-direction, such that all forces are again in equilibrium. As a consequence, one should not expect a transverse magnetoresistance.

For a magnetic field parallel to the current in x-direction, the electrons are not effected by the Lorentz force. So, also a longitudinal magnetoresistance cannot be explained by these simple assumptions.

Thus it is easily seen that this simple theory is not sufficient to describe this phenomenon. One can extend the theory by considering two different types of change carriers, such as electrons and holes. Generally, the electric field can be written as

$$\vec{E} = \rho_0 \vec{j} + \frac{e\tau}{m} \vec{B} \times \rho_0 \vec{j}$$

where ρ_0 is the zero-field resistivity and $\vec{E}_\mathrm{H} := \frac{e\tau}{m} \vec{B} \times \rho_0 \vec{j}$ is the electric field generated by the Hall effect. For both change-carrier types one obtains the same equation

$$\vec{E} = \rho_i \vec{j}_i + \frac{e\tau_i}{m_i} \vec{B} \times \rho_i \vec{j}_i$$

with $i = 1, 2$ denoting the charge-carrier type. ρ_1 (ρ_2) is the zero-field resistivity associated to the first (second) charge carrier type with the current contribution j_1 (j_2). The total current is given by

$$\vec{j}_\mathrm{tot} = \vec{j}_1 + \vec{j}_2$$

The current contributions $\vec{j}_i$ can be written as

$$\vec{j}_i = \frac{1}{\rho_i} \cdot \frac{E - \beta_i \vec{B} \times \vec{E}}{1 + \beta_i^2 B^2} = \sigma_i \cdot \frac{E - \beta_i \vec{B} \times \vec{E}}{1 + \beta_i^2 B^2}$$

where $\beta_i := \frac{e\tau_i}{m_i}$. For the total current we obtain

$$\vec{j}_\mathrm{tot} = \left(\frac{\sigma_1}{1 + \beta_1^2 B^2} + \frac{\sigma_2}{1 + \beta_2^2 B^2} \right) \vec{E} - \left(\frac{\sigma_1 \beta_1}{1 + \beta_1^2 B^2} + \frac{\sigma_2 \beta_2}{1 + \beta_2^2 B^2} \right) \vec{B} \times \vec{E}$$

To investigate the transverse magneto-resistance we connect $\vec{j}$ with the component of $\vec{E}$ in $\vec{j}$-direction:

$$\rho = \frac{\vec{j} \cdot \vec{E}}{j^2} = \frac{\frac{\sigma_1}{1+\beta_1^2 B^2} + \frac{\sigma_2}{1+\beta_2^2 B^2}}{\left(\frac{\sigma_1}{1+\beta_1^2 B^2} + \frac{\sigma_2}{1+\beta_2^2 B^2}\right)^2 + \left(\frac{\sigma_1 \beta_1 B}{1+\beta_1^2 B^2} + \frac{\sigma_2 \beta_2 B}{1+\beta_2^2 B^2}\right)^2}$$

Comparing this with the specific resistivity without magnetic field

$$\rho_0 = \frac{1}{\sigma_1 + \sigma_2}$$

we obtain after a few rearrangements

$$\frac{\Delta\rho}{\rho_0} = \frac{\sigma_1 \sigma_2 (\beta_1 - \beta_2)^2 B^2}{(\sigma_1 - \sigma_2)^2 + B^2(\beta_1 \sigma_1 + \beta_2 \sigma_2)^2}$$

with $\Delta\rho := \rho - \rho_0$. Unfortunately this formula is usually inapplicable, because the classification of the charge carriers into two types with such plain properties is an oversimplification. Furthermore this simple two-band model cannot explain the longitudinal magnetoresistance, that occurs if the magnetic field is applied parallel to the current. This effect is observed in metals as well, but requires a rather complicated explanation (see Ziman, Electrons and Phonons, 1960). However the simple model is sufficient for our considerations of the transversal magnetoresistance. $\Delta\rho$ is always positive and for $\beta_1 = \beta_2$ we obtain $\Delta\rho$, e.g. if we have only one type of charge carriers. Therefore different types of charge carriers are necesseray for magnetoresistance. They can differ in their charge, sign, relaxation time and mass. In metals a magnetoresistance is observed, so the existence of different charge carriers is proven. Assumed the different types of charge carriers have the same relaxation time, the ratio $\frac{\Delta\rho}{\rho_0}$ is a function of the product of τ and B. τ is inversely proportional to ρ_0, so we obtain

$$\frac{\Delta\rho}{\rho_0} = F\left(\frac{B}{\rho_0}\right) \tag{1}$$

This is known as Kohler's Rule. The function F depends on the type of metal only. To prevent false conclusions, it should be emphasized that this formula is just an approximation.

1.2 Thermal conductivity in magnetic fields

In metals the heat transport is carried mainly by the conduction electrons. Consider a heat flow in x direction and a transverse magnetic field in z direction, the electrons are deflected in y direction between the scattering processes due to the Lorentz force. This deflection causes a reduction of the thermal conductivity in x direction. Kohler proposed a relation for the magnetic field dependence of the thermal conductivity similar to that of the resistivity:

$$\frac{\Delta\kappa}{\kappa} = G\left(\frac{B\kappa_0}{T}\right) \tag{2}$$

where κ_0 is the thermal conductivity in zero field and G is like F a metal specific function. As well as for the resistivity, this function is an approximation.

2 Measuring methods

2.1 Resistivity

To determine the resistivity ρ a constant current I_R is applied to the sample and the voltage drop U_R caused by the sample is measured. The copper sample has a very small resistivity, so with the standard method (2-point method) the voltage drop is caused mainly by the wires and contacts. To avoid this we use the 4-point method that allows to measure the voltage nearly currentless by a voltmeter connected directly to the sample. The resistivity is then determined by

$$\rho = \frac{AU_\mathrm{R}}{I_\mathrm{R} l_\mathrm{R}} \tag{3}$$

where A is the cross-sectional area of the sample and l_R is the distance between the inner contacts.

2.2 Thermal conductivity

The lower part of the sample is thermally coupled to the sample holder, which has temperature T_0. On the upper part of the sample a heater is arranged to produce the stationary heat flow through the sample. The temperature gradient is measured by a thermo couple. Because copper is a good heat conductor, the thermal resistance of the couplings between heater and sample as well as between sample and sample holder cannot be neglected. Therefore a 4-point method is chosen similar to the measurement of the resistivity. The relation between heat flux j_x, heater power P and cross-sectional area A is given by $j_x = P/A$. The heater power is given by $P = U_\mathrm{PH} I_\mathrm{PH}$. With the temperature difference between the tips of the thermo couple ΔT and their distance l_TE the temperature gradient $\Delta T / l_\mathrm{TE}$ is given by

$$\frac{U_\mathrm{PH} I_\mathrm{PH}}{A} = \kappa \frac{\Delta T}{l_\mathrm{TE}} \tag{4}$$

For a thermo couple it is $\Delta T = U_\mathrm{TE}/S(T,B)$ with the thermopower $S(T,B)$ and the thermoelectric voltage U_TE. We obtain

$$\kappa = \frac{U_\mathrm{PH} I_\mathrm{PH} S(T,B) l_\mathrm{TE}}{U_\mathrm{TE} A} \tag{5}$$

2.3 Hall effect

If we apply a constant current I_H to the sample and a magnetic field B perpendicular to the current, we can measure the so called Hall voltage U_H perpendicular to the current and the field. The Hall constant is given by

$$A_\mathrm{H} = \frac{U_\mathrm{H} d}{B I_\mathrm{H}} \tag{6}$$

where d is the thickness of the sample.