$$\hat{\beta}_0 = \frac{S_{XY}}{S_{XX}} = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_0 \bar{x}$$

Multiple Linear Reg.

$$\vec{\beta} = (X^T X)^{-1} X^T y$$

For simple Reg. $MS_E = SS_E$

For multiple $MS_E \neq SS_E$

$$MS_E = \frac{SS_E}{\text{deg of freedom}}$$

$$= \frac{SS_E}{n-k-1} = \frac{SS_E}{n-p}$$

$$\hat{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

$$\underbrace{\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3}_{k=3}$$

$$p = 3 + 1$$

3 tests

① Marginal — T-test

(Test on single β_j).

$H_0: \beta_1 = 3$, $H_1: \beta_1 \neq 3$

$$t = \frac{\hat{\beta}_1}{se(\beta_1)}$$

Test marginal

$$\hat{Y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

t-test, if σ^2 is unknown. Use MS_E as $\hat{\sigma}^2$

If σ^2 is given, Z-test.

$$C = (X'X)^{-1} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$se(\beta_1) = \sqrt{MS_E \times C_{..}}$$

$$se(\beta_1) = \sqrt{MS_E \times C_{1+1,1+1}}$$

$$= \sqrt{MS_E \times C_{22}}$$

---

(2) . Test if all β's is zero.

F-test $\rightarrow$ ANOVA

$$\begin{cases} H_0: \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0 \\ H_1: \text{At least one } \beta_j \neq 0, \\ \quad j = 0, 1, 2, 3 \end{cases}$$

Ex. 2.4.1

Test stat.

$H_0: \beta_2 = \beta_3 = 0$, H_1: At least one $\beta_j \neq 0$, $j = 2, 3$

$$F_1 = \frac{SS_E(\beta_2 | \beta_1) / (2+1-2)}{SS_E(\beta) / (40-2-1)}$$

$$30.83 = \frac{SS_E(B) - SS_E(A)}{SS_E(B) / 37}$$

$$\Rightarrow SS_E(B) = \frac{37(220)}{30.83 + 37}$$

$$= 120.0059.$$

$$F_2 = \frac{SS_E(\beta_3 \mid \beta_1, \beta_2) / 1}{SS_E(C) / (40-3-1)}$$

$$= \frac{[SS_E(B) - SS_E(C)]}{SS_E(C) / 36} = 12$$

$$\Rightarrow SS_E(C) = 90.004$$

$$F_0 = \frac{SS_E(\beta_2, \beta_3 \mid \beta_1) / (4-2)}{SS_E(C) / (40-3-1)}$$

$$\overline{\qquad SS_E(C) \,/\, (40-3-1)}$$

$$= \frac{[SS_E(A) - SS_E(C)]/2}{SS_E(C)/36}$$

$$\Rightarrow F_0 = 25.99875.$$

Ex. 2.5.3

$$C = (X^T X)^{-1} = \begin{bmatrix} 188.9832 & & \\ & \overset{C_{22}}{0.25} & \\ & & 5.0625 \end{bmatrix}$$

$$Se(\beta_1) = \sqrt{MSE \times C_{22}} = \sqrt{0.0361 \times 0.25}$$

$$= 0.95$$

95% CI for β_1,

$$\hat{\beta}_1 \pm t_{\frac{0.05}{2}, 25-2-1} Se(\beta_1)$$

$$= 0.14 \pm 2.074 (0.95)$$

$$= [-0.05703, 0.33703].$$

ii.) $$[1 \quad 1.5 \quad 3.2] \begin{bmatrix} 188.9832 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} 1 \\ 1.5 \\ 3.2 \end{bmatrix}$$

$$= K$$

$$Se(\hat{y}_h) = \sqrt{0.0361 (1 + K)}$$

$$\hat{y}_h = [1 \quad 1.5 \quad 3.2] \begin{bmatrix} -4.04 \\ 0.14 \\ 0.45 \end{bmatrix}$$

$$= 1 \times (-4.04) + 1.5(0.14) + 3.2(0.45)$$

$$= -2.39$$

95% PI,

$$-2.39 \pm \boxed{t_{\frac{0.05}{2}, 25-2-1}} Se(\hat{y}_n)$$

2.074

(iii). Test Stat.

$$H_0: \beta_2 = 2, \qquad H_1: \beta_2 \neq 2.$$

$$T^* = \frac{\hat{\beta}_2 - 2}{Se(\beta_2)} = -3.6257$$

0.45

$$Se(\beta_2) = \sqrt{MS_E \times C_{33}}$$

$$= \sqrt{0.0361 \times 5.0625}$$

Ex. 2.6.2

$$\hat{\beta} = \begin{bmatrix} 95 \end{bmatrix}$$

$$\hat{\beta} = \begin{bmatrix} 95 \\ 15 \\ 55 \\ -1 \end{bmatrix}, \quad X_h' = [1 \quad 25 \quad 1.5 \quad 37.5]$$

$$E(Y_h) = X_h' \hat{\beta}$$

$$= [1 \quad 25 \quad 1.5 \quad 37.5] \begin{bmatrix} 95 \\ 15 \\ 55 \\ -1 \end{bmatrix}$$

$$= 515.$$

2. 99% PI on y_h,

$$X_h' (X^T X)^{-1} X_h = 0.8.$$

$$se(y_h) = \sqrt{MS_E (1 + 0.8)}$$

$$= \sqrt{14.0118(1+0.8)}$$

$MS_E = ?$ $\quad k = 3$

$$SS_E = SS_T - SS_R$$

$$= 688.2 - 213.6$$

$= 688.2 - 213.6$

$= 474.6.$

$$MS_E = \frac{SS_E}{(32+6)-3-1} = 14.0118.$$

95% PI, $\hat{Y}_h \pm t_{\frac{0.01}{2}, 32-3-1} \sqrt{14.0118(1+0.8)}$

$= [501.403, 528.5797].$

#

3. Test stat.

$H_0: \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$

$H_1:$ At least one $\beta_j \neq 0$,

$j = 0, 1, 2, 3.$

F-test

Source	Sum of Sq.	Df	Mean Sq.

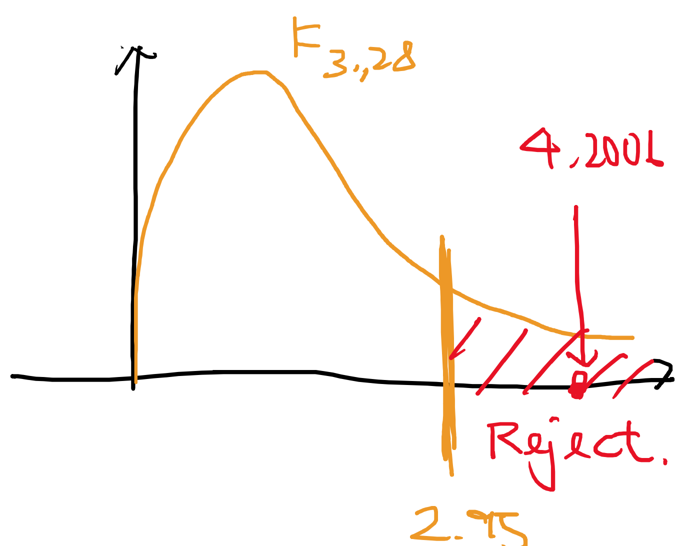
Source	Sq.	Df	Mean Sq.
Regression	213.6	3	$\frac{213.6}{3} = 71.2$
Error	688.2 − 213.6 = 474.6 (SS_E)	32 − 3 − 1 = 28	$MS_E = 16.95$
Treatment	688.2	31	

$$F^* = \frac{MS_R}{MS_E} = \frac{71.2}{16.95} = 4.2006$$

$$F_{0.05;3,28} = 2.95$$

$$\because F^* > F_{0.05;3,28} = 2.95$$

Reject H_0



4. $R^2_{Adj} = 1 - SS_E/(n-k-1)$

4. $$R^2_{Adj} = 1 - \frac{SS_E/(n-k-1)}{SS_T/(n-1)}$$

$$= 1 - \frac{474.6/(32-3-1)}{688.2/(32-1)}$$

$$R^2_{Adj} = 0.2365.$$

$$R^2 = 1 - \frac{SS_E}{SS_T} = 0.689625$$

$\because R^2_{Adj} < R^2$, No extra predictors need to be add.

5. Rate of change

$$\hat{Y} = \underset{0}{\overset{\text{constant}}{\cancel{\boxed{95}}}} + 15\underset{1}{\overset{1?}{\cancel{x_1}}} + \overset{0}{\cancel{\boxed{55x_2}}} - \overset{\text{coef}}{\cancel{x_1}}\boxed{x_2}$$

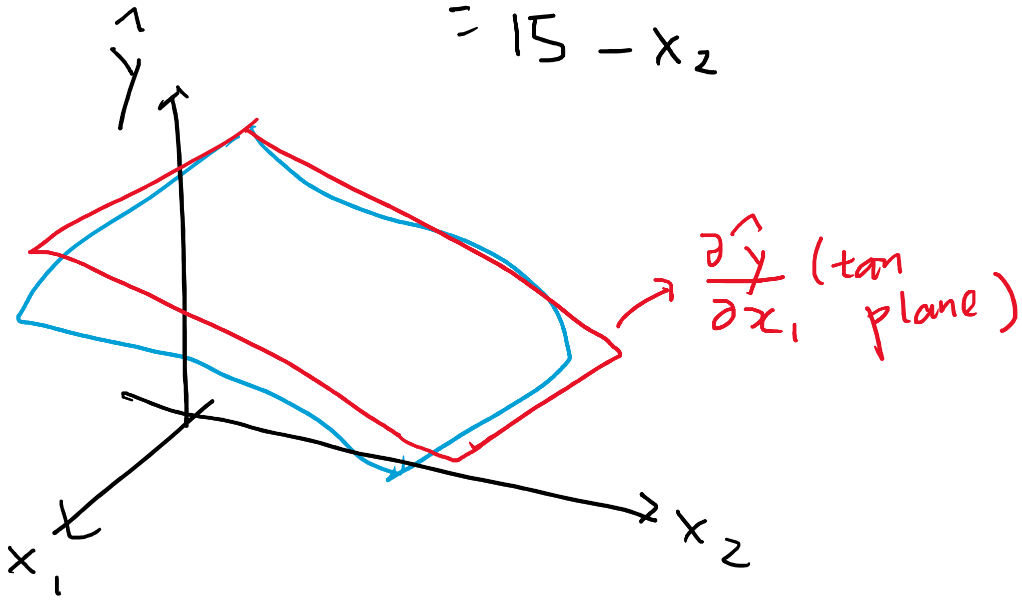
Rate of change of temparature (x_1).

$= 1$

rate of change of temperature (x_1).

$$\frac{\partial \hat{y}}{\partial x_1} = 15(1) + 0 - x_2$$

$$= 15 - x_2$$



6. Marginal Test

$H_0: \beta_3 = 0 \qquad H_1: \beta_3 \neq 0.$

σ^2 unknown, $\hat{\sigma}^2 \longrightarrow MS_E = 16.95$.

$$t^* = \frac{\hat{\beta}_3}{Se(\beta_3)} = \frac{-1}{0.35}$$

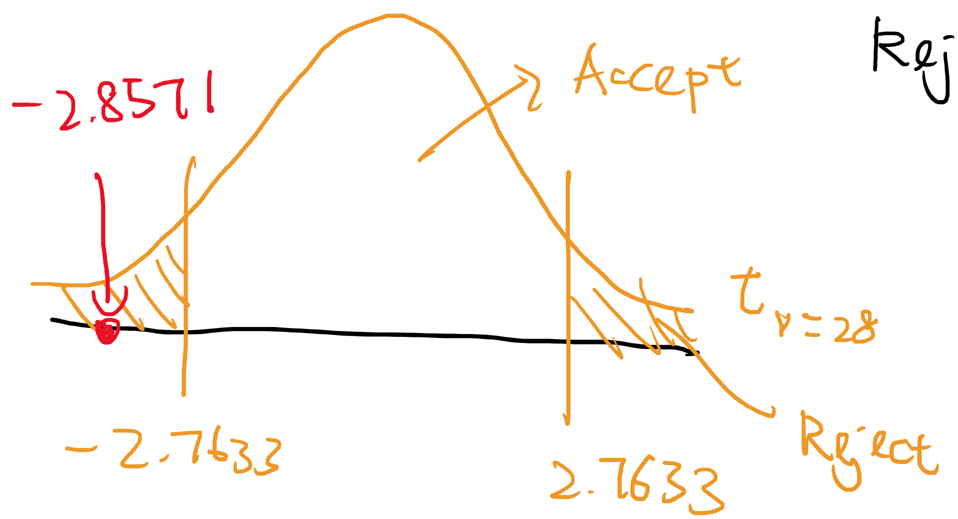
$$= -2.8571$$

At $\alpha = 0.01$

-2.8571

At $\alpha = 0.01$

$t_{\frac{0.01}{2};28} = 2.7633$



Reject H_0.

$\therefore |t^*| = |-2.8571| = 2.8571 > t_{\frac{0.01}{2};28} = 2.7633$

Reject H_0. . . .