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Terminologies

A financial derivative serves as a contract between two parties that derives its value from the value of some underlying asset(s). Examples of underlying assets includes financial assets (such as stocks or bonds), commodities (raw materials, gold), market indices (S&P500, FTSE100), interest rates (LIBOR rates), foreign currencies, etc.

1.1 Risks Management

1.1.1 Catastrophe bonds (Cat bonds)

Catastrophe bond (cat bonds) are risk-linked securities that allows the issuer to receive funding from the bond in the event of a natural disaster. The sponsors of cat bonds can be insurers, reinsurers, corporations, or government agencies.

Stock vs Options

The stocks represent shares of ownership in individual companies, while options are contracts with other investors that let you bet on which direction you think a stock price is headed. The beauty of investing a stock is simplicity: you buy a stock, betting its price will rise so you can sell at a higher price. The options can also be used to bid on market movement – Call prices increase as the underlying stock price goes up, and vice versa for put prices.

1.2 Forward contracts

A **forward contract** (or future contract) is a commitment to purchase stock at a specified **future date** (also known as **expiration date** or **maturity date**) for a certain price. The forward contracts are privately executed between two parties.

- European options can only be exercised on the expiration.
- American options can be exercise at any time during the lifetime of the option.
- Bermuda options can be only exercise on certain pre-specified date before or at the expiration date. Such options are rarely happened in stock options market but more common in swap markets.

Consider we have some simple European put/call options. At expiry time T you have the right, but not obligation, to

- Buy an asset at a prescribed price K (the exercise or strike price). This is a call option.
- Sell the asset at a prescribed price K (the exercise or strike price). This is a put option.

$$\begin{array}{cc} \text{Long} \longrightarrow \text{Buyer} & \text{Short} \longrightarrow \text{Seller} \\ \text{Call} \longrightarrow \text{to buy an asset} & \text{Put} \longrightarrow \text{to sell the asset} \end{array}$$

Without causing confusion, the terminologies are simplified below:

So "long put" refers to buying a put option.

1.2.1 Prepaid Forward Contract

1.3 Covered Writing

A covered call is a call option that is sold by an investor who owns the underlying assets.

$$\text{Covered Call Profit} = \underbrace{\boxed{-\max(S_T - K, 0)}}_{\text{Call Payoff}} + \underbrace{\boxed{c(S_0, K, T)e^{rT}}}_{\text{FV of call premium}} \tag{1.1}$$

1.4 Put-Call parity

Theorem 1.4.1: Put-Call Parity for European options

Let $c(S_0, K, T)$ and $p(S_0, K, T)$ be the premiums for a T-year European call option with strike price K.

$$c(S_0, K, T) - p(S_0, K, T) = F^P_{0,T}(S) - Ke^{-rT} \tag{1.2}$$

The put-call parity stated that the difference between the call's payoff and the put's payoff, is equal to the difference between the prepaid forward price of the underlying stock and the present value of strike price. By the law of one price, the costs must be equal.

Example 1.4.1. A 15-year 1000 par value bond bearing a 12% coupon rate payable annually is bought to yield 10% convertible continuously. A 1000-strike European call on the bond sells for 150 and expires in 15 months. Calculate the value of a 1000-strike European put on the bond that expires in 15 months.

Solution With annual interest rate $j = e^{0.10}$, the price of the bond is

$$\begin{aligned} B_0 &= 120a_{\overline{15}|j} + 1000e^{-0.10\times 15} \\ &= 120\left(\frac{1 - e^{-0.10\times 15}}{e^{0.10} - 1}\right) + 1000e^{-1.5} \\ &= 1109.54 \end{aligned}$$

By put-call parity for options on bonds, we have

$$\begin{aligned} c(1000, 1.25) &= p(1000, 1.25) + [B_0 - PV_{0,1}(\text{Coupons})] - PV_{0,1.25}(K) \\ \rightarrow 150 &= p(1000, 1.25) + [1109.54 - (1000 \times 12\%)e^{-0.10}] - 1000e^{-0.10\times 1.25} \end{aligned}$$

Solving for put price, we find $p(1000, 1.25) = 31.54$. □

1.4.1 Moneyness of options

Moneyness describes the intrinsic value of an option in its current state. It describes whether the payoff of the option would be positive if we exercised immediately.

1.4.2 Capped Options

A capped option sets the limit of maximum possible profit for its holder. The option will automatically exercises once the underlying asset closes at or beyond a specified price. Likewise, capped put options exercise if and only if the underlying asset closes at or below the predetermined level.

$$\text{Unhedged payoff} + \text{Payoff} = \text{Hedged Payoff} \tag{1.3}$$

Unlimited uncertain possibility of gain.

1.5 Future Contracts

Like a forward contract, a future contract is an agreement between two counterparts to buy/sell an asset for a prescribed price at the delivery date. In fact, futures are **exchange-traded** forward contracts.

Theorem 1.5.1: Put-Call Parity for European futures options

Let $c(F_0, K, T)$ and $p(F_0, K, T)$ be the premiums for a T-year European futures options with strike price K.

$$c(F_0, K, T) - p(F_0, K, T) = (F_0 - K)e^{-rT} \tag{1.4}$$

Note that futures usually expires a short time after time T. This is similar to ordinary put-call parity, with dividend yield $\delta = r$.

1.5.1 Foreign currencies

The exchange rate is the value of one unit of the foreign currency measured in the domestic currency.

Theorem 1.5.2: Put-Call Parity for European Foreign Currency Options

Let x be the exchange rate. with strike price K.

$$c(x_0, K, T) - p(x_0, K, T) = F_{0,T}^P(x) - Ke^{-rT} \tag{1.5}$$

where $c(x_0, K, T)$ and $p(x_0, K, T)$ are dollar-dominated call price and put price, respectively.

1.6 Spread strategies

Option spread strategies are commonly applied to minimize risk on various market outcomes using two or more options of the **same** type (either two or more calls, or two or more puts).

1.6.1 Bull Spread

The profit of the bull call spread is the profit of the long call plus the profit of the short call. In fact, the profit is the payoff of the combined position subtracts the FV of the net premium, that is,

$$
\begin{aligned}
\text{Profit} &= \max(S_T - K_2, 0) - FV(c(K_2, T)) - [\max(S_T - K_1, 0) - FV(c(K_1, T))] \\
&= \max(S_T - K_2, 0) - \max(S_T - K_1, 0) - FV[c(K_1, T) - c(K_2, T)] \\
&= \begin{cases} -FV[c(K_1, T) - c(K_2, T)] & \text{if } S_T < K_1 \\ S_T - K_1 - FV[c(K_1, T) - c(K_2, T)] & \text{if } K_1 \le S_T < K_2 \\ K_2 - K_1 - FV[c(K_1, T) - c(K_2, T)] & \text{if } S_T \ge K_2 \end{cases}
\end{aligned}
$$

Bull put spread

$$
\begin{aligned}
\text{Profit} &= \overset{\text{Buying } K_2 \text{ -strike put}}{\boxed{\max(K_1 - S_T, 0) - FV(p(K_2 - S_T, T))}} - \overset{\text{Selling } K_1 \text{ -strike call}}{\boxed{[\max(K_2 - S_T, 0) - FV(p(K_2, T))]}} \\
&= \max(K_1 - S_T, 0) - \max(K_2 - S_T, 0) - FV[p(K_1, T) - p(K_2, T)] \\
&= \begin{cases} K_1 - K_2 - FV[p(K_1, T) - p(K_2, T)] & \text{if } S_T < K_1 \\ S_T - K_2 - FV[p(K_1, T) - p(K_2, T)] & \text{if } K_1 \le S_T < K_2 \\ -FV[p(K_1, T) - p(K_2, T)] & \text{if } S_T \ge K_2 \end{cases}
\end{aligned}
$$

1.7 Volatility Speculation

Volatility speculations are important means to make profit. **Speculation** refers to strategic activities that trying to make profit from security's price movement, whereas hedging attempts to reduce the risk exposure that associated with a security's price change.

In this section we discuss the strategies that depend on the volatility of a financial market rather than the direction of the asset price change. Such strategies are: butterfly spread, straddles, and strangles.

1.7.1 Butterfly spread

A butterfly spread is a strategy that make profit from very small stock price movement. It involves in positions in options of the same type with three different strikes. Given the strike prices $0 < K_1 < K_2 < K_3$, we define λ as following:

$$\lambda = \frac{K_3 - K_2}{K_3 - K_1} \tag{1.6}$$

such that $K_2 = \lambda K_1 + (1 - \lambda) K_3$, where $0 < \lambda < 1$.

Yet another way to form a butterfly spread is to combine **bull call spread** and **bear put spread**, which means we are trading four options at the same time.

The profit of long calls in butterfly spread is

$$\begin{aligned}
\text{Profit} &= \lambda[\max(S_T - K_1, 0) - FV(c(K_1, T))] + (1-\lambda)[\max(S_T - K_3, 0) - FV(c(K_3, T))] \\
&\quad - [\max(S_T - K_2, 0) - FV(c(K_2, T))] \\
&= \lambda \max(S_T - K_1, 0) + (1-\lambda)\max(S_T - K_3, 0) - \max(S_T - K_2, 0) \\
&\quad - FV[\lambda c(K_1, T) + c(K_3, T)(1-\lambda) - c(K_2, T)]
\end{aligned}$$

1.8 Collars

A collar is a strategy that involved purchasing a put and selling the call at a higher strike price. The difference $K_2 - K_1$ is called the **collar width**. If the collar width is zero, then the collar becomes a short forward.

The cost of a collar is $p(K_1, T) - c(K_2, T)$, which can be positive or negative.

$$\begin{aligned}
\text{Profit} &= \max(K_1 - S_T, 0) - FV(p(K_1, T)) - [\max(S_T - K_2, 0) - FV(c(K_2, T))] \\
&= \max(K_1 - S_T, 0) - \max(S_T - K_2, 0) - FV[p(K_1, T) - c(K_2, T)]
\end{aligned}$$