This is a WinBUGS program for the artificial example in Chapter 11, Section 11.4.1.

Model: Finite mixtures in Structural Equation Model
Data Set Name: YO.dat
Sample Size: N=800

We assume a mixture of K subpopulations, and let L[i] denote a latent group membership indicator for sample i, pi[1:K] are the unknown mixing proportions. Bayesian estimates are obtained by using permutation sampler with the identifiability constraint mu[1,5]<mu[2,5].

```
model {
    for(i in 1:N){
        #measurement equation model
        for(j in 1:P){
            y[i,j]~dnorm(mu[i,j],psi[L[i],j])
            ephat[i,j]<-y[i,j]-mu[i,j]
        }
        mu[i,1]<-eta[i]+alp[L[i],1]
        mu[i,2]<-lam[L[i],1]*eta[i]+alp[L[i],2]
        mu[i,3]<-lam[L[i],2]*eta[i]+alp[L[i],3]
        mu[i,4]<-xi[i,1]+alp[L[i],4]
        mu[i,5]<-lam[L[i],3]*xi[i,1]+alp[L[i],5]
        mu[i,6]<-lam[L[i],4]*xi[i,1]+alp[L[i],6]
        mu[i,7]<-xi[i,2]+alp[L[i],7]
        mu[i,8]<-lam[L[i],5]*xi[i,2]+alp[L[i],8]
        mu[i,9]<-lam[L[i],6]*xi[i,2]+alp[L[i],9]

        #structural equation model
        xi[i,1:2]~dmnorm(u[1:2],phi.xi[L[i],1:2,1:2])
        eta[i]~dnorm(nu[i],psd[L[i]])
        nu[i]<-gam[L[i],1]*xi[i,1]+gam[L[i],2]*xi[i,2]
        dthat[i]<-eta[i]-nu[i]

        L[i] ~ dcat(pi[1:K])
    }# end of i

    # prior for mixture probability vector
    pi[1:K] ~ ddirch(alpha[])
    for (j in 1:K) {alpha[j]<-1}

    # priors on intercepts: for finding the identification constraints
    #alp[1,1] ~ dnorm(0.0321,0.01);        alp[2,1] ~ dnorm(0.0321,0.01)
    #alp[1,2] ~ dnorm(0.0040,0.01);        alp[2,2] ~ dnorm(0.0040,0.01)
    #alp[1,3] ~ dnorm(0.0132,0.01);        alp[2,3] ~ dnorm(0.0132,0.01)
    #alp[1,4] ~ dnorm(0.2786,0.01);        alp[2,4] ~ dnorm(0.2786,0.01)
    #alp[1,5] ~ dnorm(0.8876,0.01);        alp[2,5] ~ dnorm(0.8876,0.01)
    #alp[1,6] ~ dnorm(0.5148,0.01);        alp[2,6] ~ dnorm(0.5148,0.01)
    #alp[1,7] ~ dnorm(1.0150,0.01);        alp[2,7] ~ dnorm(1.0150,0.01)
    #alp[1,8] ~ dnorm(1.0378,0.01);        alp[2,8] ~ dnorm(1.0378,0.01)
    #alp[1,9] ~ dnorm(0.9449,0.01);        alp[2,9] ~ dnorm(0.9449,0.01)
```

```
    # priors on intercepts: for the final Bayesian solutions
    alp[1,1] ~ dnorm(0.0321,0.01);          alp[2,1] ~ dnorm(0.0321,0.01)
    alp[1,2] ~ dnorm(0.0040,0.01);          alp[2,2] ~ dnorm(0.0040,0.01)
    alp[1,3] ~ dnorm(0.0132,0.01);          alp[2,3] ~ dnorm(0.0132,0.01)
    alp[1,4] ~ dnorm(0.2786,0.01);          alp[2,4] ~ dnorm(0.2786,0.01)
    alp[1,6] ~ dnorm(0.5148,0.01);          alp[2,6] ~ dnorm(0.5148,0.01)
    alp[1,7] ~ dnorm(1.0150,0.01);          alp[2,7] ~ dnorm(1.0150,0.01)
    alp[1,8] ~ dnorm(1.0378,0.01);          alp[2,8] ~ dnorm(1.0378,0.01)
    alp[1,9] ~ dnorm(0.9449,0.01);          alp[2,9] ~ dnorm(0.9449,0.01)
    alp[2,5] ~ dnorm(0.8876,0.00001)I(alp[1,5],)
    alp[1,5] ~ dnorm(0.8876,0.00001)I(,alp[2,5])

    # priors on loadings and coefficients
    lam[1,1]~dnorm(0.4,psi[1,2]);   lam[1,2]~dnorm(0.4,psi[1,3])
    lam[1,3]~dnorm(0.8,psi[1,5]);   lam[1,4]~dnorm(0.8,psi[1,6])
    lam[1,5]~dnorm(0.4,psi[1,8]);   lam[1,6]~dnorm(0.4,psi[1,9])
    lam[2,1]~dnorm(0.8,psi[2,2]);   lam[2,2]~dnorm(0.8,psi[2,3])
    lam[2,3]~dnorm(0.4,psi[2,5]);   lam[2,4]~dnorm(0.4,psi[2,6])
    lam[2,5]~dnorm(0.8,psi[2,8]);   lam[2,6]~dnorm(0.8,psi[2,9])
    gam[1,1]~dnorm(0.2,psd[1]);     gam[1,2]~dnorm(0.7,psd[1])
    gam[2,1]~dnorm(0.7,psd[2]);     gam[2,2]~dnorm(0.2,psd[2])

    # priors on precisions
    for(j in 1:K){
        for(k in 1:P){
            psi[j,k]~dgamma(10.0, 8.0)
            sgm[j,k]<-1/psi[j,k]
        }
    }
    for(j in 1:K){
        psd[j]~dgamma(10.0, 8.0)
        sgd[j]<-1/psd[j]
    }
    phi.xi[1,1:2,1:2]~dwish(R1[1:2,1:2], 6)
    phx[1,1:2,1:2]<-inverse(phi.xi[1,1:2,1:2])
    phi.xi[2,1:2,1:2]~dwish(R2[1:2,1:2], 6)
    phx[2,1:2,1:2]<-inverse(phi.xi[2,1:2,1:2])
} #end of model
```

Data

```
list(N=800,P=9,K=2, u=c(0.0, 0.0),
    R1=structure(
        .Data= c(5.0, 0.0, 0.0, 5.0),
        .Dim= c(2,2)),
    R2=structure(
        .Data= c(5.0, 0.0, 0.0, 5.0),
        .Dim= c(2,2)),
    y=structure(
        .Data= c(paste YO.dat here),
        .Dim= c(800,9)))
```

Three different initial values

```
list(
    pi=c(0.75,0.25),
    alp=structure(
        .Data=c(0.0,0.0,0.0,0.0,0.0,1.0,1.0,1.0,1.0,0.0,0.0,0.0,0.5,1.5,0.0,1.0,1.0,1.0),
        .Dim=c(2,9)),
    lam=structure(
        .Data=c(0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0),
        .Dim=c(2,6)),
    psi=structure(
        .Data=c(1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0),
        .Dim=c(2,9)),
    psd=c(1.0, 1.0),
    gam=structure(
        .Data=c(1.0, 1.0, 1.0, 1.0),
        .Dim=c(2,2)),
    phi.xi=structure(
        .Data=c(1.0, 0.1, 0.1, 0.9, 1.0, 0.1, 0.1, 0.9),
        .Dim=c(2,2,2)))

list(
    pi=c(0.456, 0.544),
    alp=structure(
        .Data=c(-0.1001,-0.0372,-0.0562,-0.0203,0.0719,0.9614,0.9253,0.9327,0.8852,-0.1018,
    -0.1605, -0.1262,0.3071,1.4770,-0.1706,0.8305,0.9410,0.8039),
        .Dim=c(2,9)),
    lam=structure(
        .Data=c(0.347,0.351,0.675,0.647,0.327,0.433,0.742,0.754,0.248,0.337,0.826,0.761),
        .Dim=c(2,6)),
    psi=structure(
        .Data=c(0.345,0.406,0.423,0.348,0.377,0.412,0.418,0.463,0.344,0.415,0.389,0.456,0.421,
    0.490,0.399,0.380,0.385,0.395),
        .Dim=c(2,9)),
    psd=c(0.266,0.331),
    gam=structure(
        .Data=c(0.061,0.638,0.518,0.060),
        .Dim=c(2,2)),
    phi.xi=structure(
        .Data=c(0.668,0.130,0.130,0.761,0.515,0.025,0.025,0.589),
        .Dim=c(2,2,2)))

list(
    pi=c(0.572, 0.428),
    alp=structure(
        .Data=c(0.1810,0.1376,0.1279,0.2650,0.3129,1.2180,1.2150,1.1160,1.0650,0.1846,
    0.0904,0.1189,0.5696,1.7070,0.0287,1.0990,1.1730,1.0370),
        .Dim=c(2,9)),
    lam=structure(
        .Data=c(0.521,0.538,0.965,0.955,0.530,0.649,0.994,1.032,0.540,0.651,1.154,1.058),
        .Dim=c(2,6)),
    psi=structure(
        .Data=c(0.661,0.560,0.593,0.613,0.667,0.665,0.745,0.642,
    0.501,0.667,0.592,0.692,0.840,0.707,0.636,0.626,0.618,0.610),
        .Dim=c(2,9)),
    psd=c(0.597,0.625),
```

```
gam=structure(
    .Data=c(0.346,0.979,0.972,0.332),
    .Dim=c(2,2)),
phi.xi=structure(
     .Data=c(1.109,0.398,0.398,1.294,1.067,0.264,0.264,1.003),
     .Dim=c(2,2,2)))
```