

Implied Future Pricing Engine

Charles Pehlivanian

January 10, 2022

Abstract

An algorithm is presented for the computation of implied quotes in a hypothetical market closely resembling CME WTI Crude (CL) futures, ICE/IPE Brent futures (B), or any one of the US crude product markets. The algorithm produces a full set of implied quote prices and sizes using only user-entered quote information. The algorithm is modular, and each leg quote (size, price) can be computed on its own execution thread. We exploit this structure to test a cpu-distributed version, using a consumer-producer queue with worker tasks consuming a user quote and generating multiple downstream implied quotes per input.

Overview

The major futures exchanges (CMEGroup, ICE, LIFFE, EUREX) depend on an underlying implied quote calculation to give the appearance of increased liquidity and narrowed spreads in their markets. This is done by disseminating, in addition to the user-entered quotes in each market, an additional set of implied quotes. Each implied quote is linked to two to many underlying user-entered quotes, and executing against an implied quote will "unravel" the chain and produce trades in all the underlying markets. All participants face the exchange, and will receive their respective fills as if the initiating order occurred in their market.

For example, at a given point in time the user-entered bbo quotes in the SP EMini markets ESU6, ESZ6 may look like Table 1. The ESU6-ESZ6 roll market may also be active, and a purchase in this market results in a purchase of ESZ6, the December contract, and a sale of ESU6. In this case, the quotes given in Table 2. will appear in the roll market *regardless of whether there are any user-entered quotes there*. So there will at least be a bbo, if there are bbos in the respective outright markets.

It is important to note that in all cases *the purchase of the roll is price-improved to the best implied quote price*. Furthermore, in many exchange matching models, the best implied quote is not part of the market feed, or the additional implied-quote feed is served on a delayed channel, and this price-improvement is invisible to the trader until the trade report occurs. For this and the following reasons, a client-side calculation of the full set of implied prices is beneficial:

- Implied quote generation can be don client-side faster than the exchange feed;
- In some markets, implied prices are not published at all;
- One can use a larger set of inputs to generate a more accurate fair value for a security;
- One can base a faster signal generation framework on an efficient implied quote server.

We will present a high-level summary only of an algorithm for complete computation of implied quotes.

Market	BidSize	BidPrice	AskPrice	AskSize
ESU6	566	2177.25	2177.50	766
ESZ6	144	2176.50	2177.25	44

Table 1: User-entered Quotes

Market	Impl. BidSize	Impl. BidPrice	Impl. AskPrice	Impl. AskSize
ESU6-ESZ6	-144	-1.00	0.00	44

Table 2: Implied Roll Quotes

Workflow

A high-level workflow is presented. The application subscribes to the user-only (non-implied) quote feed (often on a faster channel than full implied feeds from the exchange, if those are available) for the respective exchange. In this case, a simple market consisting of two outright O_1 , O_2 , and one calendar spread S_{12} is considered. The market feed is demultiplexed to 3 channels, one for each instrument. The price, size are passed through to a user-quote object, and also to each implied outright computation (1 for each leg only). The results of those computations are fed to the respective implied quote objects, and the user may query the synchronous merged (best of user and implied quote) for each outright market. Of course the graph is significantly more complicated for the typical case of 12 or more outright, with associated calendar spreads.

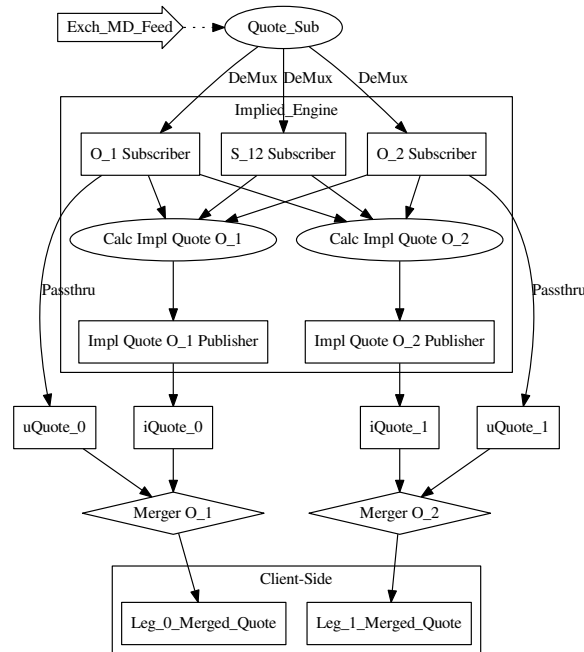


Figure 1: High-level Workflow: 2 Outright Markets, 1 Calendar Spread

Notes

1. The initial demultiplexor separates the flow of 2 leg and 1 spread product and forwards them to one of 3 quote publishers.
2. The main algorithm and most of the work is performed in the oval nodes. Here multiple leg, calendar spread quotes are consumed and an implied quote (price,size) is computed for the specified leg. There are 2 Calc nodes in this case. Care must be taken to ensure that these passes are threadsafe, and that the downstream merging is synchronized.
3. Separate user and implied quote objects (uQuote, iQuote) are maintained, and best-of or merged quotes are computed, and pushed to the client..

Implementation

The main entry point is the `ImpliedServer<N>` class template, consisting of 2 components: a `Client` object for reading of quote information via socket, an `ImpliedEngine<N>` object which is the computation engine, and a threadpool object for asynchronous calculations off of the main calc thread. Appropriate serialization and barriers are implemented within the `ImpliedEngine<N>` class to ensure thread safety. The interface can be found on https://github.com/pehlivanian/Implied_Price_Engine/

```

template<int N>
class ImpliedServer
{

```

```

public:
    ImpliedServer(bool sim_mode=true) :
        p_(std::make_unique<impl<ImpliedServer<N>>>(sim_mode)) { init_(); }
    void process() { preload_tasks_(); profiled_process_tasks_(); };

    // ...

private:
    void preload_tasks_();
    void process_tasks_();

    void init_();
    std::unique_ptr<impl<ImpliedServer>> p_;
};

template<int N>
struct impl<ImpliedServer<N>>
{
    impl(bool sim_mode) :
        sim_mode_(sim_mode),
        IE_(std::make_unique<ImpliedEngine<N>>()),
        C_(std::make_unique<Client>(8008, (char*)"0.0.0.0"))
    {}

    bool sim_mode_;
    std::unique_ptr<ImpliedEngine<N>> IE_;
    std::unique_ptr<Client> C_;
};

```

We use the Eigen library for our linear algebra engine, and custom data structures, custom allocators, fast forward_list, fast binary heap, fast customized graph search/traversal algorithms for efficiency.

Within the ImpliedEngine<N> class, the main calculation algorithm makes use of some notions from linear algebra and graph theory. A typical graph for a 12-leg problem is shown in Figure 2., without expansion.

Benchmarks

The theoretical complexity of the algorithm is $O(n^2 \log(n))$, where n is the number of legs to calculate. I don't believe we can do any better than this, for theoretical reasons, unless we constrain the problem based on ad-hoc market structure considerations (no liquidity in certain markets, so we eliminate them from the calculation, etc.). In any case, the algorithm is highly configurable and will allow for most such constraints.

We first empirically verify the algorithmic complexity by plotting empirical (single-core, single-threaded) CPU times with a theoretical fit. The times given are for a single update step consisting of

1. receiving published parsed quote update to ImpliedEngine<N> class;
2. updating user quote object for this market;

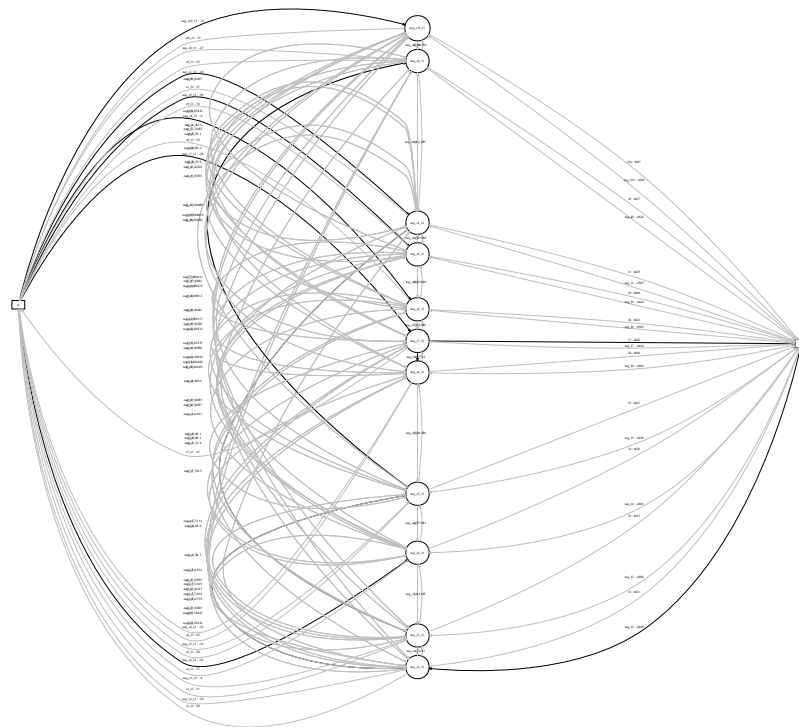


Figure 2: Setup for 12-leg Problem

3. calculating *all* implied leg quotes affected by this update (usually N legs);
4. merging user, implied quotes into single quote object.

The qualitative fit is good, with constant $C = .1223$. The fit was performed using the `scipy` orthogonal directions regression library.

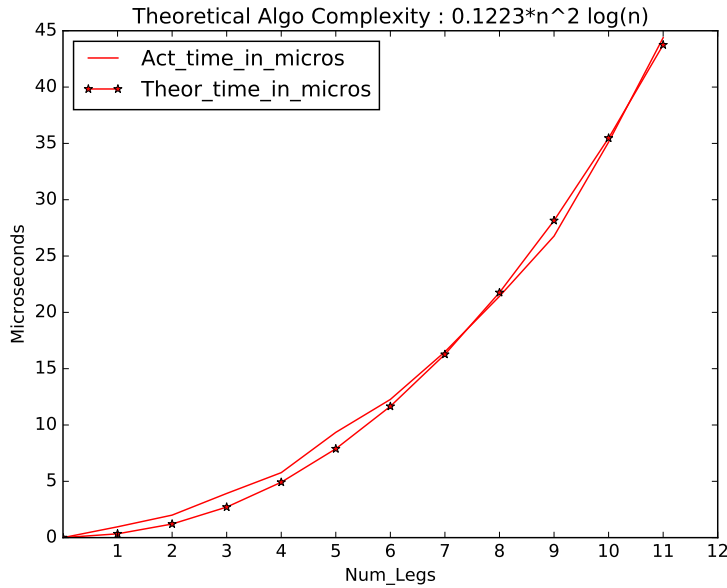


Figure 3: Algorithm running time with theoretical fit

We also can calculate the update time (time start: immediately after incoming quote is parsed, time end: full update of quote objects and task returns success; exactly the 4 steps above). The following tables show times for a single update step on a single EC2 RedHat m2.large node:

a 4-core CentOS linux commodity machine. Times are in microseconds. Note that this time includes an update of the user quote object with the incoming quote, a step that would have to be performed in any case whether we were computing implied prices or not.

Benchmark tests were run on a 2.3GHz Intel Xeon E5-2686 v4 Broadwell, 4-core machine with 16Gb RAM.

N	Average	Min	Max	Stddev	Num
0	14.563158	11	31	3.567040	9500
1	14.295474	11	30	3.323515	9500
2	14.150211	11	29	3.369054	9500
3	14.012737	11	28	2.926719	9500
4	14.050211	11	29	3.124248	9500
5	13.997158	11	28	3.082419	9500
6	13.974526	11	27	2.821791	9500
7	14.220842	11	29	3.148978	9500
8	13.908105	11	27	2.767451	9500
9	13.945158	11	27	2.911735	9500

Table 3: 12-Leg Problem, 10 Iterations

N	Average	Min	Max	Stddev	Num
0	11.265895	8	26	3.154317	9500
1	10.876737	8	24	2.717391	9500
2	10.807263	8	23	2.611048	9500
3	10.714737	8	23	2.612919	9500
4	10.792737	8	23	2.466470	9500
5	10.839158	8	23	2.601639	9500
6	10.902632	8	24	2.816397	9500
7	10.886947	8	25	2.793006	9500
8	10.790632	8	23	2.615368	9500
9	10.914737	8	24	2.721616	9500

Table 4: 11-Leg Problem, 10 Iterations

N	Average	Min	Max	Stddev	Num
0	10.218000	8	25	2.835815	9500
1	9.975474	8	23	2.480480	9500
2	9.948947	8	24	2.475615	9500
3	9.834000	8	22	2.142498	9500
4	10.079263	8	24	2.683880	9500
5	9.976737	8	24	2.604584	9500
6	10.039368	8	22	2.475212	9500
7	9.976737	8	22	2.221967	9500
8	9.970211	8	23	2.303404	9500
9	9.810105	8	22	2.097212	9500

Table 5: 10-Leg Problem, 10 Iterations

N	Average	Min	Max	Stddev	Num
0	8.095368	6	20	1.988481	9500
1	8.238316	6	18	2.100899	9500
2	8.033263	6	20	1.907523	9500
3	8.079474	6	21	2.086936	9500
4	7.972211	6	17	1.614646	9500
5	7.932211	6	18	1.664525	9500
6	8.004211	6	20	1.844572	9500
7	8.142632	6	21	2.089525	9500
8	8.077263	6	21	2.061618	9500
9	8.084421	6	20	2.041992	9500

Table 6: 9-Leg Problem, 10 Iterations

N	Average	Min	Max	Stddev	Num
0	6.198421	5	15	1.519532	9500
1	6.165895	5	15	1.528084	9500
2	6.145053	5	17	1.666477	9500
3	6.159158	5	15	1.467598	9500
4	6.061684	5	15	1.484377	9500
5	6.272000	5	17	1.800889	9500
6	6.078842	5	16	1.576353	9500
7	6.072632	5	16	1.454141	9500
8	6.256316	5	15	1.591008	9500
9	6.163158	5	15	1.569364	9500

Table 7: 8-Leg Problem, 10 Iterations

N	Average	Min	Max	Stddev	Num
0	4.422737	3	12	1.067384	9500
1	4.415579	3	12	1.096525	9500
2	4.411895	3	10	0.936406	9500
3	4.397053	3	12	1.036156	9500
4	4.316632	3	12	0.947155	9500
5	4.356000	3	12	1.034819	9500
6	4.353895	3	12	0.960325	9500
7	4.664421	3	13	1.448348	9500
8	4.301895	3	12	0.919325	9500
9	4.434211	3	12	1.131518	9500

Table 8: 7-Leg Problem, 10 Iterations

N	Average	Min	Max	Stddev	Num
0	3.025474	2	7	0.816572	9500
1	3.023053	2	7	0.795552	9500
2	2.967789	2	7	0.788788	9500
3	3.022316	2	7	0.803408	9500
4	3.050737	2	7	0.852258	9500
5	2.963895	2	7	0.751642	9500
6	2.983474	2	7	0.791071	9500
7	3.108421	2	8	0.966451	9500
8	3.275158	2	8	1.100045	9500
9	2.990526	2	7	0.799854	9500

Table 9: 6-Leg Problem, 10 Iterations

Curve Evolution

This is a typical snapshot of the user vs. implied price (no size) for a simulated data set. The simulator produces prices in legs and calendar spread contracts, maintaining no-arbitrage conditions across the curve, and producing realistic bid-ask spreads and shape throughout the curve. This snapshot represents the market at sequence number 1350.

The implied quote curve gives a more comprehensive, arguably better notion of fair value across the curve. Note that implied prices are narrower starting at about the second or third leg, matching experience in these markets.

A full evolution for a randomly selected interval of sequence numbers is given in the file *quote_evolution.pdf*.

