Developing Precautionary Reference Points for Fishery Management Using Robust Control Theory: Application to the Chesapeake Bay Blue Crab Fishery

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ABSTRACT

Most efforts to establish precautionary guidelines for fisheries management use essentially ad hoc impressions of what represents “conservative” management. Such ad hoc approaches, however, fail to take into account the magnitude of the uncertainty about a given system. One approach for guiding precautionary management is robust control in which a decision maker attempts to maximize an outcome under the assumption that conditions will be worse than expected. In this paper we apply a robust optimization approach to estimate maximum sustainable yield (MSY) and other reference points for the Chesapeake Bay Blue Crab (*Callinectes sapidus*) fishery. The approach is relatively easily implemented into standard stock assessment models that use a maximum likelihood approach to estimating model parameters. In addition, it has the advantage that a standard level of precaution could be chosen by decision makers, and this could then be applied in different fisheries with vastly different levels of data and analysis.

Introduction

Precautionary management is one of the primary tenets of modern fisheries management (Pikitch et al., 2004; Francis et al., 2007). In practice, however, managers frequently lack a way to objectively characterize the extent to which their policies are precautionary, and efforts to establish precautionary guidelines amount to essentially ad hoc impressions of what represents "conservative" management. Including a buffer between target and limit reference points for uncertainty has become a priority in federal U.S. fisheries management (Shertzer et al. 2010) and has been recommended as good practice for some time (Caddy and Mahon 1995). While many regions of the world include precaution in management, the amount of precaution being applied is often based on ad hoc approaches. For example, Restrepo et al. (1998) recommended using a target fishing mortality rate that is 75% of limit fishing mortality rate. While approaches like this include precaution, it is unclear whether that amount of precaution will be appropriate given the management objectives of the fishery. In some cases, such an approach may provide more precaution than is warranted, while in other cases it may provide too little.

In this paper we offer a new approach to developing precautionary reference points (either targets or thresholds) that applies insights from robust optimization to fisheries management. Under robust optimization, a decision maker seeks an optimal choice, taking into account his or her uncertainty about important aspects of the underlying problem. Robust optimization has long been applied in engineering (Ben-Tal and Nemirovski, 1998; Ben-Tal and Nemirovski, 1999; Ben-Tal et al., 2009) and has also been gathering attention among economists (Hansen et al., 2002; Hansen et al., 2006; Hansen and Sargent, 2007; Hansen and Sargent, 2011). Robust optimization methods typically posit a maxmin optimization problem in which a decision maker attempts to achieve the best possible outcome while acting as if the worst possible outcome will occur. In this way, the approach finds a maximum outcome (max) under a worst-case scenario (min).

There have been several applications of robust optimization to problems of natural resource management (Doyen and Béné, 2003; Xepapadeas and Roseta-Palma, 2003; Roseta-Palma and Xepapadeas, 2004; Woodward and Shaw, 2008; Gaivoronski et al., 2012; Chen et al., 2013; Woodward and Tomberlin, 2014). There are, however, two important limitations in much of the robust optimization as applied to resource management to date. First, robust optimization methods have usually been applied to highly stylized models with analytical solutions (Woodward and Shaw, 2008). While qualitatively informative, such an approach does not easily translate to applied problems such as fishery stock assessments. Second, while the idea of a worst-case scenario is clear in principle, in practice the question of how bad things could actually be is a question that usually has no objective answer. Doole and Kingwell (2010) have provided a numerical application that addresses the first problem and Woodward and Tomberlin (2014) address both problems in a fashion similar to that used below, but in a highly stylized setting.

In this paper we develop a data-driven approach to precautionary management using methods of robust optimization and apply that approach to an actual stock assessment model for the Chesapeake Bay Blue Crab (*Callinectes sapidus*) fishery. Blue Crab supports the most valuable commercial fishery in Chesapeake Bay with an average dockside value of approximately US$88 million per year during 2010-2016. New target and limit fishing mortality rate reference points were adopted following the 2011 benchmark stock assessment (Miller et al. 2011). The fishery is currently managed with a target exploitation rate on adult females, which was set arbitrarily at 75% of the estimated exploitation rate that would achieve maximum sustainable yield following the recommendations in Restrepo et al. (1998). The management agencies use a suite of regulations to achieve this target including sex-specific minimum size limits, season limits, daily harvest limits, and other regulations (Huang et al. 2015).

Methods

Robust Optimization for Fisheries Management

Traditionally, management advice generated from stock assessments can be characterized as a multi-step process (although the process may be condensed in a single modeling framework). First, the parameters of the fisheries biological model are estimated. Using these parameters, the analysts then estimate management reference points and the range of outcomes that might be achieved, often focusing on achieving maximum sustainable yield (MSY). These results often include the fishing mortality rate expected to achieve MSY (F_{MSY}), or the exploitation rate that achieves MSY (μ_{MSY}), which can be used to set a target or limit total allowable catch (TAC). MSY-based reference points are often used to provide an upper threshold for fishing (Mace 2002), and ad hoc adjustments are often made to develop conservative or precautionary targets given the imperfect understanding of the system and its stochastic nature.

To formally consider this process of including a buffer between target and upper threshold fishery management advice, suppose that the population and fishery are described by a model with a vector of parameters, θ. The best estimate of these parameters, θ^*, is often found by maximizing the likelihood function, L, i.e., given the available data, i.e., $\theta^* = \arg\max L(\theta)$. Let $MSY(\theta^*)$ be the maximum sustainable yield given the set of parameters, θ^*. This can be formally written

$$MSY(\theta^*) = \max_C C \qquad s.t. \qquad E\dot{x} = f(x, C; \theta^*) \geq 0\,, \tag{1}$$

where C is a level of harvest from the fishery, $f\left(x,C;\theta^{*}\right)$ is the instantaneous growth rate after harvest, and $E\dot{x}$ refers to the expected rate of change in the fish stock, x, given the parameters θ^{*}.

Precautionary management is, in effect, an acceptance that it may not be appropriate to simply optimize expected benefits based on the single best estimate, θ^{*}. Robust optimization formally allows the consideration of uncertainty in parameter estimates. Maxmin decision rules have a rich empirical and theoretical foundation and is a compelling way to operationalize robust optimization. Experimental studies dating back to Ellsberg (1961) have shown that individuals do not simply maximize expected benefits when they are uncertain about the underlying probability distribution, frequently adopting a maxmin decision rule. More recently, Kameda et al. (2016) found that agents regularly adopt such a rule when making decisions that affect the welfare of others and situations under uncertainty, and both of these factors are prevalent in the choices of fishery managers. Theoretically, Gilboa and Schmeidler (1989) ask what are rational choices in conditions of ambiguity, i.e. when the decision maker is uncertain about the underlying probability distribution itself and find that maxmin decision rule can be consistent with axioms of rationality. This rule is frequently referred to as robust optimization (Hansen and Sargent, 2007). Hence, a maxmin decision rule, also known as robust optimization, is both consistent with observed human behavior and is motivated based on rational choice theory.

In the current context, robust optimization would involve setting the harvest (or harvest rate) target *as if* the decision makers faced the parameters values from the set of possible values, Θ, that would give rise to the lowest level of sustainable yield. That is, for a fishery manager seeking to maximize sustainable yield, the robust optimization problem would be

$$MSY^R(\Theta) = \min_{\theta\in\Theta} \max_C C \qquad \text{s.t. } E\dot{x} = f(x, C; \theta) \geq 0 \ \forall \theta \in \Theta, \tag{2}$$

or, using (1),

$$MSY^R(\Theta) = \min_{\theta\in\Theta} MSY(\theta). \tag{3}$$

If θ^R is the parameter vector that solves (3), then $MSY(\theta^R) \leq MSY(\theta)$ for all $\theta\in\Theta$. Hence, the solution to (3) will satisfy the constraint in (2) requiring that the catch rate be at least sustainable for all $\theta\in\Theta$. The robust MSY, $MSY(\theta^R)$, could then be used to establish a target TAC or be used to calculate a target robust fishing mortality rate. This policy rule would be appropriate if policy makers are not confident in the exact value of the parameters; so they set the TAC so that it will not exceed the MSY for any set of parameter values that fall within the allowable set, Θ.

The size of the set Θ, therefore, becomes a key choice. Nilim and El Ghaoui's (2005) point out that the likelihood ratio around the estimated parameters for a system can be used to establish bounds on Θ based on levels of statistical confidence. This is similar to Hansen and Sargent's (2007) entropy measure, which seeks to identify a policy that is robust for the set of models that "are difficult to distinguish statistically from the approximating model with the amount of data at hand" (p. 16). For example, the decision maker may choose a 90% precaution level on the belief that they cannot reject the hypothesis that the true parameters are anywhere within the 90% confidence interval. In this case, the set of parameter values, Θ, would be those that lie within a 90% confidence interval around θ^*. The precaution level, however, is not a probability statement about the chance that the policy will be robust; it is more conservative than that because it is derived from a maxmin decision rule that maximizes the worst possible outcome from the set of parameter values that the decision maker wants to consider.

To implement this approach, we start with the likelihood ratio statistic for the parameter vector θ,

$$D = -2\ln\left(L(\theta)/L(\theta^*)\right), \tag{4}$$

the distribution of which is approximated by the chi-squared distribution with one degree of freedom. For a given probability level α, therefore, the set of all admissible values of θ would be those with $D \leq \chi_\alpha^2$. Equivalently, a parameter vector θ lies within the admissible range if $L(\theta)/L(\theta^*) \geq \beta^\alpha$, where $\beta^\alpha = \exp\left(-\chi_\alpha^2/2\right)$. For example, θ lies within the 90% confidence interval around the likelihood-maximizing vector if $L(\theta)/L(\theta^*) \geq 0.258$.

Figure ure 1 shows graphically how robust optimization would work in the case of a model with a single normally distributed parameter or the marginal distribution of a derived quantity, θ. The estimate, θ^*, associated with maximum likelihood coincides with the peak of the likelihood function. A policy that treats θ^* as known is not precautionary, i.e. a 0% precaution level (PL). In the figure we assume that the left side of the distribution is the pessimistic side, i.e. if θ is less than θ^*, the MSY associated with this parameter value will be lower. Hence, the robust-optimal parameters for the 60% and 90% PLs are chosen from that side with $\theta^R(90\%) < \theta^R(60\%) < \theta^*$, and $MSY^R(90\%) < MSY^R(60\%) < MSY(0^*)$. Figure 2 presents the case of a model with two parameters, θ_1 and θ_2. The concentric ellipses show the range of parameters that lie within increasingly large confidence intervals around the maximum likelihood values, θ^*. The dotted lines show the MSY values as functions of θ_1 and θ_2. The dots at the tangent points indicate the solutions to the robust optimization problem, indicating the parameter values that give rise to the lowest possible MSY within a given confidence interval.

Using this precaution level criterion for defining the set Θ, the robust optimization problem would be written

$$MSY^R(\Theta) = \min_{\theta} MSY(\theta) \qquad s.t.\, L(\theta)/L(\theta^*) \geq \beta^{\alpha}.$$

This optimization problem can be solved using the Lagrangian, $\mathcal{L}$,

$$\mathcal{L} = MSY(\theta) - \lambda\left(L(\theta) - \beta^{\alpha}\cdot L(\theta^*)\right), \tag{5}$$

where λ is the Lagrange multiplier. For well-behaved optimization problems, λ is positive and increases as the constraint becomes more binding. Hence, there will typically be an inverse and monotonic relationship between probability level, α, and λ; as α grows, increasing the precaution level, the value of λ will decrease.

Holding λ constant at an arbitrary value, the solution to (5) can also be found by minimizing over θ the function

$$\mathcal{L}' = \frac{1}{\lambda} MSY(\theta) - L(\theta) - \beta^{\alpha}\cdot L(\theta^*). \tag{6}$$

Because $\beta^{\alpha}\cdot L(\theta^*)$ is constant, dropping it from (6) will not affect the solution. The robust optimization problem can, therefore, be solved by choosing θ to *maximize* the function

$$\mathcal{L}'' = L(\theta) - \omega MSY(\theta), \tag{7}$$

where $\omega = \lambda^{-1}$.

This final objective function, (7), can be interpreted as a penalized maximum likelihood function. If ω=0, the problem reverts to a simple maximum likelihood problem with the solution

θ^*. As ω increases, the parameters that are identified will be pushed away from θ^* in the direction that minimizes $MSY(\theta)$. For a given value of ω, the associated distance away from θ^* in terms of the confidence interval can be recovered using the likelihood ratio, (4), and the χ^2 statistic. In essence, this approach provides a numerical solution to estimate confidence intervals using a profile likelihood method. While this approach would not be necessary if we were only considering an estimated parameter, it becomes useful when considering a quantity that is a function of multiple estimated parameters like MSY.

This suggests a three-step process by which it is possible to identify a range of robust-optimal policies for increasing levels of precaution.

1. With ω=0, solve for the maximum-likelihood parameter values, θ^*, and the associated value of MSY. Store the value of the likelihood function , $L(\theta^*)$.
2. Gradually increasing ω, solve for the parameter values that solve the weighted likelihood function, (7). Store the values of the likelihood function, $L(\theta^\omega)$, and the MSY associated with each value of ω.
3. For each of the stored likelihood ratios, $L(\theta^\omega)/L(\theta^*)$, use the chi-squared statistic to recover the level of precaution associated with each value of ω. The *MSY* values can then be used to infer a robust MSY for each level of precaution.

As we also show, for each MSY value, it is possible to calculate the associated fishing mortality rate, *F*, yielding a potential *F* reference point (either target or limit) as is used in most U.S. marine fisheries.

In our application below, we estimate the parameters of the robust stock assessment model for the Chesapeake Bay Blue Crab fishery with different values of the weight, ω. Because the relationship between ω and levels of precaution are difficult to anticipate, some trial and error was necessary to choose a range of values for ω that would map out precaution intervals up to 98%. In this particular application, we used values for ω between 0 and 0.385 in 0.001 increments.

Assessment Model

We applied the robust optimization approach derived above to the 2011 Chesapeake Bay Blue Crab stock assessment model (Miller et al., 2011). The Miller et al. (2011) assessment uses a sex-specific catch multiple survey analysis (SSCMSA) with sex-specific harvest data and three fishery-independent data sources. The full description of the assessment model is provided in Appendix A, and variable definitions are in Table 1. Briefly, the model is a statistically fitted population dynamics model that estimates abundance, fishing mortality, and parameters of the stock-recruitment relationship. The model includes a Ricker stock recruitment function that allows estimation of MSY and the stock size and fishing mortality rates that achieve MSY. The assessment separates the population into two age groups: pre-recruit age-0 crabs and fully recruited age-1+ crabs. Thus, the model tracks the dynamics of four stages of Blue Crab: age-0 males, age-0 females, age-1+ males and age-1+ females. Model parameters are estimated using a penalized maximum likelihood approach that fits several data time series simultaneously: harvest, winter dredge survey abundance, Maryland trawl survey catch per unit effort (CPUE), and Virginia Institute of Marine Science trawl survey CPUE.

The SSCMSA includes a sex-specific version of the Ricker stock-recruitment model to estimate age-0 abundance in the beginning of each year (Miller et al., 2011). Productivity at low abundance is a function of female abundance, while density dependence is a function of male and female abundance,

$$R_{y+1,s} = x_s a SP_{y,f} e^{-b\left(SP_{y,f} + SP_{y,m}\right)} e^{\delta_y},$$

where $R_{y,s}$ is recruitment in year y of sex $s=\{m,f\}$, x_s is sex ratio of recruits of sex s, a and b are estimated parameters of the stock-recruitment relationship, $SP_{y,s}$ is number of spawners of sex s in year y, and δ_y is a normally distributed process error. Compensatory mortality of age-0 Blue Crabs is likely driven by cannibalism from age 1+ Blue Crabs (Hines and Ruiz, 1995), which makes the Ricker model well suited for this stock.

Abundance in the age-1+ category of sex s, $N_{y+1,s}$, is estimated as the sum of age-0 recruits and age-1+ adults that survived from the year before,

$$N_{y+1,s} = N_{y,s} e^{-(M+F_{y,s})} + R_{y,s} e^{-(M+\eta F_{y,s})},$$

where M is the instantaneous natural mortality rate (from all non-fishing sources), F is the instantaneous fishing mortality rate, and η is the partial recruitment to the fishery for age-0 crabs. Natural mortality is assumed to be 0.9 yr^{-1} and is the same for age-0 and age-1+ (Miller et al., 2005, Hewitt et al., 2007). The instantaneous fishing mortality rate is estimated for each year and sex. The partial recruitment for age-0 crabs is specified outside of model fitting and is assumed to be 0.6 based on growth patterns of juvenile Blue Crabs (Miller et al., 2011).

The number of spawners is calculated by decrementing the number of age-1+ individuals at the beginning of the year by mortality that occurred before spawning,

$$SP_{y,s} = N_{y,s} e^{-\kappa(M+F_{y,s})},$$

where κ is the proportion of total mortality prior to spawning (0.37, approximately July 1). The assessment model also assumed that natural and fishing mortality followed the same seasonal patterns. We model catch using a sex-specific Baranov catch equation,

$$C_{y,s} = \frac{F_{y,s}}{F_{y,s} + M}\left(1 - e^{-(M+F_{y,s})}\right)N_{y,s} + \frac{\eta F_{y,s}}{\eta F_{y,s} + M}\left(1 - e^{-(M+\eta F_{y,s})}\right)R_{y,s},$$

where $C_{y,s}$ is catch in number of individuals in year y and sex s.

In Miller et al. (2011), the parameters of the stock assessment model were estimated using a penalized maximum likelihood approach to maximize the fit between observed and predicted indices of abundance from three surveys, total catch for 1968-1993, and sex-specific catch for 1994-2009. The same data are used here. These data include three fishery independent surveys: the Virginia Institute of Marine Science trawl survey, the Maryland trawl survey, and the winter dredge survey. The winter dredge survey is assumed to provide an absolute estimate of abundance (i.e., catchability (q) =1) for age-1+ Blue Crabs (Sharov et al. 2003). For all other survey indices of abundance, catchability is estimated. Catchability is sex-specific for the age-1+ stage but is combined for sexes for age-0 for all surveys because rapid identification of sex of small Blue Crabs in the field is prone to error.

Calculation of MSY

We calculate maximum sustainable yield (MSY) based reference points by adapting the methods of Shepherd (1982) for a sex-specific stock-recruitment model. Spawners per recruit (SPR) is calculated as the product of equilibrium age-1+ abundance and survival until spawning,

$$SPR_s = \frac{x_s e^{-\left((1+\kappa)M + \eta F_s + \kappa F_s\right)}}{1 - e^{-(M+F_s)}}.$$

Yield per recruit (YPR) is calculated by applying the Baranov catch equation to equilibrium abundance per recruit of age-1+ and age-0,

$$N_{YPR,s} = \frac{x_s e^{-\left(M + \eta F_s\right)}}{1 - e^{-(M+F_s)}},$$

and

$$YPR_s = \frac{F_s}{M + F_s}\left(1 - e^{-(M+F_s)}\right) N_{YPR,s} + \frac{\eta F_s}{M + \eta F_s}\left(1 - e^{-(M+\eta F_s)}\right) x_s.$$

Equilibrium abundance of age-1+ is calculated by rearranging the Ricker stock-recruitment function and applying the SPR for each sex,

$$N_{eq,s} = \frac{\log_e SPR_f + \log_e \alpha + \sigma_R / 2}{\beta} \times \frac{SPR_s}{SPR_f + SPR_m}.$$

Equilibrium recruitment is the quotient of sex-specific equilibrium abundance of age-1+ and SPR,

$$R_{eq} = \frac{N_{eq,s}}{SPR_s}.$$

Equilibrium catch (i.e., sustainable yield) is the product of equilibrium recruitment and YPR,

$$C_{eq,s} = R_{eq,s} YPR_s,$$

and total equilibrium catch is the sum of equilibrium catch across sexes,

$$C_{eq} = \sum_s R_{eq,s} YPR_s.$$

Estimation of robust reference points

The Blue Crab stock assessment model is implemented in AD Model Builder (Fournier et al. 2012). Any solution that failed to converge (based on the largest first partial derivative of the objective function) was excluded from later analysis. For a given set of parameters, MSY was found using a Gauss-Newton search over fishing mortality rates. We used the ratio of male:female fishing mortality from the last year of the model (2010) to estimate MSY, although it is possible to estimate MSY for any other ratio of male:female fishing mortality.

Results

In Figure 3 we present the MSY^R for a wide range of precaution levels, from 0% (i.e., using maximum likelihood parameter values) to 98%. While the estimated *MSY* that could be achieved at the MLE for the parameters of the model is nearly 500 million Blue Crabs yr^{-1}, the confidence intervals around this estimate are fairly large (coefficient of variation = 31%). As we consider higher levels of precaution, the robust parameter values diverge from the maximum likelihood values in the direction that leads to the lowest sustainable yield. At the 50% precaution level, the associated MSY^R falls about 20% to 400 million crabs yr^{-1} indicating that values within a 50% confidence interval around the maximum-likelihood values are consistent with MSY values of at least 400 million crabs. If policy makers seek a 90% precaution level, the MSY^R value falls by 40% from the base level with no precaution, to nearly 300 million crabs yr^{-1}. Thus, as the precaution level rises the MSY^R values fall substantially. On the other hand, the decline is bounded for the range of precaution levels presented. Even at the most conservative level presented, the 98% precaution level, the MSY^R value is still more than 50% of the *MSY* associated with the MLE parameters.

It is more common for fisheries to pursue a constant fishing mortality rate harvest control rule than a constant harvest control rule, so in Figure 3 we also present an alternative reference point, a robust fishing mortality rate, F^R, which is equal to the F (assuming the maximum likelihood parameter values, θ^*) that would match the corresponding MSY^R values. The MLE for F_{MSY} was approximately 0.99 yr^{-1}. The target fishing mortality rate from the stock assessment was approximately 0.65 yr^{-1} (exploitation rate = 0.33 yr^{-1}), which corresponded to about 45% precaution. The 90% precaution level for the F_{MSY} was F = 0.46 yr^{-1}. As a policy maker seeks higher levels of precaution, the fishing mortality rate declines at roughly the same rate as the decline in *MSY*. Our approach identifies how much lower they need to be to match a given level of precaution.

Discussion

We provided an alternative method to develop precautionary reference points for fisheries management based on robust control. One way of thinking about robust control as a precautionary fisheries management tool is that it can be used to estimate reference points that attempt to maximize yield while assuming a specific degree of pessimism about the future (i.e., the precaution level). Instead of applying an ad hoc adjustment, policy recommendations are expressed in terms similar to standard confidence intervals, e.g. the policy recommendation might be expressed as robust up to a 90% confidence interval, which we call a 90% precaution level. In this way, our approach has much in common with that advocated by Dankel et al. (2016) in that resulting management advice is clear and the role played by uncertainty is transparent.

The robust optimization approach proposed has several attractive features. First, it is a direct application of the maxmin decision rule that is frequently mentioned in discussion of precautionary management. Second, it is an improvement over previous fisheries applications of robust control in that the range of possible parameter values, the set Θ, is derived from the familiar notion of a statistical confidence interval. Finally, it has the advantage that a standard level of precaution could be chosen by decision makers, and this could then be applied in different fisheries with vastly different levels of data and analysis. Although the level of precaution would be consistent, decisions would tend to be less conservative where the natural resource system is better understood. However, our approach relies on being able to estimate MSY within the assessment model because the basis for the reference point is to maximize sustainable yield using precautionary estimates of stock productivity. It is possible that a similar approach could be developed for other commonly used fishery management reference points (e.g., spawning potential ratio), but a quantity to maximize is needed.

Our approach is not a replacement for more holistic evaluations of management performance and risk like management strategy evaluation (MSE; Punt et al. 2016). Rather, it is a tool that can be used to provide additional information about precautionary reference points or to develop candidate reference points for more detailed evaluations of potential management performance. Specifically, our application of a robust control approach attempts to find the MSY assuming that productivity of the stock will be less than that at the MLE. As such, it is only attempting to maximize long term yield and does not explicitly account for other potential fishery objectives. MSEs are used to explore the expected tradeoffs among multiple management objectives. In many cases, the goal of the MSE is not to find the optimal solution, but rather to identify management options that are expected to provide acceptable tradeoffs among competing objectives (Miller and Shelton 2010; Punt et al. 2016).

What precaution level should be used to make management decisions? There is no objective answer to that question, and given the tradeoffs inherent in the management choice, policy makers will have differing opinions about how much precaution is warranted. As policy makers want to reduce the probability of a poor outcome, they should choose higher precaution levels (e.g., 95% rather than the 90% in our analysis). In U.S. fisheries, managers have chosen probabilities of overfishing that could be used in the approach we developed. For example, in the U.S. mid-Atlantic region, if the stock is above its estimate of the biomass that would produce MSY for species with a typical life history, the target probability of $F>F_{MSY}$ is 40%, (Mid-Atlantic Fishery Management Council 2011). In the Blue Crab example, this would correspond to approximately a 20% precaution level. The approach we propose could be used to evaluate how alternative levels of precaution affect management choices. For example, Wiedenmann et al. (2017) evaluated several alternative levels of precaution for the Mid-Atlantic Fishery Management Council's harvest control rule and found that the effects of adopting more precaution differed among life history and data quality scenarios. We believe that the precaution levels as described in our analyses are more intuitive in what they are attempting to achieve than other approaches for calculating precautionary reference points. Simulation testing using an MSE approach would be helpful for determining the amount of precaution that most aligns with managers' goals.

As with all stock assessment models, our approach retains many of the limitations of the underlying model. Stock assessment models vary substantially in which components are assumed to be known and which are estimated. Because of the inability to include all the uncertainty in the model, the precision of estimates is often thought to be a substantial underestimate of the true uncertainty (Ralston et al., 2011; Magnusson et al. 2013). For example, in our Blue Crab application several of the parameters including natural mortality and selectivity of age-0 crabs were assumed known. Therefore, simply using the uncertainty estimated within the assessment model will underestimate the true uncertainty. However, the idea of using a maxmin criterion to attempt to identify management options that will perform as well as possible under poor conditions is a general suggestion for how to include precaution in management regardless of how the uncertainty is estimated. If the analyst believes that substantial sources of uncertainty are not included in the assessment model, then a direct application of our approach is not advised. For example, if a substantial retrospective pattern was present in the stock assessment results, then the uncertainty is likely to underestimated within the model, and our method could produce unrealistically optimistic results (although the likely bias depends on the direction of the retrospective pattern).

We used a profile likelihood approach to estimate confidence intervals in estimated reference points, but other methods are also available to estimate confidence intervals. Magnusson et al. (2013) evaluate three common approaches (Markov Chain Monte Carlo (MCMC), asymptotic standard errors (ASE), and bootstrapping) for incorporating uncertainty in stock assessments and recommend using MCMC or ASE for constructing confidence intervals for age-structured stock assessments. Our use of profile likelihood for implementing the robust optimization approach is similar to using ASE or MCMC for estimating confidence intervals. In additional analyses, we compared confidence intervals generated from our likelihood profiling and ASEs and found that assuming a lognormal distribution with the ASEs for MSY produced very similar results to the profile likelihood confidence intervals except at the extreme tails of the distribution. The approach we used to estimate profile likelihood confidence intervals worked well for our Blue Crab assessment model, but the default profile likelihood approach in ADMB failed to converge on a solution. Regardless of the method used to estimate the confidence intervals of the reference points, we feel that our approach has the advantage of a clear normative motivation and appropriately adjusts the management targets to uncertainty in the parameter values.

Other proposed approaches to precautionary management have similarities with our proposed approach. For example, Shertzer et al. (2010) proposed a method for setting precautionary catch limits, commonly called the P* approach, that is currently used to inform catch limits in several regions of the U.S. for federal fisheries management. The P* approach attempts to estimate a level of catch that achieves a specific probability of overfishing (P*; the target probability that F exceeds its upper threshold reference point). The P* approach requires estimates of the distribution of the catch (called the overfishing limit [OFL]) that would achieve the upper threshold fishing mortality rate, which is used to define overfishing for U.S. federally managed stocks. The result of the P* approach is a catch limit, which is used as an upper limit for the annual catch limits that can be recommended by the Fishery Management Councils. This approach has been implemented in several ways in different regions. For example, the Pacific Fishery Management Council and the Mid-Atlantic Fishery Management Council use the point estimates of the OFL from the assessments but use uncertainty estimates from outside analyses because uncertainty in the stock assessments is thought to be underestimated (e.g., Ralston et al. 2011). In contrast, a procedure that pairs Monte Carlo simulation with a non-parametric bootstrap (MC bootstrap) is used to estimate uncertainty and apply the P* control rule in the U.S. South Atlantic region.

Our proposed approach was designed to estimate precautionary target reference points for fishery management. However, some fishery management systems may not directly use such a reference point. Current U.S. federal management uses a series of precautionary buffers to develop catch limits such that the OFL ≥ acceptable biological catch (ABC) ≥ annual catch limit (ACL) (National Standard 1[1]). The OFL is the highest level of catch and provides a technical definition for overfishing. A buffer between the OFL and acceptable biological catch (ABC) is supposed to add precaution by accounting for "scientific" uncertainty, i.e., the uncertainty associated with the estimated reference points and stock biomass. In many regions of the U.S., a P* approach (Shertzer et al. 2010) is used to estimate an ABC that has a specific probability of overfishing. The ACL is supposed to account for "management" or implementation uncertainty. Current guidance also allows for a target at or below the ACL to further account for implementation uncertainty. This approach to adding precaution has some general similarities to our robust control approach in that it is a way to produce a precautionary buffer. However, our approach attempts to maximize sustainable yield under a precautionary estimate of stock productivity, whereas the current U.S. federal management largely attempts to avoid overfishing a prespecified fraction of the time.

For some U.S. fisheries, 75% of the limit fishing mortality reference point has been adopted as a precautionary target following guidance in Restrepo et al. (1998). Blue Crab management in Chesapeake Bay adopted a target exploitation rate that is 75% of the exploitation rate that would achieve MSY (Miller et al. 2011). The robust optimization approach provides decision makers with additional information to understand how much precaution they are exhibiting when adopting a target. Comparing the target instantaneous fishing mortality rate for Blue Crabs in Chesapeake Bay (0.66 yr^{-1} for adult females) to our results, we find that the

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[1] Available at https://www.ecfr.gov/cgi-bin/retrieveECFR?gp=&SID=3ea20ed553f359cdaf6ce0d768a9b9b6&mc=true&n=sp50.12.600.d&r=SUBPART&ty=HTML#se50.12.600_1310, Accessed 1/27/2019.

current target is at about a 45% precaution level. This level of precaution may not align with policy makers' goals for managing the fishery and may need to be revisited. For example, Blue Crab abundance has remained below its target level on average during 2012-2017 (Chesapeake Bay Stock Assessment Committee 2018), and this may be because managers are not being as precautionary as they expect given the basis of their exploitation rate target.

Methods to formally include precaution in the selection of management targets have increased recently in their use. Admittedly, any approach that relies on the estimated uncertainty from a single model relies heavily on the assumption that the model provides a "good" approximation of reality. In our approach, reducing uncertainty leads to increases in target reference points for yield and fishing mortality. Thus, the question of how to reduce uncertainty in stock assessment models is important. Reducing uncertainty is not as easy as simply adding more years of data. Adding more years of data to a common age-structured assessment approach did not reduce the uncertainty in terminal year biomass estimates (Wiedenmann et al. 2015). Therefore, if the goal is to reduce uncertainty, use of new, more informative data sets would likely be needed. In any case, the robust approach presented here will adjust the buffer between the MLE and the precautionary reference point as data and theoretical understanding evolve over time. However, within the constraints of the model or models adopted for analysis, the approach we propose in this study offers a rigorous and theoretically grounded way for policy makers to think about the amount of precaution associated with alternative management reference points. In addition, we recommend simulation testing of potential robust control reference points before implementing them in a real fishery management situation.

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Table 1: Variable and parameter definitions for the Chesapeake Bay Blue Crab stock assessment model (Miller et al. 2011)**.**

Variable	**Description**
y	Year
s	Sex
i	Index of abundance
f	Female
m	Male
k	Number of observations
x	Sex ratio at recruitment
M	Natural mortality rate
η	Partial recruitment to the fishery
κ	Proportion of mortality before spawning
τ	Proportion of mortality before Maryland trawl survey
Estimated values	
R	Recruitment
N	Adult abundance
I_R	Recruitment index of abundance
I_N	Adult index of abundance
Fundamental parameters (estimated)	
N_0	Initial adult abundance
R_0	Median recruitment
δ	Log-scale deviations from median recruitment
F	Instantaneous fishing mortality rate
α, β	S-R parameters
q	Catchability
Variance terms	
σ_R	Log-scale SD for recruitment deviations
σ_i	Log-scale SD for observation error in indices of abundance
σ_F	SD for variability in the ratio of male to female fishing mortality during 1968-2006
μ	Mean of the ratio of male to female fishing mortality during 1968-2006
Reference point variables	
YPR	Yield per recruit
N_{YPR}	Abundance per recruit for YPR calculations
SPR	Spawners per recruit
N_{eq}	Equilibrium age-1+ abundance
R_{eq}	Equilibrium age-0 abundance
C_{eq}	Equilibrium catch in numbers

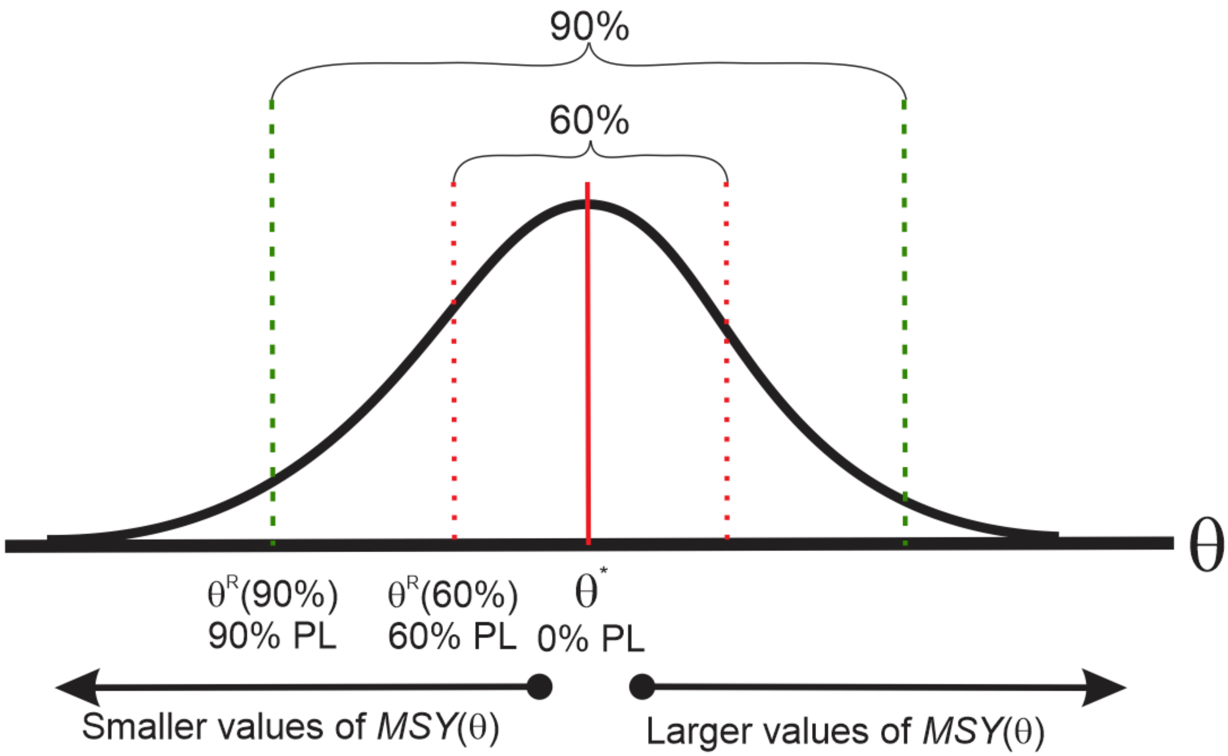

Figure 1. Example of robust estimates for the case of a single normally distributed parameter (θ), where θ^* is the maximum likelihood estimate and $\theta^R(60\%)$ and $\theta^R(90\%)$ indicate the parameter values associated with 60% and 90% precaution levels (PL), respectively . The brackets on the top of the figure indicate the width of a two-tailed confidence interval with the given percentage. MSY(θ) represents the values of the maximum sustainable yield conditional on θ.

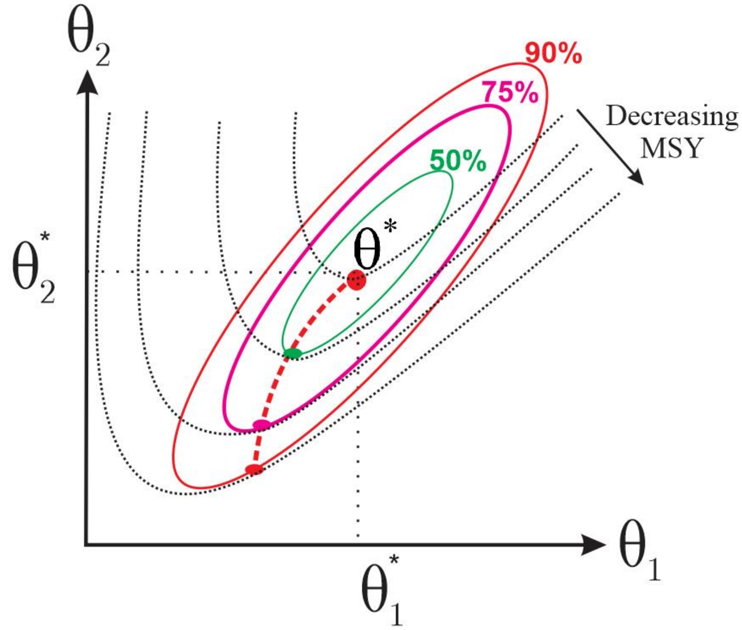

Figure 2: Example of robust optimization for the case of a two parameters (θ_1 and θ_2), where $\theta^* = \left(\theta_1^*, \theta_2^*\right)$ are the maximum likelihood estimates the ellipses represent likelihood contours. The dotted lines represent the levels of MSY that would be associated with different parameter values. The dots along the dashed line sloping downward from θ^* indicate the parameter values with the lowest MSY for any set of parameter values within a given confidence interval around $\left(\theta_1^*, \theta_2^*\right)$.

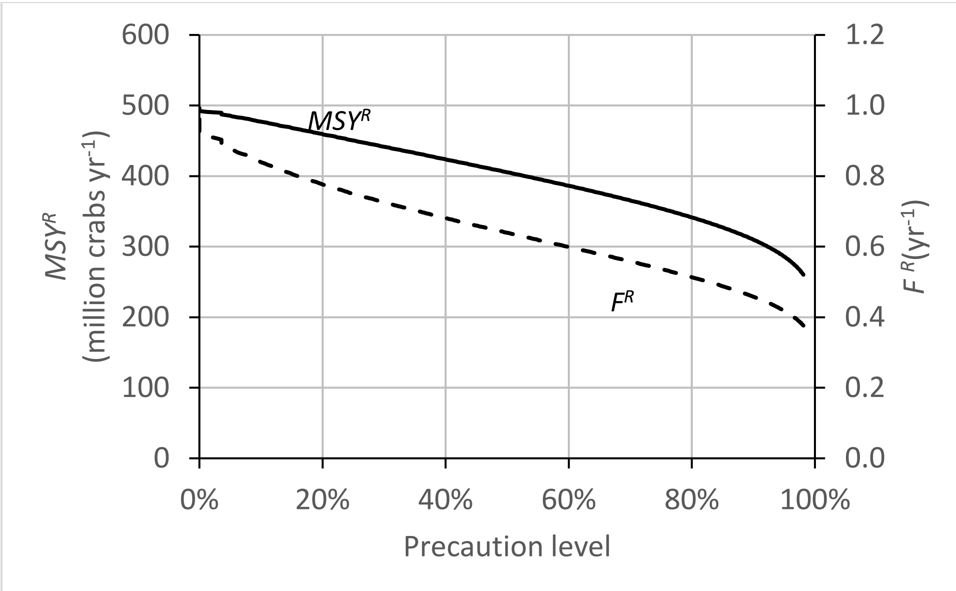

Figure 3: Robust maximum sustainable yield (MSY^R) and robust instantaneous fishing mortality rates (F^R) for different precaution levels for the Chesapeake Bay Blue Crab stock assessment model.

Appendix A. Model Details.

The stock assessment model and several sensitivity analyses are described in Miller et al. (2011). An update of the Chesapeake Bay Blue Crab stock assessment is currently underway, but not yet complete (as of early 2019). Here we provide a synopsis of the mathematical details of the Chesapeake Bay Blue Crab stock assessment model. Abundance in the age-1+ category was estimated as the sum of age-0 recruits and age-1+ adults that survived from the year before,

$$N_{y+1,s} = N_{y,s} e^{-(M+F_{y,s})} + R_{y,s} e^{-(M+\eta F_{y,s})} . \tag{A1}$$

Variable definitions are provided in Table 1. Natural mortality was assumed to be the same for age-0 and age-1+, but we also conducted sensitivity runs, which evaluated several sex-specific natural mortality rates. We used 0.9 as our assumed natural mortality rate based on the previous stock assessment for Chesapeake Bay Blue Crabs (Miller et al. 2005, Hewitt et al. 2007).

The instantaneous fishing mortality rate was estimated for each year and sex. The partial recruitment for age-0 Blue Crabs was specified at 0.6 and was the same for males and females. About 90% of age-0 Blue Crabs in the beginning of the year grow large enough to enter the fishery during that year. A partial recruitment of 0.3 was considered a lower bound based on both the amount of time age-0 crabs are vulnerable to the softshell/peeler fishery and the proportion of the catch from later months when most crabs that were age-0 in the beginning of the year have grown into the fishery. Abundance in the first year of the model, combined for age-1+ and combined for age-0, was estimated as individual parameters.

The number of spawners was calculated by decrementing the number of age-1+ at the beginning of the year by mortality that occurred before spawning, (A2) $$SP_{y,s} = N_{y,s} e^{-\kappa(M+F_{y,s})}.$$

The proportion of mortality that occurred before spawning was chosen to be 0.37 because we assumed a spawning date of July 1, and 37% of the pot effort in Maryland has occurred by July 1 on average. The assessment model also assumed that natural and fishing mortality followed the same seasonal patterns. This approach assumes that the same proportions of annual mortality occurred prior to spawning for males and females during this period.

We modeled catch using a sex-specific Baranov catch equation with partial recruitment for age-0,

(A3) $$C_{y,s} = \frac{F_{y,s}}{F_{y,s}+M}\left(1-e^{-(M+F_{y,s})}\right)N_{y,s} + \frac{\eta F_{y,s}}{\eta F_{y,s}+M}\left(1-e^{-(M+\eta F_{y,s})}\right)R_{y,s}.$$

The exploitation rate of fully selected Blue Crabs was calculated as the product of the annual mortality rate and the proportion of total mortality due to fishing,

(A4) $$u_{y,s} = \frac{F_{y,s}}{F_{y,s}+M}\left(1-e^{-(M+F_{y,s})}\right).$$

Observation Model

The model is fitted to data from three surveys: the Chesapeake Bay Blue Crab winter dredge survey (WDS), the Maryland trawl survey (MTS), and the Virginia Institute of Marine Science trawl survey (VTS). The WDS and the VTS survey are treated as beginning of the year surveys, (A5) $$\hat{I}_{R,y,s} = q_i R_{y,s},$$

(A6) $$\hat{I}_{N,y,s} = q_i N_{y,s},$$

where the surveys are assumed to have constant catchability over time. For the MTS, we treated the survey as occurring in the middle of the year, such that age-0 prerecruits from the beginning of the year were recruited to the Age-1+ category by the time of the survey,

(A7) $$\hat{I}_{N,y,s} = q_i \left(N_{y,s} e^{-\tau(M+F_{y,s})} + R_{y,s} e^{-\tau(M+\eta F_{y,s})} \right).$$

We assumed that 67% of total mortality (F+M) had occurred by the time of the MTS based on a September 1 date for the trawl survey and the cumulative amount of crab pot effort in Maryland before September 1 (Miller et al. 2011). The age-0 portion of the MTS indexes recruitment at the beginning of the next year. We assumed that the WDS provided an absolute estimate of abundance (i.e., q=1) for age-1+ Blue Crabs. For all other survey indices of abundance, catchability was estimated using the MLE approach by calculating the average difference (on the log scale) between the observed index of abundance and predicted abundance (Miller et al. 2005),

(A8) $$\log_e q_i = \frac{\sum_y \log_e I_{R,y} - \log_e R_y}{k_i}$$

for recruits, and

(A9) $$\log_e q_i = \frac{\sum_y \log_e I_{N,y,s} - \log_e N_{y,s}}{k_i}$$

for ages 1+. Catchability was sex-specific for the age-1+ stage but was combined for sexes for age-0.

Likelihood and Penalty Functions

We estimated the parameters by minimizing the objective function, which was the sum of the likelihood components for each data source and the penalties for recruitment deviations and deviations from the mean 1994-2006 ratio of male to female fishing mortality. The model is estimated using AD Model Builder (admb-project.org). We assumed lognormal observation errors for indices of abundance from the MTS, VTS, and for catch,

$$L_i = k_i \log_e(\sigma_i) + \frac{1}{2\sigma_i^2} \sum_{y \in i} \left(\log_e E_{i,y} - \log_e O_{i,y} \right)^2 , \tag{A10}$$

where E and O are estimated and observed values of the indices of abundance. The variances were assumed for each data source, and constants were ignored to simplify the equations. We assumed that the recreational crab catch, which is not reported, represented 8% of the total commercial catch and was proportionally constant over time. For the winter dredge survey, we assumed normally distributed errors with a constant coefficient of variation (CV) because of the large sample sizes in the survey (approximately 1500 stations per year),

$$L_i = \sum_{y \in i} \log_e(E_{i,y} CV_i) + \frac{1}{2\left(E_{i,y} CV_i\right)^2} \sum_{y \in i} \left(E_{i,y} - O_{i,y} \right)^2 . \tag{A11}$$

The log-scale standard deviations of catch were specified at 0.1 to indicate that catch was relatively accurate. The CVs of the WDS were estimated from design-based estimators. The average CV over time for age-1+ males and females was approximately 10%, so we assumed a 10% CV for the winter dredge survey. The log-scale SDs of the trawl surveys were iteratively tuned until the input value was approximately equal to the post-hoc value (McAllister and Ianelli 1997). Recruitment deviations followed a lognormal distribution,

(A12)
$$L_R = k_R \log_e(\sigma_R) + \frac{1}{2\sigma_R^2}\sum_y \delta_y^2$$
.

The log-scale standard deviation of recruitment was estimated during model fitting.

A penalty on the relative fishing mortality between males and females was imposed on years before sex-specific catch data were available to constrain the model from having large interannual differences in the relative fishing mortality rates,

(A13)
$$L_F = k_F \log_e(\sigma_F) + \frac{1}{2\sigma_F^2}\sum_y \left(\frac{F_{y,m}}{F_{y,f}} - \mu\right)^2 .$$

The mean and variance for the ratio of male to female fishing mortality were calculated using years during which sex-specific catch data were available, but before sex-specific management measures were imposed, 1994-2006.