

## Final Project

### ***FRAIG:*** ***Functionally Reduced And-Inverter Graph***

資料結構與程式設計  
Data Structure and Programming

12/11/2019

1

## **Functionally – Reduced – AIG (FRAIG)**

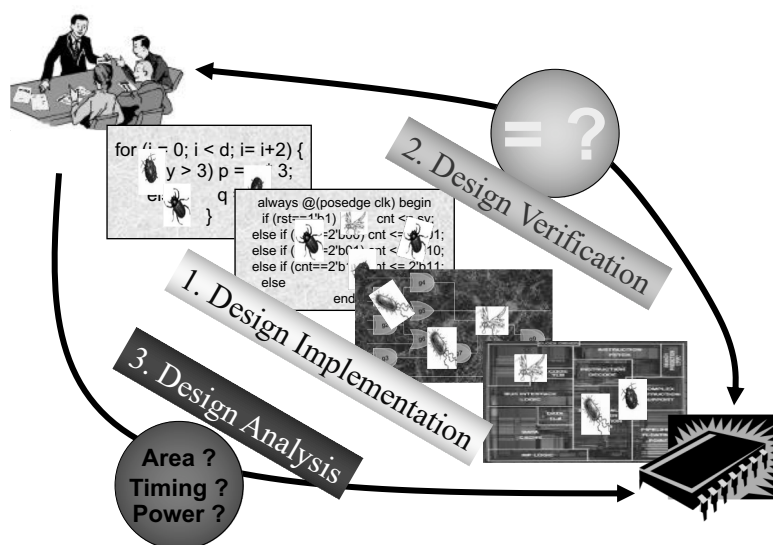
- ◆ AIG: you have learned it in HW#6
- ◆ Functionally?
  - Well, AIG represents a circuit, so it represents some Boolean function.
- ◆ Reduced?
  - Reduction on AIG → Simplifying graph
  - How to simplify AIG?
- ◆ Functionally Reduced?
  - (e.g.) Two functionally equivalent nodes can be merged together
  - (e.g.) Simplify circuit by constant propagation

2

# Electronic Design Automation (EDA)

3

## How is a “chip” designed nowadays?



4

## How/What to optimize a circuit?

### ◆ Area

- Reduce the number of gates
- Moreover, using library cells of smaller sizes  
→ but they will have weaker driving capability

### ◆ Timing

- Shorten the longest path
- Additionally, insert buffers and/or enlarge the cells to increase the driving capability

### ◆ Power

- Reduce the switching activities
- Moreover, shutdown the sub-circuit that is not currently used

## Optimization trade-offs

- ◆ In general, area, timing, and power optimizations contradict with each other
- ◆ Moreover, different stages of design flow have different granularities and complexities for circuit optimization
  - HDL (e.g. Verilog) // algorithm
  - Gate (Boolean) // logic
  - Layout (transistor) // RC network

## A simplified view of circuit optimization

### ◆HDL (Verilog)

- Architectural and algorithmic optimizations

```
always @(posedge clk) begin
  if (rst==1'b1) cnt <= sv;
  else if (cnt==2'b00) cnt <= 2'b01;
  else if (cnt==2'b01) cnt <= 2'b10;
  else if (cnt==2'b10) cnt <= 2'b11;
  else cnt <= sv;
end
```

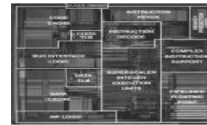
### ◆Gate (Boolean) What FRAIG focuses!!

- Minimize gate counts under reasonable timing and power constraints



### ◆Layout (transistor)

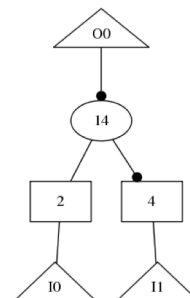
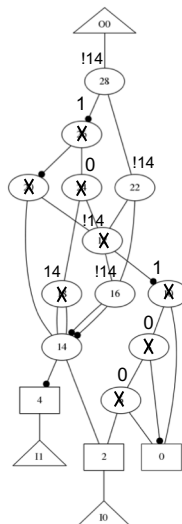
- Minimize wire length for timing and power optimizations with limited area overhead



## A simple example

### ◆sim05.aag

original:  
12 AIGs



optimized:  
1 AIG

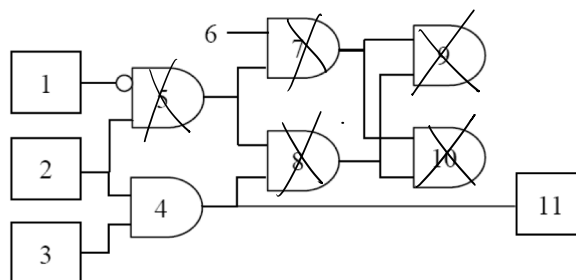
## Functionally Reduced AIG

In the final project, we will apply 4 different types of optimization techniques:

1. Unused gate sweeping
2. Trivial optimization (constant propagation)
3. Simplification by structural hash
4. FRAIG: Equivalence gate merging

## Unused Gate Sweeping

- ◆ Sweeping out those gates that are not reachable from POs.



Example: opt07.aag

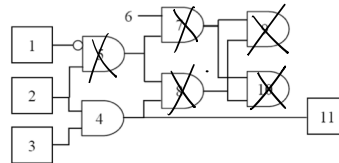
## Unused Gate Sweeping

### ◆ Command: CIRSWEEP

- Can be called whenever necessary.
- Note: do not remove unused PIs.
- After this command, all gates except for the unused PIs will be in the DFS list.
- Note: be sure to update the reporting for "CIRPrint -FLoating".

### ◆ In the previous example (cirp -fl):

- Before:
  - Defined but not used: 9 10
  - Gates with floating fanin: 7
- After:
  - Defined but not used: 1



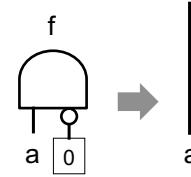
## Functionally Reduced AIG

1. Unused gate sweeping
2. Trivial optimization (constant propagation)
3. Simplification by structural hash
4. FRAIG: Equivalence gate merging

## Trivial optimization

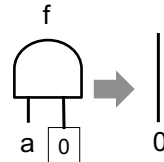
1. Fanin has constant 1

➔ Replaced by the other fanin



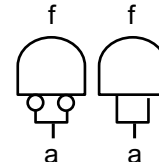
2. Fanin has constant 0

➔ Replaced with 0



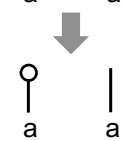
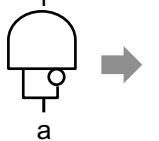
3. Identical fanins

➔ Replaced with the (fanin+phase)



4. Inverted fanins

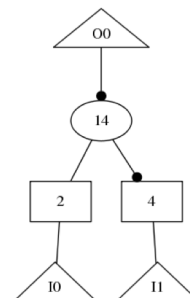
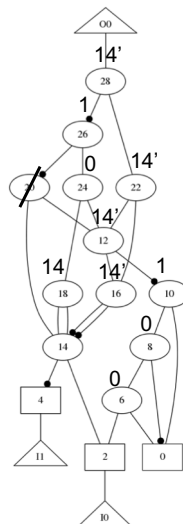
➔ Replaced with 0



## A simple example

◆ sim05.aag

original:  
12 AIGs



optimized:  
1 AIG

## Trivial optimization

- ◆ Command: CIROPTimize
  - Can be called whenever necessary
  - Scan the DFS list and perform optimization ONCE. Don't repeatedly optimize the circuit. → The latter can be achieved by calling CIROPTimize multiple times.
  - Don't perform optimization during CIRRead
- ◆ Do not remove PIs / POs
- ◆ Some UNDEF or defined-but-not-used gates may disappear!
- ◆ Some gates (with side input = constant 0) may become "defined-but-not-used".

## Functionally Reduced AIG

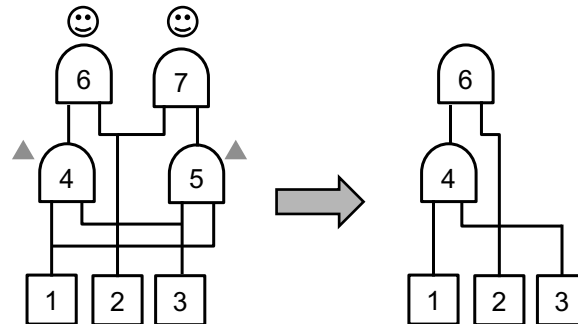
1. Unused gate sweeping
2. Trivial optimization (constant propagation)
3. Simplification by structural hash
4. FRAIG: Equivalence gate merging

$$f1 = \text{AND}(a, b) \quad f2 = \text{AND}(b, a)$$



## Structural Hash (Strash)

### ◆ Example:



## Structural Hash (Strash)

- ◆ Problem: How to identify two AIG gates in a circuit that have the same inputs?
  - [Method 1] Check for  $O(n^2)$  pairs of gates
  - [Method 2] For each gate, check its fanouts
    - How many checks?
  - [Method 3] For each gate, create hash table <fanins, this gate>
    - How many checks?
- ◆ We will pick method 3 in our project
  - You can modify your "util/myHashSet.h" to HashMap, or use `std/unordered_map`
- ◆ Although it is possible to perform strash during circuit parsing, we choose to make "strash" a separate command.
  - ➔ CIRSTRash
- ◆ Note: Order matters!! You should merge from PIs to POs (Why??)

## Structural Hash Algorithm

- ◆ `HashMap<HashKey, HashData> hash;`
  - HashKey depends on gate type & list of fanins
  - HashData is Gate\*
- What if we have only AIG?
- How about inverted match?
- ◆ `class HashKey`

```
{
    size_t operator () () const { // as hash function }
    bool operator == (const HashKey& k) const {...}
private:
    Gate *g0, *g1; size_t in0, in1;
};
```
- ◆ HashData can be `size_t`
- ◆ For `unordered_map`, need to define "hash" class

## Structural Hash Algorithm

- ◆ `for_each_gate_from_pi_to_po(gate, hash)`

```
// Create the hash key by gate's fanins
HashKey<...> k(...); // a function of fanins
size_t mergeGate;
if (hash.check(k, mergeGate) == true)
    // mergeGate is set when found
    mergeGate.merge(gate);
else hash.forceInsert(k, gate);
```
- ◆ `size_t ?` → `CirGateV`  
Create a wrapper class on top of a `size_t` !!

找到就merge還是最後一起  
merge;  
誰merge誰

## Notes about CIRSTRash

- ◆ Perform strash only on gates in DFS list
  - Do not perform strash on gates which cannot be reached from POs
  - This is to avoid those unreachable gates appearing in DFS list
- ◆ It doesn't make sense to perform strash again before doing other optimizations
  - CIRSTRash cannot be repeated called

## Maintaining Netlist Consistency

- ◆ Once circuit is simplified, some gates may become invalid.
  - How to maintain the netlist consistency?
    1. Properly re-connect fanins/fanouts
    2. Properly release memory (if necessary)
    3. Properly update the lists in CirMgr  
(Note: PI/PO lists should never be changed)

## Functionally Reduced AIG

1. Unused gate sweeping
2. Trivial optimization
3. Simplification by structural hash
4. FRAIG: Equivalence gate merging

## FRAIG: Merging equivalent gates

- ◆ Some gates are NOT structurally equivalent, but functionally equivalent.
  - Cannot be detected by strash
  - e.g.  $ab + c \equiv (a + c)(b + c)$
- ◆ How to know two gates are functionally equivalent?
  - By simulation? (If two gates have the same value)
    - ➔ Not quite possible, equivalence requires to enumerate “ALL input patterns”  
// exhaustive simulation
  - Need “formal (mathematical) proof”!!
    - ➔ But, what to prove?  $O(n^2)$  pairs?
    - ➔ By simulation!! // to check the potential equivalence

## FEC Pairs

### ◆ Functionally Equivalent Candidate (FEC)

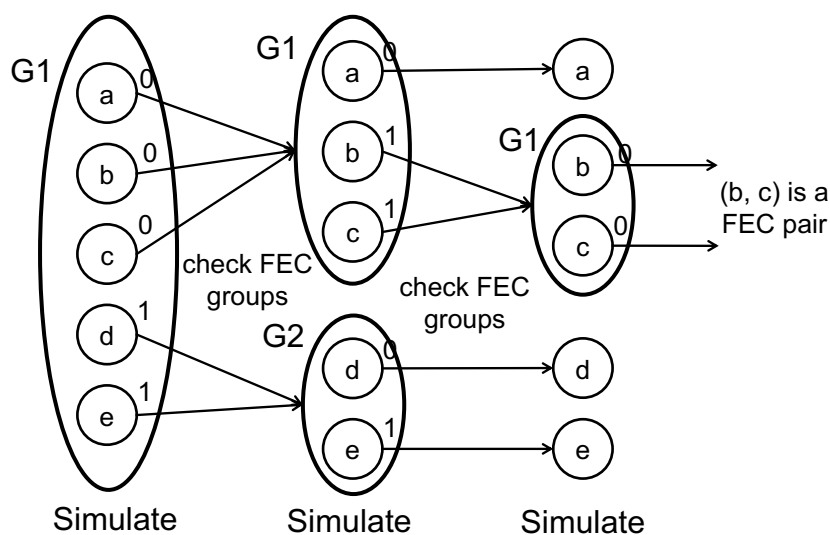
- For all simulated patterns, if two signals always have the same response, they are very likely to be equivalent.

### ◆ Properties

- Two signals can be separated if they have different simulation responses for at least ONE input pattern
- Two paired signals can be separated by simulation, but two separated signals won't get paired again
  - Singleton signal won't be in any FEC pair anymore

FEC group : vector(一堆fec group) of vector (fec group 裡會有一堆variables)

## Identify FECs by Simulation



## Simulation Algorithm

### ◆ All-gate simulation:

Perform simulation for each gate on the DFS list

- `void CirMgr::simulate() {  
    for_each_gate(gate, _dfsList) gate->simulate(); }`

### ◆ Event-driven simulation:

Perform simulation only if any of the fanins changes value

- `void CirMgr::simulate() {  
    for_each_PO(po, _dfsList) po->simulate(); }  
bool CirAigGate::simulate() {  
    Recursively simulate each fanin.  
    If (no fanin has value change) return false;  
    Simulate this gate;  
    if (value changed) return true;  
    return false;  
}`

如果pattern的改變不多→影響到的gate也少→可以少simulate一點東西

用polymorphism

## Discussions: Simulation algorithm trade-offs

### ◆ All-gate simulation or event-driven?

### ◆ Evaluation

- By operator? By if-else? By table lookup?

### ◆ To detect FEC pairs, how many simulation patterns are enough?

- Stop if no new FEC pair is found?
- (Dynamically) Controlled by “#failTimes”

### ◆ Patterns

- Single pattern? Parallel pattern?

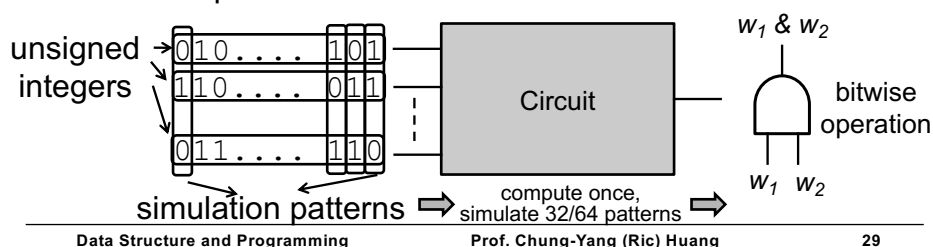
## Parallel-Pattern Simulation for FEC Identification

◆ Note: The speed overhead in bitwise operations is very small.

- Most of the programming languages (e.g. C/C++) support “bit-wise” operations (e.g. &, |, ~ in C/C++).

◆ Idea

- Using 32- or 64-bit unsigned integer to pack 32 or 64 patterns into a word



Data Structure and Programming

Prof. Chung-Yang (Ric) Huang

29

29

## How many patterns to parallelize?

◆ In practice, max parallelization will lead to the best simulation performance

- Use the max “unsigned int” to store the parallel patterns (e.g. `size_t` in C/C++)

[Discussion]

◆ Can we go beyond 32/64 bits?

- e.g. 1024-bit

◆ What are the pros and cons?

◆ How about the FEC detection rate?

Data Structure and Programming

Prof. Chung-Yang (Ric) Huang

30

30

## Identify FECs by Simulation

1. Initial: put all the signals in ONE FEC group.
2. Add this FEC group into fecGrps (list of FEC groups)
3. Randomly simulate the entire circuit
4. for\_each(fecGrp, fecGrps):  
    Hash<SimValue, FECGroup> newFecGrps;  
    for\_each(gate, fecGrp)  
        grp = newFecGrps.check(gate);  
        if (grp != 0) // existed                      頂多分出兩個 ?  
            grp.add(gate);  
        else newFecGrps.add(createNewGroup(gate));  
    CollectValidFecGrp(newFecGrps, fecGrp, fecGrps);
5. Repeat 3-4 until no new FEC Group can be identified, or efforts exceed certain limit.

Mass simulation → Identify FEC pairs

How to prove/disprove  
the equivalence of gates in an FEC pair?

Convert it into a SAT problem!!!



## Boolean Satisfiability (SAT) Problem

- ◆ Given a Boolean function  $f(X)$ , find an input assignment  $X = A$  such that  $f(A) = 1$ .
  - Satisfiable: if such an assignment is found
  - Unsatisfiable: if no assignment is possible
    - i.e. All assignments make  $f(X) = 0$
  - Undecided: can't find a satisfying assignment, but haven't exhaust the search
  - SAT Game: <https://goo.gl/9JJVmJ>
- ◆ Complexity?
  - First proven NP-complete problem by Dr. S. Cook in 1971 (Turing Award winner)

## How to prove the equivalence of FEC gates?

- ◆ In general, given two Boolean functions,  $f$ ,  $g$ , how to check if they are equivalent?
- ◆ Note:
  - SAT proves things by contraposition
    - By showing that it is *impossible* to find an assignment to make  $f \neq g$ .
    - Create a SAT problem  $F \equiv (f \neq g)$ , showing that it is unsatisfiable.
    - Note:  $f \neq g \rightarrow$  an XOR gate

Diagram illustrating a 2-to-1 multiplexer (MUX) with inputs  $f$  and  $g$ , and output  $F$ . The output is defined as  $F \equiv (f \neq g)$ . The MUX is shown with a select line and a data input line. The output  $F$  is labeled  $F \equiv (f \neq g)$ .

$(f, g)$  is an FEC pair  
 $\rightarrow$  SAT? ( $F = 1$ )

If UNSAT  $\rightarrow f = g$

If SAT  $\rightarrow f \neq g$  with an input assignment  $A$  that can distinguish  $(f, g)$  and potentially distinguish other pairs

35

## FRAIG flow

## 1. Simulation

- 36

18

## FRAIG flow

2. For each FEC pair, call Boolean Satisfiability (SAT) engine to prove their equivalence
  1. If they are equivalent, merge them together  
→ remove one of them
  2. If they are NOT equivalent, acquire the counter-example (CEX) that distinguishes them
  3. Repeat until all the FEC pairs have been proved, or enough CEXes (2.2) have been collected → Repeat “1. Simulation”

## In short...

1. Simulation identifies a group of FEC pairs
2. For each FEC pair, say (f, g), call SAT engine to check if (f != g) is satisfiable
3. If UNSAT → f = g → f can replace g
4. If SAT  
→ collect the pattern that witness (f != g)  
→ simulate again to see if it can distinguish other FEC pairs
5. Repeat 2 ~ 4  
→ So the remaining problems are: How to call SAT engine? How to create SAT proof instance?

## Boolean Satisfiability (SAT) Engine

- ◆ An engine (i.e. a program/library/function) that can prove or disprove a Boolean Satisfiability problem
  - Called a “SAT engine” or “SAT solver”
- ◆ A well-studied CS problem, but was once generally thought as an intractable problem.
  - Many practical, powerful, and brilliant ideas were brought up by EDA researchers in early 2000 → Orders of improvement
  - Made a revolutionary change on the applications of SAT

## Creating Proof Instance

- ◆ Proof instance: the formula under proof
- ◆ Conjunctive Normal Form (CNF)
  - Most modern SAT engines represent the proof instances in CNF
  - Actually a “product of sum” representation
$$(a+b+c)(a'+b'+c)(a'+b+c')(a+b'+c')$$

```
graph TD; V[Variables] --> L[Literals]; L --> C[Clauses];
```
- To be satisfied, all the clauses need to be 1

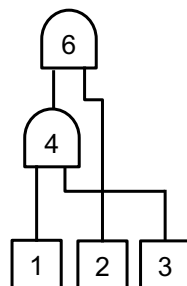
## Converting circuit to CNF

- ◆ Each gate is assigned a variable ID
- ◆ Each gate is converted to a set of CNF clauses based on its fanin variables
  - $g = \text{AND}(a, b)$ 
    1.  $a = 0 \rightarrow g = 0$                        $(a + !g)$
    2.  $b = 0 \rightarrow g = 0$                        $(b + !g)$
    3.  $a = 1 \ \& \ b = 1 \rightarrow g = 1$      $(!a + !b + g)$
- ◆ To solve  $(f = 1)$ , add a  $(f)$  clause
  - SAT engine is to check if all the clauses can be satisfiable at the same time.

## Converting circuit to CNF

### ◆ Example:

SAT [6] = 1



$(1 + !4)(3 + !4)(!1 + !3 + 4)$   
 $(4 + !6)(2 + !6)(!2 + !4 + 6)$   
 $(6)$

## Calling SAT engine

- ◆ Create a solver object
- ◆ Add clauses → proof instance
- ◆ (optional) Set proof limits
- ◆ Solve()!!
- We provide a SAT interface in “sat.h”
  
- ◆ (FYI) Incremental SAT
  - Reuse the partial learned information

可能有一個pair證很久，可能之後下面有merge過後再算會比較好算→可能看conflict數 (ex. 1000) 決定什麼時候kill去做別的 (conflict學到的東西都還會留著，不算浪費時間)

## Using SAT to prove FEC pair

1. Create a solver object

```
SatSolver solver;
solver.initialize();
```
2. Create CNF for the circuit
  - For each gate in the circuit, create a variable for it

```
solver.newVar();
```
  - For each gate in the circuit, create CNF clauses for it

```
solver.addAigCNF(v, v1, ph1, v2, ph2);
```
  - Remember to take care of CONST gate
3. Create the proof instance for  $F \equiv (f \neq g)$ 
  - Add clauses for F

```
solver.addXorCNF(FVar, fVar, fPh, gVar, gPh);
```
  - Call SAT to prove

```
solver.assumeRelease();
solver.assumeProperty(newV, true);
bool isSat = solver.assumpSolve();
getSatAssignment(solver, patterns);
```

不需要整個都證完CN取2那種

## Notes about FEC proof

### ◆ Order matters!!

- Proving from PIs to POs can greatly reduce the proof effort
- DFS or BFS?

variable id 0 by default  
是給constant 1

### ◆ Don't waste SAT-generated patterns (for $f \neq g$ )

- Pack them for parallel pattern simulation

因為有and，是0的機率  
比較高->最大的群可能  
是 (1,a,b,c,d...)  
-> abcd跟1證就好不  
用CN取2

### ◆ Many FEC pairs are actually (f, 1) or (f, 0).

- Should we do anything special for them?

### ◆ It's OK to skip some proofs. (Why?)

- Skip it or limit the proof effort (e.g. #conflicts)

### ◆ Incremental SAT

不用全部證完->不證頂多沒把他  
merge起來

### ◆ Balance between simulation and proof efforts

Data Structure and Programming

Prof. Chung-Yang (Ric) Huang

45

45

proof efforts太少 -> 太多pair要  
證，effort太多->浪費很多

## Some advices

### ◆ Please do not fall into 軍備競賽...

- Although it is possible you can implement a version that is 10X faster than mine...

### ◆ It's OK that you CANNOT finish the project.

- I don't expect many people to finish the project.
- Think: 你的電子學有拿 100 分嗎?

### ◆ Please DO NOT spend 80% time on 20% points

- e.g. parser error message, circuit optimization

### ◆ Always keep your code simple and straight!!

- Always modularize your code
- Compile and test from time to time

Data Structure and Programming

Prof. Chung-Yang (Ric) Huang

46

46

## References

- ◆ Functionally Reduced And-Inverter-Graph
  - [http://www.eecs.berkeley.edu/~alanmi/publications/2005/tech05\\_fraigs.pdf](http://www.eecs.berkeley.edu/~alanmi/publications/2005/tech05_fraigs.pdf)
- ◆ A System for Sequential Synthesis and Verification
  - <http://www.eecs.berkeley.edu/~alanmi/abc/>
- ◆ SAT solver
  - <http://www.satcompetition.org/>
  - <http://www.princeton.edu/~chaff/publication/DAC2001v56.pdf>
  - [http://www.princeton.edu/~chaff/publication/cad\\_e\\_cav\\_2002.pdf](http://www.princeton.edu/~chaff/publication/cad_e_cav_2002.pdf)