

# Topic 8

## Graph and Circuit

資料結構與程式設計  
Data Structure and Programming

11/13/2019

1

### From CS to EE? What does that mean?

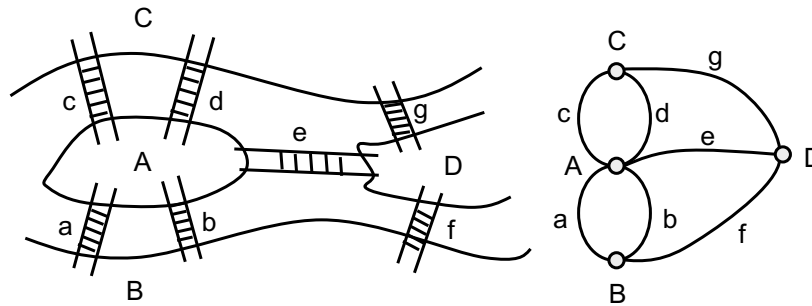
- ◆ Most people think that “Data Structure” is a CS class
  - A “must” subject for CS entrance exam
- ◆ In EE area, many problems can be either mapped as graphic problems, or resolved by graphic algorithms
  - e.g. Circuit netlist, network, communication, etc.
  - Understanding graphic data structure and algorithms will be very helpful

2

## The First Use of Graph

### ◆ Königsberg Bridge Problem

- Leonhard Euler, 1736
- Starting at one land, is it possible to walk across all bridges exactly once and returning to the starting land area?



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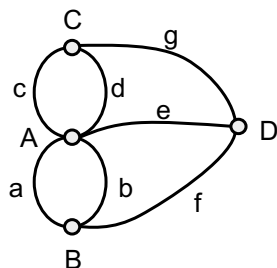
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## Eulerian Theorem

- ◆ There is a walk starting at any vertex, going through each edge exactly once and terminating at the starting vertex, iff the degree of each vertex is even.

➔ Eulerian walk



No Eulerian walk, since all 4 vertices are of odd degree.

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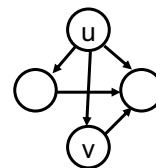
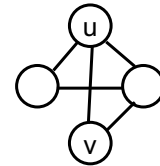
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4

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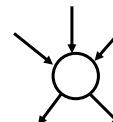
## Definition of a Graph

- ◆ A graph,  $G(V, E)$ 
  - $V$ : a finite, nonempty set of vertices  $\rightarrow V(G)$
  - $E$ : a set of pairs of vertices  
these pairs are called edges  $\rightarrow E(G)$
- 1. Undirected Graph
  - Every pair of vertices representing any edge is unordered
  - i.e.  $(u, v)$  and  $(v, u)$  represent the same edge
- 2. Directed Graph (Digraph)
  - Order of the pair of vertices matters
  - $\langle u, v \rangle$ : 'u' is the tail and 'v' is the head
  - e.g. A circuit is a directed graph



## Terminologies

- ◆ Given 2 nodes  $u, v$ , and an undirected edge  $(u, v)$ 
  - $u$  and  $v$  are called *adjacent*
  - The edge  $(u, v)$  is *incident* on vertices  $u$  and  $v$
- If  $\langle u, v \rangle$  is a directed edge
  - $u$  is *adjacent to*  $v$ , and  $v$  is *adjacent from*  $u$
- ◆ *Degree* of a vertex
  - The number of edges incident to it
- If the graph is directed
  - *In-degree*
    - The number of edges for which the vertex is the head
  - *Out-degree*
    - The number of edges for which the vertex is the tail



## Terminologies

### ◆ *Path*

- A sequence of vertices in which each vertex is adjacent to the next one
  - e.g.  $\{ n_1, e_1, n_2, e_2, n_3, e_3, \dots, e_{k-1}, n_k \}$

### ◆ *Simple path*

- All vertices in a path are distinct

### ◆ *Length of a path*

- The number of edges in a path

### ◆ *Loop (self-edge)*

- An edge with 2 identical end-points

### ◆ *Cycle*

- A path with identical start and end points

## Graph Properties

### ◆ Subgraph $G(V', E')$ of $G(V, E)$

- $V' \subseteq V; E' \subseteq E$

### ◆ Simple graph

- No loops and no two edges link the same vertex pair

### ◆ Multigraph

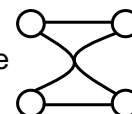
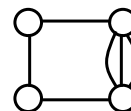
- Not simple graph

### ◆ Weighted graph

- Each edge is associated with some weight

### ◆ Hypergraph

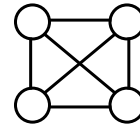
- An extension of a graph where edges may be incident to any number of vertices



## Complete Graph

### ◆ Complete graph

- Each vertex is adjacent to all the other vertices in the graph
- For complete graph with  $n$  vertices
  - $\#edges = n(n - 1) / 2$



### ◆ Clique of a graph

- Complete subgraph

### ◆ Complement $G(V', E')$ of a graph $G(V, E)$

- $V' = V; E \cap E' = \emptyset$
- $G(V, E \cup E')$  is a complete graph

## Undirected Graph Properties

### ◆ Two vertices $u$ and $v$ are said to be connected

- iff there a path from  $u$  to  $v$

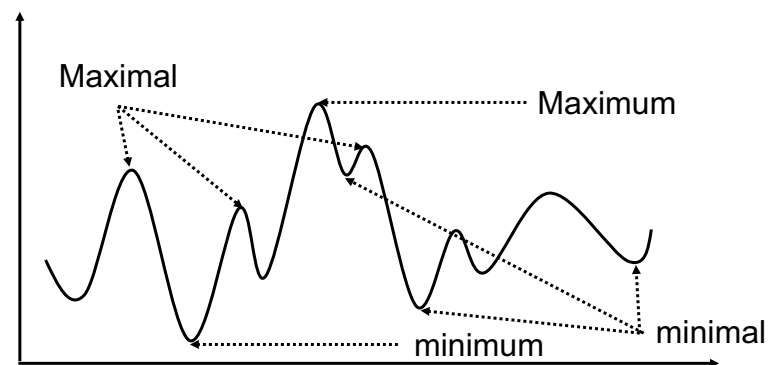
### ◆ A graph is said to be connected

- iff every vertex pair is connected
- ➔ A tree is a connected acyclic graph

### ◆ A connected component (or simply *component*) of a graph

- A maximal connected subgraph

## (FYI) Maximal vs. Maximum

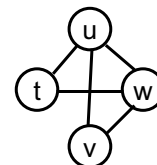


- In many problems, finding maximum/minimum is very hard
  - Finding maximal/minimal is the only possibility
- How to find a better maximal/minimal?

## Undirected Graph Properties

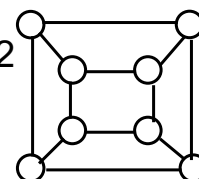
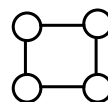
### ◆ Cutset

- A minimal set of edges whose removal from the graph makes the graph disconnected



### ◆ Bipartite graph

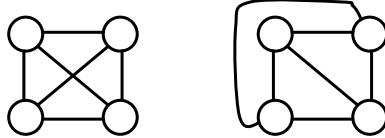
- Vertex set can be partitioned into 2 subsets such that each edge has end-points in different subsets



## Undirected Graph Properties

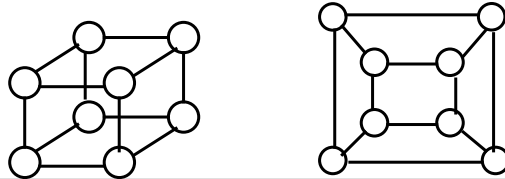
### ◆ Planar graph

- A diagram on a plane surface such that no two edges cross



### ◆ Two graphs are isomorphic

- There is a one-to-one correspondence between their vertex sets and preserves adjacency



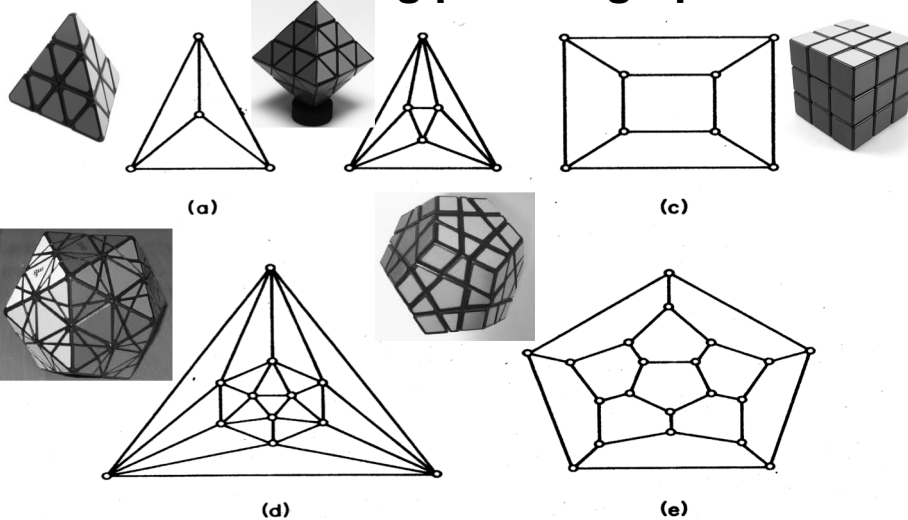
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13

13

## Some interesting planar graphs



(a) The tetrahedron; (b) the octahedron; (c) the cube; (d) the icosahedron; (e) the dodecahedron

14

## Undirected Graph Properties

- ◆ Each undirected graph can be characterized by four numbers
- 1. *Clique number*  $\omega(G)$ 
  - The cardinality of its largest clique, called *clique number*
- 2. *Chromatic number*  $\chi(G)$ 
  - The minimum number of colors needed to color the vertices, such that no edge has end-points with the same color
  - e.g. A bipartite graph is a 2-colorable graph

Property:  $\omega(G) \leq \chi(G)$

## Undirected Graph Properties

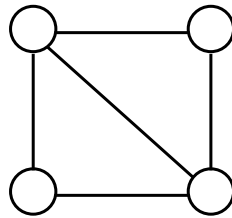
- 3. *Clique cover number*  $\kappa(G)$ 
  - A graph is said to be *partitioned into cliques* if its vertex set is partitioned into (disjoint) subsets, each one including a *clique*
  - The cardinality of a minimum clique partition is called *Clique cover number*
- 4. *Stability number*  $\alpha(G)$ 
  - A *stable set*, or *independent set*, is a subset of vertices with the property that no two vertices in the stable set are adjacent
  - The *stability number* is the cardinality of its largest stable set
  - A *coloring* of a graph is a partition of the vertices into subsets, such that each is a stable set

Property:  $\alpha(G) \leq \kappa(G)$



## Perfect Graph

- ◆ A graph is said to be perfect iff
  - $\omega(G) = \chi(G)$  (clique = chromatic)
  - $\alpha(G) = \kappa(G)$  (stability = clique covering)

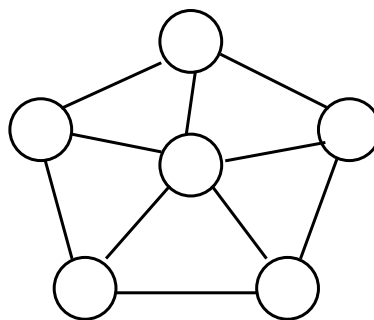


$$\omega(G) = \chi(G) = 3$$

$$\alpha(G) = \kappa(G) = 2$$

## Any graph that is NOT perfect?

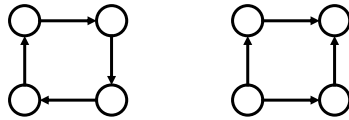
- ◆ That is:
  - Clique number  $\omega(G)$  < Chromatic number  $\chi(G)$*
  - Stability number  $\alpha(G)$  < Clique cover number  $\kappa(G)$*



## Directed Graph Properties

- ◆ A digraph is said to be **strongly connected**
  - Iff for every pair of distinct vertices  $u$  and  $v$ , there is a path from  $u$  to  $v$ , also from  $v$  to  $u$

跟一般的connected差在哪



- ◆ Strongly connected component (SCC)
  - Maximal subgraph that is strongly connected
  - If a graph is **strongly connected**, it has only one SCC
  - Linear time algorithm for finding SCCs:  
**Robert E. Tarjan**, *Depth-first search and linear graph algorithms*, SIAM Journal on Computing, 1(2):146-160, 1972.

## Directed Acyclic graph (DAG)

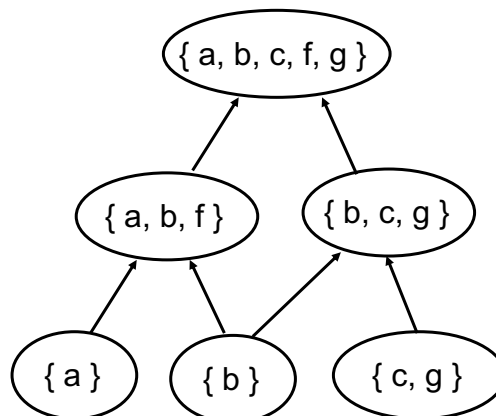
- ◆ A directed graph that has no cycle
- ◆ Can represent *partially ordered set*
  - A vertex  $v$  is a successor (descendant) of a vertex  $u$ 
    - If there is a path from  $u$  to  $v$
    - Called direct successor if the path is an edge
  - Predecessor (ancestor)
- ◆ Polar DAG
  - A DAG with 2 distinguished vertices
    - A source and a sink
  - All vertices are reachable from the source
  - Sink is reachable from all the vertices
  - A generic polar DAG may have multiple sources and sinks

## Partially vs. Totally Ordered Set

- ◆ A relation “ $\leq$ ” is a partial order on a set  $S$  if it has:
  1. Reflexivity:  $a \leq a$  for all  $a \in S$
  2. Antisymmetry:  $a \leq b$  and  $b \leq a$  implies  $a = b$ .
  3. Transitivity:  $a \leq b$  and  $b \leq c$  implies  $a \leq c$
- ◆ A relation “ $\leq$ ” is a total order on a set  $S$  if it has the above 3 properties and the following:
  4. Comparability (trichotomy law):  
For any  $a, b \in S$ , either  $a \leq b$  or  $b \leq a$

## A Partial Order Example

- ◆ The “containment” relation among the subsets of a set is a partial order



## Graphic Algorithms

- ◆ The importance of learning “graphs” is that many practical problems can be modeled and then solved by standard/well-known graphic algorithms
  1. Breadth-First Search and Depth-First Search
  2. Topological Sort
  3. Strongly Connected Component
  4. Shortest and Longest Path Algorithms
  5. Minimum Spanning Tree
  6. Maximum Flow and Minimum Cut
- ◆ Please refer to “Algorithm” book or class for more information
  - We may cover some of them if we have time...

## Graph Traversal

- ◆ In many graph (DAG) applications, it is important to go through every vertex in certain order
  - e.g. checkSum(), simulate(), etc
- ◆ Topological order
  - An order sorted by certain relationship of adjacent vertices
  - e.g.
    - For each vertex, it has higher order than all of its predecessors, and lower order than all of its successors

## Depth-First Traversal (Take 1)

```
void
Graph::dfsTraversal(const List<Node*>& sinkList)
{
    sink -> fanout
    for_each_sink(node, sinkList)
        node->dfsTraversal(_dfsList);
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for_each_predecessor(next, _predecessors)
        next->dfsTraversal(dfsList);
    dfsList.push_back(this);
}
```

Any Problem??

## Depth-First Traversal (Take 2)

```
void
Graph::dfsTraversal(const List<Node*>& sinkList)
{
    for_each_sink(node, sinkList)
        node->dfsTraversal(_dfsList);
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for_each_predecessor(next, _predecessors)
        if (!next->isMarked()) {
            next->setMarked();
            next->dfsTraversal(dfsList);
        }
    dfsList.push_back(this);
}
```

Any Problem??

## Depth-First Traversal (Take 3)

```
void
Graph::dfsTraversal(const List<Node*>& sinkList)
{
    for each sink(node, sinkList)
        node->dfsTraversal(dfsList);
    for each node(node, dfsList)
        node->unsetMarked();
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for each predecessor(next, predecessors)
        if (!next->isMarked()) {
            next->setMarked();
            next->dfsTraversal(dfsList);
        }
    dfsList.push_back(this);
}
```

Any Problem??

## Depth-First Traversal (Take 4)

- ◆ Remember: use “static data member in a class”

```
class T
{
    static -> class scope's global var
    static unsigned globalRef;
    unsigned ref;

public:
    T() : ref(0) {}
    bool isGlobalRef() { return (ref == GlobalRef); }
    void setToGlobalRef() { ref = globalRef; }
    static void setGlobalRef() { globalRef++; }
};
```

- ◆ Use this method to replace “setMarked()” functions in graph traversal problems

## Depth-First Traversal (Take 4)

```
void
Graph::dfsTraversal(const List<Node*>& sinkList)
{
    Node::setGlobalRef();
    for each sink(node, sinkList)
        node->dfsTraversal(dfsList);
}
// post order traversal
void Node::dfsTraversal(List<Node *>& dfsList)
{
    for each predecessor(next, predecessors)
        if (!next->isGlobalRef()) {
            next->setToGlobalRef();
            next->dfsTraversal(dfsList);
        }
    dfsList.push_back(this);
}
```

Any Problem??

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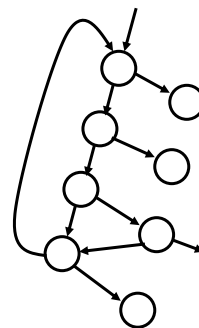
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29

目前正在走的path設成active -> 如果遇到active -> 有loop

## Depth-First Traversal (Take 5)

```
void
Graph::dfsTraversal(const List<Node*>& sinkList)
{
    Node::setGlobalRef();
    for each sink(node, sinkList)
        node->dfsTraversal(dfsList, fbList);
}
// post order traversal
void Node::dfsTraversal(
    List<Node *>& dfsList, list<NodePair>& fbList)
{
    for each predecessor(next, predecessors)
        if (!next->isGlobalRef()) {
            next->setToGlobalRef();
            next->setActive();
            next->dfsTraversal(dfsList, fbList);
            next->unsetActive();
        }
        else if (next->isActive())
            fbList.push_back(NodePair(this, next));
    dfsList.push_back(this); // not push back(next); why?
}
```



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30

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## Breath-First Traversal

```
algorithm levelOrder(TreeNode t)
    Input: a tree node (can be considered to be a
           tree)
    Output: None.

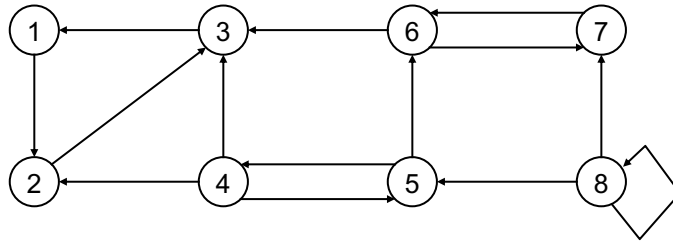
    Let Q be a Queue
    Q.enqueue(t)
    while the Q is not empty
        tree = Q.dequeue()
        Visit node tree
        if tree has a left child
            Q.enqueue(left child of tree)
        if tree has a right child
            Q.enqueue(right child of tree)
```

How about the “marked” and “loop” Issues ??

## Tarjan's Strongly Connected Components (SCC) algorithm

```
Global: idx = 0; S = empty; SCCGroup = empty;
for_each_node (v, G)
    if (v is unvisited) SCC(v);
Func SCC(v) {
    v.idx1 = v.idx2 = idx; idx++; // DFS labeling
    S.push(v); v.setVisited();
    for_each_predecessor (v, w) { // v → w
        if (w is unvisited) {
            SCC(w);
            v.idx2 = min(v.idx2, w.idx2);
        } else if (w is in S) {
            v.idx2 = min(v.idx2, w.idx1);
        }
    }
    if (v.idx1 == v.idx2) {
        SCCGroup.add(S); S.flush();
    }
}
```



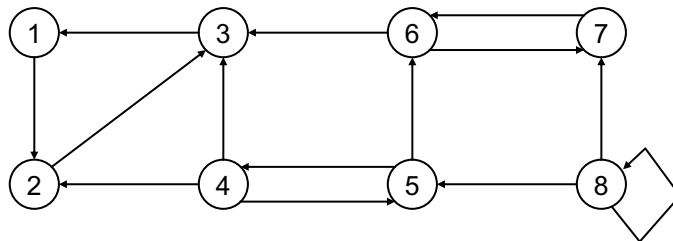


```

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Func SCC(v) {
  v.idx1 = v.idx2 = idx; idx++; // DFS labeling
  S.push(v); v.setVisited();
  for_each_predecessor (v, w) { // v → w
    if (w is unvisited) {
      SCC(w);
      v.idx2 = min(v.idx2, w.idx2);
    } else if (w is in S) {
      v.idx2 = min(v.idx2, w.idx1);
    }
  }
  if (v.idx1 == v.idx2) {
    SCCGroup.add(S); S.flush();
  }
}

```

- for 1 in G:
- 1 is unvisited, call SCC(1)
- ⇒ SCC(1)
  - 1.idx1 = 1.idx2 = idx = 0
  - idx <= 1
  - S <= {1}; 1.setVisited()
  - for 2 in 1 → 2
  - 2 is unvisited, call SCC(2)
  - ⇒ SCC(2)
    - 2.idx1 = 1.idx2 = idx = 1
    - idx <= 2
    - S <= {1, 2}, 2.setVisited()

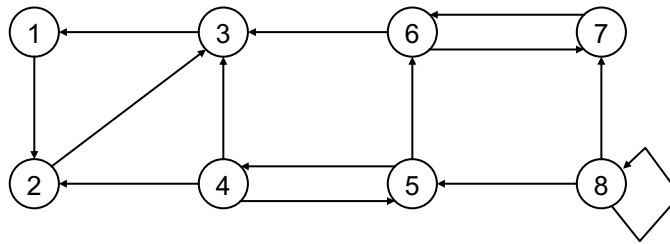


```

Global: idx = 0; S = empty; SCCGroup = empty;
for_each_node (v, G)
  if (v is unvisited) SCC(v);
Func SCC(v) {
  v.idx1 = v.idx2 = idx; idx++; // DFS labeling
  S.push(v); v.setVisited();
  for_each_predecessor (v, w) { // v → w
    if (w is unvisited) {
      SCC(w);
      v.idx2 = min(v.idx2, w.idx2);
    } else if (w is in S) {
      v.idx2 = min(v.idx2, w.idx1);
    }
  }
  if (v.idx1 == v.idx2) {
    SCCGroup.add(S); S.flush();
  }
}

```

- for 3 in 2 → 3
- 3 is unvisited, call SCC(3)
- ⇒ SCC(3)
  - 3.idx1 = 3.idx2 = 2
  - idx = 3
  - S <= {1, 2, 3}, 3.setVisited()
  - for 1 in 3 → 1
  - 1 is visited, 1 is in S
  - 3.idx2 = min(2, 0) = 0
  - end for
  - 3.idx1 != 3.idx2, return



```

Global: idx = 0; S = empty; SCCGroup = empty;
for_each_node (v, G)
    if (v is unvisited) SCC(v);
Func SCC(v) {
    v.idx1 = v.idx2 = idx; idx++; // DFS labeling
    S.push(v); v.setVisited();
    for_each_predecessor (v, w) { // v → w
        if (w is unvisited) {
            SCC(w);
            v.idx2 = min(v.idx2, w.idx2);
        } else if (w is in S) {
            v.idx2 = min(v.idx2, w.idx1);
        }
    }
    if (v.idx1 == v.idx2) {
        SCCGroup.add(S); S.flush();
    }
}

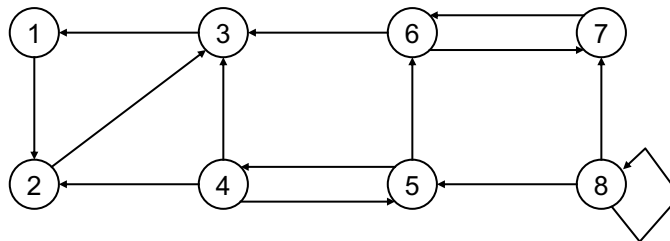
```

```

// back to SSC(2)
• 2.idx2 = min(1, 0) = 0
• end for (2)
• 2.idx1 != 2.idx2, return
// back to SSC(1)
• 1.idx2 = min(0, 0) = 0
• end for (1)
• 1.idx1 == 1.idx2,
  SCCGroup <= {{1,2,3}}, S <= {}
• return
// back to top
• for 2 in G, 2 is visited, continue
• for 3 in G, 3 is visited, continue

```

35



```

Global: idx = 0; S = empty; SCCGroup = empty;
for_each_node (v, G)
    if (v is unvisited) SCC(v);
Func SCC(v) {
    v.idx1 = v.idx2 = idx; idx++; // DFS labeling
    S.push(v); v.setVisited();
    for_each_predecessor (v, w) { // v → w
        if (w is unvisited) {
            SCC(w);
            v.idx2 = min(v.idx2, w.idx2);
        } else if (w is in S) {
            v.idx2 = min(v.idx2, w.idx1);
        }
    }
    if (v.idx1 == v.idx2) {
        SCCGroup.add(S); S.flush();
    }
}

```

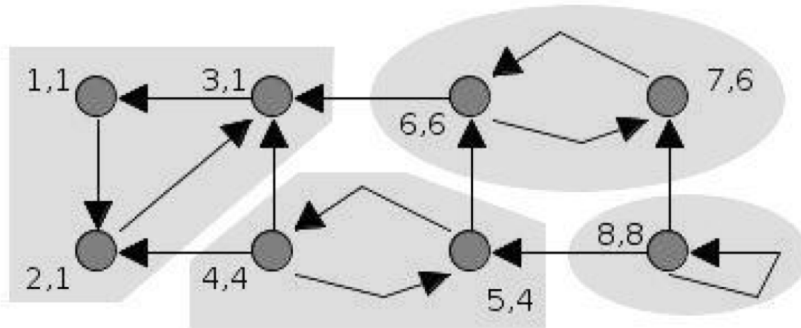
```

• for 4 in G:
• 4 is unvisited, call SCC(4)
⇒ SSC(4)
• 4.idx1 = 4.idx2 = idx = 3
• idx <= 4
• S <= {4}; 4.setVisited()
• for 2 in 4 → 2
• 2 is visited but 2 is NOT in S
• for 3 in 4 → 3
• 3 is visited but 3 is NOT in S
• for 5 in 4 → 5
• 5 is unvisited, call SSC(5)
⇒ SSC(5)...

```

36

## Tarjan's Strongly Connected Components (SCC) algorithm

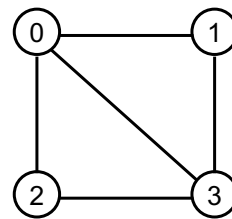


37

## Graph Implementation (1)

### ◆ Adjacency Matrix

```
class Graph
{
    bool _adjacency[n][n];
};
```



- For undirected graph  $\rightarrow$  upper triangle
- How to perform traversal?
- Difficult to implement various graphic algorithms
- Could be a sparse matrix
- Complexity can be as high as  $O(n^2)$

$$\begin{array}{c} \phantom{0} \phantom{1} \phantom{2} \phantom{3} \\ 0 \phantom{1} \phantom{2} \phantom{3} \\ 1 \phantom{2} \phantom{3} \\ 2 \phantom{3} \\ 3 \end{array} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

38

## Graph Implementation (2)

### ◆ Adjacency List

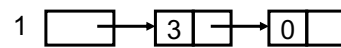
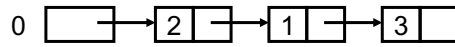
```
class Graph
```

```
{
```

```
    List<int> * _headNodes;
```

```
    int _numNodes;
```

```
};
```



.....

	0	1	2	3
0	0	1	1	1
1	1	0	0	1
2	1	0	0	1
3	1	1	1	0

- Better for sparse matrix
- Require  $n$  `_headNodes` and  $2 \cdot e$  `ListNodes`
- $(u, v)$  and  $(v, u)$  redefined
- Some operations may still be as expensive as  $O(n + e)$

## Graph Implementation (3)

### ◆ Adjacency Multilist

```
class Edge
```

```
{
```

```
    bool _visited;
```

```
    int _vertex1, _vertex2;
```

```
    Edge *_path1, *_path2;
```

```
};
```

```
class Graph
```

```
{
```

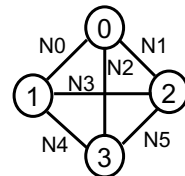
```
    Edge** _headNodes;
```

```
    int _numNodes;
```

```
};
```

	_vertex1	_vertex2	_path1	_path2
N0	0	1	N1	N3
N1	0	2	N2	N3
N2	0	3	0	N4
N3	1	2	N4	N5
N4	1	3	0	N5
N5	2	3	0	0

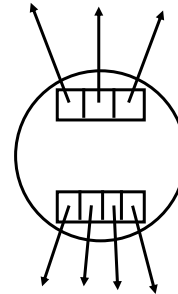
- Same memory requirement as “adjacent list” (except for `_visited` field)
- Not very intuitive to understand



## Graph Implementation (4)

- ◆ Two dynamic arrays

```
class Node
{
    Array<Node *> _successors;
    Array<Node *> _predecessors;
};
class Graph
{
    Array<Node *> _nodes;
    // Array<Node *> _sinks;
    // Array<Node *> _sources;
};
```



- Memory usage is about the same ( $n + 2 * e$ )
- A more intuitive implementation

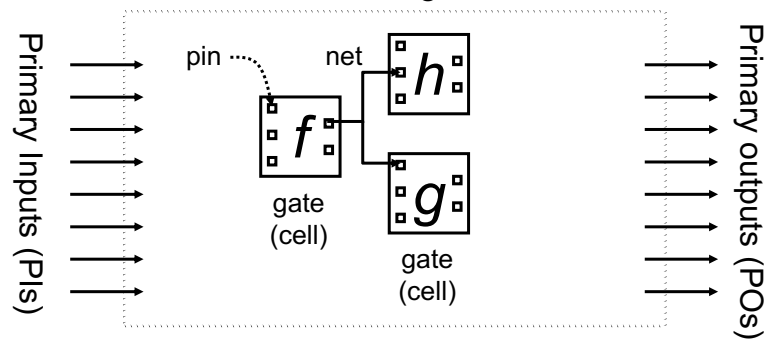
## Graph Implementation (5)

- ◆ To contain data in nodes

```
template <class T>
class Node
{
    T _data;
    Array<Node<T> *> _successors;
    Array<Node<T> *> _predecessors;
};
template <class T>
class Graph
{
    Array<Node<T> *> _nodes;
};
```

## Circuit

- ◆ A directed diagram for representing the current flow of an electronic design



- ◆  $h$  and  $g$  are  $f$ 's *fanouts*
- ◆  $f$  is  $h$ 's and  $g$ 's *fanin*

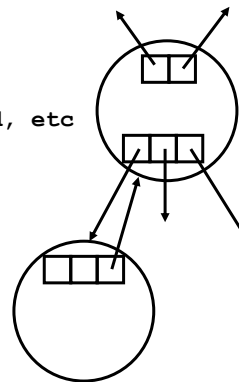
43

## Circuit Implementation (1)

- ◆ Cell-based implementation (1)

```
class Gate
{
    GateType      _type;
    GateFlag      _flag; // visited, etc
    Array<Gate *> _faninList;
    Array<Gate *> _fanoutList;
};

class Circuit
{
    Array<Gate *> _piList;
    Array<Gate *> _poList;
    Array<Gate *> _totalList;
};
```



- Gate::\_type is to distinguish different functionalities
  - Drawback: usually need a BIG switch in codes

44

## Circuit Implementation (2)

### ◆ Cell-based implementation (2)

```
class Gate
{
    GateType         _type;
    GateFlag         _flag;
    Array<Gate *>     _faninList;
    Array<Gate *>     _fanoutList;
};

class And : public Gate
{
};

class Circuit
{
    Array<Gate *>     _piList;
    Array<Gate *>     _poList;
    Array<Gate *>     _totalList;
};
```

## Virtual Functions for Different Types of Gates

```
class Gate {
    virtual bool simulate() = 0;
};

class And : public Gate {
    bool simulate() { ... }
};

=====
void Circuit::constructNetlist() {
    Gate* newGate = new And;
    ...
}

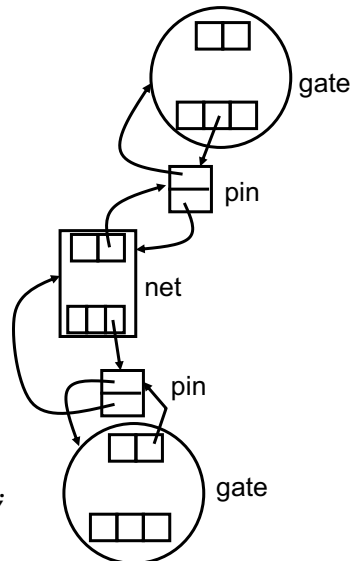
bool Circuit::simulate() {
    for_each_gate(thisGate, _totalList)
        thisGate->simulate(); // Will call And::simulate()
    ...
}
```

## Circuit Implementation (2)

◆ Net-based implementation

```

class Net
{
    Array<Pin *>    _inPinList;
    Array<Pin *>    _outPinList;
};
class Pin
{
    Gate*           _gate;
    Net*            _net;
};
class Gate
{
    GateFlag        _flag;
    Array<Pin *>    _inPinList;
    Array<Pin *>    _outPinList;
};
    
```



## Circuit Implementation (3)

◆ AND-Inverter Graph (AIG)

◆ All the Boolean functions can be represented by  
“And:  $\wedge$ ” and “Inverter:  $\neg$ ”

- e.g.  $OR(a, b) = \neg(\neg a \wedge \neg b)$

◆ As for circuit implementation, it is better to have simpler data structure

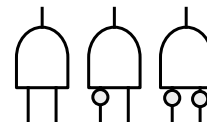
- AIG is enough
- Two classes: AndGate and InvGate?

- InvGate is kind of unnecessary...

- One class: NandGate?

- Still need an object to represent an Inverter

➔ Solution: AndGate with (optional) inverted inputs





## AIG Implementation

```
class AigGate {
    Array<AigGateV> _faninList;
    size_t _ref;
    static size_t _globalRef_s
};

class AigGateV {
    #define NEG 0x1
    AigGateV(AigGate* g, size_t phase):
        _gateV(size_t(g) + phase) { }
    AigGate* gate() const {
        return (AigGate*)(_gateV & ~size_t(NEG)); }
    bool isInv() const { return (_gateV & NEG); }

    size_t _gateV;
};
```

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49

49

## AIGER Format

- ◆ An simplified, well-accepted AIG format
  - Documents and source codes available at:  
<http://fmv.jku.at/aiger/>
- ◆ Two versions
  - ASCII format: text format ← HW #6
  - Binary format: more compact representation
  - ➔ In HW#6 and final project, we will handle ASCII format only

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50

50

## AIGER ASCII Format

- ◆ ASCII format contains several sections
  - Header
  - Inputs
  - Latches // ignored in HW#6
  - Outputs
  - ANDs
  - Symbols
  - Comments
- ◆ Except for header, any of the above sections can be omitted if it is not necessary
  - However, their relative order cannot be altered

## AIGER ASCII Format

- ◆ Header
  - [Syntax] aag M I L O A
    - aag: specify ASCII AIG format
      - [cf] aig: specify binary format
    - M: maximal variable index
    - I, L, O, A: number of inputs, latches, outputs, AND gates
  - [Example] aag 7 2 0 2 3
  - [Note]
    - Exact ONE space before M, I, L, O, A
    - “A” must be immediately followed by a “new line” char.
    - If all variables are used and there are no unused AND gates, then  $M = I + L + A$ .

## AIGER ASCII Format

### ◆ Variables and Literals

- Each input, latch, and AND gate is assigned with a distinct variable ID (i.e. an unsigned number)
  - Between  $[1, M]$
  - Variable 0 means constant FALSE.
  - The input, latch, and AND variable IDs can be arbitrary. No one is necessarily bigger/smaller than the other.
- A “literal” is a positive or negative form of a variable
  - Let  $v$  be the ID of a variable, then the literal  $(2v)$  and  $(2v+1)$  stands for the positive and negative forms of the variable, respectively
  - e.g. Literal 12 is the positive form of variable 6  
Literal 1 stands for constant TRUE

## AIGER ASCII Format

### ◆ Inputs

- [Syntax] <inputLiteralID>
- [Example] 2
- [Note]
  - Each line defines exactly one input, which is represented as a literal ID
  - Inputs are non-negative, so the literal IDs must be even numbers

### ◆ Example

```
aag 3 2 0 1 1 // header
2           // input 0
4           // input 1
```

## AIGER ASCII Format

### ◆ Latches

- [Syntax] <currStateLiteralID>  
<nextStateLiteralID>
- [Example] 8 15
- [Note]
  - Each line defines exactly one latch, which contains the current state literal ID followed by the next state ID
  - Current states are non-negative (as inputs), so their literal IDs must be even numbers
  - Next states can be inverted (as outputs), so their literal IDs can be positive or negative

## AIGER ASCII Format

### ◆ Outputs

- [Syntax] <outputLiteralID>
- [Example] 9
- [Note]
  - Each line defines exactly one output, which is represented as a literal ID
  - Outputs can be inverted, so their literal IDs can be even or odd

### ◆ Example

```
aag 3 2 0 1 1 // header
2           // input 0
4           // input 1
6           // output 0
```

## AIGER ASCII Format

### ◆ AND gates

- [Syntax] <LHS> <RHS1> <RHS2>
- [Example] 12 7 15
- [Note]
  - Each line defines exactly one AND gate, which contains the LHS literal followed by exactly two RHS literals
  - LHS literals must be even, and the RHS literals can be even or odd (i.e. non-inverted or inverted)

### ◆ Example

```
aag 3 2 0 1 1 // header
2 // input #0 (var id = 1)
4 // input #1 (var id = 2)
6 // output #0 (var id = 3)
6 2 4 // AND gate 3 = 1 & 2
```

## AIGER ASCII Format

### ◆ Symbols

- [Syntax] [ilo]<position> <symbolicName>
- [Example] i0 reset  
o1 done
- [Note]
  - Each line defines exactly one symbolic name for inputs, latches, or outputs
  - There is at most ONE symbolic name for each input, latch, or output
  - <position> denotes the position of the corresponding input/latch/output is defined in it section. It counts from 0.
  - Symbolic name can contain any printable character, except for “new line”
    - [Note] White space and numbers are allowed in names

## AIGER ASCII Format

### ◆ Comments

- [Syntax] c  
[anything]...
- [Example] c  
Game over!!
- [Note]
  - The comment section starts with a *c* character followed by a new line. The following lines are comments.

## Notes on AIGER Format

- ◆ No leading or trailing spaces in each line
- ◆ No empty line
- ◆ “New line” character must present at the end of each line
- ◆ All parsed tokens in the same line, except for comments, must be separated by exactly ONE space character
- ◆ Need to identify undefined literals and floating signals in parser

## AIGER Examples

1. Empty circuit  

```
aag 0 0 0 0 0 // header
```
2. And gate  

```
aag 3 2 0 1 1 // header
2 // input #0 (var id = 1)
4 // input #1 (var id = 2)
6 // output #0 (var id = 3)
6 2 4 // AND gate 3 = 1 & 2
```
3. Or gate  

```
aag 3 2 0 1 1
2 // input #0 (var id = 1)
4 // input #1 (var id = 2)
7 // output #0 (var id = 3)
6 3 5 // AND gate 3 = !1 & !2
```

## AIGER Examples

4. Half Adder  

```
aag 7 2 0 2 3 // header line
2 // input #0 1st addend bit 'x'
4 // input #1 2nd addend bit 'y'
6 // output #0 sum bit 's'
12 // output #1 carry 'c'
6 13 15 // AND gate #0 x ^ y
12 2 4 // AND gate #1 x & y
14 3 5 // AND gate #2 !x & !y
i0 x // symbol
i1 y // symbol
o0 s // symbol
o1 c // symbol
c // comment header
half adder // comment
```

## Some notes about HW#6

### ◆ Topic: An ALGER parser

- Parse an ALGER netlist file into a circuit data structure (a DAG)
  - Note: Error handling can be VERY complicated...  
Try to work on “good” circuits first!!

- Check for floating/undefined variables

- ~~Check for cyclic conditions~~

defined but not used : 定义了没人用  
floating : 没有被定义到直接拿来用

- Report circuit statistics

- Report gate connections

- ~~Perform logic simulations~~

print aig fanin 時，如果前面畫過了而且那個gate不是最後一個level，後面加一個(\*) 就不要再往下印了（會印不完）

- Output AIG file

63

dfs 從 P0 開始，順序從  
input0 -> input1開始 ->  
每個人的dfs應該都一樣