

# Chap 03 | Motion in one dimension

## 1. Position

Any object is situated at point O and three observers from three different places are looking for same object, then all three observers will have different observations about the position of point O and no one will be wrong. Because they are observing the object from their different positions.

Observer 'A' says : Point O is 3 m away in west direction.

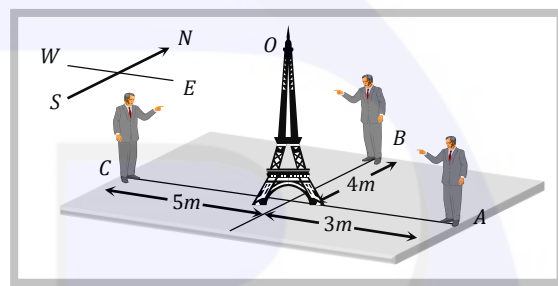
Observer 'B' says : Point O is 4 m away in south direction.

Observer 'C' says : Point O is 5 m away in east direction.

Therefore position of any point is completely expressed by two factors: Its distance from the observer and its direction with respect to observer.

That is why position is characterised by a vector known as position vector.

Let point P is in a xy plane and its coordinates are (x, y). Then position vector ( $\vec{r}$ ) of point will be  $x\hat{i} + y\hat{j}$  and if the point P is in a space and its coordinates are (x, y, z) then position vector can be expressed as  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ .



## 2. Rest and Motion

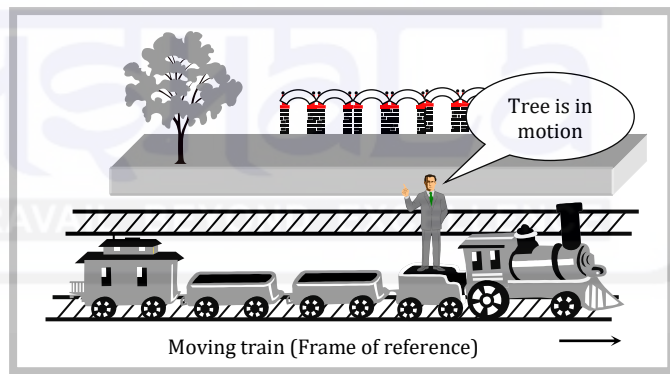
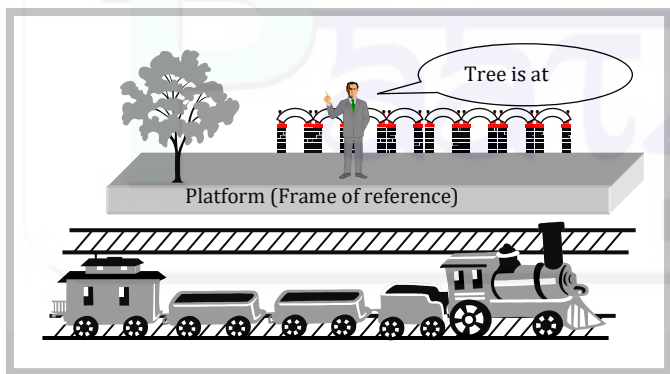
If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference : It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that tree on a platform is at rest. But when the same passenger is passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But observations are different because in first situation observer stands on a platform, which is reference frame at rest and in second situation observer moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references.



## 3. Types of Motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g.. Motion of car on a straight road. Motion of freely falling body.	e.g. Motion of car on a circular turn. Motion of billiards ball.	e.g.. Motion of flying kite. Motion of flying insect.

## 4. Particle or Point Mass

The smallest part of matter with zero dimension which can be described by its mass and position is defined as a particle.

If the size of a body is negligible in comparison to its range of motion then that body is known as a particle.

A body (Group of particles) to be known as a particle depends upon types of motion. For example in a planetary motion around the sun the different planets can be presumed to be the particles.

In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

## 5. Distance and Displacement

(1) **Distance** : It is the actual path length covered by a moving particle in a given interval of time.

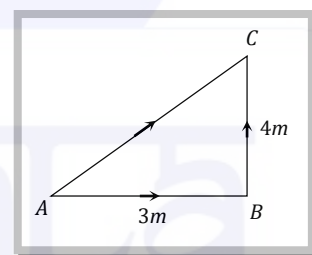
(i) If a particle starts from A and reach to C through point B as shown in the figure.

Then distance travelled by particle =  $AB + BC = 7 \text{ m}$

(ii) Distance is a scalar quantity.

(iii) Dimension :  $[M^0L^1T^0]$

(iv) Unit : metre (S.I.)



(2) **Displacement** : Displacement is the change in position vector i.e., A vector joining initial to final position.

(i) Displacement is a vector quantity

(ii) Dimension :  $[M^0L^1T^0]$

(iii) Unit : metre (S.I.)

(iv) In the above figure the displacement of the particle  $\vec{AC} = \vec{AB} + \vec{BC}$

$$\Rightarrow |\vec{AC}| = \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 90^\circ} = 5\text{m}$$

(v) If  $\vec{S}_1, \vec{S}_2, \vec{S}_3, \dots, \vec{S}_n$  are the displacements of a body then the total (net) displacement is the vector sum of the individuals.  $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \dots + \vec{S}_n$

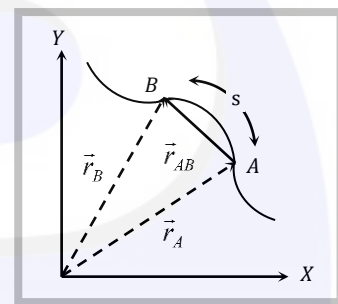
(3) **Comparison between distance and displacement :**

- (i) The magnitude of displacement is equal to minimum possible distance between two positions.  
So distance  $\geq |\text{Displacement}|$ .
- (ii) For a moving particle distance can never be negative or zero while displacement can be.  
(zero displacement means that body after motion has come back to initial position)  
i.e., Distance  $> 0$  but Displacement  $> = \text{or} < 0$
- (iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.
- (iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.
- (v) In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.
- (vi) If  $\vec{r}_A$  and  $\vec{r}_B$  are the position vectors of particle initially and finally.

Then displacement of the particle

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

and  $s$  is the distance travelled if the particle has gone through the path APB.



## Sample problems based on distance and displacement

**Problem 1.** A man goes 10m towards North, then 20m towards east then displacement is

[KCET (Med.) 1999; JIPMER 1999; AFMC 2003]

- (a) 22.5m
- (b) 25m
- (c) 25.5m
- (d) 30m

**Solution :** (a) If we take east as  $x$  - axis and north as  $y$  - axis, then displacement  $= 20\hat{i} + 10\hat{j}$

So, magnitude of displacement  $= \sqrt{20^2 + 10^2} = 10\sqrt{5} = 22.5 \text{ m}$ .

**Problem 2.** A body moves over one fourth of a circular arc in a circle of radius  $r$ . The magnitude of distance travelled and displacement will be respectively

- (a)  $\frac{\pi r}{2}, r\sqrt{2}$
- (b)  $\frac{\pi r}{4}, r$
- (c)  $\pi r, \frac{r}{\sqrt{2}}$
- (d)  $\pi r, r$

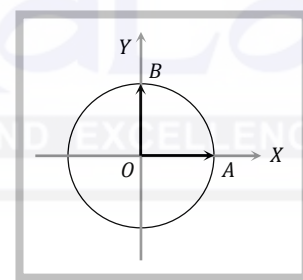
**Solution :** (a) Let particle start from A, its position vector  $\vec{r}_{OA} = r\hat{i}$

After one quarter position vector  $\vec{r}_{OB} = r\hat{j}$ .

So displacement  $= r\hat{j} - r\hat{i}$

Magnitude of displacement  $= r\sqrt{2}$ .

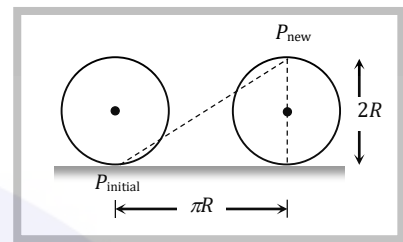
and distance  $= \text{one fourth of circumference} = \frac{2\pi r}{4} = \frac{\pi r}{2}$



**Problem 3.** The displacement of the point of the wheel initially in contact with the ground, when the wheel rolls forward half a revolution will be (radius of the wheel is  $R$ )

- (a)  $\frac{R}{\sqrt{\pi^2 + 4}}$
- (b)  $R\sqrt{\pi^2 + 4}$
- (c)  $2\pi R$
- (d)  $\pi R$

Solution : (b) Horizontal distance covered by the wheel in half revolution =  $\pi R$   
 So the displacement of the point which was initially in contact with a ground =  $\sqrt{(\pi R)^2 + (2R)^2}$   
 $= R\sqrt{\pi^2 + 4}$ .



## 6. Speed and Velocity

(1) **Speed** : Rate of distance covered with time is called speed.

- (i) It is a scalar quantity having symbol  $v$ .
- (ii) Dimension :  $[M^0L^1T^{-1}]$
- (iii) Unit : metre/second (S.I.), cm/second (C.G.S.)
- (iv) Types of speed :

(a) **Uniform speed** : When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5m) in each second. So we can say that particle is moving with uniform speed of 5 m/s.

Distance	5m	5m	5m	5m	5m	5m
Time	1 sec	1 sec	1 sec	1 sec	1 sec	1m/s
Uniform Speed	5m/	5m/	5m/s	5m/	5m/	5m/s

(b) **Non-uniform (variable) speed** : In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5m in 1<sup>st</sup> second, 8m in 2<sup>nd</sup> second, 10m in 3<sup>rd</sup> second, 4m in 4<sup>th</sup> second etc.

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.

Distance	5m	8m	10m	4m	6m	7m
Time	1 sec	1 sec	1 sec	1 sec	1 sec	1 sec
Variable Speed	5m/	8m/	10m/	4m/	6m/	7m/

(c) **Average speed** : The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} ; v_{av} = \frac{\Delta s}{\Delta t}$$

- Time average speed : When particle moves with different uniform speed  $v_1, v_2, v_3 \dots$  etc in different time intervals  $t_1, t_2, t_3, \dots$  etc respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + \dots}{t_1 + t_2 + t_3 + \dots} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

Special case : When particle moves with speed  $v_1$  upto half time of its total motion and in rest time it is moving

with speed  $v_2$  then  $v_{av} = \frac{v_1 + v_2}{2}$

- Distance averaged speed : When a particle describes different distances  $d_1, d_2, d_3, \dots$  with different time intervals  $t_1, t_2, t_3, \dots$  with speeds  $v_1, v_2, v_3, \dots$  respectively then the speed of particle averaged over the total distance can be given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + \dots}{t_1 + t_2 + t_3 + \dots} = \frac{d_1 + d_2 + d_3 + \dots}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + \dots}$$

- When particle moves the first half of a distance at a speed of  $v_1$  and second half of the distance at speed  $v_2$  then

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

- When particle covers one-third distance at speed  $v_1$ , next one third at speed  $v_2$  and last one third at speed  $v_3$ , then

$$v_{av} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

- (d) **Instantaneous speed** : It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e.,  $\Delta t \rightarrow 0$ ). Thus

$$\text{Instantaneous speed } v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- (2) **Velocity** : Rate of change of position i.e. rate of displacement with time is called velocity.

(i) It is a scalar quantity having symbol  $v$ .

(ii) Dimension :  $[M^0 L^1 T^{-1}]$

(iii) Unit : metre/second (S.I.), cm/second (C.G.S.)

(iv) Types

(a) **Uniform velocity** : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity** : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

(c) **Average velocity** : It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

(d) **Instantaneous velocity** : Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

$$\text{Instantaneous velocity } \vec{v} = \lim_{t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

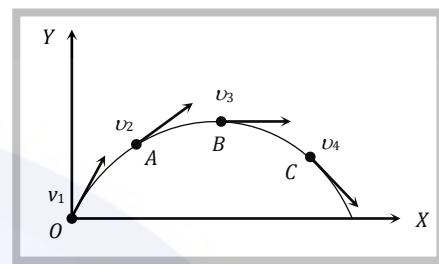
(v) **Comparison between instantaneous speed and instantaneous velocity**



(a) instantaneous velocity is always tangential to the path followed by the particle.

When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is  $v_1$ , at point A the instantaneous velocity of stone is  $v_2$ , similarly at point B and C are  $v_3$  and  $v_4$  respectively.

Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.



(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement  $\vec{x} = A_0 - A_1t + A_2t^2$

Instantaneous velocity  $\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1t + A_2t^2)$

$$\vec{v} = -A_1 + 2A_2t$$

For the given value of t, we can find out the instantaneous velocity.

e.g. for  $t = 0$ , Instantaneous velocity  $\vec{v} = -A_1$  and Instantaneous speed  $|\vec{v}| = A_1$

(vi) **Comparison between average speed and average velocity**

(a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions  $[LT^{-1}]$ .

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then  $\vec{v}_{av} = \vec{0}$  (as  $\Delta\vec{r} = 0$ ) but  $v_{av} > 0$  and finite as  $(\Delta s > 0)$ .

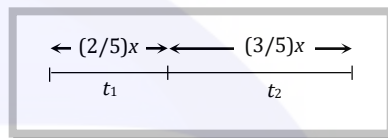
(e) For a moving body average speed can never be negative or zero (unless  $t \rightarrow \infty$ ) while average velocity can be i.e.  $v_{av} > 0$  while  $\vec{v}_{av} = \text{or} < 0$ .

## Sample problems based on speed and velocity

**Problem 4.** If a car covers  $2/5^{\text{th}}$  of the total distance with  $v_1$  speed and  $3/5^{\text{th}}$  distance with  $v_2$  then average speed is [MP PMT 2003]

- (a)  $\frac{1}{2}\sqrt{v_1 v_2}$  (b)  $\frac{v_1 + v_2}{2}$  (c)  $\frac{2v_1 v_2}{v_1 + v_2}$  (d)  $\frac{5v_1 v_2}{3v_1 + 2v_2}$

**Solution : (d)** Average speed =  $\frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{x}{t_1 + t_2}$   
 $= \frac{x}{\frac{(2/5)x}{v_1} + \frac{(3/5)x}{v_2}} = \frac{5v_1 v_2}{2v_2 + 3v_1}$



**Problem 5.** A car accelerated from initial position and then returned at initial point, then [AIEEE 2002]

- (a) Velocity is zero but speed increases (b) Speed is zero but velocity increases  
 (c) Both speed and velocity increase (d) Both speed and velocity decrease

**Solution : (a)** As the net displacement = 0  
 Hence velocity = 0 ; but speed increases.

**Note :**  $\frac{|\text{Average velocity}|}{|\text{Average speed}|} \leq 1 \Rightarrow |\text{Av. speed}| \geq |\text{Av. velocity}|$

**Problem 6.** A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. The average speed of the man over the interval of time 0 to 40 min. is equal to [AMU (Med.) 2002]

- (a) 5 km/h (b)  $\frac{25}{4}$  km/h (c)  $\frac{30}{4}$  km/h (d)  $\frac{45}{8}$  km/h

**Solution : (d)** Time taken in going to market =  $\frac{2.5}{5} = \frac{1}{2} \text{ hr} = 30 \text{ min}.$

As we are told to find average speed for the interval 40 min., so remaining time for consideration of motion is 10 min.

So distance travelled in remaining 10 min =  $7.5 \times \frac{10}{60} = 1.25 \text{ km}.$

Hence, average speed =  $\frac{\text{Total distance}}{\text{Total time}} = \frac{(2.5 + 1.25) \text{ km}}{(40 / 60) \text{ hr.}} = \frac{45}{8} \text{ km / hr}.$

**Problem 7.** The relation  $3t = \sqrt{3x} + 6$  describes the displacement of a particle in one direction where  $x$  is in metres and  $t$  in sec. The displacement, when velocity is zero, is [CPMT 2000]

- (a) 24 metres (b) 12 metres (c) 5 metres (d) Zero

**Solution : (d)**  $3t = \sqrt{3x} + 6 \Rightarrow \sqrt{3x} = (3t - 6) \Rightarrow 3x = (3t - 6)^2 \Rightarrow x = 3t^2 - 12t + 12$

$\therefore v = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - 12t + 12) = 6t - 12$

If velocity = 0 then,  $6t - 12 = 0 \Rightarrow t = 2 \text{ sec}$

Hence at  $t = 2$ ,  $x = 3(2)^2 - 12(2) + 12 = 0 \text{ metres}.$

- Problem 8.** The motion of a particle is described by the equation  $x = a + bt^2$  where  $a = 15 \text{ cm}$  and  $b = 3 \text{ cm}$ . Its instantaneous velocity at time 3 sec will be [AMU (Med.) 2000]  
 (a) 36 cm/sec (b) 18 cm/sec (c) 16 cm/sec (d) 32 cm/sec

Solution : (b)  $x = a + bt^2 \quad \therefore v = \frac{dx}{dt} = 0 + 2bt$

At  $t = 3 \text{ sec}$ ,  $v = 2 \times 3 \times 3 = 18 \text{ cm/sec}$  (As  $b = 3 \text{ cm}$ )

- Problem 9.** A train has a speed of 60 km/h for the first one hour and 40 km/h for the next half hour. Its average speed in km/h is [JIPMER 1999]  
 (a) 50 (b) 53.33 (c) 48 (d) 70

Solution : (b) Total distance travelled =  $60 \times 1 + 40 \times \frac{1}{2} = 80 \text{ km}$  and Total time taken =  $1 \text{ hr} + \frac{1}{2} \text{ hr} = \frac{3}{2} \text{ hr}$   
 $\therefore$  Average speed =  $\frac{80}{3/2} = 53.33 \text{ km/h}$

- Problem 10.** A person completes half of its his journey with speed  $v_1$  and rest half with speed  $v_2$ . The average speed of the person is [RPET 1993; MP PMT 2001]

(a)  $v = \frac{v_1 + v_2}{2}$  (b)  $v = \frac{2v_1 v_2}{v_1 + v_2}$  (c)  $v = \frac{v_1 v_2}{v_1 + v_2}$  (d)  $v = \sqrt{v_1 v_2}$

Solution : (b) In this problem total distance is divided into two equal parts. So

$$v_{av} = \frac{d_1 + d_2}{\frac{d_1}{v_1} + \frac{d_2}{v_2}} = \frac{\frac{d}{2} + \frac{d}{2}}{\frac{d/2}{v_1} + \frac{d/2}{v_2}} \Rightarrow v_{av} = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

- Problem 11.** A car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is [MP PMT 1999; CPMT 2002]

(a) 40 km/hr (b) 80 km/hr (c)  $46 \frac{2}{3} \text{ km/hr}$  (d) 36 km/hr

Solution : (d) Let total distance travelled =  $x$  and total time taken  $t_1 + t_2 = \frac{x/3}{20} + \frac{2x/3}{60}$

$$\therefore \text{Average speed} = \frac{x}{\frac{(1/3)x}{20} + \frac{(2/3)x}{60}} = \frac{1}{\frac{1}{60} + \frac{2}{180}} = 36 \text{ km/hr}$$

## 7. Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

- (1) It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)
- (2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	e.g. Motion under gravity	e.g. Projectile motion



(3) Dimension :  $[M^0L^1T^{-2}]$

(4) Unit : metre/second<sup>2</sup> (S.I.); cm/second<sup>2</sup> (C.G.S.)

(5) Types of acceleration :

(i) **Uniform acceleration** : A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : ☐ If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

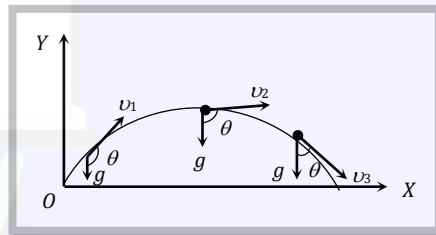
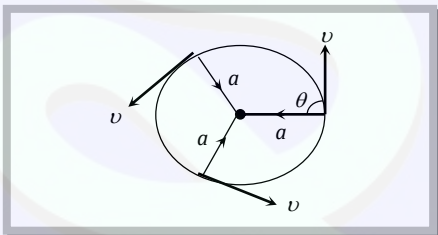
(ii) **Non-uniform acceleration** : A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

(iii) **Average acceleration** :  $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as  $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

(iv) **Instantaneous acceleration** =  $\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.



e.g. (a) In uniform circular motion  $\theta = 90^\circ$  always

(b) In a projectile motion  $\theta$  is variable for every point of trajectory.

(vi) If a force  $\vec{F}$  acts on a particle of mass  $m$ , by Newton's 2<sup>nd</sup> law, acceleration  $\vec{a} = \frac{\vec{F}}{m}$

(vii) By definition  $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$  [As  $\vec{v} = \frac{d\vec{x}}{dt}$ ]

i.e., if  $x$  is given as a function of time, second time derivative of displacement gives acceleration

(viii) If velocity is given as a function of position, then by chain rule  $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \cdot \frac{dv}{dx}$  [as  $v = \frac{dx}{dt}$ ]

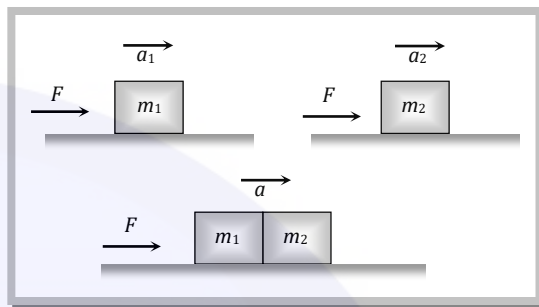
(ix) If a particle is accelerated for a time  $t_1$  by acceleration  $a_1$  and for time  $t_2$  by acceleration  $a_2$  then average

acceleration is  $a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$

(x) If same force is applied on two bodies of different masses  $m_1$  and  $m_2$  separately then it produces accelerations  $a_1$  and  $a_2$  respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that  $a$  be the acceleration of the combined system, then

$$F = (m_1 + m_2)a \Rightarrow \frac{F}{a} = \frac{F}{a_1} + \frac{F}{a_2}$$

$$\text{So, } \frac{1}{a} = \frac{1}{a_1} + \frac{1}{a_2} \Rightarrow a = \frac{a_1 a_2}{a_1 + a_2}$$



(xi) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

(xii) For motion of a body under gravity, acceleration will be equal to “g”, where g is the acceleration due to gravity. Its normal value is  $9.8 \text{ m/s}^2$  or  $980 \text{ cm/s}^2$  or  $32 \text{ feet/s}^2$ .

## Sample problems based on acceleration

**Problem 12.** The displacement of a particle, moving in a straight line, is given by  $s = 2t^2 + 2t + 4$  where  $s$  is in metres and  $t$  in seconds. The acceleration of the particle is [CPMT 2001]  
(a)  $2 \text{ m/s}^2$  (b)  $4 \text{ m/s}^2$  (c)  $6 \text{ m/s}^2$  (d)  $8 \text{ m/s}^2$

Solution : (b) Given  $s = 2t^2 + 2t + 4 \therefore$  velocity  $(v) = \frac{ds}{dt} = 4t + 2$  and acceleration  $(a) = \frac{dv}{dt} = 4(1) + 0 = 4 \text{ m/s}^2$

**Problem 13.** The position  $x$  of a particle varies with time  $t$  as  $x = at^2 - bt^3$ . The acceleration of the particle will be zero at time  $t$  equal to [CBSE PMT 1997; BHU 1999; DPMT 2000; KCET (Med.) 2000]  
(a)  $\frac{a}{b}$  (b)  $\frac{2a}{3b}$  (c)  $\frac{a}{3b}$  (d) Zero

Solution : (c) Given  $x = at^2 - bt^3 \therefore$  velocity  $(v) = \frac{dx}{dt} = 2at - 3bt^2$  and acceleration  $(a) = \frac{dv}{dt} = 2a - 6bt$ .

$$\text{When acceleration} = 0 \Rightarrow 2a - 6bt = 0 \Rightarrow t = \frac{2a}{6b} = \frac{a}{3b}$$

**Problem 14.** The displacement of the particle is given by  $y = a + bt + ct^2 - dt^4$ . The initial velocity and acceleration are respectively [CPMT 1999, 2003]  
(a)  $b, -4d$  (b)  $-b, 2c$  (c)  $b, 2c$  (d)  $2c, -4d$

Solution : (c) Given  $y = a + bt + ct^2 - dt^4 \therefore v = \frac{dy}{dt} = 0 + b + 2ct - 4dt^3$

Putting  $t = 0$ ,  $v_{\text{initial}} = b$

So initial velocity =  $b$

$$\text{Now, acceleration } (a) = \frac{dv}{dt} = 0 + 2c - 12dt^2$$

Putting  $t = 0$ ,  $a_{\text{initial}} = 2c$

**Problem 15.** The relation between time  $t$  and distance  $x$  is  $t = \alpha x^2 + \beta x$ , where  $\alpha$  and  $\beta$  are constants. The retardation is ( $v$  is the velocity) [NCERT 1982]  
(a)  $2\alpha v^3$  (b)  $2\beta v^3$  (c)  $2\alpha\beta v^3$  (d)  $2\beta^2 v^3$

Solution : (a) differentiating time with respect to distance  $\frac{dt}{dx} = 2\alpha x + \beta \Rightarrow v = \frac{dx}{dt} = \frac{1}{2\alpha x + \beta}$

$$\text{So, acceleration (a)} = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx} = \frac{-v \cdot 2\alpha}{(2\alpha x + \beta)^2} = -2\alpha \cdot v \cdot v^2 = -2\alpha v^3$$

**Problem 16.** If displacement of a particle is directly proportional to the square of time. Then particle is moving with [RPET 1999]

- (a) Uniform acceleration (b) Variable acceleration  
(c) Uniform velocity (d) Variable acceleration but uniform velocity

**Solution : (a)** Given that  $x \propto t^2$  or  $x = Kt^2$  (where K= constant)

$$\text{Velocity (v)} = \frac{dx}{dt} = 2Kt \text{ and Acceleration (a)} = \frac{dv}{dt} = 2K$$

It is clear that velocity is time dependent and acceleration does not depend on time.

So we can say that particle is moving with uniform acceleration but variable velocity.

**Problem 17.** A particle is moving eastwards with velocity of 5 m/s. In 10 sec the velocity changes to 5 m/s northwards. The average acceleration in this time is [IIT-JEE 1982]

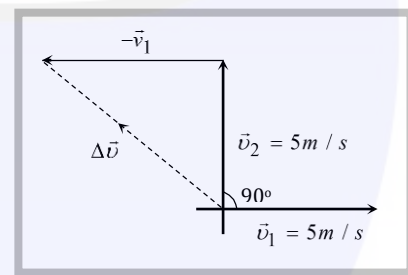
- (a) Zero (b)  $\frac{1}{\sqrt{2}} \text{ m/s}^2$  toward north-west  
(c)  $\frac{1}{\sqrt{2}} \text{ m/s}^2$  toward north-east (d)  $\frac{1}{2} \text{ m/s}^2$  toward north-west

**Solution : (b)**  $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$

$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos 90^\circ} = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$$\Delta v = 5\sqrt{2}$$

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \text{ m/s}^2 \text{ toward north-west (As clear from the figure).}$$



**Problem 18.** A body starts from the origin and moves along the x-axis such that velocity at any instant is given by  $(4t^3 - 2t)$ , where t is in second and velocity is in m/s. What is the acceleration of the particle, when it is 2m from the origin? [IIT-JEE 1982]

- (a)  $28 \text{ m/s}^2$  (b)  $22 \text{ m/s}^2$  (c)  $12 \text{ m/s}^2$  (d)  $10 \text{ m/s}^2$

**Solution : (b)** Given that  $v = 4t^3 - 2t$

$$x = \int v dt, \quad x = t^4 - t^2 + C, \text{ at } t=0, x=0 \Rightarrow C=0$$

When particle is 2m away from the origin

$$2 = t^4 - t^2 \Rightarrow t^4 - t^2 - 2 = 0 \Rightarrow (t^2 - 2)(t^2 + 1) = 0 \Rightarrow t = \sqrt{2} \text{ sec}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(4t^3 - 2t) = 12t^2 - 2 \Rightarrow a = 12t^2 - 2$$

$$\text{for } t = \sqrt{2} \text{ sec} \Rightarrow a = 12 \times (\sqrt{2})^2 - 2 \Rightarrow a = 22 \text{ m/s}^2$$

**Problem 19.** A body of mass 10 kg is moving with a constant velocity of 10 m/s. When a constant force acts for 4 sec on it, it moves with a velocity 2 m/sec in the opposite direction. The acceleration produced in it is

[MP PET 1997]

(a)  $3 \text{ m/s}^2$

(b)  $-3 \text{ m/s}^2$

(c)  $0.3 \text{ m/s}^2$

(d)  $-0.3 \text{ m/s}^2$

Solution : (b) Let particle moves towards east and by the application of constant force it moves towards west

$$\vec{v}_1 = +10 \text{ m/s} \text{ and } \vec{v}_2 = -2 \text{ m/s} . \text{ Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{\vec{v}_2 - \vec{v}_1}{t}$$

$$\Rightarrow a = \frac{(-2) - (10)}{4} = \frac{-12}{4} = -3 \text{ m/s}^2$$

## 8. Position Time Graph

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

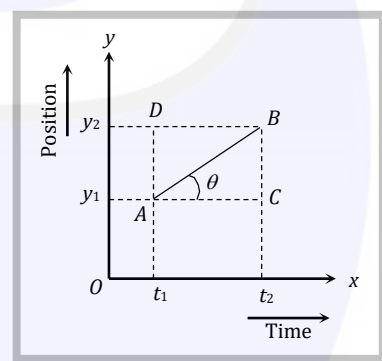
Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

Let AB is a position-time graph for any moving particle

$$\text{As Velocity} = \frac{\text{Change in position}}{\text{Time taken}} = \frac{y_2 - y_1}{t_2 - t_1} \quad \dots(i)$$

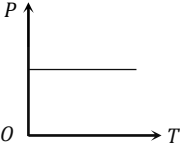
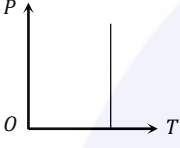
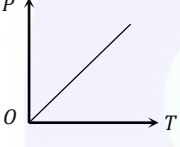
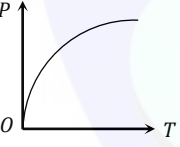
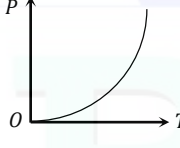
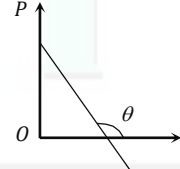
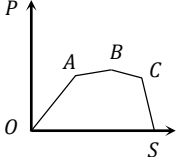
$$\text{From triangle ABC } \tan \theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{y_2 - y_1}{t_2 - t_1} \quad \dots(ii)$$

By comparing (i) and (ii)      Velocity =  $\tan \theta$   
 $v = \tan \theta$

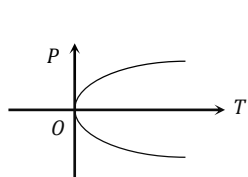


It is clear that slope of position-time graph represents the velocity of the particle.

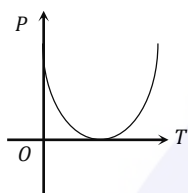
**Various position – time graphs and their interpretation**

	<p><math>\theta = 0^\circ</math> so <math>v = 0</math> i.e., line parallel to time axis represents that the particle is at rest.</p>
	<p><math>\theta = 90^\circ</math> so <math>v = \infty</math> i.e., line perpendicular to time axis represents that particle is changing its position but time does not change it means the particle possesses infinite velocity. Practically this is not possible.</p>
	<p><math>\theta = \text{constant}</math> so <math>v = \text{constant}</math>, <math>a = 0</math> i.e., line with constant slope represents uniform velocity of the particle.</p>
	<p><math>\theta</math> is increasing so <math>v</math> is increasing, <math>a</math> is positive. i.e., line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.</p>
	<p><math>\theta</math> is decreasing so <math>v</math> is decreasing, <math>a</math> is negative i.e., line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.</p>
	<p><math>\theta</math> constant but <math>&gt; 90^\circ</math> so <math>v</math> will be constant but negative i.e., line with negative slope represent that particle returns towards the point of reference. (negative displacement).</p>
	<p>Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.</p>





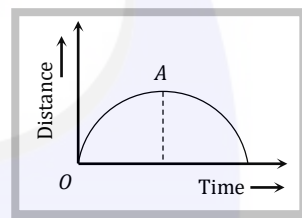
This graph shows that at one instant the particle has two positions. Which is not possible.



The graph shows that particle coming towards origin initially and after that it is moving away from origin.

**Note :** ☐ If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.

☐ For two particles having displacement time graph with slopes  $\theta_1$  and  $\theta_2$  possesses velocities  $v_1$  and  $v_2$  respectively then  $\frac{v_1}{v_2} = \frac{\tan \theta_1}{\tan \theta_2}$



## Sample problems based on position-time graph

**Problem 20.** The position of a particle moving along the x-axis at certain times is given below :

t (s)	0	1	2	3
x (m)	-2	0	6	16

Which of the following describes the motion correctly

[AMU (Engg.) 2001]

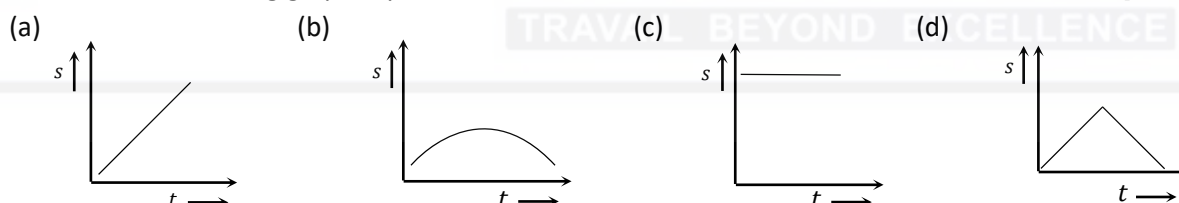
- (a) Uniform, accelerated (b) Uniform, decelerated  
(c) Non-uniform, accelerated (d) There is not enough data for generalisation

**Solution :** (a) Instantaneous velocity  $v = \frac{\Delta x}{\Delta t}$ , By using the data from the table

$v_1 = \frac{0 - (-2)}{1} = 2 \text{ m/s}$ ,  $v_2 = \frac{6 - 0}{1} = 6 \text{ m/s}$  and  $v_3 = \frac{16 - 6}{1} = 10 \text{ m/s}$  i.e. the speed is increasing at a constant rate so motion is uniformly accelerated.

**Problem 21.** Which of the following graph represents uniform motion

[DCE 1999]



**Solution :** (a) When distance time graph is a straight line with constant slope than motion is uniform.

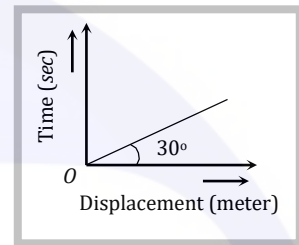
**Problem 22.** The displacement-time graph for two particles A and B are straight lines inclined at angles of  $30^\circ$  and  $60^\circ$  with the time axis. The ratio of velocities of  $v_A : v_B$  is [CPMT 1990; MP PET 1999; MP PET 2001]

- (a) 1 : 2 (b)  $1 : \sqrt{3}$  (c)  $\sqrt{3} : 1$  (d) 1 : 3

Solution : (d)  $v = \tan \theta$  from displacement graph. So  $\frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$

**Problem 23.** From the following displacement time graph find out the velocity of a moving body

- (a)  $\frac{1}{\sqrt{3}}$  m/s
- (b) 3 m/s
- (c)  $\sqrt{3}$  m/s
- (d)  $\frac{1}{3}$



Solution : (c) In first instant you will apply  $v = \tan \theta$  and say,  $v = \tan 30^\circ = \frac{1}{\sqrt{3}}$  m/s.

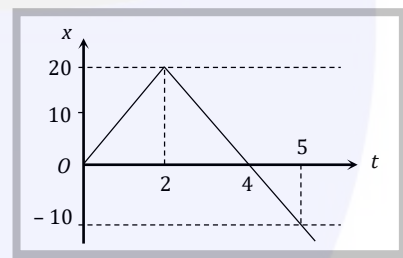
But it is wrong because formula  $v = \tan \theta$  is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So angle from time axis  $= 90^\circ - 30^\circ = 60^\circ$ .

Now  $v = \tan 60^\circ = \sqrt{3}$

**Problem 24.** The diagram shows the displacement-time graph for a particle moving in a straight line. The average velocity for the interval  $t = 0, t = 5$  is

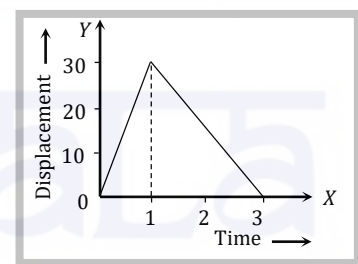
- (a) 0
- (b)  $6 \text{ ms}^{-1}$
- (c)  $-2 \text{ ms}^{-1}$
- (d)  $2 \text{ ms}^{-1}$



Solution : (c) Average velocity =  $\frac{\text{Total displacement}}{\text{Total time}} = \frac{(20) + (-20) + (-10)}{5} = -2 \text{ m/s}$

**Problem 25.** Figure shows the displacement time graph of a body. What is the ratio of the speed in the first second and that in the next two seconds

- (a) 1 : 2
- (b) 1 : 3
- (c) 3 : 1
- (d) 2 : 1



Solution: (d) Speed in first second = 30 and Speed in next two seconds = 15. So that ratio 2 : 1

## 9. Velocity Time Graph

The graph is plotted by taking time  $t$  along x-axis and velocity of the particle on y-axis.

**Distance and displacement :** The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

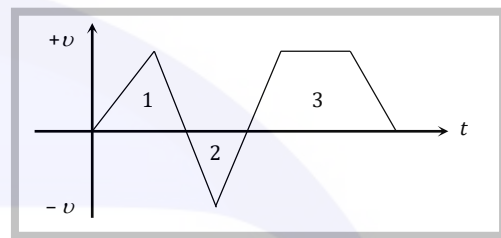
Then Total distance  $= |A_1| + |A_2| + |A_3|$

= Addition of modulus of different area. i.e.  $s = \int |v| dt$

Total displacement  $= A_1 + A_2 + A_3$

= Addition of different area considering their sign. i.e.  $r = \int v dt$

here  $A_1$  and  $A_2$  are area of triangle 1 and 2 respectively and  $A_3$  is the area of trapezium .



**Acceleration :** Let AB is a velocity-time graph for any moving particle

$$\text{As Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1} \quad \dots(i)$$

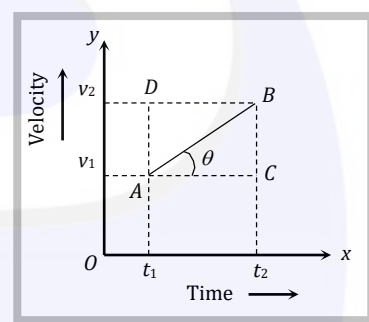
$$\text{From triangle ABC, } \tan \theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{v_2 - v_1}{t_2 - t_1} \quad \dots(ii)$$

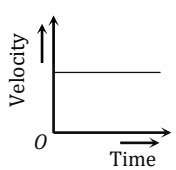
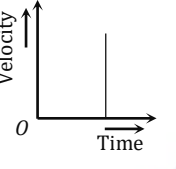
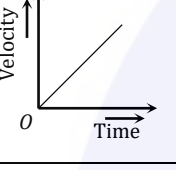
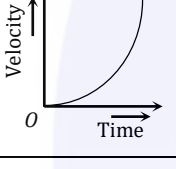
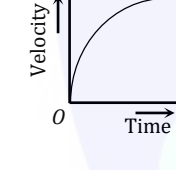
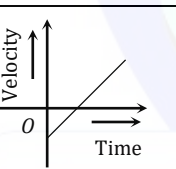
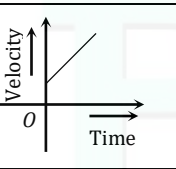
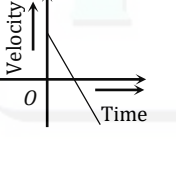
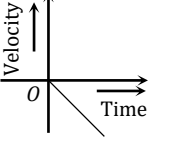
By comparing (i) and (ii)

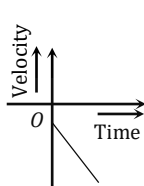
Acceleration (a) =  $\tan \theta$

It is clear that slope of velocity-time graph represents the acceleration of the particle.

**Various velocity – time graphs and their interpretation**



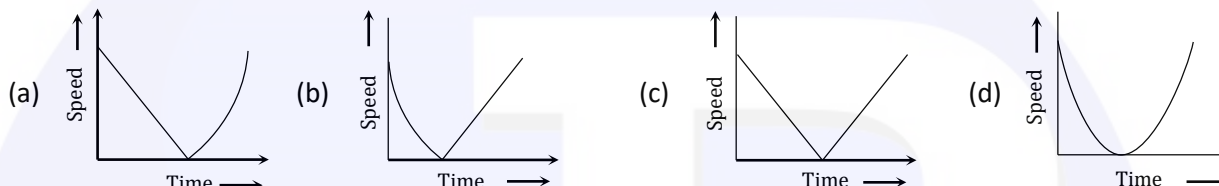
	$\theta = 0, a = 0, v = \text{constant}$ i.e., line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90^\circ, a = \infty, v = \text{increasing}$ i.e., line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	$\theta = \text{constant}, \text{ so } a = \text{constant and } v \text{ is increasing uniformly with time}$ i.e., line with constant slope represents uniform acceleration of the particle.
	$\theta \text{ increasing so acceleration increasing}$ i.e., line bending towards velocity axis represent the increasing acceleration in the body.
	$\theta \text{ decreasing so acceleration decreasing}$ i.e. line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because $\theta$ is constant and $< 90^\circ$ but initial velocity of the particle is negative.
	Positive constant acceleration because $\theta$ is constant and $< 90^\circ$ but initial velocity of particle is positive.
	Negative constant acceleration because $\theta$ is constant and $> 90^\circ$ but initial velocity of the particle is positive.
	Negative constant acceleration because $\theta$ is constant and $> 90^\circ$ but initial velocity of the particle is zero.



Negative constant acceleration because  $\theta$  is constant and  $> 90^\circ$  but initial velocity of the particle is negative.

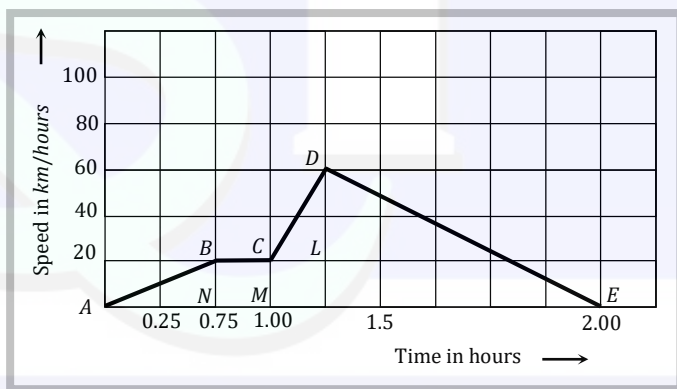
## Sample problems based on velocity-time graph

**Problem 26.** A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored [AIIMS 2003]



**Solution : (c)** In first half of motion the acceleration is uniform & velocity gradually decreases, so slope will be negative but for next half acceleration is positive. So slope will be positive. Thus graph 'C' is correct. Not ignoring air resistance means upward motion will have acceleration  $(a + g)$  and the downward motion will have  $(g - a)$ .

**Problem 27.** A train moves from one station to another in 2 hours time. Its speed-time graph during this motion is shown in the figure. The maximum acceleration during the journey is [Kerala (Engg.) 2002]

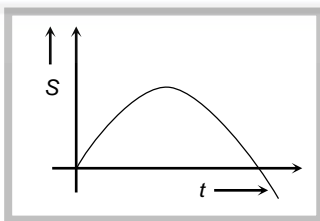


- (a)  $140 \text{ km h}^{-2}$  (b)  $160 \text{ km h}^{-2}$  (c)  $100 \text{ km h}^{-2}$  (d)  $120 \text{ km h}^{-2}$

**Solution : (b)** Maximum acceleration means maximum slope in speed – time graph.

that slope is for line CD. So,  $a_{\text{max}} = \text{slope of CD} = \frac{60 - 20}{1.25 - 1.00} = \frac{40}{0.25} = 160 \text{ km h}^{-2}$ .

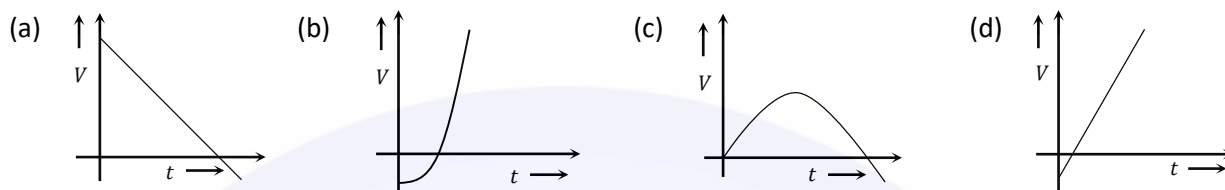
**Problem 28.** The graph of displacement v/s time is





Its corresponding velocity-time graph will be

[DCE 2001]

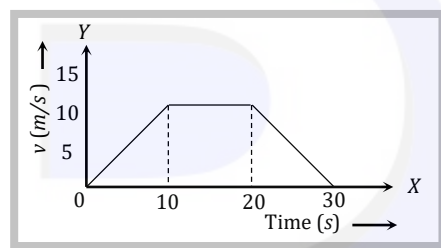


**Solution :** (a) We know that the velocity of body is given by the slope of displacement – time graph. So it is clear that initially slope of the graph is positive and after some time it becomes zero (corresponding to the peak of the graph) and then it will be negative.

**Problem 29.** In the following graph, distance travelled by the body in metres is

[EAMCET 1994]

- (a) 200
- (b) 250
- (c) 300
- (d) 400



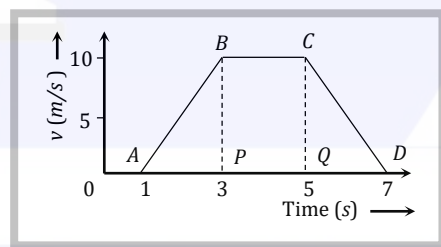
**Solution :** (a) Distance = The area under  $v - t$  graph

$$S = \frac{1}{2} (30 + 10) \times 10 = 200 \text{ metre}$$

**Problem 30.** For the velocity-time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds

[MP PMT/PET 1998; RPET 2001]

- (a)  $\frac{1}{2}$
- (b)  $\frac{1}{4}$
- (c)  $\frac{1}{3}$
- (d)  $\frac{2}{3}$



**Solution :** (b) Distance covered in total 7 seconds = Area of trapezium  $ABCD = \frac{1}{2} (2 + 6) \times 10 = 40 \text{ m}$

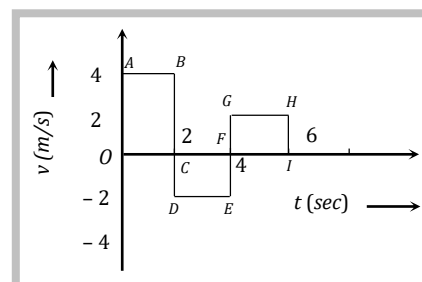
Distance covered in last 2 second = area of triangle  $CDQ = \frac{1}{2} \times 2 \times 10 = 10 \text{ m}$

$$\text{So required fraction} = \frac{10}{40} = \frac{1}{4}$$

**Problem 31.** The velocity time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 sec are respectively

[MP PET 1994]

- (a)  $8 \text{ m}, 16 \text{ m}$
- (b)  $16 \text{ m}, 8 \text{ m}$
- (c)  $16 \text{ m}, 16 \text{ m}$
- (d)  $8 \text{ m}, 8 \text{ m}$



**Solution :** (a) Area of rectangle  $ABCO = 4 \times 2 = 8 \text{ m}$

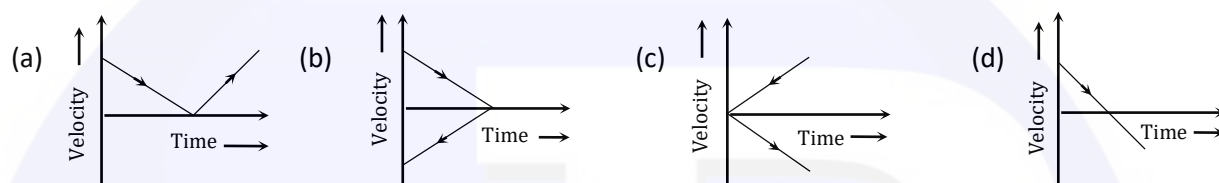
Area of rectangle  $CDEF = 2 \times (-2) = -4 \text{ m}$

Area of rectangle FGHI =  $2 \times 2 = 4$  m

Displacement = sum of area with their sign =  $8 + (-4) + 4 = 8$  m

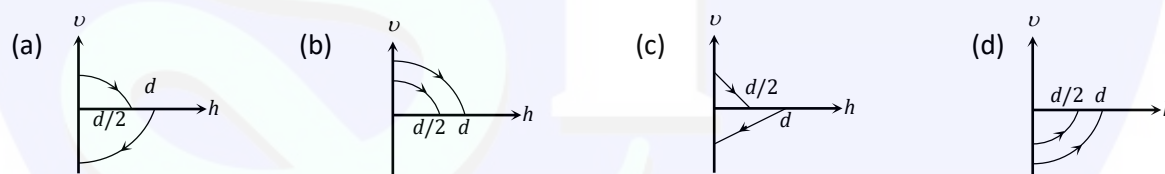
Distance = sum of area with out sign =  $8 + 4 + 4 = 16$  m

**Problem 32.** A ball is thrown vertically upward which of the following graph represents velocity time graph of the ball during its flight (air resistance is neglected) [CPMT 1993; AMU (Engg.) 2000]



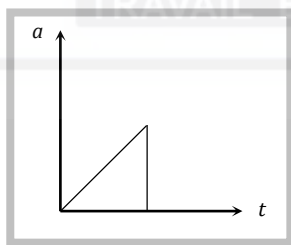
**Solution : (d)** In the positive region the velocity decreases linearly (during rise) and in negative region velocity increase linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

**Problem 33.** A ball is dropped vertically from a height  $d$  above the ground. It hits the ground and bounces up vertically to a height  $\frac{d}{2}$ . Neglecting subsequent motion and air resistance, its velocity  $v$  varies with the height  $h$  above the ground as. [IIT-JEE (Screening) 2000]

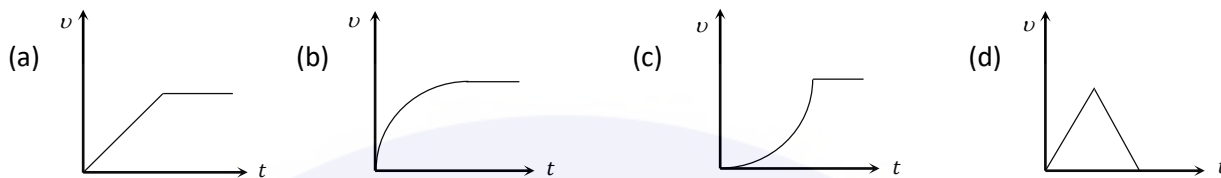


**Solution : (a)** When ball is dropped from height  $d$  its velocity will be zero. As ball comes downward  $h$  decreases and  $v$  increases just before the rebound from the earth  $h = 0$  and  $v = \text{maximum}$  and just after rebound velocity reduces to half and direction becomes opposite. As soon as the height increases its velocity decreases and becomes zero at  $h = \frac{d}{2}$ . This interpretation is clearly shown by graph (a).

**Problem 34.** The acceleration-time graph of a body is shown below –



The most probable velocity-time graph of the body is



**Solution :** (c) From given  $a - t$  graph acceleration is increasing at constant rate

$$\therefore \frac{da}{dt} = k \text{ (constant)} \Rightarrow a = kt \text{ (by integration)}$$

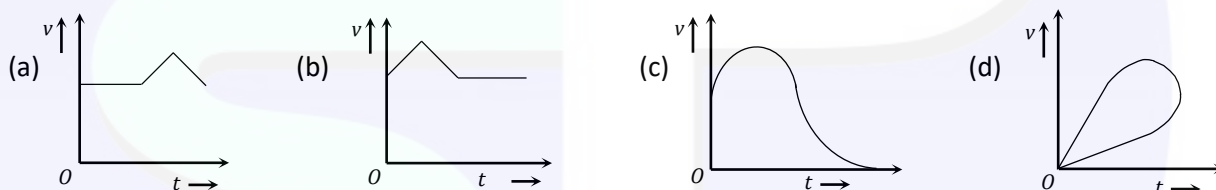
$$\Rightarrow \frac{dv}{dt} = kt \Rightarrow dv = kt dt \Rightarrow \int dv = k \int t dt \Rightarrow v = \frac{kt^2}{2}$$

i.e.,  $v$  is dependent on time parabolically and parabola is symmetric about  $v$ -axis.

and suddenly acceleration becomes zero. i.e. velocity becomes constant.

Hence (c) is most probable graph.

**Problem 35.** Which of the following velocity time graphs is not possible



**Solution :** (d) Particle can not possess two velocities at a single instant so graph (d) is not possible.

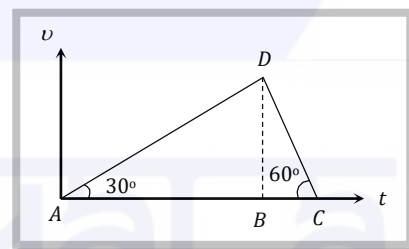
**Problem 36.** For a certain body, the velocity-time graph is shown in the figure. The ratio of applied forces for intervals AB and BC is

(a)  $+\frac{1}{2}$

(b)  $-\frac{1}{2}$

(c)  $+\frac{1}{3}$

(d)  $-\frac{1}{3}$

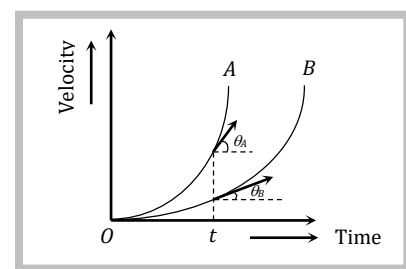


**Solution :** (d) Ratio of applied force = Ratio of acceleration

$$= \frac{a_{AB}}{a_{BC}} = \frac{\tan 30}{\tan(120)} = \frac{1/\sqrt{3}}{-\sqrt{3}} = -1/3$$

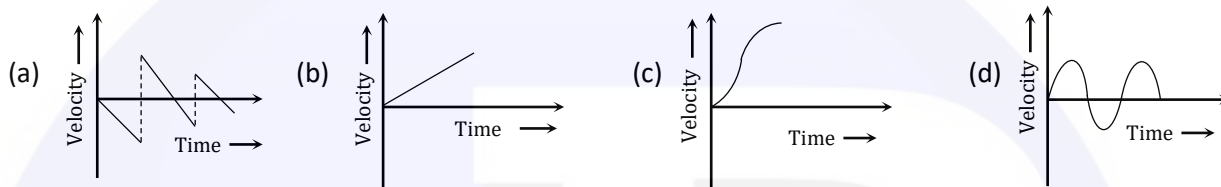
**Problem 37.** Velocity-time graphs of two cars which start from rest at the same time, are shown in the figure. Graph shows, that

- (a) Initial velocity of A is greater than the initial velocity of B
- (b) Acceleration in A is increasing at lesser rate than in B
- (c) Acceleration in A is greater than in B
- (d) Acceleration in B is greater than in A



Solution : (c) At a certain instant  $t$  slope of A is greater than B ( $\theta_A > \theta_B$ ), so acceleration in A is greater than B

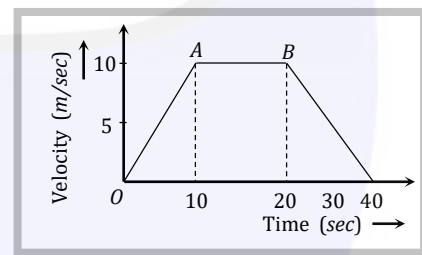
**Problem 38.** Which one of the following graphs represent the velocity of a steel ball which fall from a height on to a marble floor? (Here  $v$  represents the velocity of the particle and  $t$  the time)



Solution : (a) Initially when ball falls from a height its velocity is zero and goes on increasing when it comes down. Just after rebound from the earth its velocity decreases in magnitude and its direction gets reversed. This process is repeated untill ball comes to at rest. This interpretation is well explained in graph (a).

**Problem 39.** The adjoining curve represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec<sup>2</sup> are respectively

- (a) 1, 0, -0.5
- (b) 1, 0, 0.5
- (c) 1, 1, 0.5
- (d) 1, 0.5, 0



Solution : (a) Acceleration along OA =  $\frac{v_2 - v_1}{t} = \frac{10 - 0}{10} = 1 \text{ m/s}^2$

Acceleration along OB =  $\frac{0}{10} = 0$

Acceleration along BC =  $\frac{0 - 10}{20} = -0.5 \text{ m/s}^2$

## 10. Equations of Kinematics

These are the various relations between  $u$ ,  $v$ ,  $a$ ,  $t$  and  $s$  for the moving particle where the notations are used as :

$u$  = Initial velocity of the particle at time  $t = 0$  sec

$v$  = Final velocity at time  $t$  sec

$a$  = Acceleration of the particle

$s$  = Distance travelled in time  $t$  sec

$s_n$  = Distance travelled by the body in  $n^{\text{th}}$  sec

### (1) When particle moves with zero acceleration

- (i) It is a unidirectional motion with constant speed.
- (ii) Magnitude of displacement is always equal to the distance travelled.
- (iii)  $v = u$ ,  $s = u t$  [As  $a = 0$ ]

### (2) When particle moves with constant acceleration

- (i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.
- (ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar form

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \left( \frac{u+v}{2} \right) t$$

$$s_n = u + \frac{a}{2}(2n-1)$$

Equation of motion in vector form

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} \cdot \vec{v} - \vec{u} \cdot \vec{u} = 2\vec{a} \cdot \vec{s}$$

$$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$$

$$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

### (3) Important points for uniformly accelerated motion

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in  $t$  sec is proportional to  $t^2$  (i.e.  $s \propto t^2$ ).

So we can say that the ratio of distance covered in 1 sec, 2 sec and 3 sec is  $1^2 : 2^2 : 3^2$  or  $1 : 4 : 9$ .

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in  $n$ th sec is proportional to  $(2n-1)$  (i.e.  $s_n \propto (2n-1)$ )

So we can say that the ratio of distance covered in I sec, II sec and III sec is  $1 : 3 : 5$ .

(iii) A body moving with a velocity  $u$  is stopped by application of brakes after covering a distance  $s$ . If the same body moves with velocity  $nu$  and same braking force is applied on it then it will come to rest after covering a distance of  $n^2s$ .

$$\text{As } v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a}, \quad s \propto u^2 \quad [\text{since } a \text{ is constant}]$$

So we can say that if  $u$  becomes  $n$  times then  $s$  becomes  $n^2$  times that of previous value.

(iv) A particle moving with uniform acceleration from A to B along a straight line has velocities  $v_1$  and  $v_2$  at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

$$v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

### Sample problems based on uniform acceleration

**Problem 40.** A body A moves with a uniform acceleration  $a$  and zero initial velocity. Another body B, starts from the same point moves in the same direction with a constant velocity  $v$ . The two bodies meet after a time  $t$ . The value of  $t$  is [MP PET 2003]

(a)  $\frac{2v}{a}$

(b)  $\frac{v}{a}$

(c)  $\frac{v}{2a}$

(d)  $\sqrt{\frac{v}{2a}}$

**Solution :** (a) Let they meet after time ' $t$ '. Distance covered by body A =  $\frac{1}{2}at^2$ ; Distance covered by body B =  $vt$   
and  $\frac{1}{2}at^2 = vt \therefore t = \frac{2v}{a}$ .

**Problem 41.** A student is standing at a distance of 50metres from the bus. As soon as the bus starts its motion with an acceleration of  $1\text{ms}^{-2}$ , the student starts running towards the bus with a uniform velocity  $u$ . Assuming the motion to be along a straight road, the minimum value of  $u$ , so that the students is able to catch the bus is [KCET 2003]

(a)  $5 \text{ ms}^{-1}$

(b)  $8 \text{ ms}^{-1}$

(c)  $10 \text{ ms}^{-1}$

(d)  $12 \text{ ms}^{-1}$

**Solution :** (c) Let student will catch the bus after  $t$  sec. So it will cover distance  $ut$ .



Similarly distance travelled by the bus will be  $\frac{1}{2}at^2$  for the given condition

$$ut = 50 + \frac{1}{2}at^2 = 50 + \frac{t^2}{2} \Rightarrow u = \frac{50}{t} + \frac{t}{2} \quad (\text{As } a = 1 \text{ m/s}^2)$$

To find the minimum value of  $u$ ,  $\frac{du}{dt} = 0$ , so we get  $t = 10$  sec

then  $u = 10$  m/s.

**Problem 42.** A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is [AIIEE 2003]

- (a) 6m (b) 12m (c) 18m (d) 24m

Solution : (d)  $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} \Rightarrow s \propto u^2$  (As  $a = \text{constant}$ )

$$\frac{s_2}{s_1} = \left(\frac{u_2}{u_1}\right)^2 = \left(\frac{100}{50}\right)^2 \Rightarrow s_2 = 4s_1 = 4 \times 6 = 24 \text{ m.}$$

**Problem 43.** The velocity of a bullet is reduced from 200m/s to 100m/s while travelling through a wooden block of thickness 10cm. The retardation, assuming it to be uniform, will be [AIIMS 2001]

- (a)  $10 \times 10^4 \text{ m/s}^2$  (b)  $12 \times 10^4 \text{ m/s}^2$  (c)  $13.5 \times 10^4 \text{ m/s}^2$  (d)  $15 \times 10^4 \text{ m/s}^2$

Solution : (d)  $u = 200 \text{ m/s}, v = 100 \text{ m/s}, s = 0.1 \text{ m}$

$$a = \frac{u^2 - v^2}{2s} = \frac{(200)^2 - (100)^2}{2 \times 0.1} = 15 \times 10^4 \text{ m/s}^2$$

**Problem 44.** A body A starts from rest with an acceleration  $a_1$ . After 2 seconds, another body B starts from rest with an acceleration  $a_2$ . If they travel equal distances in the 5th second, after the start of A, then the ratio  $a_1 : a_2$  is equal to [AIIMS 2001]

- (a) 5 : 9 (b) 5 : 7 (c) 9 : 5 (d) 9 : 7

Solution : (a) By using  $S_n = u + \frac{a}{2}(2n-1)$ , Distance travelled by body A in 5th second =  $0 + \frac{a_1}{2}(2 \times 5 - 1)$

Distance travelled by body B in 3rd second is =  $0 + \frac{a_2}{2}(2 \times 3 - 1)$

$$\text{According to problem : } 0 + \frac{a_1}{2}(2 \times 5 - 1) = 0 + \frac{a_2}{2}(2 \times 3 - 1) \Rightarrow 9a_1 = 5a_2 \Rightarrow \frac{a_1}{a_2} = \frac{5}{9}$$

**Problem 45.** The average velocity of a body moving with uniform acceleration travelling a distance of 3.06 m is  $0.34 \text{ ms}^{-1}$ . If the change in velocity of the body is  $0.18 \text{ ms}^{-1}$  during this time, its uniform acceleration is [EAMCET (Med.) 2000]

- (a)  $0.01 \text{ ms}^{-2}$  (b)  $0.02 \text{ ms}^{-2}$  (c)  $0.03 \text{ ms}^{-2}$  (d)  $0.04 \text{ ms}^{-2}$

Solution : (b)  $\text{Time} = \frac{\text{Distance}}{\text{Average velocity}} = \frac{3.06}{0.34} = 9 \text{ sec}$

$$\text{and Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{0.18}{9} = 0.02 \text{ m/s}^2.$$

**Problem 46.** A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec [RPET 2000]

- (a) 8.3 m (b) 9.3 m (c) 10.3 m (d) None of above

Solution : (a) Let initial ( $t = 0$ ) velocity of particle =  $u$

for first 5 sec of motion  $s_5 = 10 \text{ metre}$ , so by using  $s = ut + \frac{1}{2}at^2$

$$10 = 5u + \frac{1}{2}a(5)^2 \Rightarrow 2u + 5a = 4 \quad \dots (i)$$

for first 8 sec of motion  $s_8 = 20 \text{ metre}$

$$20 = 8u + \frac{1}{2}a(8)^2 \Rightarrow 2u + 8a = 5 \quad \dots (ii)$$

By solving (i) and (ii)  $u = \frac{7}{6} \text{ m/s}$   $a = \frac{1}{3} \text{ m/s}^2$

Now distance travelled by particle in total 10 sec.  $s_{10} = u \times 10 + \frac{1}{2}a(10)^2$

by substituting the value of  $u$  and  $a$  we will get  $s_{10} = 28.3 \text{ m}$

So the distance in last 2 sec  $= s_{10} - s_8 = 28.3 - 20 = 8.3 \text{ m}$

**Problem 47.** A body travels for 15 sec starting from rest with constant acceleration. If it travels distances  $S_1$ ,  $S_2$  and  $S_3$  in the first five seconds, second five seconds and next five seconds respectively the relation between  $S_1$ ,  $S_2$  and  $S_3$  is [AMU (Engg.) 2000]

- (a)  $S_1 = S_2 = S_3$  (b)  $5S_1 = 3S_2 = S_3$  (c)  $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$  (d)  $S_1 = \frac{1}{5}S_2 = \frac{1}{3}S_3$

Solution : (c) Since the body starts from rest. Therefore  $u = 0$ .

$$S_1 = \frac{1}{2}a(5)^2 = \frac{25a}{2}$$

$$S_1 + S_2 = \frac{1}{2}a(10)^2 = \frac{100a}{2} \Rightarrow S_2 = \frac{100a}{2} - S_1 = 75 \frac{a}{2}$$

$$S_1 + S_2 + S_3 = \frac{1}{2}a(15)^2 = \frac{225a}{2} \Rightarrow S_3 = \frac{225a}{2} - S_2 - S_1 = \frac{125a}{2}$$

Thus Clearly  $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$

**Problem 48.** If a body having initial velocity zero is moving with uniform acceleration  $8 \text{ m/sec}^2$ , the distance travelled by it in fifth second will be [MP PMT 1996; DPMT 2000]

- (a) 36 metres (b) 40 metres (c) 100 metres (d) Zero

Solution : (a)  $S_n = u + \frac{1}{2}a(2n-1) = 0 + \frac{1}{2}(8)[2 \times 5 - 1] = 36 \text{ metres}$

**Problem 49.** The engine of a car produces acceleration  $4 \text{ m/sec}^2$  in the car, if this car pulls another car of same mass, what will be the acceleration produced [RPET 1996]

- (a)  $8 \text{ m/s}^2$  (b)  $2 \text{ m/s}^2$  (c)  $4 \text{ m/s}^2$  (d)  $\frac{1}{2} \text{ m/s}^2$

Solution : (b)  $F = ma$   $a \propto \frac{1}{m}$  if  $F$  = constant. Since the force is same and the effective mass of system becomes double

$$\frac{a_2}{a_1} = \frac{m_1}{m_2} = \frac{m}{2m}, a_2 = \frac{a_1}{2} = 2 \text{ m/s}^2$$

**Problem 50.** A body starts from rest. What is the ratio of the distance travelled by the body during the 4<sup>th</sup> and 3<sup>rd</sup> second. [CBSE PMT 1993]

- (a) 7/5 (b) 5/7 (c) 7/3 (d) 3/7

Solution : (a) As  $S_n \propto (2n-1)$ ,  $\frac{S_4}{S_3} = \frac{7}{5}$

## 11. Motion of Body Under Gravity (Free Fall)

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol  $g$ .

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ( $h \ll R$ ) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

**(1) If a body dropped from some height (initial velocity zero)**

(i) Equation of motion : Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have

$$u = 0 \quad [\text{As body starts from rest}]$$

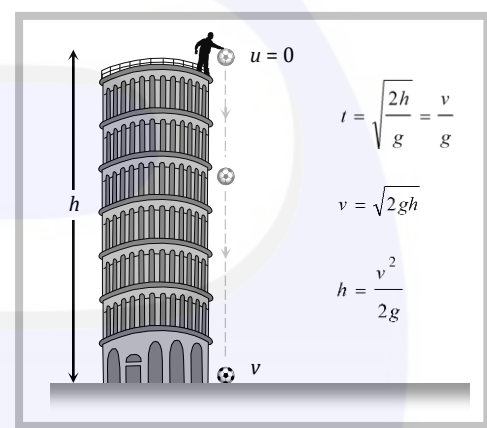
$$a = +g \quad [\text{As acceleration is in the direction of motion}]$$

$$v = g t \quad \dots(i)$$

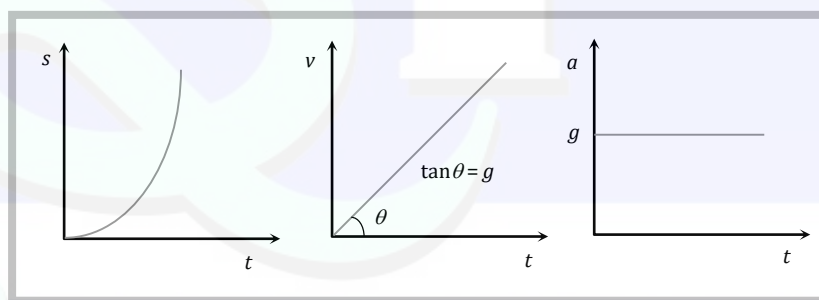
$$h = \frac{1}{2} g t^2 \quad \dots(ii)$$

$$v^2 = 2gh \quad \dots(iii)$$

$$h_n = \frac{g}{2} (2n - 1) \quad \dots(iv)$$



(ii) Graph of distance velocity and acceleration with respect to time :



(iii) As  $h = (1/2)gt^2$ , i.e.,  $h \propto t^2$ , distance covered in time  $t, 2t, 3t$ , etc., will be in the ratio of  $1^2 : 2^2 : 3^2$ , i.e., square of integers.

(iv) The distance covered in the  $n$ th sec,  $h_n = \frac{1}{2} g (2n - 1)$

So distance covered in I, II, III sec, etc., will be in the ratio of  $1 : 3 : 5$ , i.e., odd integers only.

**(2) If a body is projected vertically downward with some initial velocity**

$$\text{Equation of motion : } v = u + g t$$

$$h = ut + \frac{1}{2} g t^2$$

$$v^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2} (2n - 1)$$

**(3) If a body is projected vertically upward**

- (i) Equation of motion : Taking initial position as origin and direction of motion (i.e., vertically up) as positive  
 $a = -g$  [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity  $u$  and after time  $t$  it reaches up to height  $h$  then

$$v = u - gt; \quad h = ut - \frac{1}{2}gt^2; \quad v^2 = u^2 - 2gh; \quad h_n = u - \frac{g}{2}(2n-1)$$

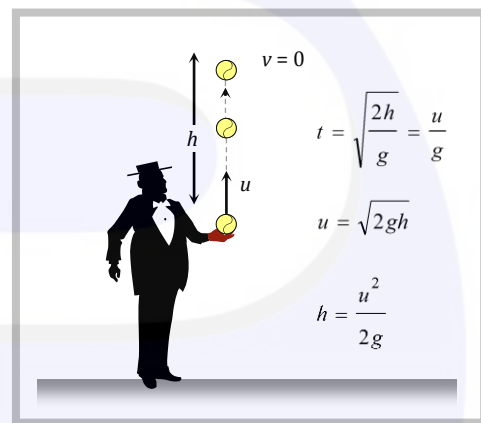
- (ii) For maximum height  $v = 0$

So from above equation

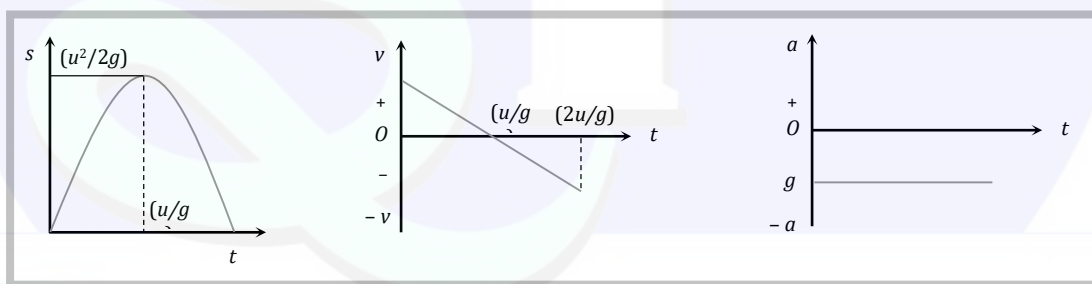
$$u = gt,$$

$$h = \frac{1}{2}gt^2$$

and  $u^2 = 2gh$



- (iii) Graph of distance, velocity and acceleration with respect to time (for maximum height) :



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

- (4) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.
- (5) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e.,  $t = \sqrt{2h/g}$  and  $v = \sqrt{2gh}$ .
- (6) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of descent ( $t_1$ ) = time of ascent ( $t_2$ ) =  $u/g$

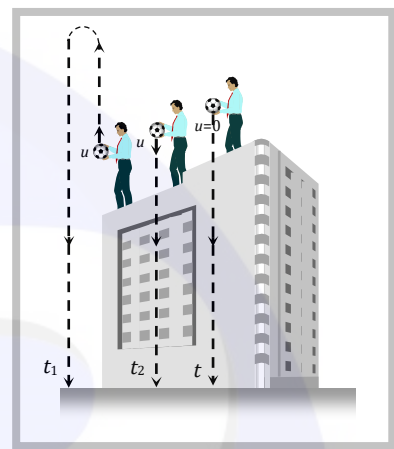
$$\therefore \text{Total time of flight } T = t_1 + t_2 = \frac{2u}{g}$$

(7) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(8) A ball is dropped from a building of height  $h$  and it reaches after  $t$  seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity  $u$  and they reach the earth surface after  $t_1$  and  $t_2$  seconds respectively then

$$t = \sqrt{t_1 t_2}$$



(9) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent.  $t_2 > t_1$

Let  $u$  is the initial velocity of body then time of ascent  $t_1 = \frac{u}{g+a}$  and  $h = \frac{u^2}{2(g+a)}$

where  $g$  is acceleration due to gravity and  $a$  is retardation by air resistance and for upward motion both will work vertically downward.

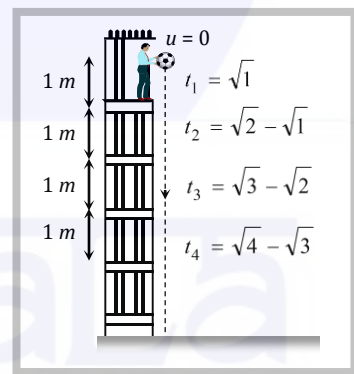
For downward motion  $a$  and  $g$  will work in opposite direction because  $a$  always work in direction opposite to motion and  $g$  always work vertically downward.

$$\text{So } h = \frac{1}{2}(g-a)t_2^2 \Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2 \Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing  $t_1$  and  $t_2$  we can say that  $t_2 > t_1$  since  $(g+a) > (g-a)$

(10) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integers i.e.

$$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), \dots, (\sqrt{4} - \sqrt{3}), \dots$$



## Sample problems based on motion under gravity

**Problem 51.** If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is ( $g = 10 \text{ m/s}^2$ ) [MP PMT 2003]

- (a) 11.25 m (b) 16.2 m (c) 24.5 m (d) 7.62 m

**Solution :** (a)  $H_{\max} = \frac{u^2}{2g} = \frac{(15)^2}{2 \times 10} = 11.25 \text{ m}$

**Problem 52.** A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is ( $g = 10 \text{ m/s}^2$ ) [MP PET 2003]

- (a) 25m (b) 45m (c) 90m (d) 125m

**Solution :** (b)  $h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5th} = \frac{10}{2}(2 \times 5 - 1) = 45 \text{ m}$ .



**Problem 53.** If a ball is thrown vertically upwards with speed  $u$ , the distance covered during the last  $t$  seconds of its ascent is [CBSE 2003]

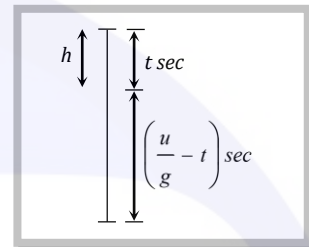
- (a)  $\frac{1}{2}gt^2$  (b)  $ut - \frac{1}{2}gt^2$  (c)  $(u - gt)t$  (d)  $ut$

Solution : (a) If ball is thrown with velocity  $u$ , then time of flight  $= \frac{u}{g}$

velocity after  $\left(\frac{u}{g} - t\right)$  sec :  $v = u - g\left(\frac{u}{g} - t\right) = gt$ .

So, distance in last 't' sec :  $0^2 = (gt)^2 - 2(g)h$ .

$$\Rightarrow h = \frac{1}{2}gt^2.$$



**Problem 54.** A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time (Given  $g = 9.8 \text{ m/s}^2$ ) [CBSE PMT 2003]

- (a) At least 0.8 m/s (b) Any speed less than 19.6 m/s  
(c) Only with speed 19.6 m/s (d) More than 19.6 m/s

Solution : (d) Interval of ball throw = 2 sec.

If we want that minimum three (more than two) ball remain in air then time of flight of first ball must be greater than 4 sec. i.e.  $T > 4 \text{ sec}$  or  $\frac{2u}{g} > 4 \text{ sec} \Rightarrow u > 19.6 \text{ m/s}$

It is clear that for  $u = 19.6$  First ball will just strike the ground (in sky), second ball will be at highest point (in sky), and third ball will be at point of projection or on ground (not in sky).

**Problem 55.** A man drops a ball downside from the roof of a tower of height 400 meters. At the same time another ball is thrown upside with a velocity 50 meter/sec. from the surface of the tower, then they will meet at which height from the surface of the tower [CPMT 2003]

- (a) 100 meters (b) 320 meters (c) 80 meters (d) 240 meters

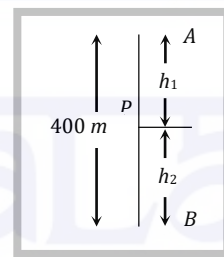
Solution : (c) Let both balls meet at point P after time  $t$ .

The distance travelled by ball A ( $h_1$ )  $= \frac{1}{2}gt^2$  .....(i)

The distance travelled by ball B ( $h_2$ )  $= ut - \frac{1}{2}gt^2$  .....(ii)

By adding (i) and (ii)  $h_1 + h_2 = ut = 400$  (Given  $h = h_1 + h_2 = 400$ .)

$\therefore t = 400 / 50 = 8 \text{ sec}$  and  $h_1 = 320 \text{ m}$ ,  $h_2 = 80 \text{ m}$



**Problem 56.** A very large number of balls are thrown vertically upwards in quick succession in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5m, the number of ball thrown per minute is (take  $g = 10 \text{ ms}^{-2}$ ) [KCET (Med.) 2002]

- (a) 120 (b) 80 (c) 60 (d) 40

Solution : (c) Maximum height of ball = 5m, So velocity of projection  $\Rightarrow u = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}$

time interval between two balls (time of ascent)  $= \frac{u}{g} = 1 \text{ sec} = \frac{1}{60} \text{ min.}$

So no. of ball thrown per min = 60

**Problem 57.** A particle is thrown vertically upwards. If its velocity at half of the maximum height is 10 m/s, then maximum height attained by it is (Take  $g = 10 \text{ m/s}^2$ ) [CBSE PMT 2001]

- (a) 8 m (b) 10 m (c) 12 m (d) 16 m

**Solution : (b)** Let particle thrown with velocity  $u$  and its maximum height is  $H$  then  $H = \frac{u^2}{2g}$

When particle is at a height  $H/2$ , then its speed is  $10 \text{ m/s}$

From equation  $v^2 = u^2 - 2gh$ ,  $(10)^2 = u^2 - 2g\left(\frac{H}{2}\right) = u^2 - 2g \frac{u^2}{4g} \Rightarrow u^2 = 200$

$\therefore$  Maximum height  $H = \frac{u^2}{2g} = \frac{200}{2 \times 10} = 10 \text{ m}$

**Problem 58.** A stone is shot straight upward with a speed of  $20 \text{ m/sec}$  from a tower  $200 \text{ m}$  high. The speed with which it strikes the ground is approximately **[AMU (Engg.) 1999]**

- (a)  $60 \text{ m/sec}$  (b)  $65 \text{ m/sec}$  (c)  $70 \text{ m/sec}$  (d)  $75 \text{ m/sec}$

**Solution : (b)** Speed of stone in a vertically upward direction is  $20 \text{ m/s}$ . So for vertical downward motion we will consider  $u = -20 \text{ m/s}$

$$v^2 = u^2 + 2gh = (-20)^2 + 2 \times 10 \times 200 \Rightarrow v = 65 \text{ m/s}$$

**Problem 59.** A body freely falling from the rest has a velocity ' $v$ ' after it falls through a height ' $h$ '. The distance it has to fall down for its velocity to become double, is **[BHU 1999]**

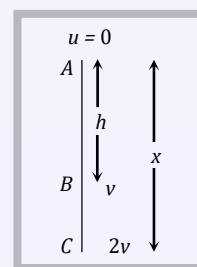
- (a)  $2h$  (b)  $4h$  (c)  $6h$  (d)  $8h$

**Solution : (b)** Let at point A initial velocity of body is equal to zero

For path AB :  $v^2 = 0 + 2gh$  ... (i)

For path AC :  $(2v)^2 = 0 + 2gx \Rightarrow 4v^2 = 2gx$  ... (ii)

Solving (i) and (ii)  $x = 4h$



**Problem 60.** A body sliding on a smooth inclined plane requires  $4$  seconds to reach the bottom starting from rest at the top. How much time does it take to cover one-fourth distance starting from rest at the top **[BHU 1998]**

- (a)  $1 \text{ s}$  (b)  $2 \text{ s}$  (c)  $4 \text{ s}$  (d)  $16 \text{ s}$

**Solution : (b)**  $S = \frac{1}{2}at^2 \Rightarrow t \propto \sqrt{s}$  (As  $a = \text{constant}$ )

$$\frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{s/4}{s}} = \frac{1}{2} \Rightarrow t_2 = \frac{t_1}{2} = \frac{4}{2} = 2 \text{ s}$$

**Problem 61.** A stone dropped from a building of height  $h$  and it reaches after  $t$  seconds on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity  $u$  and they reach the earth surface after  $t_1$  and  $t_2$  seconds respectively, then

**[CPMT 1997; UPSEAT 2002; KCET (Engg./Med.) 2002]**

- (a)  $t = t_1 - t_2$  (b)  $t = \frac{t_1 + t_2}{2}$  (c)  $t = \sqrt{t_1 t_2}$  (d)  $t = t_1^2 t_2^2$

**Solution : (c)** For first case of dropping  $h = \frac{1}{2}gt^2$ .

For second case of downward throwing  $h = -ut_1 + \frac{1}{2}gt_1^2 = \frac{1}{2}gt^2$

$$\Rightarrow -ut_1 = \frac{1}{2}g(t^2 - t_1^2) \quad \dots\dots(i)$$

For third case of upward throwing  $h = ut_2 + \frac{1}{2}gt_2^2 = \frac{1}{2}gt^2$

$$\Rightarrow ut_2 = \frac{1}{2}g(t^2 - t_2^2) \quad \dots\dots(ii)$$

on solving these two equations :  $-\frac{t_1}{t_2} = \frac{t^2 - t_1^2}{t^2 - t_2^2} \Rightarrow t = \sqrt{t_1 t_2}$ .

**Problem 62.** By which velocity a ball be projected vertically downward so that the distance covered by it in 5th second is twice the distance it covers in its 6th second ( $g = 10 \text{ m/s}^2$ ) [CPMT 1997; MH CET 2000]

- (a)  $58.8 \text{ m/s}$  (b)  $49 \text{ m/s}$  (c)  $65 \text{ m/s}$  (d)  $19.6 \text{ m/s}$

Solution : (c) By formula  $h_n = u + \frac{1}{2}g(2n-1) \Rightarrow u - \frac{10}{2}[2 \times 5 - 1] = 2 \{u - \frac{10}{2}[2 \times 6 - 1]\}$   
 $\Rightarrow u - 45 = 2 \times (u - 55) \Rightarrow u = 65 \text{ m/s}$ .

**Problem 63.** Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant [CBSE PMT 1995]

- (a) 2.50 m (b) 3.75 m (c) 4.00 m (d) 1.25 m

Solution : (b) Let the interval be  $t$  then from question

For first drop  $\frac{1}{2}g(2t)^2 = 5 \quad \dots\dots(i)$  For second drop  $x = \frac{1}{2}gt^2 \quad \dots\dots(ii)$

By solving (i) and (ii)  $x = \frac{5}{4}$  and hence required height  $h = 5 - \frac{5}{4} = 3.75 \text{ m}$ .

**Problem 64.** A balloon is at a height of 81 m and is ascending upwards with a velocity of  $12 \text{ m/s}$ . A body of 2 kg weight is dropped from it. If  $g = 10 \text{ m/s}^2$ , the body will reach the surface of the earth in [MP PMT 1994]

- (a) 1.5 s (b) 4.025 s (c) 5.4 s (d) 6.75 s

Solution : (c) As the balloon is going up we will take initial velocity of falling body  $= -12 \text{ m/s}$ ,  $h = 81 \text{ m}$ ,  $g = +10 \text{ m/s}^2$

By applying  $h = ut + \frac{1}{2}gt^2$ ;  $81 = -12t + \frac{1}{2}(10)t^2 \Rightarrow 5t^2 - 12t - 81 = 0$

$$\Rightarrow t = \frac{12 \pm \sqrt{144 + 1620}}{10} = \frac{12 \pm \sqrt{1764}}{10} \approx 5.4 \text{ sec}.$$

**Problem 65.** A particle is dropped under gravity from rest from a height  $h$  ( $g = 9.8 \text{ m/s}^2$ ) and it travels a distance  $9h/25$  in the last second, the height  $h$  is [MNR 1987]

- (a) 100 m (b) 122.5 m (c) 145 m (d) 167.5 m

Solution : (b) Distance travelled in  $n$  sec  $= \frac{1}{2}gn^2 = h \quad \dots\dots(i)$

Distance travelled in  $n^{\text{th}}$  sec  $= \frac{g}{2}(2n-1) = \frac{9h}{25} \quad \dots\dots(ii)$

Solving (i) and (ii) we get.  $h = 122.5 \text{ m}$ .

**Problem 66.** A stone thrown upward with a speed  $u$  from the top of the tower reaches the ground with a velocity  $3u$ . The height of the tower is [EAMCET 1983; RPET 2003]

- (a)  $3u^2/g$  (b)  $4u^2/g$  (c)  $6u^2/g$  (d)  $9u^2/g$

Solution : (b) For vertical downward motion we will consider initial velocity  $= -u$ .

By applying  $v^2 = u^2 + 2gh$ ,  $(3u)^2 = (-u)^2 + 2gh \Rightarrow h = \frac{4u^2}{g}$ .

**Problem 67.** A stone dropped from the top of the tower touches the ground in 4 sec. The height of the tower is about  
[MP PET 1986; AFMC 1994; CPMT 1997; BHU 1998; DPMT 1999; RPET 1999]

- (a) 80 m (b) 40 m (c) 20 m (d) 160 m

Solution : (a)  $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 4^2 = 80m.$

**Problem 68.** A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is  
[CPMT 1983; Kerala PMT 2002]

- (a) 4.9 m (b) 9.8 m (c) 19.6 m (d) 24.5 m

Solution : (d) The separation between two bodies, two second after the release of second body is given by :

$$s = \frac{1}{2}g(t_1^2 - t_2^2) = \frac{1}{2} \times 9.8 \times (3^2 - 2^2) = 24.5 \text{ m.}$$

## 12. Motion with Variable Acceleration

(i) If acceleration is a function of time

$$a = f(t) \quad \text{then } v = u + \int_0^t f(t)dt \text{ and } s = ut + \int \left( \int f(t)dt \right) dt$$

(ii) If acceleration is a function of distance

$$a = f(x) \quad \text{then } v^2 = u^2 + 2 \int_{x_0}^x f(x)dx$$

(iii) If acceleration is a function of velocity

$$a = f(v) \text{ then } t = \int_u^v \frac{dv}{f(v)} \text{ and } x = x_0 + \int_u^v \frac{v dv}{f(v)}$$

### Sample problems based on variable acceleration

**Problem 69.** An electron starting from rest has a velocity that increases linearly with the time that is  $v = kt$ , where  $k = 2 \text{ m/sec}^2$ . The distance travelled in the first 3 seconds will be  
[NCERT 1982]

- (a) 9 m (b) 16 m (c) 27 m (d) 36 m

Solution : (a)  $x = \int_{t_1}^{t_2} v dt = \int_0^3 2t dt = 2 \left[ \frac{t^2}{2} \right]_0^3 = 9 \text{ m.}$

**Problem 70.** The acceleration of a particle is increasing linearly with time  $t$  as  $bt$ . The particle starts from the origin with an initial velocity  $v_0$ . The distance travelled by the particle in time  $t$  will be  
[CBSE PMT 1995]

- (a)  $v_0 t + \frac{1}{3}bt^2$  (b)  $v_0 t + \frac{1}{3}bt^3$  (c)  $v_0 t + \frac{1}{6}bt^3$  (d)  $v_0 t + \frac{1}{2}bt^2$

Solution : (c)  $\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt = \int_{t_1}^{t_2} (bt) dt$   
 $\Rightarrow v_2 - v_1 = \left( \frac{bt^2}{2} \right)_{t_1}^{t_2}$   
 $\Rightarrow v_2 = v_1 + \left( \frac{bt^2}{2} \right)_0^t = v_0 + \frac{bt^2}{2}$   
 $\Rightarrow S = \int v_0 dt + \int \frac{bt^2}{2} dt = v_0 t + \frac{1}{6}bt^3$

**Problem 71.** The motion of a particle is described by the equation  $u = at$ . The distance travelled by the particle in the first 4 seconds [DCE 2000]

(a)  $4a$

(b)  $12a$

(c)  $6a$

(d)  $8a$

Solution : (d)  $u = at \Rightarrow \frac{ds}{dt} = at$

$$\Rightarrow s = \int_0^4 at \, dt = a \left[ \frac{t^2}{2} \right]_0^4 = 8a.$$