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Calculating the Jet Quenching Parameter from AdS/CFT

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Models of medium-induced radiative parton energy loss account for the strong suppression of high-p_T hadron spectra in $\sqrt{s_{NN}} = 200$ GeV Au-Au collisions at RHIC in terms of a single "jet quenching parameter" $\hat{q}$. The available suite of jet quenching measurements make $\hat{q}$ one of the experimentally best constrained properties of the hot fluid produced in RHIC collisions. We observe that $\hat{q}$ can be given a model-independent, nonperturbative, quantum field theoretic definition in terms of the short-distance behavior of a particular light-like Wilson loop. We then use the AdS/CFT correspondence to obtain a strong-coupling calculation of $\hat{q}$ in hot $\mathcal{N} = 4$ supersymmetric QCD, finding $\hat{q}_{\rm SYM} = 26.69\,\sqrt{\alpha_{\rm SYM} N_c}\,T^3$ in the limit in which both N_c and $4\pi\alpha_{\rm SYM} N_c$ are large. We thus learn that at strong coupling $\hat{q}$ is not proportional to the entropy density s, or to some "number density of scatterers" since, unlike the number of degrees of freedom, $\hat{q}$ does not grow like N_c^2.

Ultrarelativistic nucleus-nucleus collisions are studied at RHIC and at the LHC to determine the properties of QCD matter at extreme energy density and temperature [1, 2]. If we could do the gedanken experiment of deep inelastic scattering on the hot fluid produced in a heavy ion collision, we could learn a lot. Even though the short lifetime of the transient dense state precludes the use of such external probes, a conceptually similar method is available at RHIC and LHC energies. This method is based upon internally generated probes: energetic partons produced in rare high transverse momentum elementary interactions in the initial stage of the collision, which then interact strongly with the hot, dense fluid produced in the collision as they plough through it [3]. The characterization of the resulting medium-induced modification of high-p_T parton fragmentation ("jet quenching") and its connection to properties of the hot, dense matter that is the object of study have become one of the most active areas of research stimulated by RHIC data [3]. Models which supplement the standard perturbative QCD formalism for high-p_T hadron production with medium-induced parton energy loss successfully account for the strong (up to a factor ~ 5) suppression of hadronic spectra in $\sqrt{s_{NN}} = 200$ GeV Au-Au collisions at RHIC, its dependence on centrality and orientation with respect to the reaction plane, and the corresponding reduction of back-to-back hadron correlations [4, 5]. These models typically involve one medium-sensitive "jet quenching parameter" denoted $\hat{q}$. This parameter is usually defined only perturbatively, and is often thought of as proportional to $1/(\lambda_D^2 \lambda_{\rm MFP})$, with λ_D the Debye screening length and $\lambda_{\rm MFP}$ some perturbatively defined transport mean-free path [6]. In the present paper, we address the question of how $\hat{q}$ can be defined and calculated from first principles in nonperturbative quantum field theory, without assuming the existence of quasiparticles with a well-defined mean-free path.

There are many indications from data at RHIC and from calculations of lattice-discretized QCD thermodynamics that the quark-gluon plasma at temperatures not far above the crossover from the hadronic phase is strongly coupled. (For example, lattice calculations show that J/Ψ mesons remain bound [7]; for example, there is qualitative agreement between the degree of azimuthal anisotropy in collisions with nonzero impact parameter seen in RHIC data and in calculations assuming ideal, zero-viscosity, hydrodynamics [1, 8, 9, 10].) Lattice QCD is the prime example of a rigorous calculational method applicable in a hot, strongly coupled, gauge theory. Because it is formulated in Euclidean space, it is well-suited to calculating static thermodynamic quantities and less well-suited to calculating transport coefficients, or dynamical processes of any sort. It therefore cannot address parton energy loss itself, and cannot be used to calculate $\hat{q}$, the property of the medium that parton energy loss "measures". Complementary nonperturbative techniques are thus desirable. One such technique is the AdS/CFT correspondence, which maps nonperturbative problems in certain hot strongly coupled gauge theories onto calculable problems in a dual gravity theory [11]. This method has been used to calculate the shear viscosity in several supersymmetric gauge theories [12, 13, 14, 15, 16, 17], as well as for certain diffusion constants [18] and thermal spectral functions [19]. (See Refs. [20] for work towards a dual description of dynamics in heavy ion collisions themselves.) The best-studied example is the calculation of the shear viscosity η, where the dimensionless ratio η/s, with s the entropy density, takes on the value $1/4\pi$ in the large number of colors (N_c), large 't Hooft coupling ($\lambda \equiv g_{\rm YM}^2 N_c$) limit of any gauge theory that admits a holographically dual supergravity description, making this result "universal" [15]. Furthermore, the leading $1/\lambda$ corrections are known [16]. The results thus obtained in supersymmetric Yang-Mills theories have been argued to be relevant to the QCD matter produced at RHIC [10].

In this paper, we shall demonstrate that the problem of calculating the jet quenching parameter also lends itself to a simple reformulation in theories with a gravity dual. After formulating a nonperturbative definition of $\hat{q}$, we shall calculate $\hat{q}$ in $\mathcal{N}=4$ supersymmetric Yang-Mills theory, in the large-N_c and large-λ limit. We shall close with a comparison to RHIC data, but at present we make no conjecture for the "universality" of our result.

We begin with the gedanken experiment of DIS on hot matter in thermal equilibrium. A virtual photon γ^*, emitted by the electron, interacts with this matter. In the rest frame of the thermal matter, the incident virtual photon can be viewed as a superposition of its hadronic Fock states $|\gamma^*\rangle = |q(\mathbf{x})\,\bar{q}(\mathbf{y})\rangle + O(g)$. The lowest lying Fock state is a color singlet quark-antiquark dipole whose transverse size in a configuration space picture is proportional to the inverse of the virtuality of the scattering process in momentum space. In the high-energy limit, the scattering $\hat{S}$-matrix can be written in terms of eikonal phase factors W, which account for the precession of the partonic projectile in the color field of the medium [21]. For instance, for an incident quark of initial color α and transverse position $\mathbf{x}$, $\hat{S}|\alpha(\mathbf{x})\rangle = W^F_{\alpha\,\beta}(\mathbf{x})\,|\beta(\mathbf{x})\rangle$. Here

$$W(\mathbf{x}) = \mathcal{P}\exp\left[i\int dx^- A^+(\mathbf{x},x^-)\right] \qquad (1)$$

is the Wilson line in the representation of the projectile. The quark α propagates at fixed transverse position $\mathbf{x}$ along the light-cone direction x^- through the medium, which is described by the target color field A^+. Then, the "photoabsorption" cross section for interaction of the $|\bar{q}\,q\rangle$ Fock state of the virtual photon with the target is

$$\sigma^{DIS} = \int d^2\mathbf{x}\, d^2\mathbf{y}\, \psi(\mathbf{x}-\mathbf{y})\, \psi^*(\mathbf{x}-\mathbf{y})\, P^{q\bar{q}}_{\rm tot}(\mathbf{x},\mathbf{y})\,, \qquad (2)$$

where $\psi(\mathbf{x}-\mathbf{y})\,\psi^*(\mathbf{x}-\mathbf{y})$ determines the probability distribution of color singlet dipoles of transverse size $|\mathbf{x}-\mathbf{y}|$. The interaction probability is [21]

$$P^{q\bar{q}}_{\rm tot} \;=\; \left\langle 2 - \frac{2}{N_c}{\rm Tr}\left[W^F(\mathbf{x})\, W^{F\dagger}(\mathbf{y})\right]\right\rangle\,. \qquad (3)$$

For a color dipole with transverse size $L \equiv |\mathbf{x}-\mathbf{y}|$ which propagates along the light-cone through a medium of length L^-, the only information about the medium that enters the calculation of the photoabsorption cross section (2) is then encoded in the thermal expectation value of a closed light-like Wilson loop $\langle W^F(\mathcal{C})\rangle$, whose contour $\mathcal{C}$ is a rectangle with large extension L^- in the x^--direction and small extension L in a transverse direction. Calculating $\langle W^F(\mathcal{C})\rangle$ gives the entire nonperturbative input to the virtual photoabsorption cross-section (2).

Inspection of the calculation of the energy distribution of the medium-induced gluon radiation, the fundamental quantity of interest in jet quenching calculations, reveals that the medium-dependence enters this calculation via the expectation value of a Wilson loop in the adjoint representation with the same contour $\mathcal{C}$ [22]. The physical origin of $\langle W^A(\mathcal{C})\rangle$ in this calculation is more difficult to picture than was the case in the DIS example. Here, the Wilson loop arises in a configuration space formulation of the gluon radiation cross-section from the combination of the gluon emitted at transverse position $\mathbf{x}$ in the radiation amplitude, with the gluon at position $\mathbf{y}$ in the complex conjugate amplitude. The difference $L=|\mathbf{x}-\mathbf{y}|$ is conjugate to the transverse momentum $k_\perp$ of the emitted gluon. (See Ref. [22] for details.) In jet quenching calculations, one frequently uses the so-called dipole approximation [23], valid for small transverse distances L:

$$\langle W^A(\mathcal{C})\rangle \;\approx\; \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}\,L^-\,L^2\right]\,. \qquad (4)$$

Here, $\hat{q}$ is the "jet quenching parameter" introduced previously in Ref. [6]. (The factor of $\sqrt{2}$ in the denominator arises because the light-cone distance L^- is larger than the spatial distance travelled, conventionally used in (4), by that factor.) We simply observe that instead of seeing (4) as an approximation, we can use it as a nonperturbative definition: $\frac{1}{4\sqrt{2}}\hat{q}$ is the coefficient of the $L^-\,L^2$ term in $\log\langle W^A(\mathcal{C})\rangle$ at small L.

Consider a high energy parton for which the dominant energy loss mechanism is the radiation of gluons with large enough $k_\perp \sim 1/L$ that QCD is weakly coupled at the scale $k_\perp$. This energy loss process cannot be modelled in a conformal theory that is strongly coupled at all scales. However, the strongly coupled physics at scales proportional to T that characterizes the medium rather than the probe does enter the energy loss calculation, only via the quantity $\hat{q}$. This, and the fact that time-averaged values of $\hat{q}$ have been determined in comparison with data from RHIC in Refs. [4, 5], motivates the calculation of $\hat{q}$ in strongly coupled gauge theory plasmas.

We have calculated the thermal expectation value of the light-like Wilson loop (4) for $\mathcal{N}=4$ super Yang-Mills theory. This is a conformally invariant theory with two parameters: the rank of gauge group N_c and the 't Hooft coupling $\lambda = g^2_{\rm YM}N_c$. Its on-shell field content includes eight bosonic and eight fermionic degrees of freedom, all in the color adjoint representation. We begin by evaluating the light-like Wilson loop in the fundamental representation [26]. According to the AdS/CFT correspondence [11], in the large-N_c and large-λ limits the thermal expectation value $\langle W^F(\mathcal{C})\rangle$ can be calculated using the metric for a 5-dimensional curved space-time describing a black hole in anti-deSitter (AdS) space [24]. Calling the fifth dimension r, the black hole horizon is at some $r=r_0$ and the $(3+1)$-dimensional conformally invariant field theory itself "lives" at $r\to\infty$. The prescription for evaluating $\langle W^F(\mathcal{C})\rangle$ is that we must find the extremal action surface in the five-dimensional AdS spacetime whose boundary at $r \to \infty$ is the contour $\mathcal{C}$ in Minkowski space $R^{3,1}$. $\langle W^F(\mathcal{C})\rangle$ is then given by

$$\langle W^F(\mathcal{C})\rangle = \exp\left[-S(\mathcal{C})\right] , \tag{5}$$

with S the action of the extremal surface [24], subject to a suitable subtraction that we discuss below.

Using light-cone coordinates $x^\mu = (r, x^\pm, x^2, x^3)$, the AdS black hole metric is given by [11]

$$\begin{aligned} ds^2 &= -\left(\frac{r^2}{R^2}+f\right)dx^+dx^- + \frac{r^2}{R^2}(dx_2^2+dx_3^2) \\ &\quad +\frac{1}{2}\left(\frac{r^2}{R^2}-f\right)\left((dx^+)^2+(dx^-)^2\right)+\frac{1}{f}dr^2 \\ &= G_{\mu\nu}dx^\mu dx^\nu , \end{aligned} \tag{6}$$

where $f = \frac{r^2}{R^2}\left(1-\frac{r_0^4}{r^4}\right)$ with R the curvature radius of the AdS space. Here, the temperature T of the Yang-Mills theory is given by the Hawking temperature of the black hole, $T = \frac{r_0}{\pi R^2}$, and R and the string tension $1/2\pi\alpha'$ are related to the t'Hooft coupling by $\frac{R^2}{\alpha'} = \sqrt{\lambda}$.

We parameterize the surface whose action $S(\mathcal{C})$ is to be extremized by $x^\mu = x^\mu(\tau,\sigma)$, where $\sigma^\alpha = (\tau,\sigma)$ denote the coordinates parameterizing the worldsheet. The Nambu-Goto action for the string worldsheet is given by

$$S = \frac{1}{2\pi\alpha'}\int d\sigma d\tau\,\sqrt{\det g_{\alpha\beta}} \tag{7}$$

with $g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu$ the induced metric on the worldsheet. This action is invariant under coordinate changes of σ^α, and we can set $\tau = x^-$ and $\sigma = x_2$, taking the "short sides" of $\mathcal{C}$ with length L along the x_2 direction. Since $L^- \gg L$, we can assume that the surface is translationally invariant along the τ direction, i.e. $x^\mu = x^\mu(\sigma,\tau) = x^\mu(\sigma)$. The Wilson loop lies at constant x_3 and at constant x^+, so $x_3(\sigma) = \text{const}$, $x^+(\sigma) = \text{const}$. For the bulk coordinate r, we implement the requirement that the world sheet has $\mathcal{C}$ as its boundary by imposing $r\left(\pm\frac{L}{2}\right) = \infty$, which preserves the symmetry $r(\sigma) = r(-\sigma)$. The action (7) now takes the form

$$S = \frac{\sqrt{2}\,r_0^2\,L^-}{2\pi\alpha' R^2}\int_0^{\frac{L}{2}} d\sigma\,\sqrt{1+\frac{r'^2R^2}{fr^2}} , \tag{8}$$

where $r' = \partial_\sigma r$. The equation of motion for $r(\sigma)$ is then

$$r'^2 = \gamma^2\frac{r^2 f}{R^2} \tag{9}$$

with γ an integration constant. Eq. (9) has two solutions. One has $\gamma = 0$ and hence $r' = 0$, meaning $r(\sigma) = \infty$ for all σ: the surface stays at infinity. This solution is not of interest. The other solution has $\gamma > 0$. It "descends" from $r(\pm\frac{L}{2}) = \infty$ and has a turning point where $r' = 0$ which, by symmetry, must occur at $\sigma = 0$. From (9), the turning point must occur where $f = 0$, implying $r' = 0$ at the horizon $r = r_0$. The surface descends from infinity, skims the horizon, and returns to infinity. Note that the surface descends all the way to the horizon regardless of how small L is [27]. This is reasonable on physical grounds, as we expect $\hat{q}$ to describe the thermal medium rather than ultraviolet physics. Knowing that $r = r_0$ at $\sigma = 0$, (9) can be integrated, yielding

$$\frac{L}{2} = \frac{R^2}{\gamma}\int_{r_0}^\infty \frac{dr}{\sqrt{r^4-r_0^4}} = \frac{aR^2}{\gamma r_0} \tag{10}$$

where $a = \sqrt{\pi}\Gamma(\frac{5}{4})/\Gamma(\frac{3}{4}) \approx 1.311$. We then find

$$S = \frac{\pi\sqrt{\lambda}L^-LT^2}{2\sqrt{2}}\sqrt{1+\frac{4a^2}{\pi^2T^2L^2}} , \tag{11}$$

where we have used $r_0 = \pi R^2 T$ and $\frac{R^2}{\alpha'} = \sqrt{\lambda}$.

Recalling our discussion of the DIS example, we now note that whereas we wish to evaluate the interaction of a "bare" $|q\bar{q}\rangle$ Fock state of the virtual photon with the medium, what we have calculated above includes the "self-energy" of the high energy quark and antiquark, moving through the medium as if in the absence of each other. We perform the required subtraction upon noting that in addition to the extremal surface constructed above, there is another trivial one given by two disconnected worldsheets, each of which descend from $r = \infty$ to $r = r_0$ at constant x_2, one at $x_2 = +\frac{L}{2}$ the other at $x_2 = -\frac{L}{2}$. The total action for these two surfaces is

$$S_0 = \frac{2L^-}{2\pi\alpha'}\int_{r_0}^\infty dr\,\sqrt{g_{--}g_{rr}} = \frac{a\sqrt{\lambda}L^-T}{\sqrt{2}} . \tag{12}$$

Subtracting this self-energy, we obtain

$$S_I \equiv S - S_0 , \tag{13}$$

the L-dependent interaction between the quark pair moving through the medium. Because the quarks are moving at the speed of light, both S_0 and S are finite: no ultraviolet subtraction like that required for static quarks is needed. $\exp[-S_I]$ is the thermal expectation value of the Wilson loop (5) in the fundamental representation, and would be our final result were we actually interested in DIS. Note that S_I vanishes for $L \to 0$ as it must, since the photoabsorption probability (3) vanishes in that limit.

We now evaluate the adjoint Wilson loop (4) that determines $\hat{q}$. For $SU(N_c)$, the Wilson line in the adjoint representation can be obtained using the identity $\mathrm{Tr}W = \mathrm{tr}W\mathrm{tr}W^\dagger - 1$, where Tr and tr denote traces in the adjoint and fundamental representations, respectively. This implies that in the large-N_c limit, the adjoint Wilson loop (4) is given by $\exp\left[-2S_I\right]$. The jet quenching parameter $\hat{q}$ in (4) is determined by the behavior of $2S_I$ for small transverse distances, $LT \ll 1$. We find

$$2S_I = \frac{\pi^2}{4\sqrt{2}\,a}\sqrt{\lambda}\,T^3L^-L^2 + \mathcal{O}(T^5L^-L^4) \tag{14}$$

and can now use (4) to read off our central result:

$$\hat{q}_{\rm SYM} = \frac{\pi^2}{a}\sqrt{\lambda}\,T^3 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}\,T^3 \approx 26.69\sqrt{\alpha_{\rm SYM}N_c}\,T^3 \tag{15}$$

in $\mathcal{N}=4$ supersymmetric Yang-Mills theory in the large-N_c and large-λ limits.

Perhaps the most striking qualitative feature of our result is that at strong coupling $\hat{q}$ is proportional to $\sqrt{\lambda}$, not to the number of degrees of freedom $\sim N_c^2$. This means that at strong coupling $\hat{q}$ cannot be thought of as "measuring" either the entropy density s or what is sometimes described as a "gluon number density" or $\varepsilon^{3/4}$ as had been expected [3], since both s and the energy density ε are proportional to $N_c^2\lambda^0$. Instead, it appears that $\hat{q}$ is better thought of as a measure of T^3[28].

We would be remiss not to attempt a comparison to implications of RHIC data. Taking $N_c = 3$ and $\alpha_{\rm SYM} = \frac{1}{2}$, reasonable for temperatures not far above the QCD phase transition, we shall use $\lambda = 6\pi$ to make estimates. From (15), we find $\hat{q}$ =4.5, 10.6, 20.7 GeV^2/fm for $T = 300$, 400, 500 MeV. In a heavy ion collision, $\hat{q}$ decreases with time τ as the hot fluid expands and cools. The time-averaged $\hat{q}$ which has been determined in comparison with RHIC data is $\overline{\hat{q}} \equiv \frac{4}{(L^-)^2}\int_{\tau_0}^{\tau_0+L^-/\sqrt{2}} \tau\,\hat{q}(\tau)\,d\tau$, found to be around 5-15 GeV^2/fm [4, 5]. If we assume a one-dimensional Bjorken expansion with $T(\tau) = T_0\left(\frac{\tau_0}{\tau}\right)^{1/3}$, take $\tau_0 = 0.5$ fm, and take $L^-/\sqrt{2} = 2$ fm, the estimated mean distance travelled in the medium by those hard partons which "escape" and are detected [5], we find that to obtain $\overline{\hat{q}} = 5$ GeV^2/fm from (15) we need T_0 such that $T(1\text{ fm}) \approx 310$ MeV, only slightly higher than expected [8]. Equivalently, the $\hat{q}$ we find from (15) is slightly smaller than that suggested by RHIC data. First, this could indicate that in going from hot $\mathcal{N}=4$ to the QCD quark-gluon plasma, as the number of color adjoint degrees of freedom is decreased by a factor of 2/15 (and the color fundamental quarks are added) the jet quenching parameter *increases* slightly. Second, it could indicate that the $\overline{\hat{q}}$ extracted from data is somewhat high, either because the medium through which the energetic parton moves has a flow velocity transverse to the parton's motion or because additional sources of energy loss are also significant, for example collisions without gluon radiation or processes occurring before the medium is in local thermal equilibrium. Testing the second conclusion likely requires further experimental data; the first can be tested by evaluating $\hat{q}$ in other gauge theories with gravity duals. Unless our result turns out to be "universal", which would be interesting in its own right, such calculations would indicate the direction in which $\hat{q}$ changes as the theory is made more QCD-like. Calculating the $1/\lambda$ corrections to our result would also be informative.

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[1] K. Adcox *et al.* [PHENIX Collaboration], Nucl. Phys. A **757**, 184 (2005); B. B. Back *et al.* [PHOBOS Collaboration], Nucl. Phys. A **757**, 28 (2005); I. Arsene *et al.* [BRAHMS Collaboration]; Nucl. Phys. A **757**, 1 (2005); J. Adams *et al.* [STAR Collaboration], Nucl. Phys. A **757**, 102 (2005).

[2] F. Carminati *et al.* [ALICE Collaboration], J. Phys. G **30**, 1517 (2004); G. Baur *et al.* [CMS Collaboration], CMS-Note-2000-060; H. Takai, [for the ATLAS Collaboration] Eur. Phys. J. C **34**, S307 (2004).

[3] For reviews, see R. Baier, D. Schiff and B. G. Zakharov, Ann. Rev. Nucl. Part. Sci. **50**, 37 (2000); A. Kovner and U. A. Wiedemann, arXiv:hep-ph/0304151; M. Gyulassy *et al.*, arXiv:nucl-th/0302077; P. Jacobs and X. N. Wang, Prog. Part. Nucl. Phys. **54**, 443 (2005).

[4] K. J. Eskola *et al.*, Nucl. Phys. A **747**, 511 (2005).

[5] A. Dainese, C. Loizides and G. Paic, Eur. Phys. J. C **38**, 461 (2005).

[6] R. Baier *et al.*, Nucl. Phys. B **484**, 265 (1997).

[7] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004); S. Datta *et al.*, Phys. Rev. D **69**, 094507 (2004).

[8] P. F. Kolb and U. W. Heinz, arXiv:nucl-th/0305084.

[9] D. Teaney, Phys. Rev. C **68**, 034913 (2003).

[10] E. V. Shuryak, Nucl. Phys. A **750**, 64 (2005).

[11] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998); S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).

[12] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001).

[13] A. Buchel and J. T. Liu, JHEP **0311**, 031 (2003).

[14] P. Kovtun, D. T. Son and A. O. Starinets, JHEP **0310**, 064 (2003).

[15] A. Buchel and J. T. Liu, Phys. Rev. Lett. **93**, 090602 (2004).

[16] A. Buchel, J. T. Liu and A. O. Starinets, Nucl. Phys. B **707**, 56 (2005).

[17] D. T. Son and A. O. Starinets, JHEP **0603**, 052 (2006); J. Mas, JHEP **0603**, 016 (2006); O. Saremi, arXiv:hep-th/0601159. K. Maeda, M. Natsuume and T. Okamura, Phys. Rev. D **73**, 066013 (2006).

[18] G. Policastro, D. T. Son and A. O. Starinets, JHEP **0209**, 043 (2002).

[19] D. Teaney, arXiv:hep-ph/0602044; P. Kovtun and A. Starinets, Phys. Rev. Lett. **96**, 131601 (2006).

[20] S. J. Sin and I. Zahed, Phys. Lett. B **608**, 265 (2005); E. Shuryak, S. J. Sin and I. Zahed, arXiv:hep-th/0511199.

[21] A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001).

[22] U. A. Wiedemann, Nucl. Phys. B **588**, 303 (2000).

[23] B. G. Zakharov, JETP Lett. **65**, 615 (1997).

[24] J. M. Maldacena, Phys. Rev. Lett. **80**, 4859 (1998); S. J. Rey and J. T. Yee, Eur. Phys. J. C **22**, 379 (2001); S. J. Rey, S. Theisen and J. T. Yee, Nucl. Phys. B **527**,

171 (1998); A. Brandhuber *et al.*, Phys. Lett. B **434**, 36 (1998); J. Sonnenschein, arXiv:hep-th/0003032.

[25] N. Itzhaki *et al.*, Phys. Rev. D **58**, 046004 (1998).

[26] A generic Wilson loop in $\mathcal{N} = 4$ super Yang-Mills theory depends on six scalar fields, in addition to the gauge field [24]. For a light-like contour, the scalar field terms vanish and the $\mathcal{N} = 4$ Wilson loop coincides with (1).

[27] We have checked that the surface also has this qualitative shape, with $r = r_0$ at $\sigma = 0$ for any L, in the $\mathcal{N} = 2^*$ theory analyzed in Ref. [13] which is a deformation of the $\mathcal{N} = 4$ theory in which some fields are given masses that we take as small compared to T. This demonstrates that this feature of the solution is not specific to $\mathcal{N} = 4$.

[28] This conclusion can be strengthened upon redoing our analysis for the $(p+1)$-dimensional super-Yang-Mills theories (with 16 supercharges) living at the boundary of the geometry describing a large number of non-extremal black Dp-branes [25]. In these theories, in which λ has mass dimension $(3 - p)$, we find $\hat{q} \propto T^2 (T\sqrt{\lambda})^{2/(5-p)}$ whereas $s \propto N_c^2 \lambda^{(p-3)/(5-p)} T^{(9-p)/(5-p)}$ and $\varepsilon \propto Ts$. This means that for $p \neq 3$, $\hat{q}$, s and $\varepsilon^{3/4}$ have different T-dependence, not only different N_c-dependence.