1.

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$
$r = f(\theta)$	0	$\frac{5}{2}$	$\frac{5\sqrt{3}}{2}$

The table gives values of a polar function $r = f(\theta)$ for selected values of θ. If the value of $r = f\left(\frac{\pi}{12}\right)$ is estimated using the average rate of change of the function over the interval $0 \leq \theta \leq \frac{\pi}{6}$, which of the following is true?

(A) The estimated value would be an overestimate of the actual value by approximately 0.223.

(B) The estimated value would be an underestimate of the actual value by approximately 0.223.

(C) The estimated value would be an overestimate of the actual value by approximately 0.335.

(D) The estimated value would be an underestimate of the actual value by approximately 0.335.

```
In[1]:= N@(5 Sqrt[3]/4 - 5/2)

Out[1]= -0.334936
```

Underestimated by 0.335, (D)

2. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = \theta(\theta - 2)(\theta - 4)$, in the polar coordinate system for $0 \leq \theta \leq 4$. Which of the following statements is true?

(A) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is increasing because the values of $f(\theta)$ are negative and decreasing.

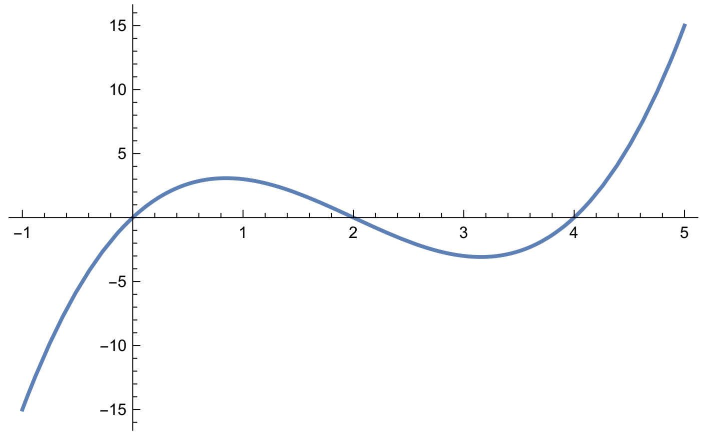
(B) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is decreasing because the values of $f(\theta)$ are negative and decreasing.

(C) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is increasing because the values of $f(\theta)$ are negative and increasing.

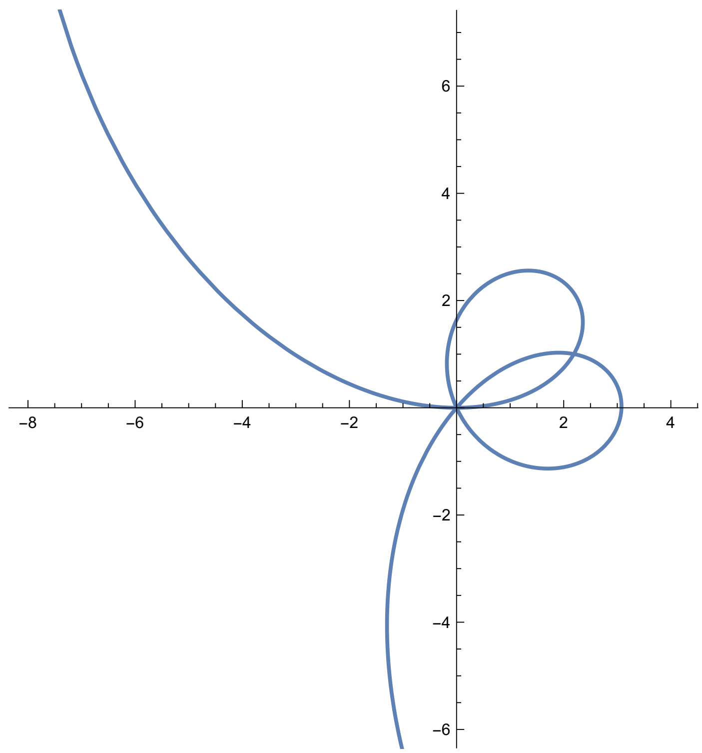
(D) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is decreasing because the values of $f(\theta)$ are negative and increasing.

In[11]:=
```
Plot[t (t - 2) (t - 4), {t, -1, 5}]
PolarPlot[t (t - 2) (t - 4), {t, -1, 5}]
```

Out[11]=



Out[12]=



negative and decreasing, (A)

3. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = 2\sin\theta - 1$, in the polar coordinate system. Which of the following descriptions is true?

(A) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

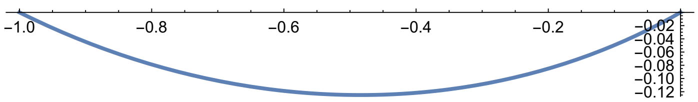
(B) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

(C) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

(D) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

```
In[5]:= PolarPlot[2 Sin[t] - 1, {t, 0, π/6}]
```

Out[5]=



r=f(θ) is clearly increasing (and negative)and the distance is the absolute value of r hence decrease, choose (B)

4. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = 4\sin(2\theta)$, in the polar coordinate system. On the interval $0 \le \theta \le 2\pi$, which of the following is true about the graph of $r = f(\theta)$?

(A) For the input values $\theta = \frac{\pi}{4}, \theta = \frac{3\pi}{4}, \theta = \frac{5\pi}{4}$, and $\theta = \frac{7\pi}{4}$, the function $r = f(\theta)$ has extrema that correspond to points that are farthest from the origin.

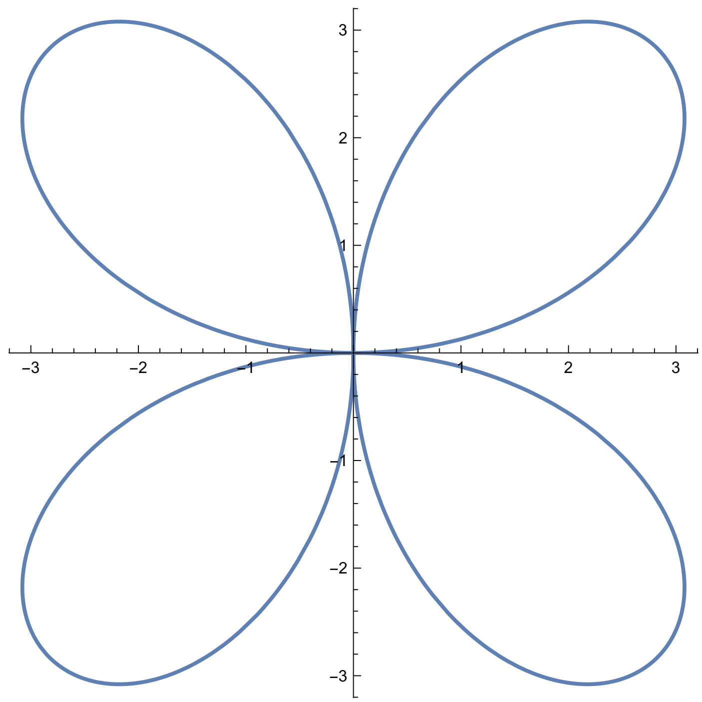
(B) For the input values $\theta = \frac{\pi}{4}, \theta = \frac{3\pi}{4}, \theta = \frac{5\pi}{4}$, and $\theta = \frac{7\pi}{4}$, the function $r = f(\theta)$ has extrema. However, only the points corresponding to $\theta = \frac{\pi}{4}$ and $\theta = \frac{5\pi}{4}$ are farthest from the origin.

(C) For the input values $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$, the function $r = f(\theta)$ has extrema that correspond to points that are farthest from the origin.

(D) For the input values $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$, the function $r = f(\theta)$ has extrema. However, only the point corresponding to $\theta = \frac{\pi}{2}$ is farthest from the origin.

In[8]:= `PolarPlot[4 Sin[2 t], {t, 0, 2 π}]`

Out[8]=



The extrema of such function occurs at θ in $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$ where the derivative of f vanishes. All these extrema correspond to farthest points from the origin. choose (A)

5.

θ	0	π	2π
$r = f(\theta)$	2	-4	3

The table gives all of the relative extrema of the polar function $r = f(\theta)$ for all input values of θ in the domain of the polar function. Which of the following statements is true about the graph of $r = f(\theta)$ in the polar coordinate system?

(A) The greatest distance between $(f(\theta), \theta)$ and the origin is 3 because this is the maximum value of all the extrema.

(B) The greatest distance between $(f(\theta), \theta)$ and the origin is 3 because this is the maximum of the absolute values of all the extrema.

(C) The greatest distance between $(f(\theta), \theta)$ and the origin is 4 because this is the maximum value of all the extrema.

(D) The greatest distance between $(f(\theta), \theta)$ and the origin is 4 because this is the maximum of the absolute values of all the extrema.

(D) by definition

6. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = \frac{3\theta^3+50}{2\theta^4+5}$, in the polar coordinate system. Which of the following is true?

(A) Because $\lim_{\theta\to\infty} f(\theta) = -\infty$, points on the graph of $r = f(\theta)$ will be arbitrarily close to the origin for sufficiently large values of θ.

(B) Because $\lim_{\theta\to\infty} f(\theta) = 0$, points on the graph of $r = f(\theta)$ will be arbitrarily close to the origin for sufficiently large values of θ.

(C) Because $\lim_{\theta\to\infty} f(\theta) = \frac{3}{2}$, points on the graph of $r = f(\theta)$ will be arbitrarily close to the polar curve $r = \frac{3}{2}$ for sufficiently large values of θ.

(D) Because $\lim_{\theta\to\infty} f(\theta) = \infty$, points on the graph of $r = f(\theta)$ will be increasingly distant from the origin for sufficiently large values of θ.

In[13]:=
```
Limit[(3 t^3 + 50)/(2 t^4 + 5), t → ∞]
```

Out[13]=
```
0
```

(which is obvious since the denominator has a superior degree)
Hence r tends to 0, choose (B)