Why Can't We Be Friends?

Samuel Jerome (sjerome)

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Introduction

Interactions with others is central to our success as a society. We form friendships, partnerships, and our own networks to progress as a civilization. The development and sustainability of cooperation among agents in a population is a phenomenon that offers much insight into its progression and hence lies at the heart of evolutionary game theory. This paper will attempt to capture the impact of a dynamic network structure on a population's development. This analysis will serve as a preliminary structure for continued research next semester. Therefore, the specific intention of this paper and project will be threefold:

1. To develop an abstract formulation of this game-theoretic setting for ease of developing future mechanisms and reaching general conclusions.
2. To develop a library to allow for the simulation of arbitrarily complex models.
3. To experiment with various mechanisms to observe the various behaviors that can evolve from a dynamic network. These experiments intend to incrementally further the current research by accounting for asymmetry in a population when considering the network update time-scales.

Motivation

My life is filled with interactions. However, the people I interact with have changed over time, and my personality has likewise changed over time. Understanding behavior in a dynamic network would therefore allow us to conjecture:

- How given mechanisms affect a person's strategy and behavior over time.
- How given mechanisms affect the network structure over time.
- In a real application, with what success rate do these models predict an agent's future strategy.
- In a real application, with what success rate do these models predict the future network structure over time.

Setting

Informal Model Description

We have a set of individuals on a network. At every time step, we advance as follows:

1. Each individual plays a simultaneous game with his or her immediate neighbors.
2. Each individual may choose to add or remove various edges at a given cost.

The analysis will primarily focus on developing mechanisms and updating rules, and determining which aspects of each promote cooperation and clustering.

Formal Model Description

Notation:

Let there be n agents, $\mathbf{V} = \{v_1, ..., v_n\}$, on a dynamic network $\mathbf{G}$, and k actions, $\mathbf{A} = \{a_1, ..., a_k\}$.

Let $u_i(s)$ be agent v_i's utility from a given strategy profile $s = \{a_i, a_{-i}\}$. Let $\mathbf{U} = \{u_1(s), ..., u_n(s)\}$.

Let us refer to a simultaneous game with available actions $\mathbf{A}$ and payoffs $\mathbf{U}$ with players $\mathbf{P} \subseteq \mathbf{V}$ as an interaction between agents $\mathbf{P}$.

Let us assume at each time step t each agent v_i has individual interactions with all agents adjacent to v_i in $\mathbf{G_t}$. We note a given agent v_i plays an action for each interaction he or she has, and we therefore refer to the set of actions v_i plays as v_i's behavior, or B_{ti}. Let $\mathbf{B_t} = \{B_{t1}, ..., B_{tn}\}$.

For completeness, for every interaction between agents v_i and v_j at time t, let the strategy profile of the interaction be s_{ijt}. The total utility of an agent v_i at time t is therefore given by: $\sum\limits_{j \in N_{G_t}(v_i)} u_i(s_{ijt})$.

Let the edge decision $D_{ti} = (E_{Ait}, E_{Rit})$ specify which edges agent v_i can add, E_{Ait}, and remove, E_{Rit}, for a given cost c_e at a given time-step t. Let $\mathbf{D_t} = \{D_{t1}, ..., D_{tn}\}$. An agent v_i decides to add or remove an edge $e \in D_{ti}$ according to some evaluation function J_i. Let $\mathbf{J} = \{J_1, ..., J_n\}$.

Let $\mathbf{H_t}$ be the history of behaviors and graphs up to and including time-step t. The edge decision updates according to some function $\mathbf{R} : \mathbf{H_t} \to \mathbf{D_t}$. The graph is then updated with $\mathbf{J} : \mathbf{H_t} \times \mathbf{D_t} \times \mathbf{G_t} \to \mathbf{G_{t+1}}$. Finally, agents then update their behavior function according to a given rule $\mathbf{F} : \mathbf{H_t} \times \mathbf{G_{t+1}} \to \mathbf{B_{t+1}}$.

The area of interest, therefore, would be:

Given an initial graph $\mathbf{G_0}$, an initial behavior ordered-set $\mathbf{B_0}$, an ordered-set of behavioral updating functions $\mathbf{F}$, an edge decision update rule $\mathbf{R}$, evaluating functions $\mathbf{J}$, and utilities $\mathbf{U}$, what will be the structure of $\mathbf{G_t}$ and the behaviors $\mathbf{B_t}$ at some later time t. More concretely, which parameters yield cooperation, and which yield both high and low clustering coefficients.

Previous Literature

There is much literature regarding games on a network, and we have even done some of these analyses in class. The primary intention of this area of research has been to understand which mechanisms promote cooperation among agents in a population.

Firstly, it was noted by Robert Trivers (1971) that direct reciprocity promotes cooperation [1] should the benefit outweigh the cost ($w > \frac{b}{c}$). This was a result that confirmed the ability of a mechanism to elicit a cooperative strategy, and was, in fact, observed empirically as well. Later work then confirmed the conditions for mechanisms involving indirect reciprocity ($q > \frac{c}{b}$) and mechanisms involving agents on a static network ($k < \frac{b}{c}$).[2]

Later work then began to consider the effect of making the network structure dynamic, and allowed agents to reciprocate with the removal and addition of an edge. This work was geared toward determining the effect of the rate at which agents may 'rewire' their local networks on cooperation. Yet, we note this research was generally extremely specific and limited to one very specific model. There are three distinct papers written:

1. Coevolution of Strategy and Structure in Complex Networks with Dynamical Linking (2006) - This paper evaluated how changing the rates at which strategies and edges are updated (time-scales) affects cooperation. If the network rarely changes and strategies change often, the problem is equivalent to a repeated prisoner's dilemma on a static network. If the network changes often and strategies rarely change, the problem reduces to a well-mixed population with a simple "rescaling" of the payoff matrix. As these time-scales are adjusted, the percentage of cooperators changes as well. [3]

2. Cooperation Prevails When Individuals Adjust Their Social Ties (2006) - Agents have the opportunity to compete to rewire an edge if they are dissatisfied. If an agent v is dissatisfied with some edge (v, w), it can only be replaced with an edge from v to a random immediate neighbor of w. Cooperation prevailed and was stable with appropriate parameters. [4]

3. Dynamic Social Networks Promote Cooperation in Experiments with Humans (2011) - This was the first paper that analyzed data empirically, as opposed to over simulation. Each player plays one strategy in $\{C, D\}$ during each time step for all neighbors, and evaluated how allowing rewiring of the network affects cooperation. An edge will be chosen randomly, and a random agent incident to that edge would have the choice to either add the edge or remove the edge should it already exist. Analysis was based on data, and cooperation prevailed and was stable with appropriate parameters.[5]

It is evident much of the literature thus far obtained with regard to dynamical networks analyzes the effect of shifting the time-scale and relative ratio of network-structure to strategy updates. I will therefore extend the analysis along the same lines — by introducing an asymmetric time-scale for each agent. I will present the hypothesis that the behaviors of the agents are dependent on the evolution of the average time-scale.

Simulations/Results/Discussion

I experimented with many simulations for several models prior to obtaining concrete results. I began simply by recreating results from class – introducing an invader into a well mixed finite population, birth-death updating, death-birth updating, etc. The results can be seen below in Figure 1. I then began experimenting with the dynamic element of the network. Following every round of interactions I first replaced a random agent by another in the population proportional to fitness to ensure selection. I then chose two random agents v_i and v_j. If an edge existed between the agents, it was removed if either v_i or v_j had defected the previous round. If the edge had not existed, it was added if v_i and v_j had never interacted in the past. This did not allow for cooperation, and the results of the simulation can be found in Figure 2a. I then shifted the time scale to rewire the network more often to give time for cooperators to remove their defecting neighbors, and found that this shift in time scale, in fact, promoted cooperation. This matched the existing literature's results nicely, and the results of the simulation can be found in Figure 2b. Finally, I included reciprocating agents (Tit-For-Tat) and found that these agents made cooperation possible even in the setting without the tipped time-scale (Figure 2c). Penultimately, I then tried a model that varied the size of the population by occasionally adding and removing vertices proportional to fitness. However, this model did not promote cooperation on its own either, as can be seen in Figure 3.

Finally, I then experimented with a model proposed by Professor Tarnita that aims to create an asymmetric time-scale among agents. The model was implemented as follows:

There is a fixed-size population of N agents. Let each agent v_i have a corresponding value, p_i, that provides a likelihood the agent will update his or her network structure. Therefore, larger p_i implies a more "social" agent, as this agent prefers to switch its network structure over its behavior. At the start of each time-step, each agent plays a simultaneous game with each of its neighbors on the network, and receives a total payoff u_i. The agents are either cooperators or defectors, and therefore cannot play multiple actions. Cooperation yields a benefit b for a cost c, whereas defecting costs $c' << c$. The fitness of each agent is then calculated to be $f_i = 1 + \delta \cdot u_i$ where δ is the selection parameter. At the end of the interactions, the network is updated as follows:

1. An individual v_i is picked at random.

2. With probability $\frac{1}{1+p_i}$ this agent undergoes a strategy change. To do so, v_i chooses another agent v_r proportional to fitness. The agent copies the strategy of v_r accurately with probability $1 - \mu$. Additionally, the agent's p_i is moved closer to the p_r by a small amount ϵ with probability $1 - \nu$. With probability ν, a new p_i is chosen at random.

3. With probability $\frac{p_i}{1+p_i}$ this agent undergoes a network update. To do so, another individual v_j is picked at random. If there exists an edge in the network between v_i and v_j, the edge is removed if v_j is a defector. If this edge does not exist, it is added if and only if both agents agree to add the edge. An agent v_k agrees to add an edge (v_k, v_l) with probability $\frac{p_k}{1+p_k}$ if and only if v_l has never been seen defecting by v_k. The probability of the second agent "accepting" the edge serves as a cost to add the edge.

I simulated the results of running this model with 50 agents, and the first set of results can be found below in Figure 4a. This model presented itself with three questions–how does p evolve, how do the strategies evolve, and how does the network evolve.

Cooperation was never promoted with the parameters I had. I therefore changed the asymmetry inherent in the model by making it more structured. I gave cooperators higher p, and defectors lower p. This initially led to high clustering, high p, and cooperation until the defectors evolved to likewise have higher p. These results can be seen in Figure 7. It is important to note this result is intuitive as well−cooperators will only form links amongst themselves when they are the only ones with high p. I then tested out differences between low and high p's–lower p tended to incite less cooperation. Though I tested the hypothesis by running many simulations, one such can be seen from the simulation run with a low p in Figure 5. This result could be explained as well, once defectors invaded the population cooperators had little way to drop the connecting edges. I then noticed that in almost all the models I had run, the average p slowly increased over time (and can be seen in almost all the simulations below). This could be explained as well–the agents that were more fit needed connections to receive payoff. To counteract this natural selection of higher p, I ensured more links do not give payoff greater than or equal to 0, by increasing c'. This did lead to a p which both increased and decreased, however cooperation was not promoted. The results of this experiment can be seen in Figure 6.

I then began experimenting with the possibility of "intelligence" evolving due to the network structure promoting cooperation. I included "smart-defectors" and "smart-cooperators" that could guess the other agent's type with some probability before adding the corresponding edge. I hypothesized selection would occur in two stages−first smart-defectors would appear, and then smart-cooperators would invade this population. However, my results did not fully confirm this hypothesis, and the results can be found below in Figure 7a. It was clear intelligence was selected for as smart-defectors did invade the population–but smart-cooperators would still die out quickly amongst the smart defectors, as though they would not be defected against, there was no way of increasing their own fitness any more than the smart-defectors unless another smart-cooperator appears *and* an edge was added between the two. This was unlikely given the mutation rate and the probability *both* agents would be picked to add an edge. Increasing the mutation rate allow several enter the population at once and they could then form clusters, but the mutation rate needed to be increased too much to maintain credibility of the model. I then experimented with changing the number of links an agent adds if it decides to undergo a network update. This would allow smart cooperators to find another should two appear, and have the effect noted earlier without the increased mutation rate. The results of this experiment can be found in Figure 7b. These results were interesting, for they showed a cycle and an ability for any agent to appear in this population. Smart-cooperators, smart-defectors, and defectors all had the ability to increase within this population. Cooperators rarely made a prolonged appearance, and were easily eliminated from the population. It should be noted smart cooperators rarely took over the entire population. This can be understood as well - for they would form clusters, and then one would change its behavior. This would have the effect of a defector invading a well mixed population, and hence the cluster could easily be broken.

Further Research and Conclusions

All of these models proved one primary result—the mechanism by which the network updates significantly influences the evolution of the agents. This project was intended to open doors for future research next semester. Firstly, it allowed for the development of a library to simulate arbitrarily complex models and make necessary changes easily. Secondly, these preliminary analyses serve as a benchmark to identify the specific behaviors a dynamic network can select. Including agent information, such as a p-sociability constant, promoted different behaviors depending on the asymmetry of the time-scale. Introducing the possibility of intelligence allowed for its development and promoted the growth of clusters and cooperation as well. Further research would be geared toward a more quantitative analysis, identifying the necessary conditions for each characteristic to be selected. The goal would be to develop a conclusion to understand which updating dynamics promote cooperation on the network in all of these models.

Acknowledgements

I would like to acknowledge Professor Tarnita for providing much guidance in the search for the previous literature and for the initial model with asymmetric time-scales.

I would also like to thank Anjali Menon and Mallika Viswanath for proof-reading this paper.

References

[1] Trivers, Robert L. "The Evolution of Reciprocal Altruism." *The Quarterly Review of Biology* 46.1 (1971): 35-57. Web.

[2] Tarnita, Corina E. "Mathematical Modeling in Biology and Medicine EEB325 - Lecture 9" [PowerPoint slides]. (2014): Retrieved from `https://blackboard.princeton.edu`.

[3] Pacheco, Jorge, Arne Traulsen, and Martin Nowak. "Coevolution of Strategy and Structure in Complex Networks with Dynamical Linking." *Physical Review Letters 97.25* (2006): Web.

[4] Santos, Francisco C., Jorge M. Pacheco, and Tom Lenaerts. "Cooperation Prevails When Individuals Adjust Their Social Ties." *PLoS Computational Biology* 2.10 (2006): E140. Web.

[5] Rand, D. G., S. Arbesman, and N. A. Christakis. "Dynamic Social Networks Promote Cooperation in Experiments with Humans." *Proceedings of the National Academy of Sciences* 108.48 (2011): 19193-9198. Web.

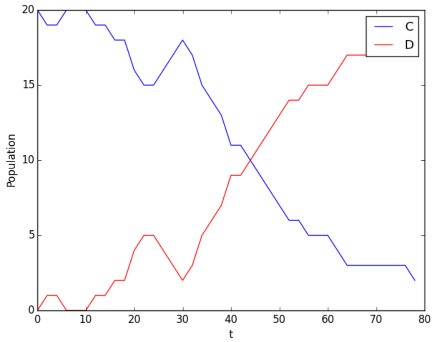
Appendix

The code to my simulations is lengthy, and hence is available publicly on GitHub (username: sjerome, direct-url: `https://github.com/sjerome/python-network-games`). To run a simulation follow the instructions in the file README.txt. The simulation will display the agents and the network after every d iterations, where d is specified in the main.py file. After T iterations the simulation will display the graphs founds below that represent the progression of the agents throughout the simulation. A sample output can be found below in Figure-8. In this example, the labels represent the type and fitness, and the color is the fitness relative to the whole population (green is high, red is low.) For a detailed explanation of the files and code, please contact me at sjerome@princeton.edu. Briefly, there are "agents" on a "network." A simulation is a rule for how to update the network after every iteration. The main.py file executes a given simulation.

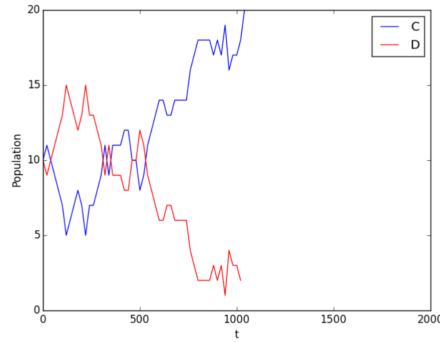
This paper represents my own work in accordance with University regulations.

Samuel Jerome

Figures and Graphs

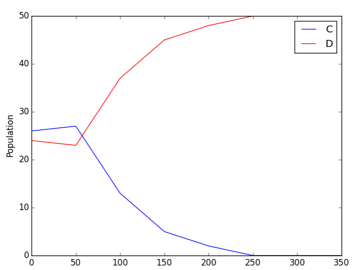


(a) Defector Invading Population

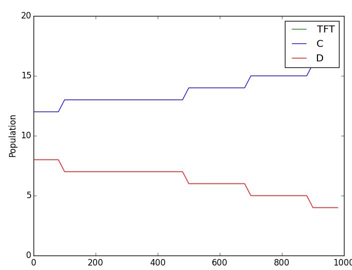


(b) Birth-Death Updating

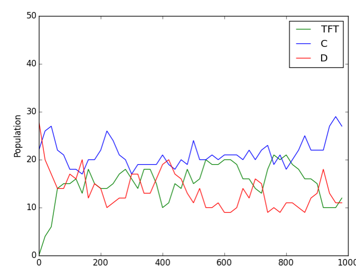
Figure 1: Models from lecture



(a) Standard Dyn. Network



(b) Time-Scaled Dyn. Network



(c) Dyn. Network With Reciprocity

Figure 2: Dynamic Network Model

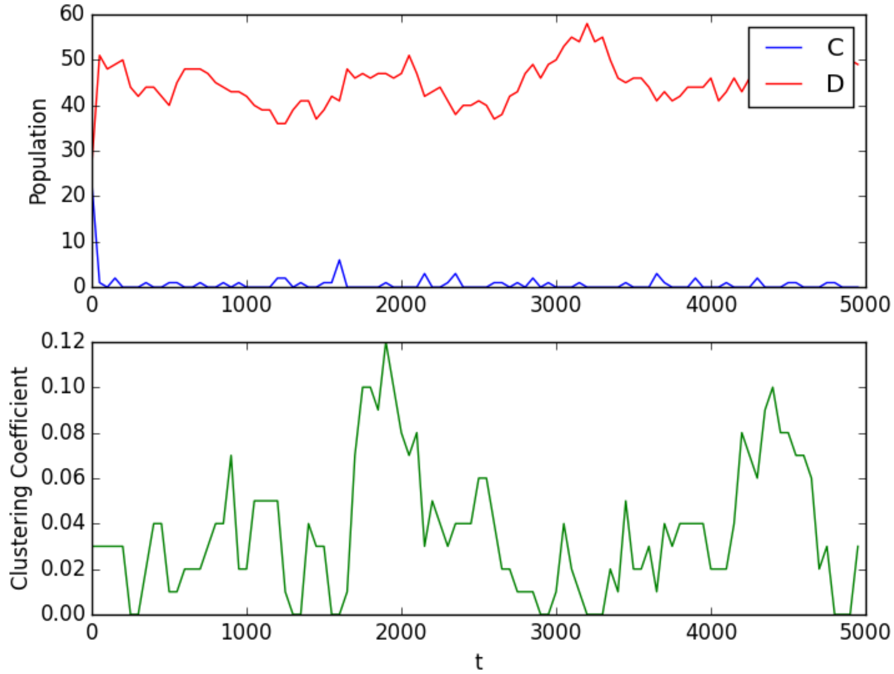
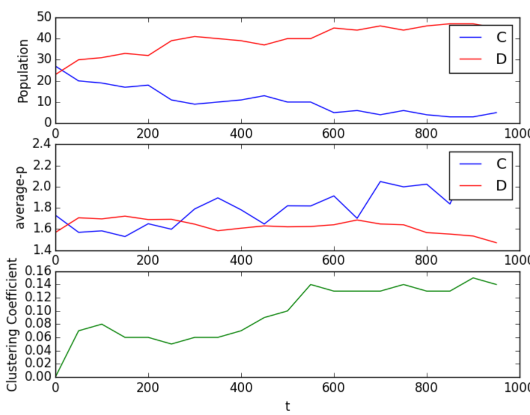
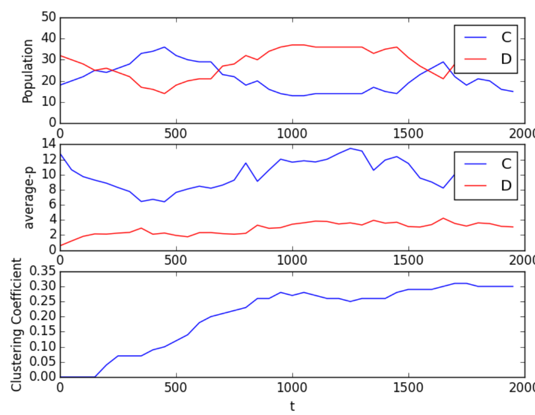


Figure 3: Variably Sized Population Dynamic Network Model



(a) Standard



(b) Cooperators have higher p

Figure 4: Asymmetric Time-Scale Model

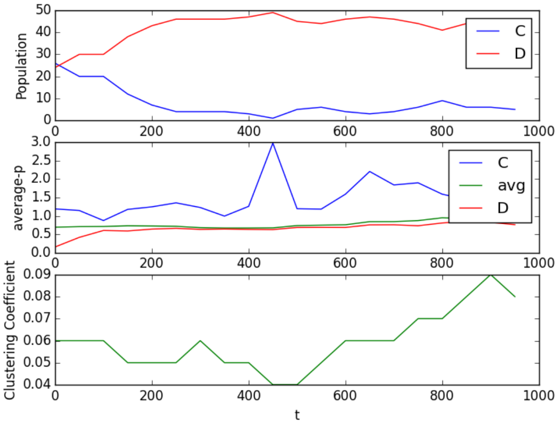


Figure 5: Asymmetric Model with Low p

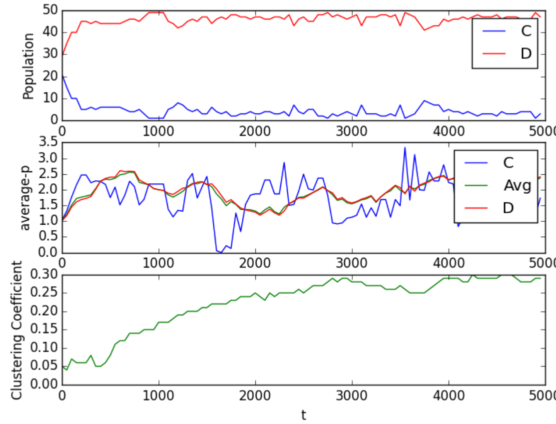
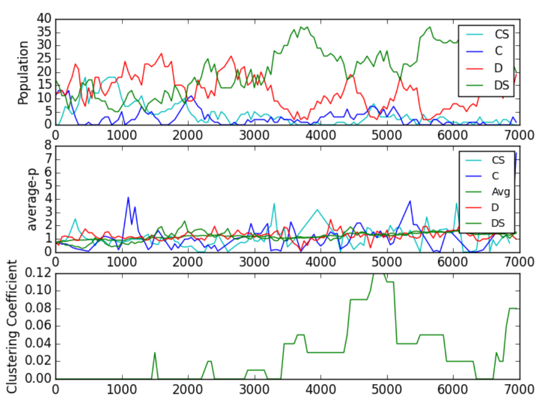
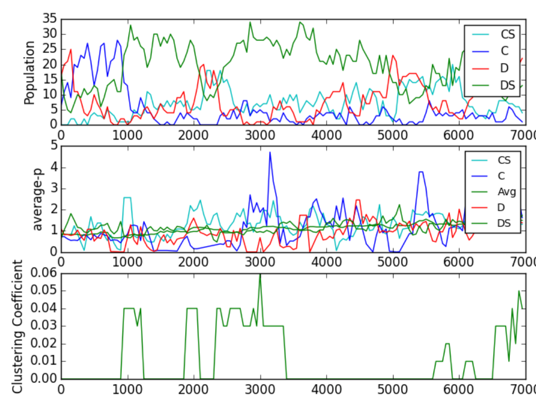


Figure 6: Increasing c' yields variable p



(a) Standard



(b) Mutants Lose All Edges

Figure 7: Asymmetric Model with intelligent cooperators and defectors

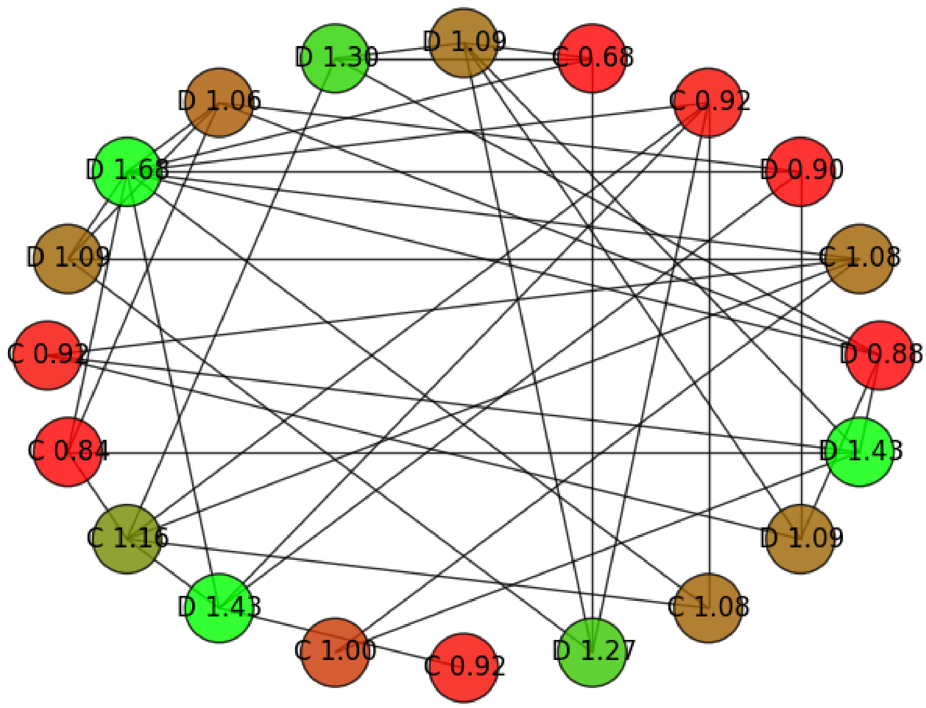


Figure 8: Example Output During Simulation