Geometrical Optics in Continuum

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Fermat's Principle

$$\delta s = 0$$

If we choose the trace connecting two points to be

$$\boldsymbol{r} = \boldsymbol{r}(t), \quad 0 \le t \le 1$$

then

$$s = \int n \, \mathrm{d}\ell = \int n \frac{\mathrm{d}\ell}{\mathrm{d}t} \, \mathrm{d}t = \int_0^1 n(\boldsymbol{r}) |\dot{\boldsymbol{r}}| \, \mathrm{d}t,$$

We define $L = n(\boldsymbol{r})|\dot{\boldsymbol{r}}|$, then

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} \nabla_{\dot{\boldsymbol{r}}} - \nabla \right) L = 0$$

We find

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(n(\boldsymbol{r}) \frac{\dot{\boldsymbol{r}}}{|\dot{\boldsymbol{r}}|} \right) - \frac{\mathrm{d}\ell}{\mathrm{d}t} \nabla n = 0$$

We define $\hat{\theta} = \dot{\boldsymbol{r}} / |\dot{\boldsymbol{r}}|$, we can get

$$\frac{\mathrm{d}}{\mathrm{d}\ell} \left(n \hat{\theta} \right) = \nabla n$$

i.e.

$$n \frac{\mathrm{d}\hat{\theta}}{\mathrm{d}\ell} = \nabla n - \hat{\theta} \frac{\mathrm{d}n}{\mathrm{d}\ell} = \left(1 - \hat{\theta}\hat{\theta}^T \right) \nabla n = \nabla_\perp n$$

also

$$\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}\ell} = \nabla_\perp n / n = \nabla_\perp \ln n$$