Pei-Ling Chiang (NUID:001533847) - Sec02

INFO 6205

Program Structures & Algorithms

Fall 2020

Assignment 5 - Parallel Sorting

Output:

For array size = 2,000,000

```
Degree of parallelism: 3
cutoff : 1000000  depth0 Avg. Time:1510ms
cutoff : 1000000  depth1 Avg. Time:1143ms
cutoff : 1000000  depth2 Avg. Time:1711ms
cutoff : 1000000  depth3 Avg. Time:1733ms
cutoff : 1000000  depth4 Avg. Time:1771ms
cutoff : 1000000  depth5 Avg. Time:1748ms
cutoff : 1000000  depth6 Avg. Time:1618ms
cutoff : 1000000  depth7 Avg. Time:1677ms
cutoff : 1000000  depth8 Avg. Time:1710ms
cutoff : 500000   depth0 Avg. Time:1129ms
cutoff : 500000   depth1 Avg. Time:1113ms
cutoff : 500000   depth2 Avg. Time:1651ms
cutoff : 500000   depth3 Avg. Time:1560ms
cutoff : 500000   depth4 Avg. Time:1600ms
cutoff : 500000   depth5 Avg. Time:1554ms
cutoff : 500000   depth6 Avg. Time:1563ms
cutoff : 500000   depth7 Avg. Time:1497ms
cutoff : 500000   depth8 Avg. Time:1563ms
cutoff : 250000   depth0 Avg. Time:1193ms
cutoff : 250000   depth1 Avg. Time:1124ms
cutoff : 250000   depth2 Avg. Time:1252ms
cutoff : 250000   depth3 Avg. Time:1548ms
cutoff : 250000   depth4 Avg. Time:1478ms
cutoff : 250000   depth5 Avg. Time:1499ms
cutoff : 250000   depth6 Avg. Time:1470ms
cutoff : 250000   depth7 Avg. Time:1548ms
cutoff : 250000   depth8 Avg. Time:1467ms
cutoff : 125000   depth0 Avg. Time:1137ms
cutoff : 125000   depth1 Avg. Time:1137ms
cutoff : 125000   depth2 Avg. Time:1227ms
cutoff : 125000   depth3 Avg. Time:1554ms
cutoff : 125000   depth4 Avg. Time:1460ms
cutoff : 125000   depth5 Avg. Time:1439ms
cutoff : 125000   depth6 Avg. Time:1425ms
cutoff : 125000   depth7 Avg. Time:1457ms
cutoff : 125000   depth8 Avg. Time:1438ms
cutoff : 62500    depth0 Avg. Time:1133ms
cutoff : 62500    depth1 Avg. Time:1164ms
cutoff : 62500    depth2 Avg. Time:1270ms
cutoff : 62500    depth3 Avg. Time:1567ms
cutoff : 62500    depth4 Avg. Time:1483ms
cutoff : 62500    depth5 Avg. Time:1408ms
cutoff : 62500    depth6 Avg. Time:1454ms
cutoff : 62500    depth7 Avg. Time:1464ms
cutoff : 62500    depth8 Avg. Time:1434ms
cutoff : 31250    depth0 Avg. Time:1141ms
cutoff : 31250    depth1 Avg. Time:1137ms
cutoff : 31250    depth2 Avg. Time:1220ms
cutoff : 31250    depth3 Avg. Time:1609ms
cutoff : 31250    depth4 Avg. Time:1525ms
cutoff : 31250    depth5 Avg. Time:1489ms
cutoff : 31250    depth6 Avg. Time:1416ms
cutoff : 31250    depth7 Avg. Time:1426ms
cutoff : 31250    depth8 Avg. Time:1462ms
cutoff : 15625    depth0 Avg. Time:1162ms
cutoff : 15625    depth1 Avg. Time:1200ms
cutoff : 15625    depth2 Avg. Time:1280ms
cutoff : 15625    depth3 Avg. Time:1537ms
cutoff : 15625    depth4 Avg. Time:1462ms
cutoff : 15625    depth5 Avg. Time:1444ms
cutoff : 15625    depth6 Avg. Time:1520ms
cutoff : 15625    depth7 Avg. Time:1586ms
cutoff : 15625    depth8 Avg. Time:1542ms
cutoff : 7812     depth0 Avg. Time:1186ms
cutoff : 7812     depth1 Avg. Time:1208ms
cutoff : 7812     depth2 Avg. Time:1372ms
cutoff : 7812     depth3 Avg. Time:1581ms
cutoff : 7812     depth4 Avg. Time:1635ms
cutoff : 7812     depth5 Avg. Time:1470ms
cutoff : 7812     depth6 Avg. Time:1525ms
cutoff : 7812     depth7 Avg. Time:1550ms
cutoff : 7812     depth8 Avg. Time:1659ms
cutoff : 3906     depth0 Avg. Time:1129ms
cutoff : 3906     depth1 Avg. Time:1146ms
cutoff : 3906     depth2 Avg. Time:1216ms
cutoff : 3906     depth3 Avg. Time:1537ms
cutoff : 3906     depth4 Avg. Time:1594ms
cutoff : 3906     depth5 Avg. Time:1435ms
cutoff : 3906     depth6 Avg. Time:1438ms
cutoff : 3906     depth7 Avg. Time:1487ms
cutoff : 3906     depth8 Avg. Time:1546ms
cutoff : 1953     depth0 Avg. Time:1173ms
cutoff : 1953     depth1 Avg. Time:1131ms
cutoff : 1953     depth2 Avg. Time:1248ms
cutoff : 1953     depth3 Avg. Time:1540ms
cutoff : 1953     depth4 Avg. Time:1464ms
cutoff : 1953     depth5 Avg. Time:1476ms
cutoff : 1953     depth6 Avg. Time:1455ms
cutoff : 1953     depth7 Avg. Time:1519ms
cutoff : 1953     depth8 Avg. Time:1556ms
```

Pei-Ling Chiang (NUID:001533847) - Sec02

For array size 4,000,000

```
Degree of parallelism: 3
cutoff：1000000 depth0  Avg. Time:2728ms
cutoff：1000000 depth1  Avg. Time:2496ms
cutoff：1000000 depth2  Avg. Time:3780ms
cutoff：1000000 depth3  Avg. Time:3292ms
cutoff：1000000 depth4  Avg. Time:3472ms
cutoff：1000000 depth5  Avg. Time:3780ms
cutoff：1000000 depth6  Avg. Time:3435ms
cutoff：1000000 depth7  Avg. Time:3579ms
cutoff：1000000 depth8  Avg. Time:3348ms
cutoff：500000  depth0  Avg. Time:2456ms
cutoff：500000  depth1  Avg. Time:2668ms
cutoff：500000  depth2  Avg. Time:2684ms
cutoff：500000  depth3  Avg. Time:4177ms
cutoff：500000  depth4  Avg. Time:3434ms
cutoff：500000  depth5  Avg. Time:3224ms
cutoff：500000  depth6  Avg. Time:3241ms
cutoff：500000  depth7  Avg. Time:3160ms
cutoff：500000  depth8  Avg. Time:3154ms
cutoff：250000  depth0  Avg. Time:2393ms
cutoff：250000  depth1  Avg. Time:2443ms
cutoff：250000  depth2  Avg. Time:2722ms
cutoff：250000  depth3  Avg. Time:3696ms
cutoff：250000  depth4  Avg. Time:3274ms
cutoff：250000  depth5  Avg. Time:3158ms
cutoff：250000  depth6  Avg. Time:3477ms
cutoff：250000  depth7  Avg. Time:3085ms
cutoff：250000  depth8  Avg. Time:3572ms
cutoff：125000  depth0  Avg. Time:2594ms
cutoff：125000  depth1  Avg. Time:3039ms
cutoff：125000  depth2  Avg. Time:2981ms
cutoff：125000  depth3  Avg. Time:4661ms
cutoff：125000  depth4  Avg. Time:3475ms
cutoff：125000  depth5  Avg. Time:3744ms
cutoff：125000  depth6  Avg. Time:3147ms
cutoff：125000  depth7  Avg. Time:3046ms
cutoff：125000  depth8  Avg. Time:3494ms
cutoff：62500   depth0  Avg. Time:2716ms
cutoff：62500   depth1  Avg. Time:3270ms
cutoff：62500   depth2  Avg. Time:3246ms
cutoff：62500   depth3  Avg. Time:3642ms
cutoff：62500   depth4  Avg. Time:3408ms
cutoff：62500   depth5  Avg. Time:3864ms
cutoff：62500   depth6  Avg. Time:3121ms
cutoff：62500   depth7  Avg. Time:3572ms
cutoff：62500   depth8  Avg. Time:3172ms
cutoff：31250   depth0  Avg. Time:2941ms
cutoff：31250   depth1  Avg. Time:2930ms
cutoff：31250   depth2  Avg. Time:2917ms
cutoff：31250   depth3  Avg. Time:3521ms
cutoff：31250   depth4  Avg. Time:3354ms
cutoff：31250   depth5  Avg. Time:3150ms
cutoff：31250   depth6  Avg. Time:3512ms
cutoff：31250   depth7  Avg. Time:3310ms
cutoff：31250   depth8  Avg. Time:3397ms
cutoff：15625   depth0  Avg. Time:3253ms
cutoff：15625   depth1  Avg. Time:2974ms
cutoff：15625   depth2  Avg. Time:2836ms
cutoff：15625   depth3  Avg. Time:3360ms
cutoff：15625   depth4  Avg. Time:3461ms
cutoff：15625   depth5  Avg. Time:3581ms
cutoff：15625   depth6  Avg. Time:3254ms
cutoff：15625   depth7  Avg. Time:4034ms
cutoff：15625   depth8  Avg. Time:4472ms
cutoff：7812    depth0  Avg. Time:3166ms
cutoff：7812    depth1  Avg. Time:2560ms
cutoff：7812    depth2  Avg. Time:2913ms
cutoff：7812    depth3  Avg. Time:3705ms
cutoff：7812    depth4  Avg. Time:3413ms
cutoff：7812    depth5  Avg. Time:3087ms
cutoff：7812    depth6  Avg. Time:3410ms
cutoff：7812    depth7  Avg. Time:3373ms
cutoff：7812    depth8  Avg. Time:3122ms
cutoff：3906    depth0  Avg. Time:2543ms
cutoff：3906    depth1  Avg. Time:2623ms
cutoff：3906    depth2  Avg. Time:2974ms
cutoff：3906    depth3  Avg. Time:3323ms
cutoff：3906    depth4  Avg. Time:3271ms
cutoff：3906    depth5  Avg. Time:3159ms
cutoff：3906    depth6  Avg. Time:3352ms
cutoff：3906    depth7  Avg. Time:3317ms
cutoff：3906    depth8  Avg. Time:3304ms
cutoff：1953    depth0  Avg. Time:2455ms
cutoff：1953    depth1  Avg. Time:2385ms
cutoff：1953    depth2  Avg. Time:2757ms
cutoff：1953    depth3  Avg. Time:3342ms
cutoff：1953    depth4  Avg. Time:5908ms
cutoff：1953    depth5  Avg. Time:4715ms
cutoff：1953    depth6  Avg. Time:3686ms
cutoff：1953    depth7  Avg. Time:3503ms
cutoff：1953    depth8  Avg. Time:3416ms
```

For array size 8,000,000

```
Degree of parallelism: 3
cutoff：1000000 depth0  Avg. Time:7208ms
cutoff：1000000 depth1  Avg. Time:5437ms
cutoff：1000000 depth2  Avg. Time:8157ms
cutoff：1000000 depth3  Avg. Time:7994ms
cutoff：1000000 depth4  Avg. Time:8009ms
cutoff：1000000 depth5  Avg. Time:7181ms
cutoff：1000000 depth6  Avg. Time:7322ms
cutoff：1000000 depth7  Avg. Time:7395ms
cutoff：1000000 depth8  Avg. Time:6636ms
cutoff：500000  depth0  Avg. Time:4930ms
cutoff：500000  depth1  Avg. Time:5012ms
cutoff：500000  depth2  Avg. Time:5281ms
cutoff：500000  depth3  Avg. Time:7074ms
cutoff：500000  depth4  Avg. Time:6961ms
cutoff：500000  depth5  Avg. Time:6766ms
cutoff：500000  depth6  Avg. Time:7315ms
cutoff：500000  depth7  Avg. Time:6625ms
cutoff：500000  depth8  Avg. Time:6576ms
cutoff：250000  depth0  Avg. Time:5351ms
cutoff：250000  depth1  Avg. Time:5611ms
cutoff：250000  depth2  Avg. Time:6043ms
cutoff：250000  depth3  Avg. Time:7053ms
cutoff：250000  depth4  Avg. Time:7055ms
cutoff：250000  depth5  Avg. Time:6376ms
cutoff：250000  depth6  Avg. Time:6867ms
cutoff：250000  depth7  Avg. Time:6373ms
cutoff：250000  depth8  Avg. Time:7085ms
cutoff：125000  depth0  Avg. Time:5483ms
cutoff：125000  depth1  Avg. Time:5049ms
cutoff：125000  depth2  Avg. Time:5735ms
cutoff：125000  depth3  Avg. Time:7145ms
cutoff：125000  depth4  Avg. Time:6659ms
cutoff：125000  depth5  Avg. Time:6515ms
cutoff：125000  depth6  Avg. Time:9504ms
cutoff：125000  depth7  Avg. Time:8840ms
cutoff：125000  depth8  Avg. Time:6933ms
cutoff：62500   depth0  Avg. Time:5496ms
cutoff：62500   depth1  Avg. Time:5197ms
cutoff：62500   depth2  Avg. Time:5901ms
cutoff：62500   depth3  Avg. Time:7715ms
cutoff：62500   depth4  Avg. Time:7549ms
cutoff：62500   depth5  Avg. Time:7523ms
cutoff：62500   depth6  Avg. Time:7319ms
cutoff：62500   depth7  Avg. Time:6786ms
cutoff：62500   depth8  Avg. Time:7599ms
cutoff：31250   depth0  Avg. Time:5716ms
cutoff：31250   depth1  Avg. Time:5428ms
cutoff：31250   depth2  Avg. Time:6621ms
cutoff：31250   depth3  Avg. Time:6890ms
cutoff：31250   depth4  Avg. Time:6666ms
cutoff：31250   depth5  Avg. Time:6631ms
cutoff：31250   depth6  Avg. Time:7109ms
cutoff：31250   depth7  Avg. Time:7417ms
cutoff：31250   depth8  Avg. Time:7505ms
cutoff：15625   depth0  Avg. Time:5340ms
cutoff：15625   depth1  Avg. Time:5727ms
cutoff：15625   depth2  Avg. Time:5831ms
cutoff：15625   depth3  Avg. Time:7339ms
cutoff：15625   depth4  Avg. Time:7301ms
cutoff：15625   depth5  Avg. Time:7614ms
cutoff：15625   depth6  Avg. Time:7610ms
cutoff：15625   depth7  Avg. Time:7361ms
cutoff：15625   depth8  Avg. Time:7869ms
cutoff：7812    depth0  Avg. Time:5361ms
cutoff：7812    depth1  Avg. Time:5729ms
cutoff：7812    depth2  Avg. Time:6036ms
cutoff：7812    depth3  Avg. Time:7578ms
cutoff：7812    depth4  Avg. Time:6839ms
cutoff：7812    depth5  Avg. Time:6514ms
cutoff：7812    depth6  Avg. Time:7002ms
cutoff：7812    depth7  Avg. Time:6489ms
cutoff：7812    depth8  Avg. Time:6946ms
cutoff：3906    depth0  Avg. Time:5055ms
cutoff：3906    depth1  Avg. Time:5429ms
cutoff：3906    depth2  Avg. Time:5701ms
cutoff：3906    depth3  Avg. Time:7788ms
cutoff：3906    depth4  Avg. Time:7138ms
cutoff：3906    depth5  Avg. Time:6987ms
cutoff：3906    depth6  Avg. Time:6920ms
cutoff：3906    depth7  Avg. Time:7655ms
cutoff：3906    depth8  Avg. Time:7535ms
cutoff：1953    depth0  Avg. Time:8248ms
cutoff：1953    depth1  Avg. Time:6797ms
cutoff：1953    depth2  Avg. Time:6525ms
cutoff：1953    depth3  Avg. Time:6757ms
cutoff：1953    depth4  Avg. Time:6834ms
cutoff：1953    depth5  Avg. Time:6858ms
cutoff：1953    depth6  Avg. Time:7312ms
cutoff：1953    depth7  Avg. Time:9873ms
cutoff：1953    depth8  Avg. Time:9675ms
```

Conclusion:

- Cutoff

I start the cutoff from 1,000,000 and make it half each time till it reaches 1953. There's no significance difference in different cutoff with the same array size and the same depth.

- Recursion depth

Recursion depth d has a relationship with the number of available threads t:

$$t = 2^d$$

Note that given cutoff and array size, the max depth possible is

lg (array size / cutoff)

Any depth larger than this max depth is meaningless because the partitioned arrays already hit the cutoff and turn into system sort.

I set all depth have to be smaller than 9, so every cases will run from depth 0 to depth 8. As I mentioned above, when the partitioned array hits the cutoff, the deeper recursion will be meaningless. For example, assume we have an array with size 2,000,000. If the cutoff is 1,000,000, according to the formula above, the max depth will be 1. That is to say we can ignore the depth larger than 1 in this case because the array has already hit the cutoff and turn to system sort. Detailed computing in Evidence.

I find that in 3 different array size cases, and with different cutoff value. As long as the depth is 0 or 1, the sorting has the greatest performance.

Evidence:

First, according to the formula max depth = lg (array size / cutoff), we can ignore the depth larger than this max depth.

Cutoff	Array Size	Max Depth
1,000,000	2,000,000	1
500,000	2,000,000	2
250,000	2,000,000	3
125,000	2,000,000	4
62,500	2,000,000	5
31,250	2,000,000	6
15,625	2,000,000	7
7,812	2,000,000	8

Cutoff	Array Size	Max Depth
1,000,000	4,000,000	2
500,000	4,000,000	3
250,000	4,000,000	4
125,000	4,000,000	5
62,500	4,000,000	6
31,250	4,000,000	7
15,625	4,000,000	8

Cutoff	Array Size	Max Depth
1,000,000	8,000,000	3
500,000	8,000,000	4
250,000	8,000,000	5
125,000	8,000,000	6
62,500	8,000,000	7
31,250	8,000,000	8

Second, as long as the depth is 0 or 1, the sorting has the greatest performance.
Take array size 2,000,000 for example.

Cutoff	Max Depth	Time(ms)
1,000,000	0	1510
1,000,000	1	1143
500,000	0	1129
500,000	1	1113
250,000	0	1193
250,000	1	1124
125,000	0	1137
125,000	1	1137
62,500	0	1133
62,500	1	1164
31,250	0	1143
31,250	1	1137
15,625	0	1162
15,625	1	1200
7,812	0	1186
7,812	1	1208

3,906	0	1129
3,906	1	1146
1,953	0	1173
1,953	1	1131