Appendix: On standard Deviations of the Parameter Estimates

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The algorithm for obtaining the variances of the estimates of parameters is outlined below.

1. For a given data set y, obtain MLE estimates $\hat{\theta}$ for the vector of unknown parameters.
2. Let $\theta = \hat{\theta}$. Generate a new data set.
3. For the data $\tilde{y}$ and parameters $\hat{\theta}$, obtain the score vector numerically. That is, given an increment h, the score is $\frac{\mathcal{G}(\theta+h)-\mathcal{G}(\theta)}{h}$, where $\mathcal{G}$ is the score vector.
4. Let $\mathcal{F}_i = \mathcal{G}\mathcal{G}'$ be the product of the score vector.
5. Repeat step 2 - 4 M times, we get $\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_M$. Get the expectation of these $\mathcal{F}_i$'s to obtain the Fisher Information Matrix $\mathcal{I}$.
6. Take the inverse of $\mathcal{I}$. This would be the asymptotic covariance matrix of the parameter estimates.

This algorithm was used in Table 1 for evaluation of the standard errors of the estimate of θ.

Table 1: Negative log-likelihood (NLL)

Period		κ	γ	μ_ξ	σ_χ	σ_ξ	ρ	λ_χ	λ_ξ	s_1	s_2	NLL
2001-2005	Init.Val	2.2500	0.7507	-2.7500	0.7575	0.7575	0.5000	-2.7500	-2.7500	0.0200	0.0200	155678
	Est	1.5123	0.0552	**0.1933**	0.3029	0.2193	0.4212	**0.0741**	**0.1035**	0.0209	0.0037	-48566
	Std.Error	0.0108	0.0026	0.1192	0.0078	0.0061	0.0190	0.1517	0.1184	4.39E-04	2.17E-05	
2005-2009	Init.Val	2.2500	0.7507	-2.7500	0.7575	0.7575	0.5000	1.7500	-2.7500	0.0200	0.0200	1639264
	Est	1.1708	**0.0010**	**0.0034**	0.2473	0.2824	0.5729	**-0.1907**	**0.0921**	0.0183	0.0032	-50741
	Std.Error	0.0092	0.0016	0.1533	0.0064	0.0078	0.0180	0.1283	0.1518	4.35E-04	1.87E-05	
2014-2018	Init.Val	0.7500	0.7507	1.7500	2.2525	0.7575	0.5000	1.7500	-2.7500	0.0200	0.0200	7878181
	Est	1.1114	**0.0011**	**0.0117**	0.2519	0.2807	0.5725	**0.2014**	**0.0936**	0.0139	0.0028	-52455
	Std.Error	0.0087	0.0016	0.1507	0.0068	0.0067	0.0180	0.1321	0.1526	3.60E-04	1.70E-05	