Appendix: On standard Deviations of the Parameter Estimates

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The algorithm for obtaining the variances of the estimates of parameters is outlined below.

1. For a given data set y, obtain MLE estimates $\hat{\theta}$ for the vector of unknown parameters.
2. Let $\theta = \hat{\theta}$. Generate a new data set.
3. For the data $\tilde{y}$ and parameters $\hat{\theta}$, obtain the score vector numerically. That is, given an increment h, the score is $\frac{\mathcal{G}(\theta+h)-\mathcal{G}(\theta)}{h}$, where $\mathcal{G}$ is the score vector.
4. Let $\mathcal{F}_i = \mathcal{G}\mathcal{G}'$ be the product of the score vector.
5. Repeat step 2 - 4 M times, we get $\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_M$. Get the expectation of these $\mathcal{F}_i$'s to obtain the Fisher Information Matrix $\mathcal{I}$.
6. Take the inverse of $\mathcal{I}$. This would be the asymptotic covariance matrix of the parameter estimates.

This algorithm was used in Table 1 for evaluation of the standard errors of the estimate of θ.

Table 1: Negative log-likelihood (NLL); standard errors are given in parentheses; n is the sample size.

Period	κ	γ	μ_ξ	σ_χ	σ_ξ	ρ	λ_χ	λ_ξ	s_1	s_2	NLL
2001-2005	1.5117	0.0558	-0.0502	0.3036	0.2201	0.4222	-4.0223	0.0093	0.0209	0.0037	-48562
(n = 993)	(0.0097)	(0.0023)	(0.0057)	(0.0205)	(0.0014)	(0.0079)	(4.70E-04)	(0.0014)	(5.20E-05)	(2.20E-05)	
2005-2009	1.2087	0.0027	-0.9515	0.2088	0.2811	0.3062	0.6292	-0.8723	0.0181	0.0032	-50717
(n = 1004)	(0.0103)	(0.0236)	(0.0236)	(0.0299)	(0.0071)	(0.0050)	(4.10E-04)	(0.0016)	(5.30E-05)	(1.80E-05)	
2014-2018	1.1293	0.0046	-3.4150	0.2441	0.2389	0.4530	-3.5956	-3.3445	0.0133	0.0029	-52450
(n = 1007)	(0.0081)	(0.0140)	(0.0222)	(0.0065)	(0.0059)	(0.0140)	(5.70E-05)	(0.0018)	(3.20E-04)	(1.50E-05)	