## Parameters STDs

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The algorithm for obtaining the variances of the estimates of parameters is outlined below.

- 1. For a given data set y, get MLE estimates  $\hat{\theta}$  for unknown parameters  $\theta$ .
- 2. Let  $\theta = \hat{\theta}$ . Generate a new data set.
- 3. For the data  $\tilde{y}$  and parameters  $\hat{\theta}$ , obtain the score vector numerically. That is, given an increment h, the score is  $\frac{\mathcal{G}(\theta+h)-\mathcal{G}(\theta)}{h}$ .
- 4. Let  $\mathcal{F}_i = \mathcal{GG}'$  be the product of the score vector and its transpose.
- 5. Repeat step 2 4 M times, we get  $\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_M$ . Get the expectation of these  $\mathcal{F}_i$ 's. to obtain the Fisher Information Matrix  $\mathcal{I}$ .
- 6. Take the inverse of  $\mathcal{I}$ . This would be the variance-covariance matrix.

This algorithm was used in Table 1 for evaluation of the standard errors of the estimates of the parameters.

Table 1: Estimation output: NLL for negative log-likelihood; standard errors are given in parentheses

| Period    | κ                  | $\gamma$           | $\mu_{\xi}$         | $\sigma_\chi$      | $\sigma_{\xi}$     | ρ                  | $\lambda_\chi$        | $\lambda_{\xi}$     | $s_1$                | $s_2$                | NLL    |
|-----------|--------------------|--------------------|---------------------|--------------------|--------------------|--------------------|-----------------------|---------------------|----------------------|----------------------|--------|
| 2001-2005 | 1.5117<br>(0.0097) | 0.0558<br>(0.0023) | -0.0502<br>(0.0057) | 0.3036<br>(0.0205) | 0.2201<br>(0.0014) | 0.4222<br>(0.0079) | -4.0223<br>(4.70E-04) | 0.0093<br>(0.0014)  | 0.0209<br>(5.20E-05) | 0.0037<br>(2.20E-05) | -48562 |
| 2005-2009 | 1.2087<br>(0.0103) | 0.0027<br>(0.0236) | -0.9515<br>(0.0236) | 0.2088<br>(0.0299) | 0.2811<br>(0.0071) | 0.3062<br>(0.0050) | 0.6292<br>(4.10E-04)  | -0.8723<br>(0.0016) | 0.0181<br>(5.30E-05) | 0.0032<br>(1.80E-05) | -50717 |
| 2014-2018 | 1.1293<br>(0.0081) | 0.0046<br>(0.0140) | -3.4150<br>(0.0222) | 0.2441<br>(0.0065) | 0.2389<br>(0.0059) | 0.4530<br>(0.0140) | -3.5956<br>(5.70E-05) | -3.3445<br>(0.0018) | 0.0133<br>(3.20E-04) | 0.0029<br>(1.50E-05) | -52450 |