1. (Detecting defects)

(a)

Proof. Let D to denote the event that at least one defective cookie is found. Then

$$
\begin{aligned}
Pr[D] &= 1 - (1-p)^k & (1)\\
&\geq 1 - (1-\alpha)^k & (2)\\
&\geq 1 - (1-\alpha)^{\frac{\ln 100}{\alpha}} & (3)\\
&\geq 1 - (e^{-\alpha})^{\frac{\ln 100}{\alpha}} & (4)\\
&\geq 1 - e^{\ln 1/100} & (5)\\
&\geq 1 - 0.01 = 0.99 & (6)
\end{aligned}
$$

Note that from (1) to (2), we have used the inequality $p \geq \alpha$; from (2) to (3), we have used the inequality $k \geq \frac{\ln 100}{\alpha}$; and from (3) to (4), we have used the inequality $1 - \alpha \leq e^{-\alpha}$.

(b) Solution: Let p_i be the proportion of detective cookies the i^{th} worker baked; Let D_i be the event that if $p_i \geq \alpha$, then at least one detective cookie is found for that person; Let D be the event that for all unreliable workers, at least one defective cookie is found.

$$Pr[D] = \sum_{i=1}^{n} Pr[] \tag{7}$$