1. (Geometric distribution)

(a) (**In parallel**) Solution: From the description of the problem, we know that $X_1 \sim Geom(p1)$ and $X_2 \sim Geom(p2)$. We can write the two distributions as follows,

$$Pr[X_1 = i] = (1 - p_1)^{i-1} p_1 \text{ for } i \geq 1$$
$$Pr[X_2 = i] = (1 - p_2)^{i-1} p_2 \text{ for } i \geq 1$$

For $X = \min\{X_1, X_2\}$, we can consider an imaginary new machine M, for each run, it is fails when either M_1 or M_2 fails. Thus, the new machine will have a failure probability $p = 1 - (1 - p_1)(1 - p_2)$. Thus, $X \sim Geom(1 - (1 - p_1)(1 - p_2))$.

$$Pr[X = i] = \Big[(1 - p_1)(1 - p_2)\Big]^{i-1}\Big(1 - (1 - p_1)(1 - p_2)\Big) \text{ for } i \geq 1.$$

(b) (**007 stype**) Solution: Let X_1 and X_2 denote the number of times that James Bond will choose the air-conditioning duct and the sewer pipe respectively before choosing the unlocked door, and $X = X_1 + X_2$ is the number of wrong choices before choosing the unlocked door. Then $(X + 1) \sim Geom(1/3)$.

$$\begin{aligned} Pr[X = i] &= (1 - 1/3)^i (1/3) \\ &= (2/3)^i (1/3) \\ \mathbb{E}(X) &= 1/(1/3) - 1 = 2 \end{aligned}$$

Since the air-conditioning and the sewer pipe have an equal probability of being chosen, then $\mathbb{E}(X_1) = \mathbb{E}(X_1) = \mathbb{E}(X)/2 = 1$. Then the expected time needed before choosing the door is $2\mathbb{E}(X_1) + 5\mathbb{E}(X_2) = 7$ hours.