

### 3. (Randomized Routing on the Hypercube)

(a)

*Proof.* In Phase 1 of the algorithm, assuming we are using bit-fixing algorithm. Let  $P_i$  and  $P_j$  be two paths that was separated at step  $k$ , which means that at their point of separation, they share bits from bit 1 to bit  $k - 1$ , and different in bit  $k$ . Assuming first that they joined at a later step  $l > k$ , then based on the bit-fixing algorithm, the first  $l$  bits should be the same, which contradicts the assumption that they different at bit  $k < l$ . Thus, they could not rejoin after they separates.

(b) Solutions: No. (a) doesn't imply that for any two packets  $i$  and  $j$ , there is at most one node such that  $i$  and  $j$  are waiting in queue.

Just consider the case that after the two bits joined at step  $k$ , and later at step  $l$ , they separate ways. If their targets share bits from  $k$  to  $l - 1$  and that  $l - k \geq 2$ , then they could wait in queue at nodes from  $k$  all the way to  $l - 1$ , which is more than one node.