2. (Random vectors)

(a) Solutions: $\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{d} a_i b_i$. Let random variable $X_i = a_i b_i$, so $\sum_{i=1}^{d} X_i = \mathbf{a} \cdot \mathbf{b}$. It is obvious that $Pr[X_i = 1/d] = Pr[-1/d] = 1/2$. Thus, $\mathbf{E}[X_i] = 0$. Using Hoeffding's Bounds,

$$
\begin{aligned}
Pr[|\mathbf{a} \cdot \mathbf{b}| > 1/10] &\leq Pr[|\mathbf{a} \cdot \mathbf{b}| \geq 1/10] && (1)\\
&= Pr[|\sum_{i=1}^{d} X_i| \geq 1/10] && (2)\\
&= Pr\big[|\frac{1}{d}\sum_{i=1}^{d} X_i| \geq 1/(10d)\big] && (3)\\
&\leq 2e^{-2d(\frac{1}{10d})^2/\frac{2}{d}^2} && (4)\\
&= 2e^{-d/200} && (5)
\end{aligned}
$$

(b) Solutions: Let $\mathbf{v}_i$ be the i_{th} vector chosen and I_{ij} where $i < j$ be the indicator random variable that $\mathbf{v}_i$ and $\mathbf{v}_j$ are not 1/10-close to being orthogonal. From (1), we have shown that for any $i < j$, $Pr[|\mathbf{v}_i \cdot \mathbf{v}_j| > 1/10] = Pr[I_{ij} = 1] \leq 2e^{-d/200}$. Using union bound

$$
\begin{aligned}
Pr\big[\sum_{i,j} I_{ij} > 0\big] &\leq \sum_{i<j} Pr[I_{ij} = 1] && (6)\\
&\leq \sum_{i<j} 2e^{-d/200} && (7)\\
&\leq \binom{k}{2} 2e^{-d/200} && (8)\\
&= k(k-1)e^{-d/200} && (9)
\end{aligned}
$$

Let the fail probability in (9) to be less or equal than a constant δ, $k(k-1)e^{-d/200} \leq \delta$, then we can solve for $k = O(e^{d/200})$.