1. Probability Review

Thirteen cards are drawn without replacement from a standard deck of 52 cards. What is the probability that:

a) they are all spades?

$$Pr[all\ spades] = 1 \Big/ \binom{52}{13} \tag{1}$$

$$\approx 1.57 \times 10^{-12} \tag{2}$$

b) they are all black?

$$Pr[all\ black] = \binom{26}{13} \Big/ \binom{52}{13} \tag{3}$$

$$= 0.00001637854 \tag{4}$$

c) they are not all of one color, given that none of the cards is an ace?

$$Pr[not\ of\ one\ color \mid none\ is\ ace] = \left(\binom{48}{13} - \binom{24}{13} - \binom{24}{13} \right) \Big/ \binom{48}{13} \tag{5}$$

$$= 0.9999741236 \tag{6}$$

d) none of the cards is an ace and none is a heart?

$$Pr[none\ ace\ and\ none\ heart] = \binom{36}{13} \Big/ \binom{52}{13} \tag{7}$$

$$= 0.00363896103 \tag{8}$$

e) there are 5 cards of one suit and 8 card of another suit?

$$Pr[five\ of\ one\ suit\ and\ eight\ of\ another\ suit] = \left(\binom{4}{1}\binom{13}{5} \times \binom{3}{1}\binom{13}{8} \right) \Big/ \binom{52}{13} \tag{9}$$

$$= 0.00003130079 \tag{10}$$