1. (Detecting Defects)

(a)

Proof. Let D to denote the event that at least one defective cookie is found. Then

$$
\begin{aligned}
Pr[D] &= 1 - (1-p)^k && (1)\\
&\geq 1 - (1-\alpha)^k && (2)\\
&\geq 1 - (1-\alpha)^{\frac{\ln 100}{\alpha}} && (3)\\
&\geq 1 - (e^{-\alpha})^{\frac{\ln 100}{\alpha}} && (4)\\
&\geq 1 - e^{\ln 1/100} && (5)\\
&\geq 1 - 0.01 = 0.99 && (6)
\end{aligned}
$$

Note that from (1) to (2), we have used the inequality $p \geq \alpha$; from (2) to (3), we have used the inequality $k \geq \frac{\ln 100}{\alpha}$; and from (3) to (4), we have used the inequality $1 - \alpha \leq e^{-\alpha}$.

(b) Solution: Let p_i be the probability that worker i will have a defect, D be the event that if all unreliable workers are caught and let F_i be the event that the test fails to catch worker i.

$$
\begin{aligned}
Pr[\cup_{i=1}^n F_i] &\leq \sum_{i=1}^{n} Pr[F_i] && (7)\\
Pr[D] &= 1 - Pr[\cup_{i=1}^n F_i] && (8)\\
&\geq 1 - \sum_{i=1}^{n} Pr[F_i] && (9)\\
&= 1 - \sum_{i=1}^{n} (1-p_i)^k && (10)\\
&\geq 1 - n(1-\alpha)^k && (11)
\end{aligned}
$$

From (10) to (11), we've used the fact that $p_i \geq \alpha$ if worker i is unreliable. In order for $Pr[D]$ to be at least 99%, then

$$1 - n(1-\alpha)^k \geq 99\%$$

Solving for k,

$$
\begin{aligned}
1 - n(1-\alpha)^k &\geq 0.99 && (12)\\
n(1-\alpha)^k &\leq 0.01 && (13)\\
(1-\alpha)^k &\leq 0.01/n && (14)\\
k &\geq \log_{1-\alpha}(0.01/n) && (15)
\end{aligned}
$$