1. (Chess board)

(a) Solution: The events A and B are not independent. $P(A \cap B) = 0 \neq P(A)P(B) = 1/64 \times 1/64$.

(b) Solution: The events A and B are independent. $P(A \cap B) = 1/4 = P(A)P(B) = 1/2 \times 1/2$.

(c) Solution: The events A and B are independent. $P(A \cap B) = 16/64 = 1/4 = P(A)P(B) = 1/2 \times 1/2$.

(d) Solution: The events A, B and C are not independent. Assuming the orientation is fixed, and the first square is black, then all the squares located in even numbered rows as well as even numbered columns are black, thus

$$P(A \cap B \cap C) = 0$$

$$\begin{aligned} P(A)P(B)P(C) &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \\ &\neq P(A \cap B \cap C) \end{aligned}$$

The above proof is also true when we choose the orientation of the board such that the first square is white in which case $P(A \cap B \cap C) = 1 \neq P(A)P(B)P(C)$.