2. (Multiple-choice test)

(a) Solution: Let K and $\overline{K}$ be the events that Adam knows and doesn't know the answer respectively, and events C and $\overline{C}$ be the events that Adam answer correctly and incorrectly respectively. Using Bayes's Rule:

$$
\begin{aligned}
Pr[K|C] &= \frac{Pr[C|K]Pr[K]}{Pr[C|K]Pr[K] + Pr[C|\overline{K}]Pr[\overline{K}]} && (1)\\
&= \frac{p \times 1}{p \times 1 + (1-p) \times 1/m} && (2)\\
&= \frac{mp}{(m-1)p+1} && (3)
\end{aligned}
$$

(b) Solution: $Pr[K|C] = \frac{mp}{(m-1)p+1} = \frac{5 \times 0.6}{(5-1) \times 0.6+1} \approx 0.88$.

(c) Solution: In addition to notations from (a), let E and N be the event that Bella can eliminate but two answers, and the event that Bella doesn't know the answer, then using Bayes's rule,

$$
\begin{aligned}
Pr[K|C] &= \frac{Pr[C|K]Pr[K]}{Pr[C|K]Pr[K] + Pr[C|E]Pr[E] + Pr[C|N]Pr[N]} && (4)\\
&= \frac{1 \times p_1}{1 \times p_1 + 0.5 \times p_2 + \frac{1}{m} \times (1 - p_1 - p_2)} && (5)\\
&= \frac{2mp_1}{2mp_1 + mp_2 + 2(1 - p_1 - p_2)} && (6)
\end{aligned}
$$

(d) Suppose $m = 5$ and $p_2 = 0.1$, then (6) simplifies to $\frac{10p_1}{10p_1+2.3-2p_1}$, we can solve for $p_1 = 0.69$.