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each element in the correct bucket.
- concatenate the lists.

If $r > n$?

Theorem: Suppose that n divides r.
if we choose n integers u.i.d from range r. then can sort in expected $O(n)$ time

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m balls into n bins

The probability that bin 1 is empty:

$$\left(1-\frac{1}{n}\right)^{m} \approx (5) \; e^{-\frac{m}{n}}$$

Probability p_r that bin 1 has r balls is

$$P_r = \binom{m}{r}\left(\frac{1}{n}\right)^{r}\left(1-\frac{1}{r}\right)^{m-r}.$$

$$= \frac{1}{r!}\left(\frac{m}{n}\cdot\frac{m-1}{n}\cdots\frac{m-r+1}{n}\right)\left(1-\frac{1}{n}\right)^{m-r}.$$

$$\approx \frac{1}{r!}\left(\frac{m}{n}\right)^{r}\cdot e^{-\frac{m}{n}},$$

$$P_r \approx \frac{\mu^{r}\cdot e^{-\mu}}{r!} \qquad \text{where } \mu = \frac{m}{n}$$

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Poisson Random variable.

Theorem:
Let X & Y be independent RV. with mean μ_X & μ_Y.
$\Rightarrow X+Y$ is a Poisson RV with mean $\mu_X + \mu_Y$.

The Poisson Approximation

m balls into n bins u-i-r.
For $i \in [n]$, let $X_i^{(m)}$ = # of balls in Bin i
$Y_i^{(m)} \sim$ Poisson with $\mu \leq \frac{m}{n}$.

where $Y_i^{(m)}$ are mutually indep.

Suppose we condition the Poisson distribution on producing exactly k balls. Then it is the same as the distribution resulting from throwing k balls to n bins.

Theorem: The distribution of $(Y_1^{(m)}, \ldots Y_n^{(m)})$ conditioned on $\sum Y_i^{(m)} = k$ is the same as $(X_1^{(k)}, \ldots X_n^{(k)})$ regardless of m.

Proof Consider $k_1, \ldots k_n$ satisfying

$$\sum_{i \in [n]} k_i = k.$$

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$$\Pr\left[\left(X_1^k, \ldots X_n^k\right) = (k_1, \ldots k_n)\right] \text{ is.}$$

$$= \frac{\binom{k}{k_1, k_2 \ldots k_n}}{n^k} \longrightarrow \text{multinomial coefficient}$$

$$= \frac{\frac{k!}{k_1! \, k_2! \cdots k_n!}}{n^k}$$

$$\Pr\left[Y_1^{(m)}, \ldots Y_n^{(m)} = (k_1, \ldots k_n)\right] \Big| \sum_{i \in [n]} Y_i^{(m)} = k\Big)$$

$$= \frac{\Pr\left[Y_1^{(m)} = k_1 \cap Y_2^{(m)} = k_2 \wedge \ldots Y_n^{(m)} = k_n\right]}{\Pr\left[\sum_{i \in n} Y_i^{m} = k\right]}$$

$$= \frac{\prod_{i \in [n]} \cancel{e^{-\frac{m}{n}}} \cdot \left(\frac{\cancel{m}}{n}\right)^{k_i} / k_i!}{\cancel{e^{-m}} \cdot \cancel{m^k} / k!}$$