CS 537: *Randomness in Computing*
Boston University

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Problems

1. **(Number of Min-Cuts)** Our proof of MU's Theorem 1.8 (bounding the error of the randomized min-cut algorithm) proved a slightly stronger statement: for any min-cut set, the algorithm returns that set with probabilty at least $\frac{2}{n(n-1)}$. Use this fact to upper bound the number of possible min-cut sets in a graph.

$Pr[\text{any min-cut set}] \le 1$

k mincut set $= C_1 \cdots C_k$

$Pr[\bigcup_{i=1}^{k} \text{return } C_i] = \sum_{i=1}^{k} Pr[\text{return } C_i] \ge k \frac{2}{n(n-1)}$

$k \frac{2}{n(n-1)} \le 1$

$k \le \frac{n(n-1)}{2} = \binom{n}{2}$

2. **(Minimum 3-Way Cut-Sets)** Define a *3-way cut-set* as a set of vertices whose removal creates the graph into 3 or more connected components. Explain how we can use our randomized 2-way min-cut algorithm to solve this problem, and bound its probability of error. [*Note: the analysis is not easy. Start by figuring out why the previous analysis doesn't work.*]