On the max-cut of sparse random hypergraphs

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(*joint work with* Dmitry Shabanov)

We consider the problem of estimating maximum q-cut in a random k-uniform sparse hypergraph $H(n,k,p)$. Recall that the max-q-cut of a hypergraph $H=(V,E)$ is the maximum number of edges which can be colored properly with q colors. The problem of estimating MAX-q-CUT$(H(n,k,p))$, the maximum q-cut of $H(n,k,p)$, is the most interesting in the sparse case, when the expected number of edges in a linear function of n, i.e. $p\binom{n}{k}=cn$ for fixed $c>0$. Our main result proves the existence of a limit constant for max-q-cut of $H(n,k,p)$ and gives some estimates for it.

Theorem 1. *Let $H(n,p,k)$ be a k-uniform random hypergraph with n vertices and probability $p(n)=\frac{c\cdot n}{\binom{n}{k}}$ for each edge, where $c>0$ is fixed. Then there exists $\gamma(c,k,q)$ such that*

$$\frac{\text{MAX-q-CUT}(H(n,k,p))}{n}\xrightarrow{\mathrm{P}}\gamma(c,k,q).$$

Theorem 2. *In the context of 1, with probability tending to 1 as $n\to+\infty$,*

$$\text{MAX-q-CUT}(H(n,p,k))\leqslant n\cdot\left(c\left(1-q^{1-k}\right)+\sqrt{c}\cdot A_1(k)+o(\sqrt{c})\right)+o(n);$$

$$\text{MAX-q-CUT}(H(n,p,k))\geqslant n\cdot\left(c\left(1-q^{1-k}\right)+\sqrt{c}\cdot A_2(k)+o(\sqrt{c})\right)+o(n);$$

where

$$A_1=\sqrt{\frac{(q^{k-1}-1)\cdot\log q\cdot 2}{q^{2k-2}}};\quad A_2=\frac{2\cdot\sqrt{2\ln q}}{k+1}\cdot\sqrt{\frac{k}{q^{k-1}}}.$$