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Topic: Homework Three
Date: {{date:MMM d, YYYY}}
Course: C241

Question 1 (a)

Claim: $(P \land Q) \to R, P \land S, \neg\neg Q \vdash R$

PF:

Assume $P \land S$ and $\neg\neg Q$,
This would imply Q (dbl. neg)
This would imply P and S (simp.)
Now $P \land Q$ (conj.)
Since $(P \land Q)$ is true, R is true (appl.) ■

Question 1 (b)

Claim: $(K \lor L) \to N, K \land M \vdash N \land M$

PF:

Assume $K \land M$,
This would imply M and K (simp.)
($K \lor L$) now holds (weak.)
Which implies N (appl.)
From N, we derive $N \land M$ (conj.) ■

Question 1 (c)

Claim: $A \land \neg\neg B \vdash B \lor (A \to \neg C)$

PF:

Assume $A \land \neg\neg B$
This would imply A and $\neg\neg B$ (simp.)
$\neg\neg B$ would imply B (dbl. neg)
B would imply $B \lor (A \to \neg C)$ (weak.) ■

Question 1 (d)

Claim: $(A \wedge B) \to C, B, A \wedge \neg D \vdash C \wedge \neg D$

PF:

Assume $A \wedge \neg D$ and B
This would imply A and $\neg D$ (simp.)
$(A \wedge B)$ holds (conj.)
Which implies C (appl.)
C would then imply $C \wedge \neg D$ (conj.) ■

Question 1 (e)

Claim $(A \wedge B) \to C, B \vdash (A \wedge \neg D) \to (C \wedge \neg D)$

PF: (Direct Proof)

Assume $(A \wedge B)$ and $\neg D$
This implies A and B (simp.)
And implies C (appl.)
With $\neg D$, we see that $(A \wedge \neg D)$ (conj.)
Aswell as $(C \wedge \neg D)$ (conj.)
Which concludes $(A \wedge \neg D) \to (C \wedge \neg D)$ (dir. pf) ■

Question 1 (f)

Claim: $\neg P \to (Q \wedge R) \vdash (\neg P \wedge S) \to (R \wedge S)$

PF:

Assume $\neg P$ and S
$\neg P$ would imply $(Q \wedge R)$ (appl.)
Which would imply Q and R (simp.)
S would imply $(\neg P \wedge S)$ (conj.)
And would also imply $(R \wedge S)$ (conj.)
These truths would imply $(\neg P \wedge S) \to (R \wedge S)$ (dir. pf)

Question 1 (g)

Claim: $X \wedge (X \to (Z \wedge Y)) \vdash X \wedge Y$

PF: Direct Proof

Assume $X \wedge (X \to (Z \wedge Y))$
Then X and $(X \to (Z \wedge Y))$ (simp.)
Since X, this would imply $(Z \wedge Y)$ (appl.)

Which would then imply Y and Z (simp.)
Since X and Y, $(X \wedge Y)$ holds (conj.) ■

Question 1 (h)

Claim: $\vdash (X \wedge (X \rightarrow (Z \wedge Y))) \rightarrow (X \wedge Y)$

PF:

Assume $(X \wedge Y)$
This would imply X and Y (simp.)
We can then imply $(Z \wedge Y)$ (appl.)
Which would verify $X \rightarrow (Z \wedge Y)$ (dir. pf)
Now we can say that $X \wedge (X \rightarrow (Z \wedge Y))$ (conj.)
Which concludes $(X \wedge (X \rightarrow (Z \wedge Y))) \rightarrow (X \wedge Y)$ (dir. pf) ■

Question 1 (i)

Claim: $P \wedge Q \vdash \neg(P \rightarrow \neg Q)$

PF:

Assume $(P \rightarrow \neg Q)$, and $P \wedge Q$
$P \wedge Q$ implies P and Q (simp.)
And P would imply $\neg Q$ (appl.)
Q and $\neg Q$ cannot stand, therefore we get $\neg(P \rightarrow \neg Q)$ (condic.) ■

Question 1 (j)

Claim: $(W \wedge X) \rightarrow \neg Y, X \vdash \neg(W \wedge Y)$

PF:

Assume $(W \wedge Y)$ and X
This implies W and Y (simp.)
And now $(W \wedge X)$ (conj.)
Which implies $\neg Y$ (appl.)
Y and $\neg Y$ cannot stand, therefore we get $\neg(W \wedge Y)$ (condic.) ■

Question 2

Claim: $P \rightarrow \neg Q, \neg Q \vdash \neg\neg P$

PF:

Assume $\neg Q$, $\neg(P \to \neg Q)$, and $\neg\neg P$

We can see P (dbl. neg)

Now $P \to \neg Q$ (dir pf.)

Which implies $(P \to \neg Q) \wedge \neg(P \to \neg Q)$ (conj.)

Since this is always false, the claim is disproven