Topic: homework 4
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Course: C241

Question one

(a) - CLAIM: $A \to \neg B, \neg\neg B \vdash \neg(A \to B)$

PF:

This claim is false because truth assignment A = false B = false indicates that the premises are true, while the conclusion is false.

(b) - CLAIM: $\vdash (M \to \neg N) \to \neg(M \wedge N)$

PF:

Assume $(M \to \neg N)$
    Suppose $(M \wedge N)$ (towards contradiction)
        This would imply M and N (simp.)
        M would indicate $\neg N$ (appl.)
        Since N and $\neg N$ cannot stand, $\neg(M \wedge N)$ holds (condic.)
■

(c) - CLAIM: $Z \to A, \neg A, \vdash \neg Z$

PF:

Assume $Z \to A$ and $\neg A$
    Assume Z (towards contradiction)
        This would imply A (appl.)
        Since A and $\neg A$ cannot stand, $\neg Z$ (condic.)
■

(d) - CLAIM: $Z \to A, \neg A, A \to B \vdash \neg Z \wedge \neg B$

PF:

This Claim is false because with assignment A = False, Z = False, and B = True, the propositions will be satisfied, but the conclusion will not be.

(e) - CLAIM: $Z \to A, B \to A, \neg A \vdash \neg Z \wedge \neg B$

PF:

Assume $\neg A$, $B \to A$, and $Z \to Z$
    Suppose B (towards contradiction)
        This would imply A (appl.)
        Since A and $\neg A$ conflict, $\neg B$ (condic.)

Suppose Z (towards contradiction)
This would indicate A
Since A and $\neg A$ conflict, $\neg Z$ (condic.)
Because $\neg B$ and $\neg Z$, we can conclude $\neg B \wedge \neg Z$
■

(f) - CLAIM: $P \vee (Q \wedge \neg\neg R), P \rightarrow R \vdash R$

PF:

Assume $P \vee (Q \wedge \neg\neg R)$ and $P \rightarrow R$
Case one: Assume P
This would imply R (appl.)
Case two: Assume $Q \wedge \neg\neg R$
This would imply Q and $\neg\neg R$ (simp.)
Which would then imply R (dbl. neg)
R is true in both cases, therefore R (cases)
■

(g) - CLAIM: $(F \wedge G) \vee (H \rightarrow I), H \vdash G \vee I$

PF:

Assume $(F \wedge G) \vee (H \rightarrow I)$ and H
Case one: Assume $(F \wedge G)$
This would imply F and G (simp.)
Which would imply $G \vee I$ (weak.)
Case two: Assume $(H \rightarrow I)$
This would imply I due to H (appl.)
Which would imply $G \vee I$ (weak.)
$G \vee I$ is true in both cases, therefore $G \vee I$ (cases)
■

(h) - CLAIM: $X \rightarrow Y \vdash (X \vee Z) \rightarrow Y$

PF:

This claim is incorrect because truth assignment X = false, Y = false, Z = true will satisfy the propositions but not the conclusion.

(i) - CLAIM: $J \rightarrow K \vdash (J \vee \neg\neg K) \rightarrow K$

PF:

Assume $J \rightarrow K$
Suppose $J \vee \neg\neg K$
Case one: Suppose J
This would indicate K (appl.)
Case two: Suppose $\neg\neg K$
This would indicate K (dbl. neg)
Both cases give K, therefore K
With both $J \vee \neg\neg K$ and K, we conclude $(J \vee \neg\neg K) \rightarrow K$ (dir. pf)
■

(j) - CLAIM: $(A \wedge C) \rightarrow D, A \wedge \neg B \vdash \neg(A \rightarrow B) \wedge (C \rightarrow D)$

PF:

Assume $(A \wedge C) \rightarrow D$ and $A \wedge \neg B$
From assumption two, we conclude A and $\neg B$ (simp.)
    Assume C
    We can say $C \wedge A$ (conj.)
    Which then concludes D (appl.)
From the above subproof, we see $C \rightarrow D$ (dir. pf)
    Now Assume $A \rightarrow B$
    With A, we imply B (appl.)
Because B and $\neg B$ cannot stand, we conclude $\neg(A \rightarrow B)$ (condic.)
With both $C \rightarrow D$ and $\neg(A \rightarrow B)$, we see $\neg(A \rightarrow B) \wedge (C \rightarrow D)$ (conj.)

Question two

(a) - Line two is incorrect because $\vee - Elim$ must be applied through a subproof, which this proof does not do.

(b) - Line six is incorrect because you cannot reuse a subproof formula outside of the subproof as part of a direct proof.

(c) - This proof is valid.

(d) - Line 7 is incorrect because they are incorrectly introducing the $\neg$ on the wrong formula.

(e) - This proof is valid.

(f) - This proof is valid, though it is awful to read.

Question three

(a) - CLAIM: $A \rightarrow (B \wedge C) \equiv (A \rightarrow B) \wedge (A \rightarrow C)$

PF::
subproof claim: $A \rightarrow (B \wedge C) \vdash (A \rightarrow B) \wedge (A \rightarrow C)$
Assume $A \rightarrow (B \wedge C)$ and A
This would imply $B \wedge C$ (appl.)
Which would imply B and C (simp.)
With A and C, we can say $A \rightarrow C$ (dir. pf)
With A and B, we can say $A \rightarrow B$ (dir. pf)
Which concludes $(A \rightarrow B) \wedge (A \rightarrow C)$
■

subproof2 claim: $(A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow (B \wedge C)$
Assume $(A \rightarrow B) \wedge (A \rightarrow C)$ and A
This implies $A \rightarrow B$ and $A \rightarrow C$
Since A, B (appl.)
Since A, C (appl.)
B and C imply $B \wedge C$ (conj.)

Conclusively, $A \to (B \land C)$
■

(b) - CLAIM: $A \to (B \land C) \equiv (A \to B) \land (A \to C)$

EQUIVALENCE PF::
$A \to (B \land C) \equiv (A \to B) \land (A \to C)$
$\neg A \lor (B \land C) \equiv (\neg A \lor B) \land (\neg A \lor C)$ (impl.)
$(\neg A \lor B) \land (\neg A \lor C) \equiv (\neg A \lor B) \land (\neg A \lor C)$ (distr. of $\lor$ over $\land$)
■

Question four

(a) - CLAIM: $(A \land B) \to C \equiv (A \to C) \land (B \to C)$

NOT LOGICALLY EQUIVALENT: When A=True, C=False, and B=False, both equations have different truth values.

(b) - CLAIM: $(A \to B) \to C \equiv (\neg A \to C) \land (B \to C)$

PF:

$\neg(\neg A \lor B) \lor C \equiv (A \lor C) \land (\neg B \lor C)$ (impl.)
$(A \land \neg B) \lor C \equiv (A \lor C) \land (\neg B \lor C)$ (de morgan)
$(A \lor C) \land (\neg B \lor C \equiv (A \lor C) \land (\neg B \lor C))$ (distr. of $\lor$ over $\land$)
■

(c) - CLAIM: $\neg((W \land \neg X) \to (\neg Y \lor Z)) \equiv (\neg W \lor X) \land (Y \land \neg Z)$

NOT LOGICALLY EQUIVALENT: When W=True, X=True, Y=True, and Z=False, both equations have different truth values

(d) - CLAIM: $\neg((W \land \neg X) \to (\neg Y \lor Z)) \equiv (Y \land \neg Z) \land (W \land \neg X)$

PF:

$\neg((\neg(W \land \neg X) \lor (\neg Y \lor Z))) \equiv (Y \land \neg Z) \land (W \land \neg X)$ (impl.)
$\neg((\neg W \lor X) \lor (\neg Y \lor Z))) \equiv (Y \land \neg Z) \land (W \land \neg X)$ (de morgan)
$\neg(\neg W \lor X) \land \neg(\neg Y \lor Z) \equiv (Y \land \neg Z) \land (W \land \neg X)$ (de morgan)
$(W \land \neg X) \land (Y \land \neg Z) \equiv (Y \land \neg Z) \land (W \land \neg X)$ (de morgan)
$(Y \land \neg Z) \land (W \land \neg X) \equiv (Y \land \neg Z) \land (W \land \neg X)$ (commutativity of $\land$)
■

(e) - CLAIM: $(M \to N) \land (\neg M \to N) \equiv N$

PF:

$(\neg M \lor N) \land (M \lor N) \equiv N$ (impl.)
$N \lor (M \land \neg M) \equiv N$ (distributivity of $\lor$ over $\land$)
$N \lor (\bot) \equiv N$ (negation)
$N \equiv N$ ($\lor$ identity)

Question five

CLAIM: $\vdash P \lor \neg P$

PF: