MATH 237 Online Calculus 3 for Honours Mathematics **Spring 2024**

Practice exam

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1.(20 points)

(i) (10 points) Determine the limit of the following function at $(0,0)$:

$$f(x,y) = \frac{xy}{x+y}.$$

(ii) (10 points) Determine the differentiability of the following function at $(0,0)$:

$$f(x,y) = \begin{cases} \frac{x^4+y^4}{x^2+y^2} + 1 & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

2.(25 points)

(i) (10 points) If $z = f(x,y)$, $y = g(x)$, find $\frac{dz}{dx}$.

(ii) (15 points) Find the first- and second-degree Taylor polynomials for the following function at the given point:

$$f(x,y) = (x+y)\sin(x-y), \text{ at } (\pi,\pi).$$

3.(25 points)

(i) (10 points) Find and classify the critical points of the function $f(x,y) = xye^{x+2y}$.

(ii) (15 points) Find the maximum and minimum of the function $f(x,y) = x^3 - 3x + y^2 + 2y$ on the region bounded by the lines $x = 0, y = 0, x + y = 1$.

4.(25 points)

(i) (10 points) Let $C = \left\{(x,y,z) \mid z \geqslant \sqrt{x^2+y^2}, x^2+y^2+(z-1)^2 \leqslant 1\right\}$ be a region in $\mathbb{R}^3$. Give descriptions of the region in spherical coordinates and cylindrical coordinates.

(ii) (15 points) Evaluate

$$\iiint_D (x^2+y)dV$$

where D is the region bounded by $x + y + z = 2, z = 2, x = 1$ and $y = x$.

5.(5-10 points) Miscellaneous problem. This problem will be proof-based, similar to the last problem in Written Assignment and Midterm. There will be bonus points in the final.