Segmentation of the Left Heart Ventricle in Ultrasound Images Using a Region Based Snake

Matilda Landgren, Niels Christian Overgaard and Anders Heyden

Centre of Mathematical Sciences, Lund University, Sweden

ABSTRACT

Ultrasound imaging of the heart is a non-invasive method widely used for different applications. One of them is to measure the blood volume in the left ventricle at different stages of the heart cycle. This demands a proper segmentation of the left ventricle and a (semi-) automated method would decrease intra-variability as well as workload. This paper presents a semi-automated segmentation method that uses a region based snake. To avoid any unwanted concavities in the segmentations due to the cardiac valve we use two anchor points in the snake that are located to the left and to the right of the cardiac valve respectively. For the possibility of segmentations in different stages of the heart cycle these anchor points are tracked through the cycle. This tracking is based both on the resemblance of a region around the anchor points and a prior model of the movement in the y-direction of the anchor points. The region based snake functional is the sum of two terms, a regularizing term and a data term. It is our data term that is region based since it involves the integration of a two-dimensional subdomain of the image plane. A segmentation of the left ventricle is obtained by minimizing the functional which is done by continuously reshaping the contour until the optimal shape and size is obtained. The developed method shows promising results.

Keywords: Segmentation, region based snake, ultrasound, left heart ventricle

1. INTRODUCTION

In this section a short background about ultrasound images in general will be given. Then some related work will be presented and finally a motivation for our approach is given.

1.1 Ultrasound Images

Ultrasound imaging is routinely used for a number of examinations in different fields such as cardiology, gynaecology, obstetrics, neurology, urology etc. When it is used for imaging the heart, it is called echocardiography. It can be used for a number of different examinations: heart development, cardiac structure and function as well as changes in normal physiologic states and pathologic conditions. Left ventricular function can for example be obtained via two-dimensional ultrasound images by computing the ejection fraction. This is done by using Simpson's biplane rule for estimating the end-diastolic and end-systolic left ventricle volumes.[1]

The ultrasound frequencies used are in the range 2–10 MHz. When the sound travels from a tissue with a certain density to a tissue with a different density (e.g. from myocardium to blood), some of the sound waves are reflected. The reflected waves are received by the transducer where they are turned back to electrical energy and then displayed as an image. Depending on where on the body the user positions the transducer, different views of the heart can be obtained. Some of the most common are: long axis view of the left atrium and the left ventricle, short axis views of the heart in planes from the base of the heart to the apex and the four chamber view, see Figure 1.[1]

Further author information:
M.L.: E-mail: matilda@maths.lth.se
N.C.O.: E-mail: nco@maths.lth.se
A.H.: E-mail: heyden@maths.lth.se

Medical Imaging 2013: Image Processing, edited by Sebastien Ourselin, David R. Haynor,
Proc. of SPIE Vol. 8669, 866945 · © 2013 SPIE · CCC code: 1605-7422/13/$18
doi: 10.1117/12.2001284

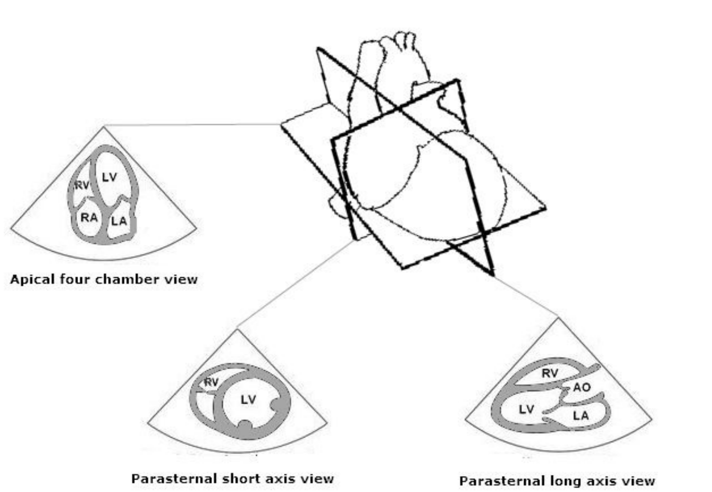


Figure 1: Schematic image of the different views used in echocardiography. *Image reprinted with permission of Amra Jujic, Skåne University Hospital, Sweden.*

1.2 Related Work

There has been quite a lot of work done on segmentation of the ventricles in the heart in different views. The properties of ultrasound images almost demands custom-made methods for this application. Some of these methods are presented below.

Kucera et al. proposed a method with a region based external force in their early work on segmentation of the left ventricle.[2] This force is used in an active contour 3D model with time as one dimension. Their method is fairly reliable on both short axis and long axis views of the heart. Sarti et al. also used a region based approach in their segmentation model where they incorporate the a priori knowledge of the statistical distribution of grey levels.[3] The level set method is used to drive the curve evolution to achieve a maximum likelihood segmentation of the target, with respect to the statistical distribution law of image pixels. When comparing the area enclosed by the resulting contours from this method with manually outlined contours the correlation is excellent. A region based segmentation has also been done by Boukerroui et al. in their adaptive segmentation algortithm.[4] They use a weighting function that takes both local and global statistics into account during the segmentation process. The results of the segmentation of the left ventricle shows good results when compared with manual outlines by a medical expert.

Mishra et al. use an active contour model when segmenting the left ventricle in short axis view.[5] They solve the optimization problem using a Genetic Algorithm (GA) and the performance is comparable with inter-observer variations. A multiscale approach to the contour optimization is done by Mignotte and Meunier.[6] Their external energy in the snake energy function is also region based. They show some segmentation results for short axis views that are qualitatively good. Chen et al. constructed a geometric active contour model with a shape and an intensity prior.[7] The results of applying the method to a two chamber view are promising.

Bosch et al. developed an Active Appearance Motion Model (AAMM) that were used to do a segmentation of the left ventricle.[8] This is an extension of Active Appearance Models (AAM) and they did an automated segmentation over the full heart cycle. Their results are comparable with inter-observer variations. Mitchell et al. did a fully three dimensional AAM with time as one of the dimensions in the ultrasound images.[9] They got a correlation coefficient of 0.79 when comparing the area of the left ventricle defined by their method and an observer respectively.

Several other methods have also been evaluated for this segmentation problem, including artificial neural networks[10,11], a fuzzy multiscale edge detector[12] and a Kalman filter based tracking method.[13] All of these methods give acceptable segmentations of the left ventricle in long axis and/or short axis views.

1.3 Motivation for Our Method

The purpose of this work has been to develop a semi-automated segmentation method that segment the left ventricle in ultrasound images of the heart. A region based snake is used for the segmentation and it is specially designed to overcome some of the problems that are encountered in ultrasound images. Two anchor points are used in the snake and a tracking algorithm with a prior model for the movement of these from frame to frame are constructed. The proposed method performs a segmentation over the full cardiac cycle with only an initialization in the first frame.

2. METHODS

The methods used in our segmentation algorithm will be described in this section. First some necessary user inputs are presented and the tracking algorithm for the anchor points is described. Then the classical snake model is explained as well as a region based snake model with anchor points. The section ends with a short step by step overview of the algorithm.

2.1 User Inputs

At the beginning of the algorithm, the user has to provide with some inputs that are presented here. In the first frame the user should annotate two points – one on each side of the cardiac valve, cf. Figure 2. These are denoted $\mathbf{a}^* = (a_1^*, a_2^*)$ and $\mathbf{b}^* = (b_1^*, b_2^*)$ and are called anchor points, see the next section for more details. The user should also, by cycling through the frames, indicate the vertical position, p, of the cardiac valve when the heart is at most contracted. This value is used for computing the mean position of the cardiac valve as

$$h = \frac{\frac{a_2^* + b_2^*}{2} + p}{2}, \tag{1}$$

which is needed later for the prior model. To get the correct number of steps in the prior model, the user also needs to specify the length of one heart cycle.

2.2 Anchor Points

Since the cardiac valve can cause the snake to get stuck with a concavity in the cranial part of the left ventricle, two anchor points are used – one on each side of the valve, cf. Figure 2. This means that no segmentation is done over the valve, instead a straight line is drawn between the anchor points. Since the segmentation is done in several frames, it is desirable to track the anchor points in the entire cycle in order to reduce the interaction with the user who annotates them in the first frame. The known anchor points in the previous frame are denoted $\mathbf{a} = (a_1, a_2)$ and $\mathbf{b} = (b_1, b_2)$. It will be described below how anchor point $\mathbf{a}$ is tracked from one frame to the next, anchor point $\mathbf{b}$ is tracked in the same way. We search for anchor point $\mathbf{a}'$ in the current frame in a bounding box of size 25×25 pixels around the position of anchor point $\mathbf{a}$ in the previous frame. The tracking is based on the following three parts.

- **Image resemblance**
  It is reasonable to assume that the intensity distribution is fairly identical around an anchor point from one frame to the next. To get a value of how good the resemblance is, the sum of squared error is computed as

$$R = \sum_{i=1}^{N} (L_i - C_i)^2, \tag{2}$$

  where L_i is the ith pixel in a 5×5 window around the previous anchor point $\mathbf{a}$ and C_i is the ith pixel in a 5×5 window around the current, not yet determined, anchor point $\mathbf{a}'$.

- **Distance**
  It is very likely that an anchor point does not take a major leap from one frame to another and hence a measure for this is included too

$$D = \beta|\mathbf{a}' - \mathbf{a}| = \beta\sqrt{(a_1' - a_1)^2 + (a_2' - a_2)^2}, \tag{3}$$

  where β is a constant.

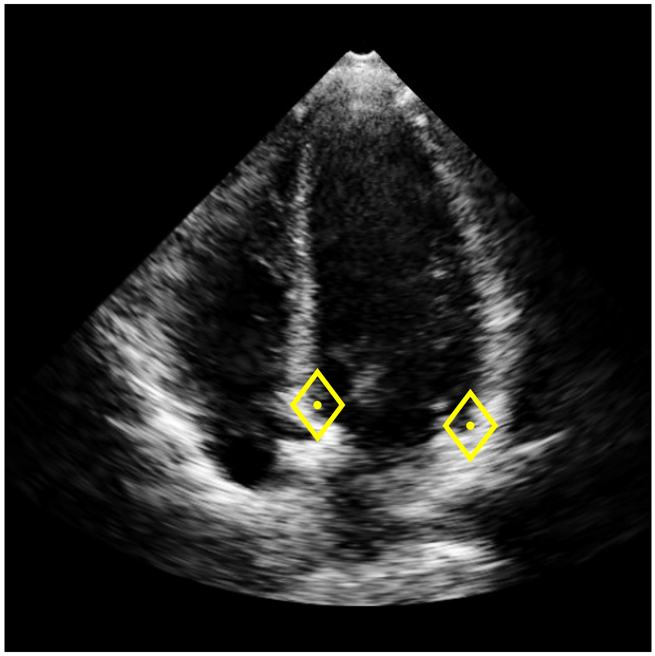


Figure 2: Two anchor points marked with a dot inside a diamond.

- **Prior model**
  A prior model of the movement in the vertical direction of the anchor points has been constructed. By manually tracking two anchor points in three patients a polynomial model, $\mathbf{m}$, of degree 5 that describes the movement in one cycle as a deviation from the mean position of the anchor points is obtained. The model can be seen in Figure 3. The comparison is then done by computing the following expression:

  $$P = \gamma |a_2' - \mathbf{M}(t)|^2, \tag{4}$$

  where γ is a constant, $\mathbf{M}(t) = h + \mathbf{m}(t)$ (where h is defined in (1)) and t is an index for the current frame.

To find the anchor point in the next frame the following expression is minimized

$$\min_{\mathbf{a}'} \{M = R + D + P\} \tag{5}$$

to get the position of the anchor point $\mathbf{a}'$ in the current frame.

2.3 Snake Model

In the first two paragraphs of this section the classical snake model is recalled together with its extension to region based data terms. In the third paragraph it is explained how the region based model is adapted to our problem.

2.3.1 The Classical Snake Model

The snake model was first introduced in 1987 by Kass, Witkin and Terzopolous[14] as a means of synthezising the noisy filter-response coming from an edge-detector into a coherent delineation of a perceived edge in the image, such as the boundary separating two image regions with distinct grey level characteristics.

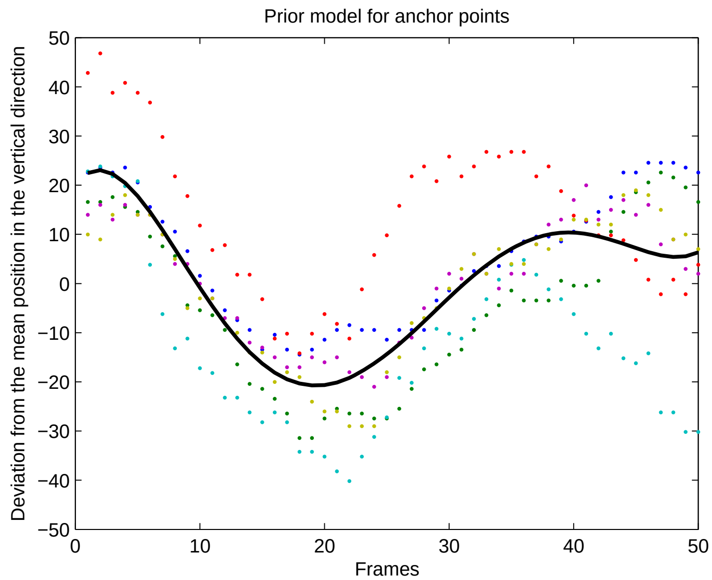


Figure 3: The prior model of the movement in the vertical direction of the anchor points as the solid line and the data that the model is based on is marked with dots.

Let us recall the classical snake model: A grey scale image I is a real-valued function $I : \Omega \to [0, 1]$ defined for every point (pixel) $\mathbf{x} = (x, y)$ in the image domain $\Omega \subset \mathbf{R}^2$. The latter is usually a rectangle. The corresponding function value $I(\mathbf{x})$, is called the grey level at the pixel $\mathbf{x}$. Mathematically, a snake is given by a differentiable parametrized curve $\mathbf{u} : [0, 1] \to \mathbf{R}^2$, where each point of the curve $\mathbf{u}(s) = (x(s), y(s))$, $0 \le s \le 1$ is a pixel in the image domain Ω. With each snake is a snake energy associated, given by

$$E[\mathbf{u}] = \alpha \int_0^1 \frac{1}{2} |\mathbf{u}'(s)|^2 \, ds + \int_0^1 g(\mathbf{u}(s)) \, ds. \tag{6}$$

Here the first term is a regularisation term and the second one is a data term. The function $g(\mathbf{x}) : \Omega \to \mathbf{R}$ is an edge map. A typical example of an edge map is

$$g(\mathbf{x}) = \frac{\epsilon}{\sqrt{\epsilon^2 + |\nabla I(\mathbf{x})|^2}}, \tag{7}$$

where $\epsilon > 0$ is a parameter. The regularisation term is smaller for shorter curves than for longer curves. At the same time the data term is smaller when the snake $\mathbf{u}(s)$ is situated near an edge, where image gradients are large. Therefore the best delineation of an edge is defined, by the snake model, as the curve $\mathbf{u}^*$ for which the snake energy is minimized,

$$E[\mathbf{u}^*] \le E[\mathbf{u}] \quad \text{for all admissible curves } \mathbf{u}.$$

The set of curves which are deemed admissible depends on the application at hand. In their original paper Kass, Witkin and Terzopolous[14] considered the delineation of incoherent edges from noisy edge-detection signals. They considered as admissible all differential curves $\mathbf{u} : [0, 1] \to \mathbf{R}^2$ with fixed end-points $\mathbf{u}(0) = \mathbf{a}$ and $\mathbf{u}(1) = \mathbf{b}$, specified by the user. The minimizer $\mathbf{u}^*$ of E in this class of curves then gives the image edge through the points $\mathbf{a}$ and $\mathbf{b}$ and consistent with the image data. In other applications, such as segmentation of an image into foreground and background, one may consider as admissible the set of all simple closed differentiable curves in the plane. The curve which optimizes the snake energy then defines the boundary between foreground and background.

2.3.2 Region Based Snake Models

A region based snake model is an active contour where the segmentation is driven by the statistical properties of the image data inside and outside the contour. This is in contrast to the classical snake model where the contour is controlled by edge-forces derived from the shift in grey levels at perceived edges in the image. One of the most commonly used region based snake models is one in which the grey levels at each pixel in the image I is assumed to follow a Gaussian distribution with a common variance σ^2 but with two distinct mean values: μ_0 if the pixel belongs to the background and μ_1 if it belongs to the object. The snake problem then becomes the optimization of the functional,

$$E[\mathbf{u}, \mu_0, \mu_1] = \alpha \int_0^1 \frac{1}{2} |\mathbf{u}'(s)|^2 \, ds + \int_{\mathrm{ext}(\mathbf{u})} (I(\mathbf{x}) - \mu_0)^2 \, d\mathbf{x} + \int_{\mathrm{int}(\mathbf{u})} (I(\mathbf{x}) - \mu_1)^2 \, d\mathbf{x}, \tag{8}$$

where $\mathrm{ext}(\mathbf{u})$ denotes the set of points (pixels) $\mathbf{x}$ on the outside of the contour given by $\mathbf{u}$, and $\mathrm{int}(\mathbf{u})$ the set of pixels inside the contour. This model can be seen as the snake formulation of the piecewise constant Mumford-Shah model[15] with two regions. This and similar region based snake models have been studied by Tsai in his thesis,[16] see also Tsai et al.[17] The functional $E[\mathbf{u}, \mu_0, \mu_1]$ has to be minimized both with respect to the mean grey levels μ_0, μ_1 and the active contour $\mathbf{u}$. It follows directly from the definition of E that, for any fixed contour $\mathbf{u}$, the optimal mean grey levels correspond to the mean intensity of the image I on the outside and the inside of the contour, respectively. The optimal active contour is found by applying a gradient descent PDE associated with E to an initial contour supplied by the user.

Region based snake models, such as the above, can be rewritten (up to a fixed additive constant) in the form

$$E[\mathbf{u}, \mu_0, \mu_1] = \alpha \int_0^1 \frac{1}{2} |\mathbf{u}'(s)|^2 \, ds + \int_{\mathrm{int}(\mathbf{u})} V(\mathbf{x}) \, d\mathbf{x}, \tag{9}$$

where $V : \Omega \to \mathbf{R}$ is a potential function derived from the data-term. For the piecewise constant Mumford-Shah model above one can take $V(\mathbf{x}) = (I(\mathbf{x}) - \mu_1)^2 - (I(\mathbf{x}) - \mu_0)^2$. This form is practical if we want to derive the gradient descent equations of motion of our active contour. The idea of the gradient descent procedure is to gradually change the shape of the snake, hence the name *active contour*, in such a manner that its energy steadily decreases. This is described mathematically by a snake $\mathbf{u} = \mathbf{u}(s, t)$ which depends on a (fictitious) time parameter $t \geq 0$. The motion of the snake is then dictaded by the gradient descent PDE, which can be written formally as

$$\begin{cases} \frac{\partial}{\partial t}\mathbf{u} = -\nabla E[\mathbf{u}], \\ \mathbf{u}(\cdot, 0) = \mathbf{u}_0(\cdot). \end{cases} \tag{10}$$

Here $\nabla E[\mathbf{u}]$ is the L^2-gradient of the functional E computed at the contour $\mathbf{u}$. Using methods from the calculus of variations it is possible to show that L^2-gradient for the region based snake at the contour $\mathbf{u} = \mathbf{u}(s)$ is

$$\nabla E[\mathbf{u}](s) = -\alpha \mathbf{u}''(s) + V(\mathbf{u})\hat{\mathbf{u}}'(s), \quad 0 \leq s \leq 1. \tag{11}$$

Here $\hat{\mathbf{u}}'(s) = (-u_2'(s), u_1'(s))$ is the $\pi/2$ radians counter-clockwise rotation of the tangent vector $\mathbf{u}'(s)$ of the contour. As usual, $\mathbf{u}'(s) = \partial \mathbf{u}(s)/\partial s$. Therefore the gradient descent PDE for the minimisation of the region based snake model becomes,

$$\begin{cases} \frac{\partial}{\partial t}\mathbf{u} = \alpha \mathbf{u}''(s) - V(\mathbf{u})\hat{\mathbf{u}}'(s), \\ \mathbf{u}(\cdot, 0) = \mathbf{u}_0(\cdot), \end{cases} \tag{12}$$

where $\mathbf{u}_0$ is the initial contour. This PDE describes a time-evolution of the contour $\mathbf{u}$ which we follow until convergence,

$$\mathbf{u}_*(s) = \lim_{t \to \infty} \mathbf{u}(s, t), \tag{13}$$

in which case $\mathbf{u}_*$ satisfies the Euler equation $0 = \nabla E[\mathbf{u}_*] = -\alpha \mathbf{u}_*''(s) + V(\mathbf{u})\hat{\mathbf{u}}_*'(s)$, so that $\mathbf{u}_*$ is a strong local minimum of the functional E.

For the region based snake given by E, the evolution defined by the descent PDE (12) has an interesting further property, namely that the contour always moves perpendicular to itself. Therefore the quantity $\frac{1}{2}|\mathbf{u}_*'(s)|^2$ is constant along the optimal contour, i.e., for all s, $0 \leq s \leq 1$. If the model is discretizised, this property ensures that the control points of the contour will remain equidistant along the contour throughout its evolution.

2.3.3 Region Based Snake Model with Anchor Points

In this paper a region based snake model with two fixed anchor points $\mathbf{a}$ and $\mathbf{b}$ is considered. The required closed contour is the union of two arcs. The first arc is the snake $\mathbf{u} : [0, 1] \to \Omega$ running counter-clockwise, inside the left ventricle, from one anchor point $\mathbf{u}(0) = \mathbf{a}$ to the other anchor point $\mathbf{u}(1) = \mathbf{b}$. The second arc is formed by the straight line running from $\mathbf{b}$ to $\mathbf{a}$, thus completing the contour. In our application we use the simple data term,

$$V(\mathbf{x}) = I(\mathbf{x}) - \mu, \tag{14}$$

where the constant μ, $0 < \mu < 1$, is chosen by the user for optimal performance. If we take $\mu = 1/2$, our model corresponds to the piecewise constant Mumford-Shah model with the fixed choice of grey levels $\mu_0 = 0$ inside the snake (supposed to be inside the left ventricle where the image is black) and $\mu_1 = 1$ outside the snake. The snake's initial position $\mathbf{u}_0(s)$ is supplied by the user (or by the solution to a previous instance of the same problem). The corresponding snake energy is minimized by evolving the snake according to the PDE,

$$\begin{cases} \frac{\partial}{\partial t}\mathbf{u}(s,t) = \alpha \mathbf{u}''(s,t) - V(\mathbf{u}(s,t))\hat{\mathbf{u}}'(s,t), \quad t > 0, 0 < s < 1, \\ \mathbf{u}(0,t) = \mathbf{a}, \quad \text{and} \quad \mathbf{u}(1,t) = \mathbf{b} \quad \text{for all } t > 0, \\ \mathbf{u}(s,0) = \mathbf{u}_0(s) \quad \text{for } 0 \leq s \leq 1. \end{cases} \tag{15}$$

This problem is solved iteratively using a semi-implicit finite-difference discretization of the derivatives. The algorithm is terminated when the change in the solution, from one iteration to the next, is less than a given tolerance.

2.4 Algorithm

The segmentation of the left ventricle follows these steps:

1: The user annotates two anchor points, one at each side of the cardiac valve. The length of one cardiac cycle is also indicated as well as the position of the cardiac valve when the heart is contracted.

2: The anchor points are tracked through the cycle using the method described in section 2.2.

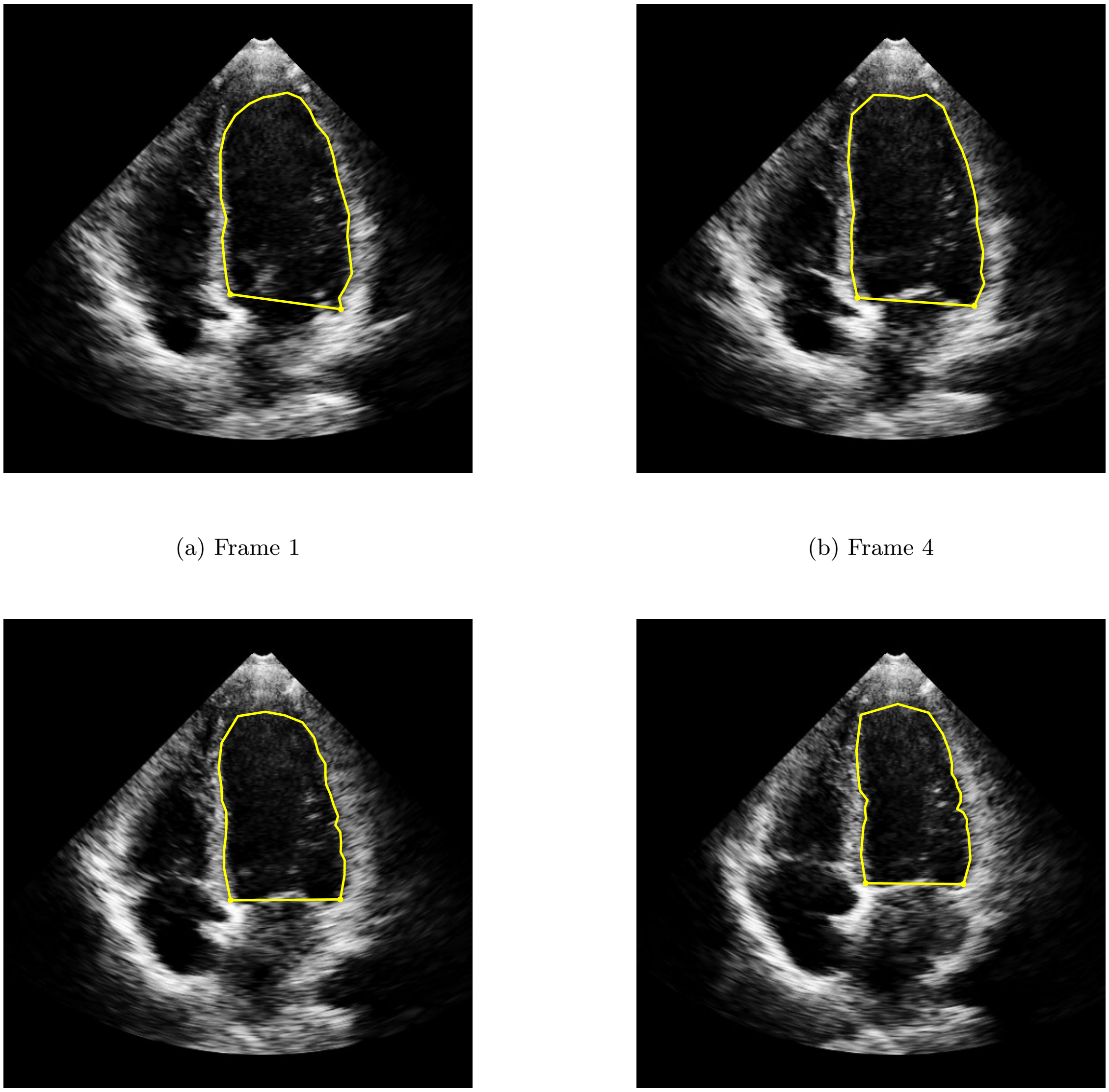
3: The segmentation begins in the first frame where an initialization of the snake is done. The region based snake, described in section 2.3.3, iterates until a tolerance level is reached and a segmentation is obtained in that frame.

4: For the next frame the segmentation in the last frame is used as initialization for the snake and then it iterates until the tolerance level is obtained.

5: Then item number 4 is repeated for the rest of the heart cycle.

3. RESULTS

The proposed method has been tested on a sequence of ultrasound images and the results from some frames can be seen in Figure 4. Both the tracking of the anchor points as well as the segmentations of the left ventricle can be studied here. The method was implemented using MATLAB. The computation time for performing the segmentation in each frame in one cardiac cycle is approximately 40 seconds when running on a 3.20 GHz Windows PC with 64 Gb of RAM.



(a) Frame 1 (b) Frame 4

(c) Frame 8 (d) Frame 12

Figure 4: The anchor points and resulting segmentations at different frames.

4. DISCUSSION & CONCLUSIONS

As can be seen in Figure 4 our algorithm captures the edges of the left ventricle quite well for different levels of heart contraction. The choice of a region based snake seems to be good.

The tracking of the anchor points also seems to work well. The use of a prior model turned out to be necessary when testing the tracking algorithm without it. With no prior model the anchor points have a tendency to wander away or not follow the cardiac valve back when the heart relaxes.

One problem with our method is that the left ventricle stretches out to the apex of the heart, which is located in the top of the images in Figure 4, and this is not captured so well by our method. The reason is the high intensity levels in that part of the image that is due to the closeness to the transducer. One idea to overcome this problem is to insert an anchor point in the apex to force the snake to stay there. This is something that will be investigated in the future.

ACKNOWLEDGMENTS

This work was funded by the European Research Council through the programme GlobalVision. The project ENGROSS, funded by the Swedish Foundation for Strategic Research, has also contributed to this work.

REFERENCES

1. Lohr, J. L. and Sivanandam, S., "Introduction to echocardiography," in [*Handbook of Cardiac Anatomy, Physiology and Devices*], Springer Science+Business Media, LLC (2009).
2. Kucera, D. and Martin, R. W., "Segmentation of sequences of echocardiographic images using a simplified 3D active contour model with region-based external forces," *Comput. Med. Imag. Graph.* **21**(1), 1–21 (1997).
3. Sarti, A., Corsi, C., Mazzini, E., and Lamberti, C., "Maximum likelihood segmentation of ultrasound images with rayleigh distribution," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **52**(6), 947–960 (2005).
4. Boukerroui, D., Baskurt, A., Noble, J. A., and Basset, O., "Segmentation of ultrasound images - multiresolution 2D and 3D algorithm based on global and local statistics," *Pattern Recognit. Lett.* **24**(4–5), 779–790 (2003).
5. Mishra, A., Dutta, P., and Ghosh, M., "A GA based approach for boundary detection of left ventricle with echocardiographic image sequences," *Image Vis. Comput.* **21**, 967–976 (2003).
6. Mignotte, M. and Meunier, J., "A multiscale optimization approach for the dynamic contour-based boundary detection issue," *Comput. Med. Imag. Graph.* **25**(3), 265–275 (2001).
7. Chen, Y., Huang, F., Tagare, H., and Rao, M., "A coupled minimization problem for medical image segmentation with priors," *Int. J. Comput. Vis.* **71**(3), 259–272 (2007).
8. Bosch, J. G., Mitchell, S. C., Lelieveldt, B. P. F., Nijland, F., Kamp, O., Sonka, M., and Reiber, J. H. C., "Automatic segmentation of echocardiographic sequences by active appearance motion models," *IEEE Trans. Med. Imag.* **21**(11), 1374–1383 (2002).
9. Mitchell, S. C., Bosch, J. G., Lelieveldt, B. P. F., van der Geest, R. J., Reiber, J. H. C., and Sonka, M., "3-D active appearance models: Segmentation of cardiac MR and ultrasound images," *IEEE Trans. Med. Imag.* **21**(9), 1167–1178 (2002).
10. Binder, T., Süssner, M., Moertl, D., Strohmer, T., Baumgartner, H., Maurer, G., and Porenta, G., "Artificial neural networks and spatial temporal contour linking for automated endocardial contour detection on echocardiograms: A novel approach to determine left ventricular contractile function," *Ultrasound Med. Biol.* **25**(7), 1069–1076 (1999).
11. Rekeczky, C., Tahy, Á., Végh, Z., and Roska, T., "CNN based spatio-temporal nonlinear filtering and endocardial boundary detection in echocardiography," *Int. J. Circuit Theory Appl.* **27**(1), 171–207 (1999).
12. Setarehdan, S. K. and Soraghan, J. J., "Automatic cardiac LV boundary detection and tracking using hybrid fuzzy temporal and fuzzy multiscale edge detection," *IEEE Trans. Biomed. Eng.* **46**(11), 1364–1378 (1999).
13. Jacob, G., Noble, J. A., Behrenbruch, C., Kelion, A. D., and Banning, A. P., "A shape-space-based approach to tracking myocardial borders and quantifying regional left-ventricular function applied in echocardiography," *IEEE Trans. Med. Imag.* **21**(3), 226–238 (2002).
14. Kass, M., Witkin, A., and Terzopoulos, D., "Snakes: Active contour models," in [*First International Conference on Computer Vision*], 259–268 (1987).
15. Mumford, D. and Shah, J., "Optimal approximation by piecewise smooth functions and associated variational problems," *Comm. Pure and Appl. Math.* **42**, 577–684 (1989).
16. Tsai, A., *Curve Evolution and Estimation-Theoretic Techniques for Image Processing*, PhD thesis, Massachusetts Institute of Technology (2000).
17. Tsai, A., Jezzi, A. J., and Willsky, A. S., "Curve-evolution implementation of the mumford-shah functional for segmentation, image denoising, interpolation and magnification," *IEEE J. Image Processing* **10**(8), 1169–1186 (2001).